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PARAMETRIZATION OF STABILIZING CONTROLLERS FOR SYSTEMS WITH MULTIPLE I/O DELAYS

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Abstract: The parametrization of stabilizing controllers for systems with multiple delays is derived. The approach is to reduce the problem to an equivalent finite dimensional stabilization problem. The resulting controller consists of two blocks: a rational block and a non-rational block with finite impulse response (FIR), both of which are implementable.

Keywords: Time-delay systems, Youla-Kučera parametrization

1. INTRODUCTION

In the mid 70s, Youla (Youla *et al.*, 1976) et. al. and Kučera (Kučera, 1975) independently derived the parametrization of all stabilizing controller for finite dimensional systems. Based on a priori knowledge of one stabilizing controller, they described all stabilizing controllers parametrized by the set of all stable proper transfer functions. Although it has the reputation of being an underutilized tool for practical control system design, the Youla-Kučera parametrization has been employed in addressing many control problems, including H_∞ and H_2 problems (Anderson, 1998).

The problem of stabilizing time-delay systems has been considered for many years. Smith (Smith, 1957) managed to reduce the stabilization problem of stable SISO plants with a delay to a finite dimensional stabilization problem. The resulting controller later became known as the Smith predictor. Over the years, there have been a lot of modifications to the Smith predictor. These include modifying the Smith predictor to an observer-predictor form which has better disturbance rejection properties than the original Smith predictor and modifications to allow unstable plants. For a review of the Smith predictor and its modifications, the reader is advised to consult (Palmor, 1996) and the references therein. The Smith predictor has also been extended to the case of MIMO plants with multiple delays. The design and analysis of the multivariable dead-time compensators are discussed in (Palmor and Halevi, 1983).

However, in all of the above cases, the controller is proposed beforehand and then its stabilizing property is proved. The problem of characterizing all stabilizing controllers is not considered.

The Youla-Kučera parametrization may be obtained via coprime factorization of the plant. Kamen, Khargonekar, and Tannenbaum (Kamen *et al.*, 1986) developed a method to construct coprime factorizations of systems with commensurate time-delays. Using this result the Youla-Kučera parametrization may readily be obtained. However, the paper does not elaborate on the state-space realization of the coprime factors.

Mirkin and Raskin (Mirkin and Raskin, 1999) came up with an elegant way to transform the problem of stabilizing systems with a single i/o delay to a finite dimensional stabilization problem. A state-space parametrization of all stabilizing controllers may then be obtained using standard finite dimensional systems theory. The result is then used to solve H_2 control problems and robust stabilization problems.

In this paper, the work in (Mirkin and Raskin, 1999) is extended to systems with multiple i/o delays, i.e. different delays in each i/o channel of the controller.

Following (Mirkin and Raskin, 1999), the approach is to reduce the problem to an equivalent finite dimensional stabilization problem. In the equivalent problem, the plant is rational and there is no delay component. A mapping between the signals of the two closed loop systems is then formulated. Based on this mapping, we may use results from finite dimensional systems theory to derive the the parametrization of stabilizing controllers for systems with multiple delays.

It is shown that the resulting controller consists of two blocks: a rational block and a non-rational block with finite impulse response (FIR), both of which are implementable.

Recently, we learned that Raskin in Chapter 8 of her PhD thesis (Raskin, 2000) also proposed a similar solution the problem we consider in this paper.

Notation A transfer matrix F is said to be stable if $F \in H_\infty$, and bistable if both F and F^{-1} are stable.

It is proved (Weiss, 1994) that a transfer matrix is causal if and only if it is proper. Here a transfer matrix F is said to be proper if there exists $\rho \in \mathbb{R}$ such that

$$\sup_{\mathrm{Re}(s)>\rho} \|F(s)\| < \infty. \tag{1}$$

In addition, F is said to be strictly proper if there is $\rho \in \mathbb{R}$ such that

$$\lim_{s\to\infty, \mathrm{Re}(s)>\rho} \|F(s)\| = 0. \tag{2}$$

The familiar (lower) linear fractional transformation (LFT) of two transfer matrices $M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}$ and U of appropriate dimension is defined as

$$F_\ell(M; U) := M_{11} + M_{12}U(I - M_{22}U)^{-1}M_{21}. \tag{3}$$

2. PROBLEM FORMULATION

We consider the control system in Figure 1, where we have multiple delays in both the input and output of the controller $K(s)$.

We assume that the rational plant has a stabilizable and detectable realization of the form

$$P(s) = \begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{bmatrix} = \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & 0 \end{array}\right] \tag{4}$$

connected with a proper controller $K(s)$, and two multiple delay matrices Λ_y and Λ_u. The delay matrices are of the form

$$\Lambda_y = \mathrm{diag}(e^{-sh_{y1}}, e^{-sh_{y2}}, \ldots, e^{-sh_{ym}}) \tag{5}$$

$$\Lambda_u = \mathrm{diag}(e^{-sh_{u1}}, e^{-sh_{u2}}, \ldots, e^{-sh_{up}}) \tag{6}$$

where m and p are dimension of y and u, respectively.

The aim is to find a parametrization of all proper controllers that internally stabilize the control system of Figure 1. Internal stability is defined to be the stability of the transfer functions from (v_1, v_2, w) to (y, u, z).

Without loss of generality, D_{22} is assumed to be zero. Indeed, if D_{22} is nonzero, we may first compute the controller for the case where it is zero and recover the controller for the nonzero case by connecting $\Lambda_y D_{22} \Lambda_u$ as a feedback to the controller.

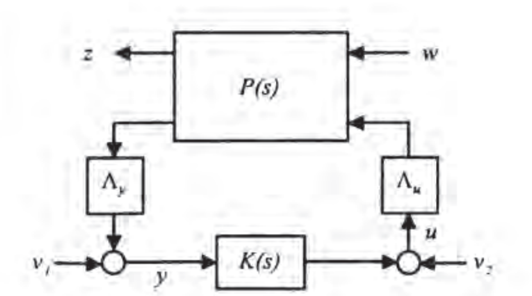

Fig. 1. The standard control system with multiple delays in stabilization setup

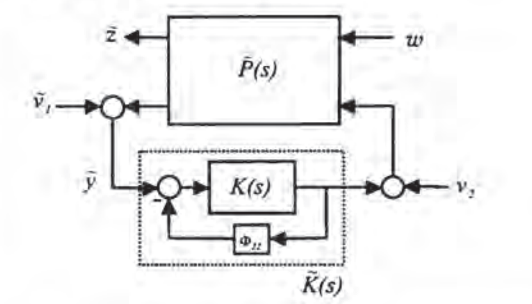

Fig. 2. The equivalent finite dimensional control system

3. EQUIVALENT FINITE DIMENSIONAL SYSTEM

Following the procedure in (Mirkin and Raskin, 1999), we develop an algorithm for transforming the control system of Figure 1 to an equivalent finite dimensional control system.

Let us first define transfer matrices Φ_{12}, Φ_{21}, and Φ_{22} such that

$$P_{12}(s)\Lambda_u = \tilde{P}_{12}(s) - \Phi_{12}(s) \tag{7}$$

$$\Lambda_y P_{21}(s) = \tilde{P}_{21}(s) - \Phi_{21}(s) \tag{8}$$

$$\Lambda_y P_{22}(s)\Lambda_u = \tilde{P}_{22}(s) - \Phi_{22}(s) \tag{9}$$

holds, with $\tilde{P}_{12}$, $\tilde{P}_{21}$ and $\tilde{P}_{22}$ rational, and Φ_{12}, Φ_{21}, and Φ_{22} stable.

The next two lemmas show that the control system in Figure 1 may be transformed into an equivalent finite dimensional system in Figure 2 where

$$\tilde{P}(s) = \begin{bmatrix} P_{11}(s) & \tilde{P}_{12}(s) \\ \tilde{P}_{21}(s) & \tilde{P}_{22}(s) \end{bmatrix}. \tag{10}$$

Here the two control systems are equivalent in the sense that there is a one to one relation between the signals in the two systems.

First we show that the controller $\tilde{K}$ in Figure 2 is well defined. Afterwards we show that the configuration in Figure 1 and Figure 2 have an equivalent internal stability property, i.e. internal stability of one configuration is equivalent to internal stability of the other.

Lemma 1. Let $\tilde{P}_{22}$ and Φ_{22} as in (9), with $\tilde{P}_{22}$ rational and Φ_{22} stable. Then

$$\tilde{K}(s) = (I + K(s)\Phi_{22}(s))^{-1}K(s) \qquad (11)$$

is well-defined and proper if and only if K(s) is proper.

Proof. First notice that $\tilde{P}_{22}$ and Φ_{22} are strictly proper. Now suppose K is proper, then $K\Phi_{22}$ is strictly proper and thus the inverse in (11) is well-defined and proper. It follows that $\tilde{K}$ is well-defined and proper. For the converse, suppose $\tilde{K}$ is well-defined and proper, then necessarily the inverse in (12) is well-defined and proper. This implies that K is proper. ■

We see that the relation between K and $\tilde{K}$ is a bijection. In fact, we can infer K from $\tilde{K}$ using the equation

$$K(s) = (I - \tilde{K}(s)\Phi_{22}(s))^{-1}\tilde{K}(s). \qquad (12)$$

Lemma 2. The controller K internally stabilizes the system in Figure 1 if and only if $\tilde{K}$ defined in (11) internally stabilizes the system in Figure 2.

Proof. By absorbing the delay matrices to the plant, we obtain Figure 3(a). Now, observe Figure 3(a)-(d). In this figure, it is shown that the setup in Figure 1, through block diagram manipulations, can be transformed into the setup in Figure 2 with $\tilde{v}_1 = v_1 - \Phi_{22}v_2 - \Phi_{21}w$, $\tilde{z} = z + \Phi_{12}u$, and $\tilde{y} = y + \Phi_{22}(u - v_2)$.

The output signals $(\tilde{y}, u, \tilde{z})$ of Figure 2 may be expressed in terms of the output signals (y, u, z) and the input signals (v_1, v_2, w) of Figure 1 by

$$\begin{bmatrix} \tilde{y} \\ u \\ \tilde{z} \end{bmatrix} = \begin{bmatrix} I & \Phi_{22} & 0 \\ 0 & I & 0 \\ 0 & \Phi_{12} & I \end{bmatrix} \begin{bmatrix} y \\ u \\ z \end{bmatrix} + \begin{bmatrix} 0 & -\Phi_{22} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ w \end{bmatrix} \qquad (13)$$

and conversely we have

$$\begin{bmatrix} y \\ u \\ z \end{bmatrix} = \begin{bmatrix} I & -\Phi_{22} & 0 \\ 0 & I & 0 \\ 0 & -\Phi_{12} & I \end{bmatrix} \begin{bmatrix} \tilde{y} \\ u \\ \tilde{z} \end{bmatrix} + \begin{bmatrix} 0 & \Phi_{22} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{v}_1 \\ v_2 \\ w \end{bmatrix}. \qquad (14)$$

We see that all transfer matrices in the above equations are stable. There is also a bistable mapping between the input signals of the two closed loop systems, as shown by the following equations

$$\begin{bmatrix} \tilde{v}_1 \\ v_2 \\ w \end{bmatrix} = \begin{bmatrix} I & -\Phi_{22} & -\Phi_{21} \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ w \end{bmatrix} \qquad (15)$$

$$\begin{bmatrix} v_1 \\ v_2 \\ w \end{bmatrix} = \begin{bmatrix} I & \Phi_{22} & \Phi_{21} \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} \tilde{v}_1 \\ v_2 \\ w \end{bmatrix}. \qquad (16)$$

The bistable mappings allow us to prove the lemma. Let us denote H and $\tilde{H}$ as the transfer functions from $[v_1, v_2, w]^T$ to $[y, u, z]^T$ in Figure 1 and from $[\tilde{v}_1, v_2, w]^T$ to $[\tilde{y}, u, \tilde{z}]^T$ in Figure 2, respectively, i.e.

$$\begin{bmatrix} y \\ u \\ z \end{bmatrix} = H \begin{bmatrix} v_1 \\ v_2 \\ w \end{bmatrix} \qquad (17)$$

$$\begin{bmatrix} \tilde{y} \\ u \\ \tilde{z} \end{bmatrix} = \tilde{H} \begin{bmatrix} \tilde{v}_1 \\ v_2 \\ w \end{bmatrix}. \qquad (18)$$

Suppose K stabilizes the control system of Figure 1, then H is stable. Substituting (14) and (16) to (17) we obtain the following

$$\begin{bmatrix} \tilde{y} \\ u \\ \tilde{z} \end{bmatrix} = \begin{bmatrix} I & \Phi_{22} & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} \left(H \begin{bmatrix} I & \Phi_{22} & \Phi_{21} \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} - \begin{bmatrix} 0 & \Phi_{22} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right) \begin{bmatrix} \tilde{v}_1 \\ v_2 \\ w \end{bmatrix} \qquad (19)$$

showing that $\tilde{H}$ is stable. Hence $\tilde{K}$ stabilizes $\tilde{P}$.

The converse can easily be proved in a similar manner. Hence, we have that K stabilizes the control system of Figure 1 if and only if $\tilde{K}$ stabilizes the control system of Figure 2. ■

Lemma 2 results in the following corollary, which is useful for obtaining the conditions for the existence of a stabilizing controller.

Corollary 3. Define

$$\hat{P}(s) = \begin{bmatrix} P_{11} & P_{12}\Lambda_u \\ \Lambda_y P_{21} & \Lambda_y P_{22}\Lambda_u \end{bmatrix}, \qquad (20)$$

then $\tilde{P}(s)$ is stabilizable by a proper controller if and only if $\hat{P}(s)$ is stabilizable.

Proof. The corollary follows immediately from Lemma 2. ■

Constructing $\tilde{P}_{12}$, $\tilde{P}_{21}$, and $\tilde{P}_{22}$ Using Lemma 2, we can use standard stability theory to obtain the Youla-Kučera parametrization. What remains is to construct rational $\tilde{P}_{12}$, $\tilde{P}_{21}$, and $\tilde{P}_{22}$ such that (7, 8, 9) holds, Φ_{12}, Φ_{21}, and Φ_{22} are stable.

First recall the rational plant $P(s)$ and the delay matrices Λ_u and Λ_y:

$$P(s) = \begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{bmatrix} = \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & 0 \end{array}\right], \qquad (21)$$

$$\Lambda_y = \mathrm{diag}(e^{-sh_{y1}}, e^{-sh_{y2}}, \dots, e^{-sh_{ym}}), \qquad (22)$$

$$\Lambda_u = \mathrm{diag}(e^{-sh_{u1}}, e^{-sh_{u2}}, \dots, e^{-sh_{up}}). \qquad (23)$$

We propose the following candidate for $\tilde{P}_{12}$, $\tilde{P}_{21}$, and $\tilde{P}_{22}$:

$$\tilde{P}_{12}(s) = \left[\begin{array}{c|c} A & \tilde{B}_2 \\ \hline C_1 & 0 \end{array}\right] \qquad (24)$$

$$\tilde{P}_{21}(s) = \left[\begin{array}{c|c} A & B_1 \\ \hline \tilde{C}_2 & 0 \end{array}\right] \qquad (25)$$

$$\tilde{P}_{22}(s) = \left[\begin{array}{c|c} A & \tilde{B}_2 \\ \hline \tilde{C}_2 & 0 \end{array}\right] \qquad (26)$$

where

$$\tilde{C}_2 = \begin{bmatrix} C_{2,1}e^{-Ah_{y1}} \\ C_{2,2}e^{-Ah_{y2}} \\ \vdots \\ C_{2,m}e^{-Ah_{ym}} \end{bmatrix} \qquad (27)$$

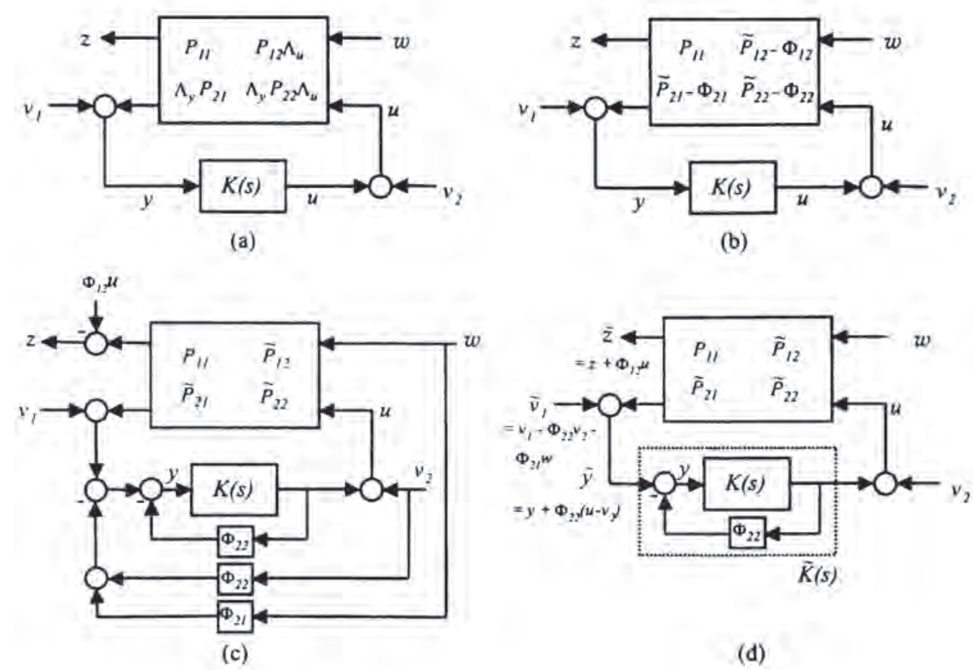

Fig. 3. The proof of Lemma 2

$$\bar{B}_2 = \left[\, e^{-Ah_{u1}} B_{2,1} \;\; e^{-Ah_{u2}} B_{2,2} \;\; \ldots \;\; e^{-Ah_{up}} B_{2,p} \,\right]. \tag{28}$$

Note that we have a freedom to choose the D matrix of $\bar{P}_{12}$, $\bar{P}_{21}$, and $\bar{P}_{22}$. The proof of Proposition 4 shows that any value will satisfy the requirement.

Proposition 4. Let $\bar{P}_{12}$, $\bar{P}_{21}$, and $\bar{P}_{22}$ be given by (24), (25), and (26), respectively. Then (7, 8, 9) holds, and Φ_{12}, Φ_{21}, and Φ_{22} are stable.

Proof. Here we only prove for the case of $\bar{P}_{22}$, since the case of $\bar{P}_{12}$ and $\bar{P}_{21}$ can be proved in a similar manner. In addition, $\bar{P}_{12}$ and $\bar{P}_{21}$ do not appear in the final formula of the parametrization.

The impulse response matrix $H_{\bar{P}_{22}}$ of $\bar{P}_{22}$ is given by

$$H_{\bar{P}_{22}} = \mathbb{1}(t)\bar{C}_2 e^{At}\bar{B}_2 = \mathbb{1}(t)\begin{bmatrix} C_{2,1}e^{A(t-h_{y1}-h_{u1})}B_{2,1} & \ldots & C_{2,1}e^{A(t-h_{y1}-h_{up})}B_{2,p} \\ \vdots & & \vdots \\ C_{2,m}e^{A(t-h_{ym}-h_{u1})}B_{2,1} & \ldots & C_{2,m}e^{A(t-h_{ym}-h_{up})}B_{2,p} \end{bmatrix} \tag{29}$$

To show that $\bar{P}_{22}$ satisfies (7), we compute the impulse response matrix $H_{\Lambda_y P_{22}\Lambda_u}(t)$ of $\Lambda_y P_{22}(s)\Lambda_u$ as follows.

$$P_{22}(s) = \left[\begin{array}{c|c} A & B_2 \\ \hline C_2 & 0 \end{array}\right] \tag{30}$$

so that its impulse response matrix is

$$H_{P_{22}}(t) = \mathbb{1}(t)\begin{bmatrix} C_{2,1}e^{At}B_{2,1} & \ldots & C_{2,1}e^{At}B_{2,p} \\ \vdots & \ddots & \vdots \\ C_{2,m}e^{At}B_{2,1} & \ldots & C_{2,m}e^{At}B_{2,p} \end{bmatrix} \tag{31}$$

It follows that the (i,j)-th component of the impulse response matrix of $\Lambda_y P_{22}(s)\Lambda_u$ is given by

$$\left(H_{\Lambda_y P_{22}\Lambda_u}(t)\right)_{i,j} = \mathbb{1}(t - h_{yi} - h_{uj})C_{2,i}e^{A(t-h_{yi}-h_{uj})}B_{2,j} \tag{32}$$

Since $\Phi_{22}(s) = \bar{P}_{22}(s) - \Lambda_y P_{22}(s)\Lambda_u$, its impulse response is given by

$$H_{\Phi_{22}}(t) = H_{\bar{P}_{22}}(t) - H_{\Lambda_y P_{22}\Lambda_u}(t) \tag{33}$$

The (i,j)-th component of $H_{\Phi_{22}}(t)$ is then given by

$$H_{\Phi_{22}}(t)^{i,j} = \left(\mathbb{1}(t) - \mathbb{1}(t - h_{yi} - h_{uj})\right) C_{2,i}e^{A(t-h_{yi}-h_{uj})}B_{2,j} \tag{34}$$

We see that $H_{\Phi_{22}}(t)$ is a finite impulse response matrix with its (i,j)-th component having support on $[0, h_{yi} + h_{uj}]$. It follows that $\Phi_{22} \in H_\infty$, in fact Φ_{22} is an entire function of s. ■

Hence, the realization of $\bar{P}$ is given by

$$\bar{P}(s) = \begin{bmatrix} P_{11}(s) & \bar{P}_{12}(s) \\ \bar{P}_{21}(s) & \bar{P}_{22}(s) \end{bmatrix} = \left[\begin{array}{c|cc} A & B_1 & \bar{B}_2 \\ \hline C_1 & D_{11} & 0 \\ \bar{C}_2 & 0 & 0 \end{array}\right] \tag{35}$$

4. YOULA-KUČERA PARAMETRIZATION

To obtain the parametrization of all stabilizing controller $K(s)$ for the LFT in figure 1, Lemma 1 and 2 suggest that we first compute the stabilizing controller $\bar{K}(s)$ for the plant $\bar{P}(s)$, and then use (12), i.e. adding a positive feedback with feedback gain matrix Φ_{22}, to obtain $K(s)$. To verify the existence of a stabilizing controller we use Corollary 3, which results in the following: the system in Figure 1 is stabilizable if and only if the pair $(A, \bar{B}_2)$ is stabilizable and the pair $(\bar{C}_2, A)$ is detectable.

Note that this condition is equivalent to the condition that the pairs (A, B_2) and (C_2, A) are stabilizable

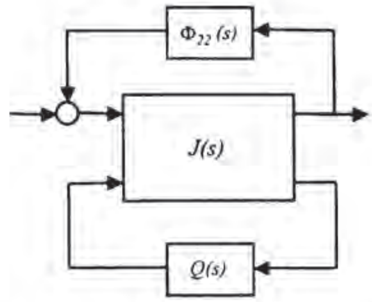

Fig. 4. Youla-Kučera parametrization for control systems with multiple delays

and detectable, respectively. To see this, consider the system described by (A, B_2, C_2) with input u, output y and state x. The pair (C_2, A) is detectable if and only if the following holds: If y is zero for all time and $u = 0$, then x asymptotically goes to zero. This fact still holds even if we have delays in the output. Hence the condition that (C_2, A) is detectable is equivalent to the condition that $(\tilde{C}_2, A)$ is detectable. Using duality of detectability and stabilizability, we arrive at the same conclusion for the equivalence of the stabilizability of the pairs (A, B_2) and $(A, \tilde{B}_2)$.

The above discussion results in the following theorem.

Theorem 5. Consider the control system of Figure 1. Recall that the plant is of the form

$$P(s) = \begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{bmatrix} = \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & 0 \end{array}\right]. \quad (36)$$

There exists a proper controller $K(s)$ such that the control system of Figure 1 is internally stable if and only if the pair (A, B_2) is stabilizable and the pair (C_2, A) is detectable.

Furthermore, let F and H be state feedback gain matrices such that $(A - \tilde{B}_2 F)$ and $(A - H\tilde{C}_2)$ are asymptotically stable with

$$\tilde{C}_2 = \left[e^{-A^T h_{y1}} C_{2,1}^T \ \ldots \ e^{-A^T h_{ym}} C_{2,m}^T \right]^T \quad (37)$$

$$\tilde{B}_2 = \left[e^{-A h_{u1}} B_{2,1} \ \ldots \ e^{-A h_{up}} B_{2,p} \right] \quad (38)$$

where $B_{2,i}$ and $C_{2,i}$ are the i-th column of B_2 and i-th row of C_2, respectively.

Then all proper controllers that internally stabilize the control system in Figure 1 are parameterized as described in Figure 4, where

$$J(s) = \left[\begin{array}{c|cc} A - \tilde{B}_2 F - H\tilde{C}_2 & H & \tilde{B}_2 \\ \hline -F & 0 & I \\ -\tilde{C}_2 & I & 0 \end{array}\right] \quad (39)$$

$$\Phi_{22}(s) = \left[\begin{array}{c|c} A & \tilde{B}_2 \\ \hline \tilde{C}_2 & 0 \end{array}\right] - \Lambda_y \left[\begin{array}{c|c} A & B_2 \\ \hline C_2 & 0 \end{array}\right] \Lambda_u \quad (40)$$

and with $Q(s) \in H_\infty$ but otherwise arbitrary.

Proof. By corollary 3, the stabilizability of the control system in Figure 1 is equivalent to the stabilizability of $\tilde{P}(s)$. The latter is stabilizable if and only if the pair (A, B_2) is stabilizable and (C_2, A) is detectable.

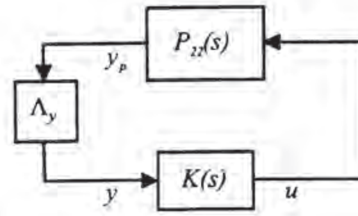

Fig. 5. Equivalent stabilization problem with $\Lambda_u = I$

Applying Lemma A.4.5 of (Green and Limebeer, 1995) to the realization of $\tilde{P}(s)$ defined in (35), we obtain the parametrization of all stabilizing controller $\tilde{K} = F_\ell(J, Q)$, where J is given by (39) and Q is any stable transfer function.

By Lemma 2, all stabilizing controller K for the control system in Figure 1 is obtained through equation (12) , i.e. by adding a positive feedback with feedback gain matrix Φ_{22} to $\tilde{K}$. This results in the controller in Figure 4. Finally, Lemma 1 ensures that K is proper. ∎

5. HOW THE CONTROLLER WORKS

In this section, an insight into how the controller works is presented. We show that in the absence of the external input w and the input delay matrix Λ_u, the central controller estimates the state of the undelayed plant.

Assuming that a stabilizing controller K exists, it is known that a controller internally stabilizes a rational plant P if and only if it stabilizes the plant $(2, 2)$-th part. Using Lemma 2, it may be shown that this is still true in the presence of i/o multiple delay matrices. This configuration with $\Lambda_u = I$ is depicted in Figure 5. Since it is a lot simpler than the original configuration with the full plant, let us analyze how the controller works in this configuration.

If we apply the parametrization of Theorem 5 to the delayed plant $\Lambda_u P_{22}$ and then set the parameter Q to zero, we obtain the central controller. It has the form a feedback interconnection between a rational transfer function and Φ_{22}. The rational part is given by

$$J_c = \left[\begin{array}{c|c} A - B_2 F - H\tilde{C}_2 & H \\ \hline -F & 0 \end{array}\right] \quad (41)$$

It may be shown that J_c may be written in an observer-based form consisting an observer $O(s)$ and a stabilizing feedback F. The observer is of the form

$$O(s) = \left[\begin{array}{c|cc} A - H\tilde{C}_2 & H & B_2 \\ \hline I & 0 & 0 \end{array}\right]. \quad (42)$$

Figure 6 shows the resulting closed-loop system.

Observe Figure 6. If we denote x as the state of P_{22}, then the undelayed measurements are given by $y_p = C_2 x$. Keeping in mind that $\Lambda_y P_{22} + \Phi_{22} = \tilde{P}_{22}$, we see that the dotted part of Figure 6 is actually $\tilde{P}_{22}$, driven by the same input u that drives P_{22}. Since P_{22} and $\tilde{P}_{22}$ have the same A and B matrices, they share the same state variable x. In fact, the measurements that are fed to the observer O are given by $y = \tilde{C}_2 x$.

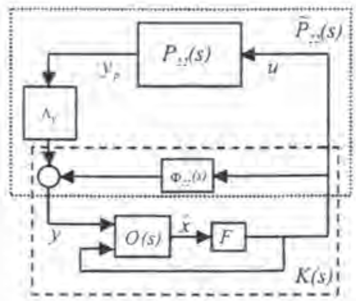

Fig. 6. The central stabilizing controller for $\Lambda_y P_{22}$ in an observer-based form

The dynamics of the error between x and the output of the observer $\hat{x}$ are given by the familiar equation

$$\dot{x} - \dot{\hat{x}} = (A - H\bar{C}_2)(x - \hat{x}). \tag{43}$$

Since $(A - H\bar{C}_2)$ is Hurwitz, the error goes exponentially to zero as time goes to infinity. Hence, $\hat{x}$ is an estimate of the state of the undelayed plant P_{22}.

From the above discussion, we may interprete the rationale of the controller as follows. Since only the delayed version of the output $y_p = C_2 x$ is available, the controller compensates the delayed measurements using the FIR block Φ_{22} to obtain $y = \bar{C}_2 x$. Using the fact that $(\bar{C}_2, A)$ is detectable, the controller estimates the state x and uses it to stabilize the plant.

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