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SELF-SYNCHRONIZATION AND CONTROLLED SYNCHRONIZATION OF DYNAMICAL SYSTEMS

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Abstract

A general definition of synchronization of dynamical systems is given capturing features of both self-synchronized systems and systems synchronized by means of control. It has been demonstrated for important special cases of "master-slave" and coupled systems that synchronizing control may be designed using feedback linearization or passification methods.

Keywords Nonlinear dynamics, nonlinear control, synchronization.

1 Introduction

Starting with the work of C.Huygens [13] synchronization phenomena attracted attention of many researchers. The development of small parameter and averaging methods by H.Poincare [23], B. Van der Pol [26], N.N. Bogolyubov [9] in the first half of the 20th century allowed for a better understanding and theoretical explanation of the mechanism of self-synchronization [3, 4], phenomenon which has numerous applications, see, e.g. [4, 15]. Motivated by the study of chaotic phenomena (see, e.g. [25],[16]) recent years have exhibited an increase in the interest in synchronization. Synchronization in chaotic systems was discussed for instance in [1, 22, 10, 5].

Recently, specialists from (nonlinear) control theory turned attention to the study of controlled synchronization. Incomplete information about the system parameters has been taken into account (adaptive and robust synchronization [11, 12, 18]) as well as incomplete information of the state of the system (observer-based synchronization [19, 20]). However, there is still a strong need for unified definitions of synchronization which would capture pecularities of both self-synchronization and controlled synchronization and which also would allow to rigorously pose and systematically solve various synchronization problems. Such definitions are proposed in Section 2 of the present paper and an example of synchronization of two feedback linearizable oscillators is considered. Section 3 deals with the problem of coupled systems synchronization by means of linear feedback.

2 Definitions of synchronization

Synchronization in its most general interpretation means correlated or corresponding in time behavior of two or more processes. According to [17]: "to synchronize" means to concur or agree in time, to proceed or to operate at exactly the same rate. Below we formalize the above description and also formulate a "controlled" version.

To this end consider k dynamical systems

$$S_i = \{T, U_i, X_i, Y_i, \phi_i, h_i\}, \quad i = 1, \dots, k$$

where T is common set of time instances, U_i, X_i, Y_i are sets of inputs, states and outputs, respectively; $\phi_i : T \times X_i \times U_i \rightarrow X_i$ are transition maps, $h_i : T \times X_i \times U_i \rightarrow Y_i$ are output maps. (We use one of

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the standard definitions of dynamical system, see e.g. [21, 14]).

First consider the case when all U_i are just singletons, i.e. inputs are not present and may be omitted from formulations.

Suppose l functionals $g_j : \mathcal{Y}_1 \times \mathcal{Y}_2 \times \ldots \times \mathcal{Y}_k \times T \rightarrow \mathrm{I\!R}^1, j = 1, \ldots, l$, are given. Here $\mathcal{Y}_i$ are the sets of all functions from T into Y_i, i.e. $\mathcal{Y}_i = \{y : T \rightarrow Y_i\}$.

In the sequel, we take as time set T either $T = \mathrm{I\!R}_{\geq 0}$ (continuous time) or $T = Z_{\geq 0}$ (discrete time). For any $\tau \in T$ we then define σ_τ as the *shift operator*, i.e $\sigma_\tau : \mathcal{Y}_i \rightarrow \mathcal{Y}_i$ is given as $(\sigma_\tau y)(t) = y(t + \tau)$ for all $y \in \mathcal{Y}_i$ and all $t \in T$. We are now prepared to define synchronization.

Definition 2.1 *We call the solutions $x_1(\cdot), \ldots, x_k(\cdot)$ of the systems $\Sigma_1, \ldots, \Sigma_k$ with initial conditions* $x_1(0), \ldots, x_k(0)$ synchronized with respect to the functionals $g_1, \ldots, g_l$ *if*

$$g_j(\sigma_{\tau_1} y_1(\cdot), \ldots, \sigma_{\tau_k} y_k(\cdot), t) \equiv 0, j = 1, \ldots, l \tag{2.1}$$

is valid for all $t \in T$ and some $\tau_1, \ldots, \tau_k \in T$, where $y_i(\cdot)$ denotes the output function of the system Σ_i: $y_i(t) = h(x_i(t), t), t \in T, i = 1, \ldots, k$.

We say that solutions $x_1(\cdot), \ldots, x_k(\cdot)$ of the systems $\Sigma_1, \ldots, \Sigma_k$ with initial conditions $x_1(0), \ldots, x_k(0)$ are approximately synchronized with respect to the functionals $g_1, \ldots, g_l$, *if there are an $\varepsilon > 0$ and $\tau_1, \ldots, \tau_k \in T$ such that*

$$|g_j(\sigma_{\tau_1} y_1(\cdot), \ldots, \sigma_{\tau_k} y_k(\cdot), t)| \leq \varepsilon, j = 1, \ldots, l \tag{2.2}$$

for all $t \in T$.

The solutions $x_1(\cdot), \ldots, x_k(\cdot)$ of the systems $\Sigma_1, \ldots, \Sigma_k$ with initial conditions $x_1(0), \ldots, x_k(0)$ are asymptotically synchronized with respect to the functionals $g_1, \ldots, g_l$, *if for some $\tau_1, \ldots, \tau_k \in T$*

$$\lim_{t \rightarrow \infty} g_j(\sigma_{\tau_1} y_1(\cdot), \ldots, \sigma_{\tau_k} y_k(\cdot), t)) = 0, j = 1, \ldots, l \tag{2.3}$$

If the synchronization phenomena is achieved for all initial conditions $x_1(0), \ldots, x_k(0)$ it is possible to say that the systems $\Sigma_1, \ldots, \Sigma_k$ are synchronized (in the appropriate sense with respect to the given functionals). In the case of asymptotic synchronization it is also possible to define the basins of the initial conditions which yield synchronization. In the sequel, we will only consider the case when the synchronism is achieved for all initial conditions.

Although this definition is rather general, it can be further generalized. For example in many practical problems the time shifts $\tau_i, i = 1, \ldots, k$ are not constant but tend to constant values, so called "asymptotic phases". In this case, instead of the shift operator for each output function $y_i(\cdot)$, it is convenient to consider the time varying shift operator defined as follows

$$(\sigma_{\tau_i}) y(t) = y(t'_i(t))$$

where $t'_i : T \rightarrow T$, $i = 1, \ldots, k$ are homeomorphisms (continuous functions having continuous inverses) such that

$$\lim_{t \rightarrow \infty} (t'_i(t) - t) = \tau_i \tag{2.4}$$

In [1], instead of (2.4) the milder condition $\lim_{t \rightarrow \infty} t'_i(t)/t = 1$ is proposed which, however allows for infinitely large phase shifts.

In many practical synchronization problems the spaces $\mathcal{Y}_i$ are identical $\mathcal{Y}_i = \mathcal{Y}$ and the functionals $\{g_{jsr}\}$ are chosen to compare similar characteristics of different systems, e.g.:

$$g_{j_{sr}}(y_s(\cdot), y_r(\cdot)) = \mathrm{dist}(J_j(\sigma_{\tau_s} y_s(\cdot)), J_j(\sigma_{\tau_r} y_r(\cdot))),$$

where $r, s = 1, \ldots, k$, $j = 1, \ldots, l$ and $J_j : \mathcal{Y} \rightarrow \mathcal{J}_j$, are some mappings (synchronization indices) which map the (output) trajectory $y_i(\cdot)$ of each system $\Sigma_1, \ldots, \Sigma_k$, into some metric space $\mathcal{J}_j$. In this case we will talk about *synchronization with respect to the indices $\{J_j\}$* . The specific choice of the synchronization indices depends on the essence of the mathematical, physical or engineering problem. The same is valid for the phase shifts τ_i which may be fixed in some problems and may be arbitrary in others. Naturally, the possibility of efficient solution of the synchronization problems depends crucially on the chosen functionals and/or indices.

Remark 1. Note that instead of the set of the functionals it is always possible to take one functional which expresses the same synchronization phenomenon, for example one can take the functional G as follows

$$G(y_1(\cdot), \ldots, y_k(\cdot), t) = \sum_{j=1}^{l} g_j^2(y_1(\cdot), \ldots, y_k(\cdot), t),$$

In many practical cases the sets U_i, X_i, Y_i are finite-dimensional vector spaces and the systems S_i can be described by ordinary differential equations. First consider the simpliest case of disconnected systems without inputs:

$$S_i: \quad \frac{dx_i}{dt} = F_i(x_i, t), \tag{2.5}$$

where $F_i, i = 1, \ldots, k$ are some vector fields. Sometimes synchronization may occur in disconnected systems (2.5) (e.g. all precise clocks are synchronized in the freequency sense). This case will be refered to as *natural synchronization*. The most interesting and important case, however, seems synchronization of interconnected systems. In this case the system models are augmented with interconnections and look as follows:

$$\begin{cases} \frac{dx_i}{dt} = F_i(x_i, t) + \tilde{F}_i(x_0, x_1, \ldots, x_k, t), \quad i = 1, \ldots, k \\ \frac{dx_0}{dt} = F_0(x_0, x_1, \ldots, x_k, t) \end{cases} \tag{2.6}$$

where the vector field F_0 describes the dynamics of the interconnection system, $\tilde{F}_i$ are vector fields of the interconnections.

A remarkable and widely used observation is that the synchronization may exist, i.e. identity (2.1) may be valid in the interconnected system (2.6) without any external action, i.e. without inputs. In this case the system (2.6) is called *self-synchronized with respect to the functionals* $g_1, \dots, g_l$. Similar definitions are introduced for approximate and asymptotic self-synchronization. Usually in this case the systems $S_1, \dots, S_k$ are autonomous.

In many cases important for applications the interconnections between the systems $S_1, \dots, S_k$ are weak, for instance when (2.6) can be represented as follows

$$\begin{cases} \frac{dx_i}{dt} = F_i(x_i, t) + \mu \tilde{F}_i(x_0, x_1, \dots, x_k, t), \quad i = 1, \dots, k \\ \frac{dx_0}{dt} = F_0(x_0, x_1, \dots, x_k, t) \end{cases} \tag{2.7}$$

where μ is a small parameter. Therefore finding conditions for self-synchronization in systems with small interactions is of special interest. Such conditions were found for a large class of dynamical systems (2.7) with time-periodic functions F_i in the right hand sides [3, 4]. However, in many cases self-synchronization is not observed and the question arises: is it possible to affect, i.e. to control the systems in such a way that the goal (2.2) or (2.3) can be achieved?

The above definitions do not yet include the possibility of controlling the system. Assume for simplicity that all S_i, $i = 0, \dots, k$ are smooth finite dimensional systems, described by differential equations with a finite-dimensional input, i.e.

$$\begin{cases} \frac{dx_i}{dt} = F_i(x_i, t) + \tilde{F}_i(x_0, x_1, \dots, x_k, u, t), \quad i = 1, \dots, k \\ \frac{dx_0}{dt} = F_0(x_0, x_1, \dots, x_k, u, t) \end{cases} \tag{2.8}$$

where $u = u(t) \in \mathrm{I\!R}^m$ is the input (control variable) which has physical meaning.

The problem of *controlled synchronization with respect to the functionals* $g_j, j = 1, \dots, l$ (respectively, *controlled asymptotic synchronization with respect to the functionals* $g_j, j = 1, \dots, l$) is to find a control u as a feedback function of the states $x_0, x_1, \dots, x_n$ and time providing that (2.1) (respectively, (2.2), (2.3)) hold for the closed loop system.

Sometimes the goal can be ensured without measuring any variables of the systems, for instance by time-periodic forcing. In this case control function u does not depend on system states and the problem of finding such a control is called an *open loop controlled (asymptotic) synchronization problem.*

However, the most powerful approach assumes the possibility of measuring the states or some function of the system variables. Finding a control function in this case is called a *closed loop* or *feedback (asymptotic) synchronization problem.*

The simpliest form of feedback is *static feedback* where the controller equation is as follows

$$u(t) = \mathcal{U}(x_0, x_1, \dots, x_k, t) \tag{2.9}$$

for some function $\mathcal{U} : \mathrm{I\!R}^{n_0} \times \mathrm{I\!R}^{n_1}, \dots, \mathrm{I\!R}^{n_k} \times \mathrm{I\!R} \to \mathrm{I\!R}^m$

A more general form is *dynamic state feedback*:

$$\frac{dw}{dt} = W(x_0, x_1, \dots, x_k, w, t) \tag{2.10}$$

$$u(t) = \mathcal{U}(x_0, x_1, \dots, x_k, w, t) \tag{2.11}$$

with $w \in \mathrm{I\!R}^\nu$, $W : \mathrm{I\!R}^{n_0} \times \mathrm{I\!R}^{n_1} \times \dots \times \mathrm{I\!R}^{n_k} \times \mathrm{I\!R}^\nu \times \mathrm{I\!R} \to \mathrm{I\!R}^\nu$, $\mathcal{U} : \mathrm{I\!R}^{n_0} \times \mathrm{I\!R}^{n_1} \times \dots \times \mathrm{I\!R}^{n_k} \times \mathrm{I\!R}^\nu \times \mathrm{I\!R} \to \mathrm{I\!R}^m$,

Now the problem of *state feedback synchronization* can be posed as follows.

Find a control law (2.9), (or (2.10), (2.11)) ensuring the asymptotic synchronization (2.3) in the closed loop system (2.8), (2.9) (or respectively, (2.8), (2.10), (2.11)).

Remark 2. Since it is worth speaking about controlled synchronization only in cases when (self-) synchronization (2.1) does not occur, the inclusion of a static or dynamic state feedback (2.9) or (2.10), (2.11) will only lead after some transient behavior to (2.1). We therefore will only be concerned with the feedback (asymptotic) synchronization (2.3).

In a variety of practical problems complete information about the states of the systems $S_0, S_1, \dots, S_k$ is not available and only some *output variables*

$$y_s, \quad s = 1, \dots, r,$$

are available for using in the control law. In case when the S_i are smooth finite-dimensional systems the problem of output feedback synchronization can be posed as follows: find controller equations

$$u(t) = \mathcal{U}(y_1, \dots, y_r, t) \tag{2.12}$$

(or with dynamic equation:

$$\frac{dw}{dt} = W(y_1, \dots, y_r, w, t) \quad) \tag{2.13}$$

$$u(t) = \mathcal{U}(y_1, \dots, y_r, w, t) \tag{2.14}$$

with $w \in \mathrm{I\!R}^\nu$, $y_s \in \mathrm{I\!R}^{p_s}$ $W : \mathrm{I\!R}^{p_1} \times \dots \times \mathrm{I\!R}^{p_s} \times \mathrm{I\!R}^\nu \times \mathrm{I\!R} \to \mathrm{I\!R}^\nu$, $\mathcal{U} : \mathrm{I\!R}^{p_1} \times \dots \times \mathrm{I\!R}^{p_s} \times \mathrm{I\!R}^\nu \times \mathrm{I\!R} \to \mathrm{I\!R}^m$, such that the goal (2.3) in system (2.8), (2.12) (or (2.8), (2.13), (2.14)) is achieved.

To illustrate the previous definition we will discuss a simple, but instructive example of static state feedback synchronization.

Example 1. Feedback synchronization of two arbitrary second order oscillators.

Consider a second order system

$$S_1 : \quad \ddot{x}_1 = f_1(x_1, \dot{x}_1, t) \tag{2.15}$$

together with a controlled second order system

$$S_2: \quad \ddot{x}_2 = f_2(x_2, \dot{x}_2, t) + u \tag{2.16}$$

Typically S_1 and the uncontrolled version of S_2 could represent the dynamics of a pendulum, or a periodically forced Duffing equation, or a periodically forced Van der Pol equation. Assume it is desirable to have that the system S_2 asymptotically synchronizes with the first system S_1, that is we require

$$\lim_{t\to\infty} [(x_2(t), \dot{x}_2(t))^T - (x_1(t), \dot{x}_1(t))^T] = (0,0)^T \tag{2.17}$$

by means of a well chosen feedback controller fo u. Note that in this setting it is logical to look for asymptotic – preferably even exponential – synchronization since we may not have that S_1 and S_2 would start at the initial time $t = 0$ in the same initial states. A static state feedback controller that solves the above problem is given by

$$u = f_1(x_1, \dot{x}_1, t) - f_2(x_2, \dot{x}_2, t) - k_d(\dot{x}_2 - \dot{x}_1) - k_p(x_2 - x_1) \tag{2.18}$$

The error dynamics, with $e = x_2 - x_1$ then reads as

$$\ddot{e} + k_d\dot{e} + k_p e = 0 \tag{2.19}$$

which is exponentially stable for all positive k_d, k_p. The feedback controller (2.18) thus achieves the synchronization goal (2.17), no matter what the original systems S_1 and S_2 were: for instance we may achieve synchronization of a forced Van der Pol oscillator and a forced Duffing oscillator.

Obviously, the controller (2.18) might not be feasible if the part of state information of both S_1 and S_2 is missing. In that case more subtle control schemes have to be designed – like observer-controller combinations developed in [19, 20] or alternative methods may be invoked.

3 Synchronization of two coupled Lorenz systems

Let us discuss the synchronization phenomenon between two coupled Lorenz systems. Consider the following system:

$$\begin{cases} \dot{x}_1 = \sigma(y_1 - x_1) + u \\ \dot{y}_1 = rx_1 - y_1 - x_1 z_1 \\ \dot{z}_1 = -bz_1 + x_1 y_1 \end{cases} \qquad \begin{cases} \dot{x}_2 = \sigma(y_1 - x_2) - u \\ \dot{y}_2 = rx_2 - y_2 - x_2 z_2 \\ \dot{z}_2 = -bz_2 + x_2 y_2 \end{cases} \tag{3.1}$$

where $u \in \mathbb{R}$ is the control input which has to be chosen.

We pose the synchronization problem in the following way: find the control u as a function of measurable variables which, as we assume here, are x_1 and x_2 and that ensures asymptotic synchronization with respect

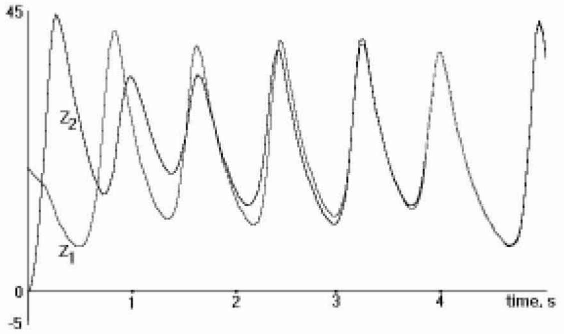

Figure 1: Synchronization of two Lorenz systems. Comparision of time histories obtained from 1st and 2nd system, $\lambda_2 = 0.2$

to the functions:

$$\begin{aligned} g_1(x_1, x_2) &= \|x_1 - x_2\| \\ g_2(y_1, y_2) &= \|y_1 - y_2\| \\ g_3(z_1, z_2) &= \|z_1 - z_2\| \end{aligned}$$

Consider the following linear output feedback controller;

$$u = -\gamma(x_1 - x_2)$$

where $\gamma > 0$ is a synchronization gain also refered in the literature to as a coupling constant. Let us prove that the overall system can be synchronized if γ exceeds some threshold value.

First let us show that all trajectories of the overall system are bounded. Consider the following scalar function:

$$W = (x_1^2 + y_1^2 + (z_1 - \sigma - r)^2 + x_2^2 + y_2^2 + (z_2 - \sigma - r)^2)/2$$

Calculate its time derivative;

$$\begin{aligned} \dot{W} &= -(\gamma + \sigma)x_1^2 - y_1^2 \\ &- b\left(z_1 + \frac{\sigma + r}{2}\right)^2 + b(\sigma + r)^2/2 + 2\gamma x_1 x_2 \\ &- (\gamma + \sigma)x_2^2 - y_2^2 - b\left(z_2 + \frac{\sigma + r}{2}\right)^2 \\ &+ b(\sigma + r)^2/2 \end{aligned}$$

Notice that $-\gamma x_1^2 + 2\gamma x_1 x_2 - \gamma x_2^2 \le 0$ for all x_1, x_2 as long as $\gamma \ge 0$. Notice also that if $\gamma \ge 0$ then the equation $\dot{W} = 0$ determines an ellipsoid in $\mathbb{R}^6$ outside which $\dot{W} \le 0$. Therefore all trajectories of the composite system are bounded. Moreover all trajectories tend to the following sphere:

$$x_1^2 + x_2^2 + y_1^2 + y_2^2 + (z_1 - \sigma - r)^2 + (z_2 - \sigma - r)^2 = K^2(\sigma + r)^2 \tag{3.2}$$

Indeed, to prove this pick some K such that inside of sphere (3.2) we have $\dot{W} \le 0$, that is

$$K^2 = \frac{1}{4} + \frac{b}{4}\max\{\frac{1}{\sigma}, 1\}$$

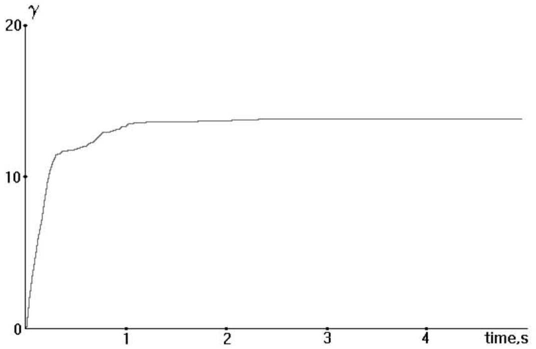

Figure 2: Synchronization of the two Lorenz systems. Adjustment of the synchronization gain γ, $\lambda_2 = 0.2$

Using (3.2) we can easily calculate upper bounds of the following values:

$$s_1 = \overline{\lim_{t\to\infty}} |z_1(t)| = \overline{\lim_{t\to\infty}} |z_2(t)| \le (K+1)(\sigma + r)$$

$$s_2 = \overline{\lim_{t\to\infty}} |y_1(t)| = \overline{\lim_{t\to\infty}} |y_2(t)| \le K(\sigma + r)$$

Introduce error vector in the standard manner: $e = (e_x,\ e_y,\ e_z)^T = (x_1 - x_2,\ y_1 - y_2,\ z_1 - z_2)^T$ and consider the following Lyapunov function:

$$V(e_x, e_y, e_z) = \frac{1}{2}(e_x^2/\sigma + e_y^2 + e_z^2)$$

Its derivative along trajectories of the overall system satisfies:

$$\dot{V} = -(1 + 2\gamma/\sigma)e_x^2 + (1 + r - z_2)e_x e_y - e_y^2 + y_2 e_x e_z - be_z^2$$

Since we have already proved that $z_2(t)$ and $y_2(t)$ are bounded it is seen that it is possible to choose γ large enough such that $\dot{V}$ is negative definite as a function of the error vector, therefore $\lim_{t\to\infty} V(e_x(t), e_y(t), e_z(t))$ exists. Furthermore, from the integrability of V on the infinite time interval, continuity of V in e and uniform continuity of $e(t)$ in t (this continuity is uniform in view of the boundedness of the error vector) and applying Barbalat's lemma [24], we obtain that the composite system is asymptotically synchronized with respect to function V provided that γ is greater than some threshold value:

$$\gamma \ge \bar{\gamma}, \quad \text{where} \quad \bar{\gamma} = \sigma\left(\frac{(1 + r + s_1)^2 + s_2^2/b}{8}\right)$$

It is also clear that asymptotic synchronization with respect to function V is equivalent to the asymptotic synchronization with respect to the set of functions g_1, g_2, g_3 and therefore we have solved the problem as it has been stated.

We have proved that the asymptotic synchronization can be achieved if one take γ large enough. From a practical point of view this result is not always satisfactory since determination of the threshold value of the coupling constant requires to know parameters of the two Lorenz systems. Thus it is interesting to find an adaptive algorithm which tunes γ until synchronization occurs. In order to achieve adaptive synchronization the following adaptation algorithm can be applied:

$$\gamma(t) = \gamma_0 + \lambda_1(x_1(t) - x_2(t))^2 + \lambda_2 \int_0^t (x_1(s) - x_2(s))^2 ds$$

where $\lambda_1 \ge 0, \lambda_2 > 0$. To prove that this algorithm ensures synchronization one can calculate the time derivative of the following scalar function:

$$V_2(e_x, e_y, e_z, \gamma) = V(e_x, e_y, e_z) + \frac{1}{2}(\gamma - \lambda_1 e_1^2 - 2\bar{\gamma})^2/\lambda_2$$

We carried out a computer simulation to show synchronization effect between two Lorenz systems. Parameters of simulation were chosen as follows: $\sigma = 10, b = 8/3, r = 28,\ \lambda_1 = 0, \lambda_2 = 0.2,\ x_1(0) = 10, y_1(0) = 2, z_1(0) = 20,\ x_1(0) = -10, y_1(0) = -2, z_1(0) = 0,\ \gamma_0 = 0$. Fig. 1 shows the transient process in the coupled system. Obviously the goal of synchronization is achieved. Figure 2 shows how the adaptive algorithm changes the synchronization gain γ.

4 Conclusion

An attempt is made to give a fairly general definition of synchronization corresponding to intuition encompassing most of the known definitions and applications, and capturing peculiarities of both self-synchronization and controlled synchronization. The general definition was illustrated by a number of examples.

Based on the introduced definitions a practical problem of synchronization of vibrating actuators [6, 7] including cases of both self-synchronization and controlled synchronization also can be considered, see [8].

The presented general control synchronization problem allows to formulate and solve the control design problem for various dynamical systems. Note that using for instance a speed-gradient method for nonlinear adaptive control [11, 12] the above problems can also be solved in an adaptive setting when some or all parameters of the controlled system are unknown. In a similar way a robustness analysis in the spirit of [18] is possible.

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