Phase-lag effects in oscillatory sheet flow

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Abstract

It is often assumed that in sheet flow conditions the transport rate in oscillatory flow behaves quasi-steady, i.e. showing a direct relation with the instantaneous flow velocity (e.g. Ribberink et al., 1994). In this paper it will be shown that this assumption is not always valid.

Therefore a new semi-unsteady sand transport model is developed which takes into account phase-lag effects of the sediment on the net transport rate. In order to verify this new semi-unsteady model and two existing quasi-steady models, new experiments were performed in the Large Oscillating Water Tunnel (LOWT) of WL | DELFT HYDRAULICS. Together with earlier experiments these measurements form a data set of net sand transport rates of uniform sand for three different grain sizes (D_{50} = 0.13; 0.21 and 0.32 mm) in combined wave-current sheet flow conditions.

The verification shows that phase-lag effects become important for a combination of fine sand, large flow velocities and small wave periods. Under these conditions the quasi-steady models cannot predict the behaviour of the net transport rates correctly and the predictions of the new semi-unsteady model show much better agreement.

Introduction

Sheet flow corresponds to conditions of high velocities when ripples are washed out and the bed becomes flat. In those conditions the majority of the sand transport takes place in a thin, high-concentrated layer close to the bed, i.e. the sheet flow layer.

Because of the small thickness of this layer it is generally assumed that the response time of the sand transport process to changes in flow conditions is small compared to the wave period. If that is the case, it can be expected that the time-dependent sediment transport rate depends directly on the instantaneous flow velocity or bed shear stress. This assumption is applied in all quasi-steady models.

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If the response time of sediment particles is not small compared to the wave period, the sediment concentration may lag behind the velocity. This will be caused by the fact that both the sediment entrainment from the bed into the flow and the settling of the particles back to the bed takes time. The latter depends on the settling time of a particle and on the height to which the particle is entrained, which is expected to be determined by the flow velocity.

Therefore it is expected that phase-lags will become important if sediment is entrained relatively high into the flow (large oscillatory velocities) and slowly settles down to the bed (fine sand) while the available fall time is short (small wave period). Moreover, it is expected that if phase-lags occur, the net transport rates are reduced. This can be explained as follows: If the transport behaves quasi-steady, the net (wave-averaged) transport rate under asymmetric oscillatory flow will always be in direction of the largest velocity, due to the non-linear relation between velocity and sand transport rate. If the sediment concentration lags behind the flow velocity, part of the sediment that is picked-up under a certain half wave cycle, may still be entrained into the flow and transported in opposite direction during the successive half wave cycle.

A new semi-unsteady model is developed to take into account the effect of phase-lags on the net transport rates. The model is called semi-unsteady, because it accounts for phase-lag effects, without describing the complete time-dependent velocity and concentration profiles. Apart from the new *semi-unsteady* model, also two existing *quasi-steady* models are presented, in order to compare the differences between these two types of models. The three transport models are verified against experimental data.

The set-up of the experiments and the measured net transport rates are presented first. Next the two existing quasi-steady models are described shortly and the new semi-unsteady model is presented. Finally, the behaviour of the measured net transport rates is discussed in relation with the predictions of the three transport models.

New experiments

Two new sets of sand transport experiments were carried out with uniform sand of different grain sizes. The experiments were performed in the Large Oscillating Water Tunnel (LOWT) at WL | DELFT HYDRAULICS from October 1996 to January 1997. The mean grain sizes of the two sands were 0.32 and 0.21 mm for series I and J respectively. The experiments are a follow up of the previous experimental series with 0.21 mm sand (Series E: Katopodi et al., 1994) and 0.13 mm sand (Series H: Janssen and Ribberink, 1996 and Janssen et al., 1997). The measurements of these four series can be considered as one consistent data set on sediment transport under combined wave-current flow in the sheet flow regime.

The LOWT of WL | DELFT HYDRAULICS is a large-scale facility that allows experiments to be performed at full scale (1:1). It consists of a large U-shaped tube, with a long horizontal test section and two vertical cylindrical risers. One of them is open to the air; the other riser contains a steel piston. The piston sets the water in motion and induces an oscillating water motion in the test section. The test section is 14 m long, 0.3 m wide and 1.1 m high. A 0.3 m thick sand bed can be brought into the test section, leaving 0.8 m for the oscillating water flow above the bed.

Underneath both risers a sand trap is constructed. The range of oscillatory velocities is 0.2-1.8 m/s; the range of periods is 4-15 s.

A recirculation flow system for the generation of a net current is connected to both cylindrical risers. The maximum superimposed net current velocity in the test section is about 0.5 m/s. A third sand trap is constructed in this recirculation system. A picture of the LOWT is given in Figure 1.

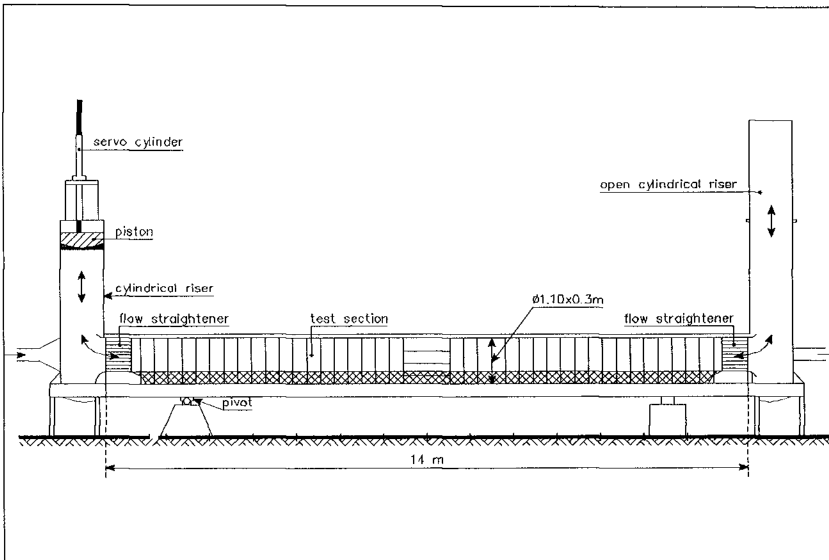

Figure 1: Large Oscillating Water Tunnel

The characteristics of the three sands, used in the experimental series are:

Series H:	$D_{10} = 0.10$ mm,	$D_{50} = 0.13$ mm,	$D_{90} = 0.18$ mm
Series E/J:	$D_{10} = 0.15$ mm,	$D_{50} = 0.21$ mm,	$D_{90} = 0.32$ mm
Series I:	$D_{10} = 0.22$ mm,	$D_{50} = 0.32$ mm,	$D_{90} = 0.46$ mm

The test conditions consisted of different combinations of a sinusoidal oscillatory flow and a net current. A condition is characterised by the wave period T (s), the velocity amplitude u_a (m/s) and the mean current velocity u_m (m/s) measured at 10 cm above the bed. For the present series I and J the conditions are mainly the same as in the previous experimental series with unsieved dune sand with D_{50}= 0.21 mm (Series E) and fine sand with D_{50}= 0.13 mm (Series H).

For every condition net (wave-averaged) transport rates were measured, together with the flow velocity at 10 cm above the bed. Net transport rates were derived from measured bed level changes and the weight of the sand, collected in the traps.

The measured net transport rates are based on measurements over the full width of the tunnel. However, the velocities are measured in the centreline of the tunnel. Due to boundary layer effects the net current velocities in the centreline are somewhat higher than the width-averaged values. Therefore the net transport rates are corrected such that they correspond to the measured velocities in the centreline of the tunnel (see e.g. Van der Hout, 1997). Table 1 presents the measured wave periods and flow velocities, together with the corresponding net transport rates, expressed in m^2/s, i.e. volume of sand per unit width per second.

Table 1: Measured wave periods, flow velocities and net transport rates

Test	D_{50} (mm)	T (s)	u_a (m/s)	u_m (m/s)	$\langle q_s \rangle$ (10^{-6} m^2/s)
H2	0.13	7.2	0.68	0.23	18.8
H3	0.13	7.2	0.93	0.24	34.9
H4	0.13	7.2	1.09	0.25	40.0
H5	0.13	7.2	1.30	0.24	51.7
H6	0.13	7.2	1.47	0.24	65.5
H7	0.13	7.2	0.49	0.42	15.6
H8	0.13	7.2	0.67	0.43	47.4
H9	0.13	7.2	0.94	0.43	85.7
H24	0.13	4.0	0.68	0.24	12.8
H44	0.13	4.0	1.06	0.25	9.0
H212	0.13	12.0	0.68	0.23	19.9
H412	0.13	12.0	1.09	0.24	97.1
J1	0.21	7.20	1.06	0.24	46.3
J2	0.21	7.20	1.28	0.25	74.4
E2	0.21	7.22	1.47	0.23	111.8
J3	0.21	7.20	0.46	0.41	9.0
J4	0.21	7.20	0.65	0.41	25.3
E4	0.21	7.23	0.95	0.44	84.4
J5	0.21	4.00	1.04	0.24	29.2
J6	0.21	12.0	1.09	0.23	49.2
I1	0.32	7.2	1.47	0.26	94.0
12	0.32	7.2	1.70	0.25	152.3
13	0.32	7.2	0.65	0.42	23.6
I4	0.32	7.2	0.92	0.42	53.3
15	0.32	7.2	1.50	0.45	193.7

For conditions E1–E4, H6, H9, and I1 also time-dependent measurements were carried out: During the wave cycle both flow velocities and sediment concentrations at different levels above the sand bed were measured. Moreover, video recording were taken to determine the bed level variation during the wave cycle. These time-dependent measurements are not included in this paper (see e.g. Katopodi et al., 1994; Ribberink et al. 1994; Janssen et al., 1997 and Janssen and Van der Hout, 1997).

Quasi-steady model of Bailard (1981)

Bailard applied the theoretical energy consideration of Bagnold (1963) to determine the sand transport rate. He assumed that the sediment transport rate is proportional to the available fluid power, which is equal to the work done by the fluid, i.e. the absolute value of the fluid shear stress times the velocity.

The model consists of a bed load and a suspended load component. Each component includes a term that depends on the bed slope. The bed slope terms are not included here, because the sand bed in the experiments is horizontal. The equation reads:

$$q_s(t) = \frac{c_f}{(s-1)g}\left(\frac{\varepsilon_b u^3(t)}{\tan\varphi} + \frac{\varepsilon_s \left|u^3(t)\right| u(t)}{w_{fall}}\right) \qquad (1)$$

Here q_s is the sediment transport rate, t is time, c_f is a friction factor, s is the relative density ($s = \rho_s/\rho$ with ρ_s the density of the sediment and ρ the density of water), g is the gravity acceleration, u is the horizontal velocity, φ is the angle of internal friction and w_{fall} is the fall velocity. The coefficients ε_b (= 0.1) and ε_s (= 0.02) are efficiency factors for the bed load and the suspended load transport.

In the present study the friction factor is calculated as a combined wave-current friction factor, as described by Ribberink (1998). The bed roughness height is considered to be equal to the grain diameter, i.e. $k_s = D_{50}$.

Quasi-steady model of Ribberink (1998)

The quasi-steady model of Ribberink is a bed load model. However, Ribberink considers all transport in the sheet flow layer as bed load. In sheet flow conditions the majority of the transport is transported inside the sheet flow layer, which means that the total transport will only be slightly larger than the bed load component, defined in this way.

Ribberink assumed the sand transport rate to be proportional to the difference between the actual time-dependent bed shear stress and the critical bed shear stress. The bed shear stress is expressed in terms of the (dimensionless) Shields parameter:

$$\theta(t) = \frac{\tau_b(t)}{\rho(s-1)gD_{50}} \qquad (2)$$

Here τ_b is the time-dependent bed shear stress and D_{50} is the mean grain diameter. The sand transport rate is normalised by the parameter $\sqrt{(s-1)gD_{50}^3}$. This gives the following expression for the sand transport rate:

$$q_s(t) = m\sqrt{(s-1)gD_{50}^3}\left(\left|\theta(t)\right| - \theta_{cr}\right)^n \frac{\theta(t)}{\left|\theta(t)\right|} \qquad (3)$$

The values of the coefficients m and n are based on many data from laboratory and field experiments with steady and oscillatory flows: m = 11, n = 1.65.

New semi-unsteady model

As mentioned in the introduction, phase-lag effects are expected for fine sand, large oscillatory velocities and small wave periods. Moreover, phase-lag effects are expected to reduce the net transport rate. Therefore a new-semi unsteady model is developed which predicts the same net transport rates as the quasi-steady model of Ribberink if phase-lag effects are small and smaller net transport rates if phase-lag effects become important.

This is realised by introducing a correction factor r to the calculated net transport rates of the model of Ribberink. The correction factor r is equal to 1.0 if no phase-lag effects occur and decreases for increasing phase-lag effects.

The correction factor r is defined as the ratio of the net sand transport rate, including phase-lag effects (*real* net transport rate) to the net sand transport rate when phase-lag effects are neglected (*equilibrium* net transport rate). These transport rates are calculated as follows:

$$q_s(t) = \int_0^h u_\infty(t) * c(z,t)\, dz \tag{4}$$

Here h is the water depth, $u_\infty(t)$ is the periodic velocity outside the wave boundary layer, (free-stream velocity) and c is the sediment concentration. The time-dependent sediment concentration profile c(z,t) is derived from an advection-diffusion equation. Nielsen (1979) showed that this equation can be solved analytically if a constant sediment mixing coefficient ε_s is used. The advection-diffusion equation reads:

$$\frac{\partial c}{\partial t} = \frac{\partial}{\partial z}\left[w_{fall} c + \varepsilon_s \frac{\partial c}{\partial z} \right] \tag{5}$$

The *equilibrium* sand transport rate is equal to the product of the free-stream velocity and the equilibrium concentration profile, while the *real* sand transport rate is equal to the product of the free-stream velocity and the real concentration profile.

To derive the time-dependent *equilibrium* concentration profile, it is assumed that thc concentration profile adjusts itself instantaneously to changes in flow velocity. This corresponds to a solution of the advection-diffusion equation for which the term $\partial c/\partial t$ is set zero. The time-dependent *real* concentration profile is derived without this assumption, which corresponds to the solution of the complete advection-diffusion equation, i.e. including the term $\partial c/\partial t$.

The bottom boundary condition for the equilibrium concentration profiles is based on the assumption that thc bottom concentration is instantaneously related to the flow velocity. Using a coefficient a and exponent b this can be expressed as follows:

$$c(0,t) = a\left| u(t)^b \right| \tag{6}$$

For the real concentration profiles the bottom boundary condition is based on the assumption that the *pick-up rate* of sediment is directly related to the instantaneously velocity. This implies that the concentration *gradient* is instantaneously related to the flow velocity:

$$\left.\frac{\partial c}{\partial z}\right|_{z=0} = -\frac{w_{fall}}{\varepsilon_s} a \left| u(t)^b \right| \quad (7)$$

The advection-diffusion equation only has an analytical solution if b is even. In the present study b = 2 is chosen, giving a transport rate proportional to u^3. This is close to the value of 3.3 in the model of Ribberink (1998), which results for negligible values of the critical Shields parameter, as is the case in sheet flow conditions.

Applying all these considerations results in the following expressions for the real and the equilibrium sand transport rate, i.e. $q_{s,r}$ and $q_{s,eq}$, respectively:

$$q_{s,r}(t) = \left[\sum_{k=0}^{N} u_k \cos(k\omega t)\right]\left[\sum_{k=0}^{2N} \frac{\varepsilon_s}{w_{fall}} \frac{\hat{c}_{bk}}{\left(P_k^2 + Q_k^2\right)^{\frac{3}{2}}} \left[P_k \cos(k\omega t + \varphi_k) + Q_k \sin(k\omega t + \varphi_k)\right]\right] \quad (8)$$

$$q_{s,eq}(t) = \left[\sum_{k=0}^{N} u_k \cos(k\omega t)\right]\left[\sum_{k=0}^{2N} \frac{\varepsilon_s}{w_{fall}} \hat{c}_{bk} \cos(k\omega t)\right] \quad (9)$$

With:

$$\hat{c}_{b0} = a\left(u_0^2 + \tfrac{1}{2}u_1^2\right) \quad (10)$$

$$\hat{c}_{b1} = a\left(2u_1u_0\right) \quad (11)$$

$$\hat{c}_{b2} = a\left(\tfrac{1}{2}u_1^2\right) \quad (12)$$

$$P_k = \frac{1}{2} + \left[\frac{1}{16} + \left(\frac{k\varepsilon_s\omega}{w_{fall}^2}\right)^2\right]^{\frac{1}{4}} \cos\left(\tfrac{1}{2}\alpha_k\right) \quad (13)$$

$$Q_k = \left[\frac{1}{16} + \left(\frac{k\varepsilon_s\omega}{w_{fall}^2}\right)^2\right]^{\frac{1}{4}} \sin\left(\tfrac{1}{2}\alpha_k\right) \quad (14)$$

$$\alpha_k = \arctan\left(\frac{4k\varepsilon_s\omega}{w_{fall}^2}\right) \quad (15)$$

$$\varphi_k = \arctan\left(-\frac{Q_k}{P_k}\right) \quad (16)$$

Here ω is the angular frequency of the wave (= $2\pi/T$, with T the wave period) and u_k is the k^{th} harmonic of the horizontal velocity. In the present analysis only u_0 and u_1 are considered (sinusoidal oscillatory flow plus a net current), because all experimental conditions consist of sinusoidal oscillatory flow combined with a net current.

As mentioned before, the correction factor r is defined as the ratio of thc *net* real sand transport rate to the *net* equilibrium sand transport rate. These net sand transport rates can be determined by averaging Eqs.(8) and (9) over time.

From these cquations it can be seen that the difference between the equilibrium and the real transport rate (and thus the value of the correction factor) is fully detcrmined by $\varepsilon_s\omega/w^2_{fall}$, called the phase-lag parameter p. The ratio of sediment mixing coefficient to fall velocity ε_s/w_{fall} can be considered as a characteristic length δ to which particles are entrained. Therefore the phase-lag parameter can be written as:

$$p = \frac{\varepsilon_s \omega}{w_{fall}^2} = \frac{\delta \omega}{w_{fall}} = 2\pi \frac{\delta}{w_{fall} T} \tag{17}$$

In order to calculate the phase-lag parameter p and thus the real and equilibrium sand transport rates and the correction factor r, either the value of the sediment mixing coefficient (ε_s) or the characteristic height to which particles are entrained (δ) must be known.

In the present study δ is assumed to be equal to the thickness of the sheet flow layer δ_s. The latter is defined as the distance between the top of the non-moving sand bed during the wave cycle and the level where the time-averaged concentration is equal to 8 vol%. Values of δ_s were derived from concentration profiles, as measured in the LOWT of WL | DELFT HYDRAULICS (e.g. Katopodi et al., 1994; Janssen et al., 1997; Janssen and Van der Hout, 1997).

It was found that the fine sand (D_{50} = 0.13 mm) behaved systematically different than the two coarser sands (D_{50} = 0.21 and 0.32 mm sand). Therefore two equations have been derived for the sheet flow layer thickness as a function of the maximum Shields parameter:

$$\frac{\delta_s}{D_{50}} = 4.5\,(7.5\theta_{max} + 0.90) \qquad \text{for } D_{50} = 0.13\text{mm}$$

$$\frac{\delta_s}{D_{50}} = 2.9\,(4.5\theta_{max} + 0.065) \qquad \text{for } D_{50} \geq 0.21\text{mm} \tag{18}$$

The maximum Shields parameter θ_{max} is based on the amplitude of the oscillatory velocity and a wave friction factor, using $k_s = D_{50}$.

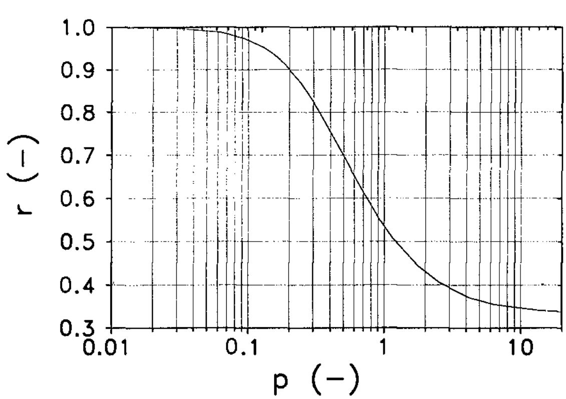

Figure 2: Reduction factor of equilibrium sand transport (r) as a function of phase-lag parameter (p) for a situation of sinusoidal oscillatory flow and a net current

Figure 2 shows the value of the reduction factor r as a function of the phase-lag parameter p for a situation of a sinusoidal oscillatory velocity and a net current, i.e. for $u(t) = u_0 + u_1\cos(\omega t)$. This figure shows that the reduction factor decreases for increasing values of p, which means that the difference between the real and the equilibrium net transport rates is large for large values of p. This can be explained as follows: The value of p is large for large values of the height to which particles are entrained (δ) and small values of the fall velocity (w_{fall}) and the wave period (T). As explained in the introduction, this corresponds to large phase-lag effects.

Behaviour of measured net transport rate and verification of sand transport models

In order to study the behaviour of the measured net transport rates and to verify the predictions of the three sand transport models, the measured and computed net transport rates are plotted as function of the grain size, the amplitude of oscillatory velocity and the wave period.

Grain size influence

Figure 3 shows the net sand transport rate as a function of the grain size for the following flow condition (H6, E2, I1):

$T = 7.2$ s
$u_a = 1.5$ m/s
$u_m = 0.25$ m/s

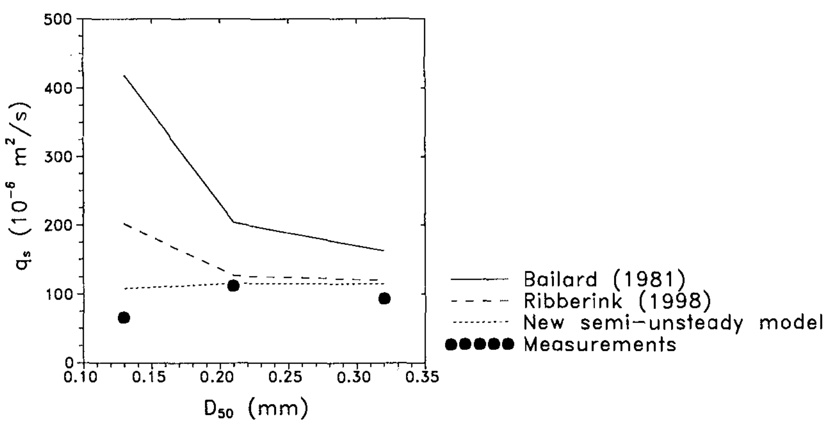

Figure 3: Measured and computed net sand transport rate as a function of grain size

The figure shows that the two quasi-steady models predict increasing net transport rates for decreasing grain size. This is in qualitative agreement with the measurements for medium and coarse sand. The magnitudes of the net transport rates for medium and coarse sand are predicted quite well by the model of Ribberink (1998) and overpredicted by the model of Bailard (1981).

The situation for fine sand is very different: The measurements show decreasing net transport rates for a grain size decrease from 0.21 to 0.13 mm. This may be explained by relatively large phase-lag effects due to the small settling velocity of the fine sand and the large oscillatory velocity, resulting in relatively large entrainment heights of the sediment particles.

In the new-semi unsteady model the small settling velocity and large oscillatory velocity result in a large value of the phase-lag parameter p, corresponding to a small value of the reduction factor r. That is why the net transport rate for the fine sand, predicted by the new semi-unsteady model is much smaller than for the model of Ribberink. Consequently, the new semi-unsteady model agrees much better with the measured net transport rate of fine sand.

For the medium and the coarse sand the predicted net transport rates of the new semi-unsteady model are almost equal to those of the quasi-steady model of Ribberink, indicating that phase-lag effects are small.

Flow velocity influence

The three panels of Figure 4 show the net sand transport rate as a function of the amplitude of the oscillatory velocity for three different grain sizes. In all cases the wave period is equal to 7.2 s and the net current velocity is equal to 0.25 m/s.

This figure shows that the two quasi-steady models predict increasing net transport rates for increasing oscillatory velocities. Again this is in qualitative agreement with the measurements for medium and coarse sand. For these two sand types the magnitudes of the measured net trans-

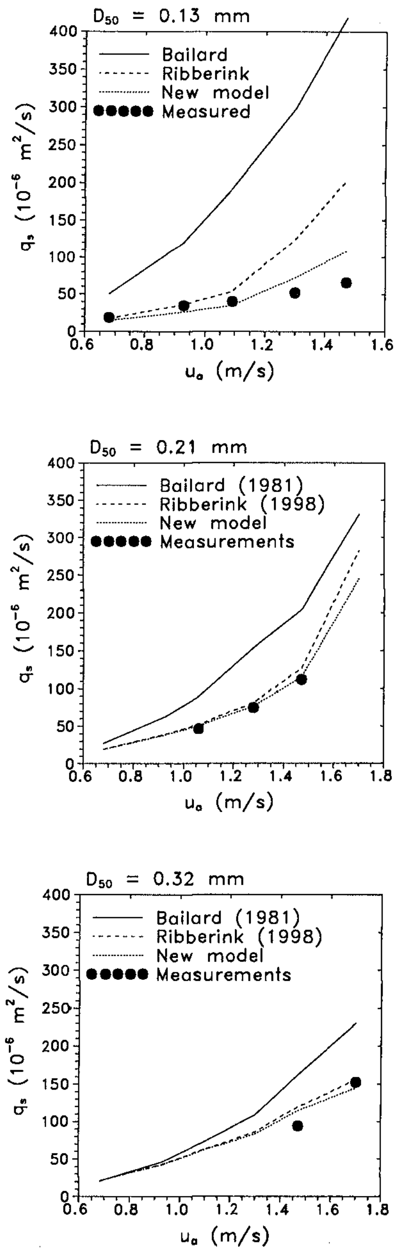

Figure 4: Measured and computed net sand transport rate as a function of amplitude of oscillatory velocity.

port rates are again overpredicted by the model of Bailard and predicted quite well by the model of Ribberink. Moreover, the semi-unsteady model gives almost the same results as the model of Ribberink for these two cases, indicating that phase-lag effects are not very important.

Again the situation is different for the fine sand: For small oscillatory velocities, the model of Ribberink predicts the measured net transport rates quite well. The model of Bailard largely overpredicts the measured net transport rates. The new semi-unsteady model gives almost the same results as the model of Ribberink, indicating that phase-lag effects are small. This can be explained by the fact that particles are not entrained very high into the flow for these low velocities.

However, for increasing oscillatory velocities, the increase in measured net transport rates is much smaller than predicted by the quasi-steady models, which may again be explained by increasing phase-lag effects. This is confirmed by the fact that the difference between the predictions of the new semi-unsteady model and the model of Ribberink increases for increasing oscillatory velocities, indicating that phase-lag effects become indeed more important. The predictions are thus again improved by including phase-lag effects.

Wave period influence

Figure 5 shows the net sand transport rate as a function of the wave period:

- The upper panel shows the results for *fine* sand and a relatively *low* oscillatory velocity of 0.7 m/s.
- The middle panel shows the results for *fine* sand and a relatively *large* oscillatory velocity of 1.1 m/s.
- The lower panel shows the results for *medium* sand and a relatively *large* oscillatory velocity of 1.1 m/s.

The net current velocity is equal to 0.25 m/s in all cases. All figures are plotted at the same scale to allow intercomparison between the three plots.

Figure 5 shows that the two quasi-steady models predict slightly increasing net transport rates for decreasing wave periods, due to the increase in wave friction factor. This is in qualitative agreement with the measurements for:

- Fine sand, $u_a = 0.7$ m/s, $T \geq 7.2$ s
- Medium sand, $u_a = 1.1$ m/s, $T \geq 7.2$ s

For these conditions the magnitudes are overpredicted by the model of Bailard and predicted quite well by the model of Ribberink.

For the other conditions the measurements show decreasing net transport rates for decreasing wave periods. This may again be explained by phase-lag effects, which become larger for shorter wave periods.

For fine sand with an oscillatory velocity of 0.7 m/s (upper panel) the measurements show that a decrease in wave period from 7.2 to 4 s leads to a decrease in net transport rate of about 30%. This can be explained by phase-lag effects: despite the relatively small oscillatory velocity, the wave period is so small and the sand so fine that phase-lag effects still occur.

The new semi-unsteady model predicts a similar phase-lag effect, which therefore results in improved predictions, compared to the model of Ribberink.

A phase-lag effect also seems to be present in the measurements for medium sand with an oscillatory velocity of 1.1 m/s and a wave period of 4 s (lower panel). The new semi-unsteady model predicts a much smaller phase-lag effect than observed in the measurements, indicated by the very small reduction in net transport rate, compared to the model of Ribberink.

For the conditions with fine sand and an oscillatory velocity of 1.1 m/s (middle panel) the measured net transport rate for a wave period of 7.2 s is predicted somewhat better by the new semi-unsteady model than by the model of Ribberink. Apparently phase-lag effects do occur for this condition.

For a wave period of 4 s the measured net transport rate is strongly reduced, compared to the transport rate for a wave period of 7.2 s. This can be explained by large phase-lag effects. The new semi-unsteady model agrees indeed much better with the measurement than the model of Ribberink. However, the measured net transport rate is still overpredicted, indicating that the phase-lag effects are somewhat larger than predicted by this model. This may be partly caused by the fact that the quasi-steady model predicts increasing net transport rates for decreasing wave periods, due to the increase in wave friction factor. It is not clear whether this is true. Only for the condition with a wave period of 12 s the predicted net transport rate does not agree at all with the measurement.

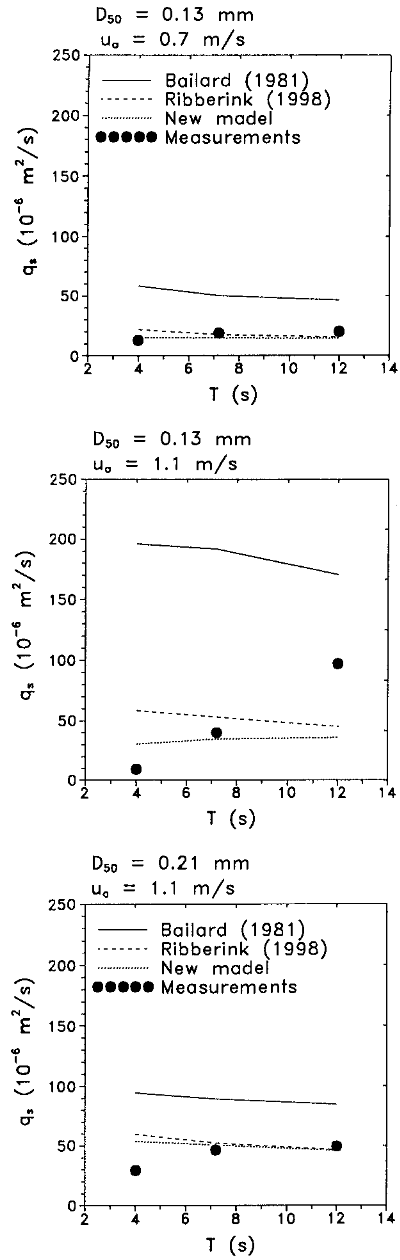

Figure 5: Measured and computed net sand transport rate as a function of wave period.

A different mechanism, which has not received any attention yet, may be responsible for this strong overprediction of the model, i.e. the limited pick-up rate of sand from the bed.

Video recordings of the bed level variation during the wave cycle show that several layers of sand grains are eroded from the bed. The thickness of the total eroded layer appears to be larger for increasing wave periods. Also, the concentration measurements in the sheet flow layer show that the sheet flow layer thickness is larger for a longer wave period. Existing expressions for the thickness of the sheet flow layer (e.g. Wilson, 1989; Sumer et al., 1996), as well as the formulae used in this study (Eq.(18)), assume that the thickness of the sheet flow layer only depends on the Shields parameter, which slightly *decreases* for increasing wave periods, due to the decrease in wave friction factor.

Both observations indicate that, at least for large velocities and fine sand, the amount of sand entrained into the flow and thus the sand transport rate depends on the wave period. The phenomenon of limited pick-up rate is not included in the presently used quasi-steady and unsteady sand transport models and needs further experimental investigation.

Conclusions

It is often assumed that in sheet flow conditions the response time of sediment particles is small with respect to the wave period, because the majority of the sand is transported in the thin sheet flow layer, close to the bed. This quick sediment response would result in a quasi-steady behaviour of the sand transport rate, i.e. a direct relation between the instantaneous sand transport rate and the instantaneous flow velocity.

However, measurements in a LOWT show that for fine sand, large oscillatory velocities and small wave periods the net transport rate is smaller than what could be expected from a quasi-steady behaviour. This may be explained by the presence of phase-lag effects: For large oscillatory velocities the sediment is entrained relatively high into the flow. If the entrained sand is very fine it settles down to the bed slowly and if the wave period is small this will result in a relatively large phase-lag between velocity and concentration. This means that sediment, which is entrained during the positive half wave cycle, can be transported in opposite direction during the successive half wave cycle, resulting in a reduction in *net* sand transport rate.

The fact that even in sheet flow conditions phase-lags effects can occur is explained by a new semi-unsteady model. This new model was developed using the bed load model of Ribberink and a phase-lag correction factor, which was based on an analytical solution of the advection diffusion equation for sediment concentration (Nielsen, 1979).

The phase-lag effects can be characterised by a phase-lag parameter p, defined as:

$$p = \frac{\varepsilon_s \omega}{w_{fall}^2} = \frac{\delta_s \omega}{w_{fall}} = 2\pi \frac{\delta_s}{w_{fall} T} \qquad (19)$$

Here ε_s is the sediment mixing coefficient, ω is the angular frequency of the wave ($=2\pi/T$, with T the wave period), δ_s is the sheet flow layer thickness and w_{fall} is the fall velocity of the sediment. It turned out that for $p > 0.8$ phase-lag effects become significant (i.e. reductions in net transport rates, compared to the quasi-steady behaviour, of 40% or more).

In general the quasi-steady model of Ribberink (1998) agrees quite well with the measurements. The model of Bailard overpredicts the measured net transport rates. However, generally the behaviour is predicted qualitatively correctly.

Only for fine sand, large oscillatory velocities and small wave periods the behaviour is different than predicted by the two quasi-steady models and both the model of Bailard and the one of Ribberink overpredict the measured net transport rates. For these conditions the new semi-unsteady model shows a better agreement with the measurements.

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