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STOCHASTIC SIGNAL CONVERSION: LEARNING INTERACTIVELY

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Abstract: The aim of the course signal conversion is to learn designing a hybrid measurement and data processing system with a specified overall signal-to-noise ratio. The noise is caused by various error sources in the measurement chain: bandwidth limitation, sampling, quantisation and reconstruction. The education method is based on simulation tools, used interactively by the students to learn quickly the consequences of changing design parameters on the quality of the output signal

Keywords: stochastic signals, signal conversion, conversion errors, interactive learning.

1. INTRODUCTION

The main topic of this course is teaching students to choose the appropriate parameters values in a measurement chain in order to guaranty the quality of the processed data in terms of power relations. Figure 1 shows a general scheme of the signal processing functions, including errors provoked by the system parts. The main subjects of the course are related with these functions [1].

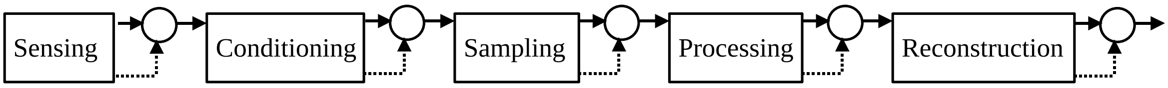

Figure 1. Signal propagation through the measurement chain. Dashed lines indicate errors added to the signal.

The aim is how to make estimations and recommendations for the best combination of anti-aliasing filters, sampling rate, digital processing algorithms and reconstruction methods. The course consists of an electronic textbook, working interactively with simulation toolboxes. We have developed these toolboxes for this course, based on MathCad, as help tools for exploring graphically the subjects of the course. In section 2 the education method is described shortly. In sections 3, 4 and 5, some subjects of the course are described. Each of these sections starts with a short introduction, followed by exercises in the corresponding simulation toolbox, and concluded with some theoretical aspects.

2. EDUCATION METHOD

The education method is based on interactions between the course elements on one hand (shown in the upper half of figure 2) and the corresponding toolboxes on the other hand as indicated in the grey area of this figure. Vertical arrows symbolise these interactions.

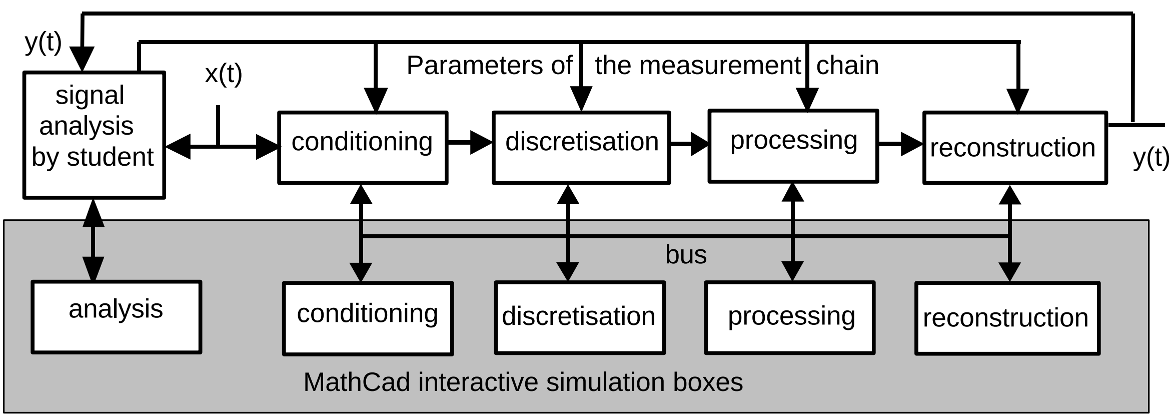

Figure 2. Scheme of the interactive education method. $x_n(t)$: measurement signal including noise; $y_n(t)$: reconstructed signal including noise.

Based on the analysis of stochastic measurement signals x(t), students learn to adjust the parameters of the measurement chain functions in a correct way, in view of the accuracy requirements of the final result. The resulting signal y(t) is analysed and compared with x(t). From the analysis of the error signal y(t)-x(t), students may change particular parameters of the measurement chain, in order to get a minimal error. The effect of each parameter change on the signal property can be explored graphically and numerically using the corresponding toolboxes. These toolboxes are activated simply by clicking the figures in the electronic text book. In this way, students can perform experiments by changing parameters: the propagation of these changes to the output signal can be explored immediately. Observing the simulated output y(t) (or its frequency spectrum), students learn to evaluate the correctness of the chosen signal processing parameter values.

An anti-aliasing filter, for example, can suppress high frequency noise but, at the same time, also high-frequency components in the measurement signal. So, interactively, students discover how to choose the best parameters for a required signal-to-noise ratio. Moreover, the associated formal expressions can be interpreted more easily.

3. SUBJECT 1: STOCHASTIC SIGNALS

First, the students have to determine quantitatively the signal properties during propagation through the measurement chain. They learn to estimate signal characteristics in terms of power spectral densities and correlation functions, for a given realisation of a stochastic signal. With the simulation box, such signal characteristics are displayed graphically, while numerical calculations can be made too. In this box, a stochastic signal with a certain power frequency characteristic is supposed to be generated by applying white noise to a so-called shape filter.

3.1 Using the stochastic signal box

Next figures are created using the stochastic signal analysis box. The box allows simple adjustment of various signal parameters. The results of these changes are displayed in the time domain as well as in the frequency domain.

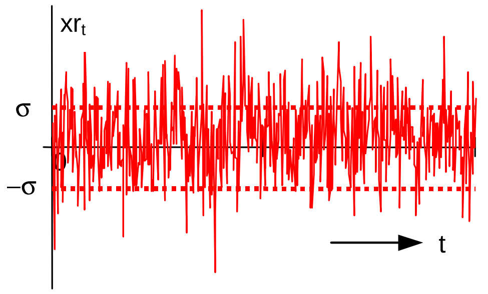

Figure 3. One realisation of normal distributed white noise. Parameters to be adjusted: mean μ (=0); standard deviation σ (=1)

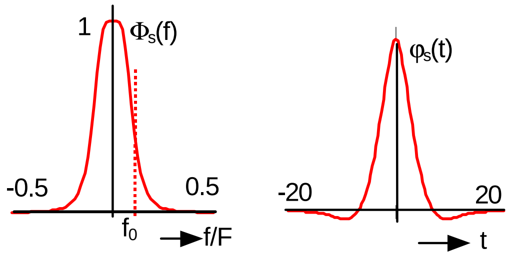

Figure 4. $\Phi_s(f)$: power frequency characteristic; $\varphi_s(t)$: correlation function of the shape filter

First, the parameters of a random number generator are defined, resulting in a white noise signal with a specified distribution (figure 3). Next, by applying a shape filter with a certain order P and cut-off frequency f_0 (figure 4) a random signal with specified spectral characteristics is obtained. Such exercises stimulate visual experience in the impact of stochastic signal parameters. Moreover, this procedure of shaping stochastic signals allows the use of analytical expressions about power densities. Figure 5 shows such a shaped signal in time domain, whereas figure 6 is the averaged spectrum over 50 realisations.

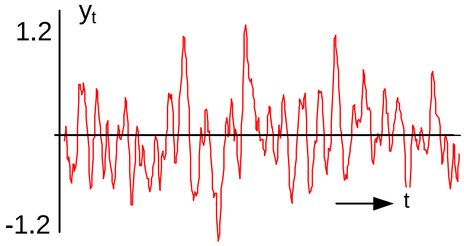

Figure 5. One realisation of noise, which is shaped by the filter of figure 4.

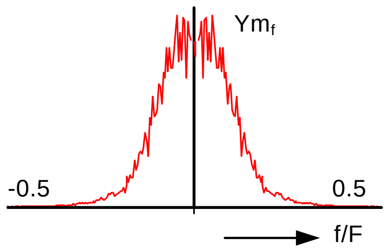

Figure 6. Mean power frequency characteristic of 50 realisations.

By comparing figures 6 and 4, students may observe that the power spectral density function of the filtered white noise is, indeed, determined by the frequency characteristic of the shape filter. Using this box, students will get a better understanding of the mathematical relations and their limited use in time domain as well as in frequency domain.

3.2 Theory of stochastic signal characteristics

After the exercises of section 3.1 students should be able to understand the expressions about the power spectral density function $\Phi_{xx}(i\omega)$ and the correlation function $\varphi_{xx}(\tau)$, which are mutually related by the Fourier transforms:

$$\varphi_{xx}(\tau) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \phi_{xx}(i\omega) e^{i\omega\tau} d\omega$$

$$\phi_{xx}(i\omega) = \int_{-\infty}^{+\infty} \varphi_{xx}(\tau) e^{-i\omega\tau} d\tau$$

Physically, both functions reflect the rate of change of the time signals.

4. SUBJECT 2: SAMPLING OF STOCHASTIC SIGNALS

In this part of the course, students learn to choose correct values for the sampling frequency and the number of quantisation levels. To do so, estimations of sampling errors should be made first. In this section, sampled signals are described in time and frequency domain. In order to estimate the sampling errors in terms of power, a definition of aliasing errors is introduced. Using the sampling box, students can simulate sampling of signals and display their frequency spectra.

4.1 Using the sampling box

In figure 7, a realisation of a sampled normally distributed stochastic signal is simulated. Figure 8 shows the associated frequency spectrum. The characteristics of the shape filter are: order 1, cut off frequency 10 Hz; the sampling rate is100 Hz.

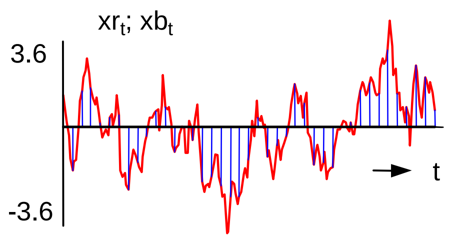

Figure 7. Sampled stochastic signal. Bars indicate the samples.

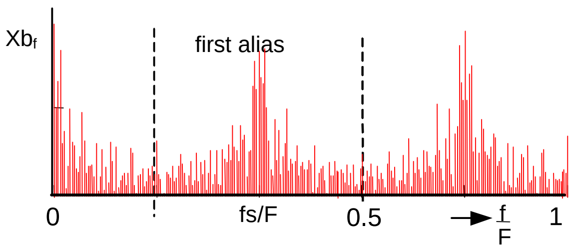

Figure 8. Amplitude spectrum of the sampled signal.

Observing figure 7, students may notice that a correct reconstruction of the original signal from the samples is rather difficult. The fine-structure of the signal shape is missing. Further, in figure 8, a periodic frequency spectrum is observed, with a period equal to the sampling rate. The part around 0 Hz is equivalent to the original spectrum, whereas around n.f_s AM versions of the original spectrum appear. These frequency bands overlap each other, causing aliasing errors. Now, in this toolbox the order of the shape filter of figure 4 can be increased, for instance to P=2, resulting in a smaller region of overlap in the frequency domain (figure 10). The toolbox responds with a smoother stochastic signal in the time domain (Figure 9).

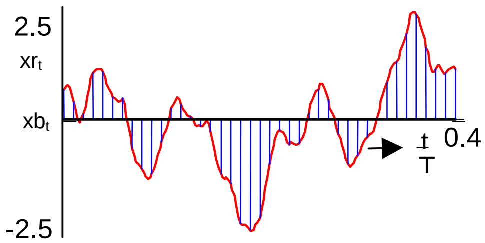

Figure 9. As figure 7, but now P=2

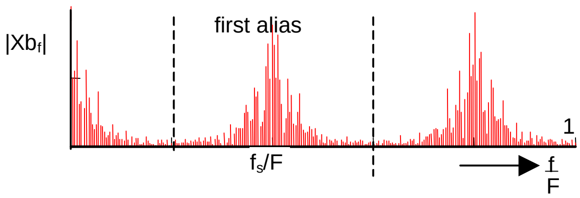

Figure 10. Amplitude spectrum of the sampled signal from figure 9

From these exercises it can be concluded that suppression of the higher frequency components in the analogue signal results in a better separation of the aliases of the sampled signal.

After these experiments the theory can be understood more easily. This theory describes the sampling process and the errors in terms of power.

4.2 Theory of sampled signals

Evaluating the exercises, students have seen graphically that the sampled signal $x_s(t)$ is a multiplication of the analogue signal x(t) with the periodic sampling function $D(t,T_s)$, where T_s is the sampling time. The frequency spectrum is periodic with the sampling rate $f_s=1/T_s$. After these exercises students should be able to understand the sampling process analytically. The sampling process is given by the expression:

$$x_S(t) = x(t).D(t,T_S) = x(t).\sum_k \delta(t-k.T_S)$$

The sampled signal $x_s(t)$ can be written as an infinite sum of AM harmonic signals:

$$x_S(t) = x(t).\sum_k \exp(2\pi jkf_S)$$

In the frequency domain the power spectral density function $\Phi_{XX}^{SS}(f)$ of a sampled signal is a periodic function of $\Phi_{xx}(f)$, the power spectrum of the analogue signal x(t):

$$\Phi_{XX}^{SS} = \sum_k \Phi_{XX}(f-k.f_S)$$

The period equals $f_s=1/T_s$.

In order to estimate the accuracy of the sampled signal, the aliasing error P_A is introduced, defined as:

$$P_A = \frac{1}{P_X} . \int_{\frac{fs}{2}}^{\infty} \Phi_{XX}(f).df$$

Graphically, P_A is the part of the surface area under curve Φ_o from $f_s/2f_0$ (figure 11). A second error is introduced by the tail of the first alias Φ_1, overlapping Φ_0.

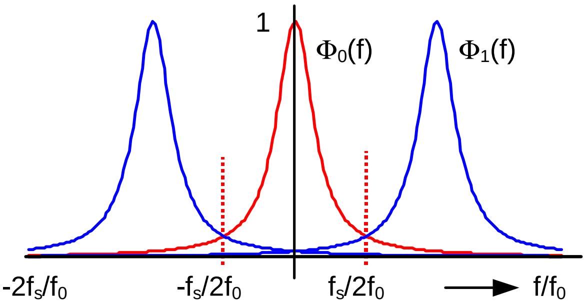

Figure 11. Frequency images of a sampled first order signal; cut-off frequency f_0; sample frequency $f_s=6.4\ f_0$.

From these expressions it follows that the error decreases with increasing order P of the power spectrum. Now, the simulation box can be opened again to see the shape of the aliasing errors graphically.

4.3 Return to the sampling box.

With the sampling box both errors can be simulated to show their contribution in time domain. For example, we consider a sampled square wave signal (figure 12). The original signal is reconstructed by an ideal low pass filter of half the sampling rate. Figures 13 and 14 show the aliasing errors separately, for this case. Frequency components above $f_s/2$ are suppressed completely, but this causes a lower rate of change (figure 13).

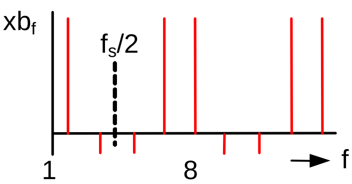

Figure 12. Spectrum of a sampled 1 Hz square wave. Sampling rate is $f_s=8$ Hz.

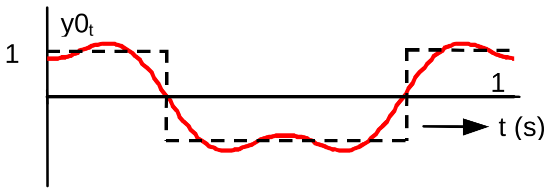

Figure 13. Reconstruction result by ideal filter at $f_s/2$; the dashed line indicates the original signal.

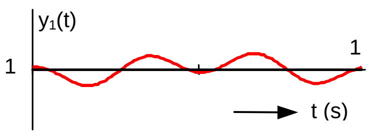

Figure 14. Error signal from first alias.

The error $y_1(t)$ due to the first alias (frequencies below $f_s/2$), is shown in figure 14. With the aid of this simulation box, students get feeling for aliasing errors of various types of signals for different signal-processing situations. In this way, signal error estimations, formula’s and graphics come close together.

5. SUBJECT 3: RECONSTRUCTION ERRORS

In this section, the aim is to determine the best reconstruction filter. The final decision about the best reconstruction can be made in relation with other system- and signal parameters. Using the reconstruction simulation box, students learn to make good estimations of reconstruction errors and find the best reconstruction with regard to the signal-to-noise ratio.

5.1 Using the signal reconstruction box

Figure 15 shows one realisation of a low frequent stochastic signal. Again, the bars represent the analogue signal. $|Xs_f|$ in figure 16 displays the amplitude frequency characteristic of a realisation of a sampled signal; in the same figure, the characteristic Hr(f) of a possible reconstruction filter has been drawn: its parameters are order P_r=2 and cut-off frequency $f_r=0.5f_0$. The student has to find out whether this is a proper choice.

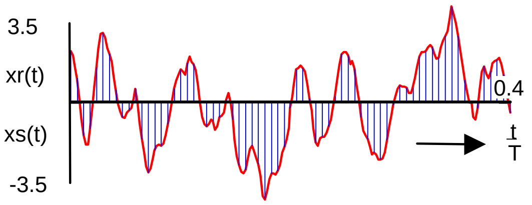

Figure 15. Line presentation: stochastic signal; shape filter has order P_s=2.

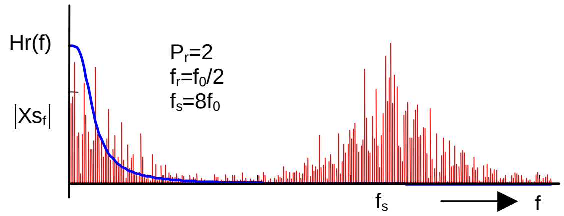

Figure 16. Amplitude frequency characteristic of the sampled signal from figure 15

Obviously, from figure 16, the low pass filter suppresses the frequency components around the sampling frequency; however, also part of the original signal band will be suppressed: its cut-off frequency f_r is chosen too low. The resulting error of omission ε_o is the rms value of the difference between the original $x_r(t)$ and the reconstructed signal y(t).

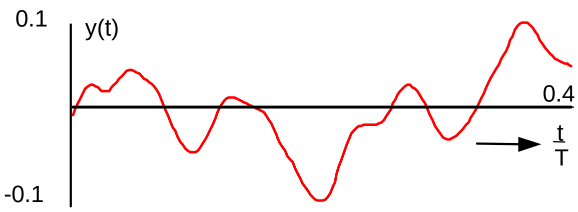

Figure 17. Result of the reconstruction filter of figure 16.

The error of omission could also be estimated by directly applying the analogue signal to the reconstruction filter.

Another error, the error of commission ε_c, is caused by insufficient second alias suppression, that occurs when the cut-off frequency of the reconstruction filter is chosen too high (figure 18). Its effect is a "ripple" on the time signal, as shown in figure 19.

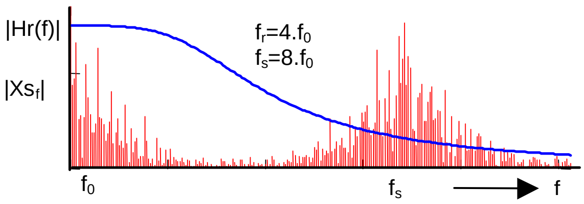

Figure 18. A too wide band reconstruction filter.

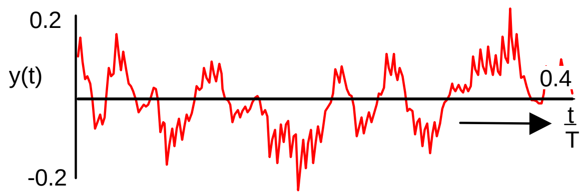

Figure 19. "Ripple" caused by insufficient suppression of the first alias.

Increasing the filter bandwidth f_r will decrease the error of omission and increase the error of commission. Based on these considerations, students can estimate the optimal reconstruction filter, given the stochastic signal parameters and the sampling rate. In this way, formulas and estimations can be connected to the physical reality.

5.2 Theory of optimisation of reconstruction errors

The choice of the optimal reconstruction can also be found theoretically, based on the power spectrum density of the sampled stochastic signal. In the next figures two situations are drawn. Figure 20 shows the power transfer function PHr(f) of a reconstruction filter with order 2 and cut-off frequency $f_r=5f_0$; the sampling rate $f_s=5f_0$. Figure 21 shows a filter with a lower cut-off frequency.

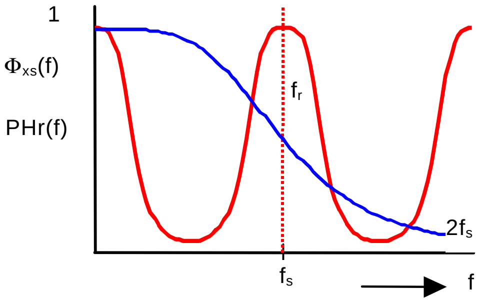

Figure 20. Power spectrum $\Phi_{xs}(f)$ of 2nd order data with cut-off frequency f_0.

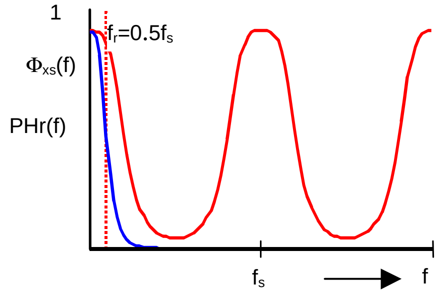

Figure 21. Application of the reconstruction filter with $f_r=f_0/2$.

Based on the experiments with the reconstruction box students can expect a too large error of commission in case of the filter of figure 20. On the other hand, the filter of figure 21 suppresses the high frequencies of the original signal too much. The power of the total reconstruction error depends on the cut-off frequency f_r, the parameters of the reconstruction filter $Hr(f,f_r)$ and the power spectral density $\Phi_{yy}(f,f_r)$ of the reconstructed signal y.

The power of the interference noise is:

$$\varepsilon_c^2(fr) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \left|Hr(f,fr)\right|^2 . \sum_{k=1}^{\infty} \Phi_{xx}(f-kfs)df$$

The power of the error of omission is:

$$\varepsilon_o^2(fr) = \sigma_x^2 + \sigma_y^2(fr) - \int_{-\infty}^{+\infty} 2\mathrm{Re}\{Hr(f,fr)\}\Phi_{xx}(f)df$$

The power of the total error is:

$$\varepsilon_r^2 = \varepsilon_c^2 + \varepsilon_o^2$$

Using these expressions in the reconstruction box, students can find the best reconstruction filter.

In figure 22 both errors are given as a function of the filter cut-off frequency relative to the signal cut-off frequency. Obviously, the optimum is at $f_r=3.5f_0$.

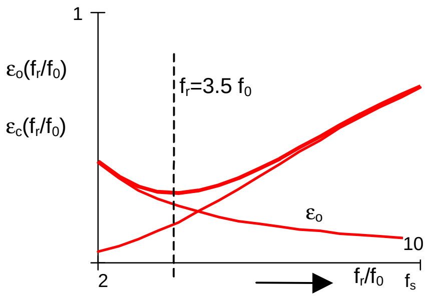

Figure 22. $\Phi_{yy}(f,f_r)$: relative errors and relative total reconstruction error as function of the relative cut-off frequency f_r/f_0 at a given sampling frequency.

6. CONCLUSION

Using several MathCad simulation boxes, students can look for the best combination of stochastic signal parameters, sampling rate and reconstruction parameters. Sometimes it will be unavoidable to limit the bandwidth of the stochastic signal before sampling. This will result in a lower reconstruction error. At the other hand, bandwidth limitation will also attenuate the high frequencies of the original signal. The method of signal conditioning before sampling is effective in case of a low frequency signal and high frequency noise.

This interactive method provides a course which can be done in the class room as well as on an individual base. The preparation time for the examination appears to be decreased significantly, because experience and knowledge of theory go together. This method gives students more feeling for the relationship between the signal processing parameters and the estimated conversion noise.

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