On the Determination of the Maximum Turnable State of a Part

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Abstract: Machining planning is an important task and requires experienced personnel and directly affects the cost of a product. Computer aided methods are increasingly being developed and used to assist in the planning task. We describe a new concept, the Maximum Turnable State of a part, and algorithms for its computation. The concept is intuitive and relates to an intermediate state of every turned part. By recognizing it explicitly, and using it in the machining planning, efficiency can be increased. We describe the concept, its use, algorithms for its determination and some examples.

Keywords: machining planning, maximum turned state, mill-turns, computer-aided process planning

1. INTRODUCTION

In this paper, we consider the Maximum Turnable State (MTS) - a novel concept in product design and manufacturing process planning. This concept was recently introduced by Yip-Hoi and Dutta [Yip-Hoi et al., 1998] and shown to be of practical use in the process planning tasks involving machining on mill-turn machines. Figure 1 illustrates a common mill/turn configuration.

The concept of MTS is simple. Consider a generic final part (FP) that is composed of planar (i.e. milled) and non-planar (i.e. turned) surfaces. The MTS is that intermediate state of the part beyond which no more turning operations can be done (without gouging the milled surfaces). By definition, the MTS is a revolute intermediate state. We provide a mathematical definition: A three dimensional object is said to be a spherical cylinder, if it has a central axis which is a line segment, and every cross-section perpendicular to this axis is a circle with its center on the central axis. The diameters of the cross-sections may vary. Given a final part which is a three dimensional object, its Maximum Turnable state is defined to be the smallest volume spherical cylinder containing it.

Knowing the MTS of any part can be of use in many instances. For example, by a boolean subtraction of the MTS from the initial workpiece (bar stock) one can obtain the total volume to be turned, and can directly generate the cutter path. Turning is more

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efficient than milling (w.r.t material removal rates) and therefore this strategy leads to efficient fabrication. On the design side, the MTS can be used effectively to determine near "net-shaped" workpieces that correspond to several different parts. That is, for a part family consisting of say M different parts, the MTS can be used to determine N ($N << M$) intermediate turned parts that can then be machined to yield the full part family (of M parts). This enables the effective utilization of turning resources, when unused, to turn the initial workpieces to the N intermediate states. As needed, the appropriate intermediate state can be further machined (possibly some additional turning and milling) to realize the final part. These N intermediate states/shapes can be produced either by turning or even by casting, depending upon the best utilization of the available resources.

Manufacturing process planning under such an environment (of intermediate stages of a part) is quite different. Feature extraction for machining is now driven by the MTS and not by machinable volumes.

The efficient determination of the MTS for a general part is quite complex. If one assumes a part axis (e.g. chosen by the designer), as in [1], the problem simplifies. However, for a complex part, it is not easy, or visually possible, to determine the best axis for a part.

In this paper, we shall describe optimization methods for determining the best part axis for computing the MTS of a part. In Section 2 we formulate the problem explicitly and describe a conceptual algorithm for determining the MTS of a general FP (without holes). Section 3 deals with the case that the FP is a (convex) polyhedron. A numerical algorithm for the calculation of the MTS is given. Section 4 deals with non-convex objects and Section 5 with holed objects. Section 6 concludes with some remarks on the structure of the MTS-problem and comments on future developments.

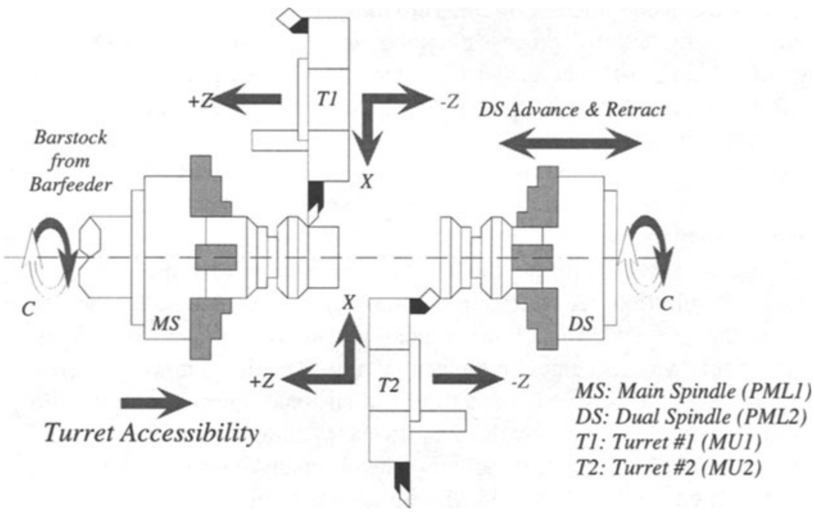

Figure 1 *A common Mill/Turn configuration*

2. A GENERAL DESCRIPTION OF THE MTS-PROBLEM

We begin with a more explicit description of the problem.

MTS-problem: Let FP be a 3D final part. Consider the machine axis (of a Mill/Turn) and a 2D profile containing that axis such that the body obtained by revolving the profile around the axis contains the final part. We want to determine such a body with minimal volume. (A body with minimizing volume is called the MTS of FP.)

This formulation leads to the following conceptual method for solving the MTS-problem for the case that FP does not have holes (or holes are neglected).

First, we choose a representation of the machine axis ax: Given a center point $c = (c_1, c_2, c_3) \in \mathbb{R}^3$ and a direction $d(\phi, \varphi) = (\cos\phi\cos\varphi, \sin\phi\cos\varphi, \sin\varphi)$, $\phi, \varphi \in [-\frac{\pi}{2}, \frac{\pi}{2}]$, the axis is defined by the line trough c along the direction $d(\phi, \varphi)$ (see Figure 2),

$$\text{ax:} \qquad ax(\gamma) = c + \gamma(\cos\phi\cos\varphi, \sin\phi\cos\varphi, \sin\varphi), \quad \gamma \in \mathbb{R}\,. \tag{1}$$

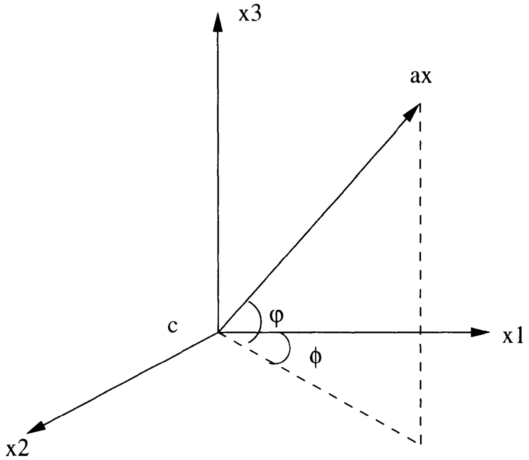

Figure 2 *The machine-axis ax.*

Assume the axis, i.e. the vector (c, ϕ, φ), is fixed. The 2D profile of revolution of FP is calculated as follows. For fixed γ we consider the plane $pl(\gamma)$ through the axis point $ax(\gamma)$, perpendicular to the direction $d(\phi, \varphi)$. This plane is given by

$$pl(\gamma) = \{x \in \mathbb{R}^3 \mid (x - c)^T d(\phi, \varphi) = \gamma\}\,. \tag{2}$$

Here, for vectors $x, y \in \mathbb{R}^3$, $x^T y$ denotes the dotproduct, $x^T y = x_1 y_1 + x_2 y_2 + x_3 y_3$ and by $||x - y|| = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2 + (x_3 - y_3)^2}$ we define the distance between x and y.

The slice $sl(\gamma)$ is the intersection of the plane $pl(\gamma)$ with FP (see Figure 3). The value $b(\gamma)$ of the 2D profile function is the maximum distance between the axis point $ax(\gamma)$ and the points in the slice $sl(\gamma)$,

$$b(\gamma) = \max\{||x - ax(\gamma)|| \mid x \in sl(\gamma)\}\,. \tag{3}$$

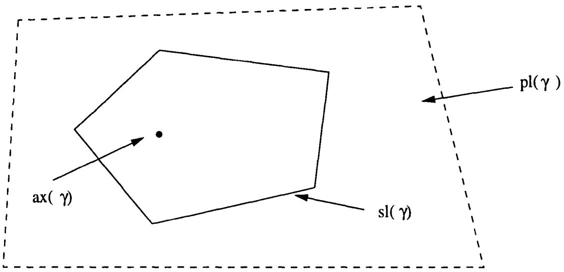

Figure 3 *The slice $sl(\gamma)$*

In the sequel, γ_b and γ_e are the minimum and maximum values of γ, respectively, such that the plane $pl(\gamma)$ intersects the final part. The value γ_b can be calculated by solving the optimization problem

$$(\mathrm{P}_b): \qquad \min\ x^T d(\phi, \varphi) \qquad \text{subject to} \quad x \in \mathrm{FP}\,. \tag{4}$$

Let $\bar{x}$ be a solution of (P_b), then γ_b is given by $\gamma_b = (\bar{x} - c)^T d(\phi, \varphi)$. The value γ_e is found by solving the problem (P_e), obtained by replacing minimize by maximize in (P_b).

The final part is contained in the body obtained by revolving the profile function $b(\gamma)$, $\gamma \in [\gamma_b, \gamma_e]$ around the axis ax (see Figure 4). In fact, this body is the maximum turnable state of FP for this fixed axis. Using the formula $r^2\pi$ for the area of a circle with radius r, the volume $\mathrm{vol}(c, \phi, \varphi)$ of this body (for fixed (c, ϕ, φ)) is given by

$$\mathrm{vol}(c, \phi, \varphi) = \pi \int_{\gamma_b}^{\gamma_e} (b(\gamma))^2\, d\gamma\,. \tag{5}$$

Solving the MTS-problem is equivalent with solving the following optimization problem.

$$(\mathrm{P}_{\mathrm{MTS}}): \quad \text{find } c = (c_1, c_2, c_3) \text{ and } \phi, \varphi \in [-\tfrac{\pi}{2}, \tfrac{\pi}{2}] \text{ such that } \mathrm{vol}(c, \phi, \varphi) \text{ is minimal}\,. \tag{6}$$

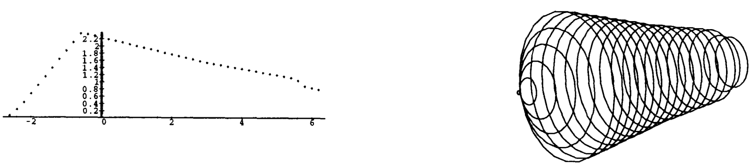

Figure 4 *a) The profile $b(\gamma)$.* *b) The MTS for a fixed axis.*

3. THE MTS OF A CONVEX POLYHEDRON

In this section we discuss the computation of the MTS of a polyhedral part. We assume that the surfaces of FP are defined by planes, $x^T a_j = b_j,\ j \in J = \{1, \dots, m\}$, with (normal) vectors $a_j \in \mathbb{R}^3$ and $b_j \in \mathbb{R}$. Then, the part is given as the intersection of the corresponding half-spaces,

$$FP = \{x \in \mathbb{R}^3 \mid x^T a_j \le b_j,\ j \in J\}. \tag{7}$$

Such a body is called a polyhedron. A vertex v of the polyhedron FP is a point on the surface of FP defined as the intersection of three (independent) defining planes, i.e.

$$v \text{ is a vertex} \iff v \in \text{ FP and } v^T a_{j_1} = b_{j_1},\ v^T a_{j_2} = b_{j_2},\ v^T a_{j_3} = b_{j_3},$$

with distinct $j_1, j_2, j_3 \in J$ and linearly independent vectors $a_{j_1}, a_{j_2}, a_{j_3}$. Let $V = \{v_1, \dots, v_l\}$ be the vertices of FP. An edge of the polyhedron is a line segment on the surface of FP given by the intersection of two planes. Any edge can be represented as the line segment between two vertices v_i, v_j of FP,

$$E_{ij}: \qquad E_{ij}(t) = v_i + t(v_j - v_i), \quad t \in [0, 1] \quad (\text{abbreviation } \overline{v_i\, v_j})\,. \tag{8}$$

We give an example.

Example 1 FP is given by the inequalities (see Figure 5):

$$\begin{array}{ccc} x_2 \le 1 & x_3 \le 1 & \frac{2}{3}x_1 + \frac{1}{3}x_3 \le 1 \\ -x_2 \le 1 & -x_3 \le 1 & -\frac{2}{3}x_1 + \frac{1}{3}x_3 \le 1 \end{array}$$

The vertices and edges of this object are:

$$\begin{array}{llll} v_1 = (1, 1, 1) & v_2 = (2, 1, -1) & v_3 = (-1, -1, 1) & v_4 = (-2, -1, -1) \\ v_5 = (-1, 1, 1) & v_6 = (-2, 1, -1) & v_7 = (1, -1, 1) & v_8 = (2, -1, -1) \end{array}$$

and $E_{17}, E_{15}, E_{37}, E_{53}, E_{34}, E_{56}, E_{78}, E_{12}, E_{46}, E_{48}, E_{82}, E_{26}$.

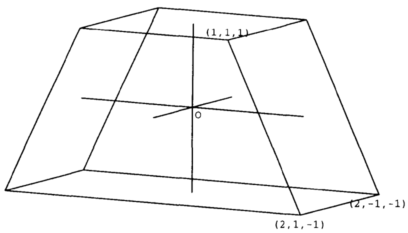

Figure 5 *A polyhedral final part.*

We now explain how the profile function $b(\gamma)$ can be calculated in the polyhedral case. For fixed γ, we have to consider the slice $sl(\gamma)$. In the polyhedral case, the slice is a 2 dimensional (convex) polygon (see Figure 3), the vertices of which are given by the intersections of the edges of FP with the plane $pl(\gamma)$. For a given edge $\overline{v_i v_j}$ we have to proceed as follows: Compute the intersection point of the line $v_i + t(v_j - v_i)$, $t \in I\!R$ with $pl(\gamma)$,

$$(v_i + t(v_j - v_i) - c)^T d(\phi, \varphi) = \gamma \qquad \text{or} \qquad \bar{t} = \frac{\gamma - (v_i - c)^T d(\phi, \varphi)}{(v_j - v_i)^T d(\phi, \varphi)}$$

If $\bar{t}$ lies in the interval [0, 1] then, $p := v_i + \bar{t}(v_j - v_i)$ is a vertex of the polygon $sl(\gamma)$. (For the case that $(v_j - v_i)$ is perpendicular to $d(\phi, \varphi)$ and $v_i \in sl(\gamma)$, the whole edge $\overline{v_i v_j}$ makes part of the slice $sl(\gamma)$ and both vertices v_j and v_i are vertices of the polygon.)

Let $P = \{p_1, \ldots, p_k\}$, $k = k(\gamma)$, be the set of vertices of the two dimensional polygon $sl(\gamma)$. Then, the maximum distance $b(\gamma)$ in (3) is given by

$$b(\gamma) = \max_{i=1,\ldots,k} \{||p_i - ax(\gamma)||\} .$$

The values γ_b and γ_e are calculated as follows. For polyhedral FP the solutions of the optimization problems P_b and P_e (cf. (4)) are attained at vertices of FP. Hence, to determine γ_b (and similarly γ_e), we compute a solution v_b of

$$\min_{v \in V} v^T d(\phi, \varphi) \qquad \text{and put } \gamma_b := (v_b - c)^T d(\phi, \varphi) . \tag{9}$$

Finally, we give an algorithm for calculating the volume $\mathrm{vol}(c, \phi, \varphi)$ numerically for fixed (c, ϕ, φ): Choose a natural number N, a mesh size $\Delta = \frac{\gamma_e - \gamma_b}{N}$ and define the grid points $\gamma_s = \gamma_b + \Delta \cdot s$, $s = 0, \ldots, N$. Then, calculate the values $b(\gamma_s)$ and approximate the profile $b(\gamma)$ on any interval $[\gamma_s, \gamma_{s+1}]$ by the line segment joining $b(\gamma_s)$ and $b(\gamma_{s+1})$ (piecewise linear interpolation). An approximation of the volume $\mathrm{vol}(c, \phi, \varphi)$ (cf. (5)) is given by

$$\mathrm{vol}^a(c, \phi, \varphi) = \pi \sum_{s=0}^{N-1} \int_{\gamma_s}^{\gamma_{s+1}} \left(b(\gamma_s) + \frac{b(\gamma_{s+1}) - b(\gamma_s)}{\Delta} (\gamma - \gamma_s) \right)^2 d\gamma . \tag{10}$$

The algorithm for finding the MTS of a polyhedral FP can now be summarized as follows.

Algorithm 1 *(Calculation of the MTS of a polyhedral FP)*

1. Compute the set V of vertices and the set E of edges of FP.

2. The problem of finding the MTS is the problem of finding the values (c, ϕ, φ) that minimizes the function $\mathrm{vol}^a(c, \phi, \varphi)$. For any given (c, ϕ, φ) we have described above how to compute this function approximately. Using it as the function evaluation subroutine, an appropriate algorithm can be used to find (c, ϕ, φ) that minimizes $\mathrm{vol}^a(c, \phi, \varphi)$.

4. MTS OF NON-CONVEX OBJECTS

In general, machined parts are non-convex. This section deals with non-convex bodies without holes. Again we restrict ourselves to parts with (finitely many) planar surfaces. Such an object can be described as the union of finitely many polyhedra, where each polyhedron is of the form (7). For brevity, we assume that FP is given as the union of two polyhedra FP^1 and FP^2,

$$\mathrm{FP} = \mathrm{FP}^1 \cup \mathrm{FP}^2 \quad \text{with} \quad \begin{array}{l} FP^1 = \{x \in I\!R^3 \mid x^T a_j^1 \leq b_j^1,\ j = 1, \ldots, m_1\} \\ FP^2 = \{x \in I\!R^3 \mid x^T a_j^2 \leq b_j^2,\ j = 1, \ldots, m_2\} \end{array}$$

The MTS of FP can be calculated by modifying the method in Section 3 as follows. (The generalization to the case of more polyhedra will be evident.): For fixed axis ax (1), i.e. for fixed (c, ϕ, φ), we can calculate the values γ_b^1, γ_e^1 and γ_b^2, γ_e^2 as well as the profile functions $b^1(\gamma)$ and $b^2(\gamma)$ for FP^1 and FP^2 as in Section 3. Then, the corresponding values γ_b, γ_e and the profile function $b(\gamma)$ for $\mathrm{FP} = \mathrm{FP}^1 \cup \mathrm{FP}^2$ are defined by

$$\gamma_b = \min\{\gamma_b^1, \gamma_b^2\}, \ \gamma_e = \max\{\gamma_e^1, \gamma_e^2\}, \quad b(\gamma) = \max\{b^1(\gamma), b^2(\gamma)\}, \ \ \gamma \in [\gamma_b, \gamma_e] .$$

Here, it is assumed that $b^1(\gamma)$ is defined as zero for $\gamma \notin [\gamma_b^1, \gamma_e^1]$ and similarly $b^2(\gamma)$. With these modifications the MTS of FP can be computed similar to Algorithm 1.

5. THE MTS OF HOLED OBJECTS

In this section we briefly discuss the calculation of the MTS for objects with holes. We are referring to objects with holes that possess at least one approach direction, from outside, for machining, as shown in Figure 7. For simplicity we assume that the final part FP is given as the 'set-valued' difference of an outer polyhedron FP^1 and an polyhedron FP^2 representing the hole,

$$\mathrm{FP} = \mathrm{FP}^1 \setminus \mathrm{FP}^2 , \quad \begin{array}{l} FP^1 = \{x \in I\!R^3 \mid x^T a_j^1 \leq b_j^1,\ j = 1, \ldots, m_1\} \\ FP^2 = \{x \in I\!R^3 \mid x^T a_j^2 \leq b_j^2,\ j = 1, \ldots, m_2\} \end{array} , \quad \mathrm{FP}^2 \subset \mathrm{FP}^1$$

For holed objects we have to calculate two different profile functions. A function $b^1(\gamma)$ corresponding to FP^1 and the profile function $b^2(\gamma)$ for the hole FP^2. We describe how this can be done.

Again, let the axis ax be fixed, i.e. the vector (c, ϕ, φ) is fixed. We calculate the values γ_b^1, γ_e^1 for FP^1 as in (9). Let $\gamma \in [\gamma_b^1, \gamma_e^1]$ be fixed. Again, we define the slices $sl(\gamma) = pl(\gamma) \cap FP$, $sl^1(\gamma) = pl(\gamma) \cap FP^1$ and $sl^2(\gamma) = pl(\gamma) \cap FP^2$. For the calculation of the function values $b^1(\gamma)$, $b^2(\gamma)$ we have to distinguish between different cases. For simplicity we will only discuss one of them.

The Case that $ax(\gamma) \notin$ FP and the slice $sl(\gamma)$ is given by two (piecewise linear) closed curves ($sl(\gamma)$ is double-connected) such that $ax(\gamma) \in \mathrm{FP}^2$ (see Figure 6): In contrast to $b^1(\gamma)$ (cf. (3)) the value $b^2(\gamma)$ is given by the minimum distance,

$$b^2(\gamma) = \min\{||x - ax(\gamma)|| \mid x \in sl^2(\gamma)\}.$$

In the polyhedral case, this value can be calculated explicitly. Let $p_1^2, \dots, p_{k_2}^2$, $k_2 = k_2(\gamma)$, be the vertices of the 2D polygon $sl^2(\gamma)$ calculated according to Section 3. We assume that these vertices are numbered in such a way that the segments

$$\overline{p_{i+1}^2\, p_i^2} = \{p_i^2 + t(p_{i+1}^2 - p_i^2),\ t \in [0, 1]\}$$

$i = 1, \dots, k_2$ define the boundary of $sl^2(\gamma)$ (put $p_{k_2+1}^2 := p_1^2$). Now we calculate the projections q_i of $ax(\gamma)$ onto the segments $\overline{p_{i+1}^2\, p_i^2}$ (points of minimal distance):

$$\bar{t} := \frac{(p_i^2 - ax(\gamma))^T (p_{i+1}^2 - p_i^2)}{(p_{i+1}^2 - p_i^2)^T (p_{i+1}^2 - p_i^2)} \quad \text{and} \quad q_i := \begin{cases} p_i^2 + \bar{t}(p_{i+1}^2 - p_i^2) & \text{if } \bar{t} \in [0, 1] \\ p_i^2 & \text{otherwise} \end{cases}$$

Then, with the vertices $p_1^1, \dots, p_{k_1}^1$, $k_1 = k_1(\gamma)$, of $sl^1(\gamma)$ we have

$$b^2(\gamma) = \min_{i=1,\dots,k_2} \{||q_i - ax(\gamma)||\} \quad \text{and} \quad b^1(\gamma) = \max_{i=1,\dots,k_1} \{||p_i^1 - ax(\gamma)||\} \tag{11}$$

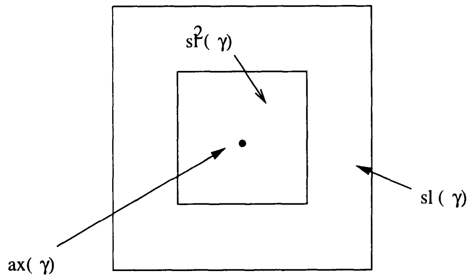

Figure 6 *The slice $sl(\gamma)$ of a holed object*

To calculate the volume of the MTS for a given axis (c, ϕ, φ), we distinguish between two different cases. The case A that the axis does not have any access to the hole FP^2 and the case B that ax enters or leaves the part FP^1 via FP^2.

To decide which case occurs we determine the points $ax(\hat{\gamma}_b)$ and $ax(\hat{\gamma}_e)$ where the axis enters and leaves FP^1 by solving the (one-dimensional linear) problem

$$\begin{matrix}\hat{\gamma}_b \\ \hat{\gamma}_e\end{matrix} := \begin{matrix}\min \\ \max\end{matrix} \{\gamma \mid ax(\gamma) \in FP^1\} = \{\gamma \mid (c + \gamma d(\phi, \varphi))^T a_j^1 \le b_j^1, j = 1, \dots, m_1\}$$

Finally $\mathrm{vol}^a(c, \phi, \varphi)$ is calculated according to the Cases A or B.

Case A, $ax(\hat{\gamma}_b) \notin FP^2$ and $ax(\hat{\gamma}_e) \notin FP^2$: In this case the profile function $b^1(\gamma)$ is calculated for the polyhedron FP^1 as in Section 3 and the volume is approximated by formula (10) with b replaced by b^1.

Case B, $ax(\hat{\gamma}_b) \in FP^2$ or $ax(\hat{\gamma}_e) \in FP^2$ (or both): Then, we determine the profile functions $b^1(\gamma)$, $b^2(\gamma)$ (as e.g. in (11)) on a discretization of $[\gamma_b^1, \gamma_e^1]$ and we calculate the volumes $\mathrm{vol}_1^a(c, \phi, \varphi)$, $\mathrm{vol}_2^a(c, \phi, \varphi)$ for these functions $b^1(\gamma)$, $b^2(\gamma)$ using formula (10). The MTS of the holed object is given approximately by the formula

$$\mathrm{vol}^a(c, \phi, \varphi) = \mathrm{vol}_1^a(c, \phi, \varphi) - \mathrm{vol}_2^a(c, \phi, \varphi) \; .$$

The calculation of the MTS of holed objects of a more complex structure is a topic of ongoing research and will be presented elsewhere.

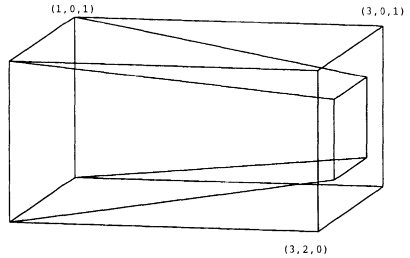

Figure 7: *Holed object*

6. FINAL REMARKS AND RECOMMENDATIONS

We comment on the structure of the MTS-problem. One could guess that for convex parts the optimal axis defining the MTS passes through the center of gravity of the object. In general this is not the case as is clear from the following example.

Example 2 Let FP be the half-cylinder,

$$FP = \{(x_1, x_2, x_3) \mid 0 \le x_1 \le 10, \; x_2 \le 0, \; x_2^2 + x_3^2 \le 1\}$$

It is easy to see that the x_1-axis is the optimal axis (i.e. for example $c = 0$ and $\phi = \varphi = 0$.) In fact, any plane intersecting the object (without intersecting the surfaces corresponding to $x_1 = 0$ and $x_1 = 10$) leads to a slice $\mathrm{sl}(\gamma)$ with value $b(\gamma)$ greater than or equal to 1. This value is minimal (equal 1) only for the x_1 axis. Hence, the optimal axis does not only avoid the center of gravity $(5, -\frac{1}{3}, 0)$ but does not even meet an inner point of FP.

In Section 2 we have parameterized all possible axes by $(c_1, c_2, c_3, \phi, \varphi)$ (cf. (1)). Consequently, we regard the MTS-problem (6) as a minimization problem in 5 variables. However the problem is only 4-dimensional. To see this, consider an (optimal) axis given by $(\hat{c}_1, \hat{c}_2, \hat{c}_3, \hat{\phi}, \hat{\varphi})$. Then it is obvious that any center point c on this axis will lead to the same profile and the same MTS (only γ_b, γ_e are shifted). So, in this parameterization, the optimal value is never unique and this parameterization can not be used to solve (P_{MTS}) by some continuous method (e.g. steepest descent-, Newton-type methods). To indicate that the problem is actually a 4 dimensional problem we can argue as follows. Any axis (which is a candidate for an optimal one) can be seen as a line intersecting the surface of FP at two points. These two points can vary independently on the two-dimensional surface of FP, i.e. we have a 4 dimensional problem. To parameterize the (P_{MTS})-problem by 4 parameters we can restrict ourselves to center points c in a fixed plane. However, this plane has to be chosen in such a way that the optimal axis (to be determined) is not 'parallel' to this plane (i.e. the optimal axis is not perpendicular to the normal vector $d(\phi, \varphi)$). Nearly 'parallel' situation would lead to a bad condition of the optimization problem.

We end up with comments on future developments.

1. The method as described in this paper has been implemented and examples have been calculated in [Wilharms, 1998]. However until now, the minimization procedure has been implemented as a simple discretization method. To make the algorithm more efficient, the minimization problem (P_{MTS}) should be solved by some continuous method using derivatives (e.g. a conjugate gradient method). A difficulty here is the fact, that the profile function $b(\gamma)$ as a function of (c, ϕ, φ) need not be differentiable at all parameter points (c, ϕ, φ). Note, that even in the polyhedral (convex) case the function $\mathrm{vol}^a(c, \phi, \varphi)$ is not a convex function in general. In particular, local (non-global) minima may occur.

2. Further investigations are necessary in the case of holed objects. In particular, FP's with different holes should be considered.

3. In a next step, the algorithm could be extended to the case of objects bounded by spherical and ellipsoidal surfaces.

4. The following generalization could be of interest (MTS of k axes). In this paper, we have used that FP was only machined once. What will happen if FP can be machined k times? i.e. what is the MTS of FP when k different axis can be used?

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