# Extraction of Singular Points from Directional Fields of Fingerprints

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## Abstract

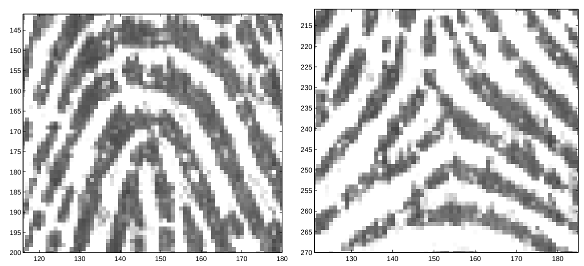
*Biometric identification is an emerging subject in applications like high-security wireless access and secure transactions across computer networks. Fingerprints are easy to use and provide relatively good performance. Furthermore, fingerprint sensors are cheap and can be integrated easily in wireless hardware.*

*In this paper, methods are presented for the estimation of a high resolution directional field from fingerprints. It is shown how, from the directional field, very accurate detection of the singular points and the orientations of those points can be obtained. These estimates can for instance be used for accurate registration (alignment) of two fingerprints in a fingerprint verification system.*

## 1. Introduction

Biometric identification is an emerging subject in applications like high-security wireless access and secure transactions across computer networks. The use of features directly connected to someone's body significantly decreases the probability of fraud.

Among many other biometric features like iris, face, voice, hand geometry, retina, etc., fingerprints are easy to use and provide relatively good performance. Another reason for the increasing popularity of the use of fingerprints for identification is the relatively low price of fingerprint sensors. Smart cards with built-in fingerprint sensors are already available on the market, and the sensors can be integrated easily in wireless hardware.



(a) Core (b) Delta

**Figure 1: Examples of singular points in a fingerprint.**

As can be seen from Figure 1, a fingerprint is built from *ridge-valley* structures. In this figure, the ridges are black and the valleys are white. When using fingerprints for recognition systems, the ridge-valley structures are the main source for the information to be extracted from the fingerprints.

## 2. Directional field estimation

The *directional field* (DF) describes the coarse structure, or basic shape, of a fingerprint. The DF is defined as the local orientation of the ridge-valley structures. The DF is for instance used for classification of fingerprints. In [1], a method is presented for the estimation of a high-resolution DF. The main results are repeated here.

The method is based on the gradient vector $[G_x(x, y) \;\; G_y(x, y)]^T$ of the gray-scale image $I(x, y)$,

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which is defined by:

$$\left[ \begin{array}{c} G_x(x,y) \\ G_y(x,y) \end{array} \right] = \nabla I(x,y) = \left[ \begin{array}{c} \frac{\partial I(x,y)}{\partial x} \\ \frac{\partial I(x,y)}{\partial y} \end{array} \right] \qquad (1)$$

The DF is, in principle, perpendicular to the gradients. However, the gradients are orientations at pixel scale, while the DF describes the orientation of the ridge-valley structures. This requires a much coarser scale in which local fluctuations do not have influence. Therefore, the DF can be derived from the gradients by performing some *averaging* operation on the gradients, involving pixels in some neighborhood [2].

Gradients cannot simply be averaged in some local neighborhood, since opposite gradient vectors will then cancel each other, although they indicate the same ridge-valley orientation. A solution to this problem is to double the angles of the gradient vectors before averaging. Then, opposite gradient vectors will point in the same direction and therefore will reinforce each other, while perpendicular gradients will cancel. After averaging, the gradient vectors have to be converted back to their single-angle representation. The main ridge-valley orientation is perpendicular to the direction of the average gradient vector. This method was proposed by [3] and was adopted in some way for the estimation of the DF of fingerprints by various researchers.

In the version of algorithm used in this paper, not only the angle of the gradients is doubled, but also the length of the gradient vectors is squared, as if the gradient vectors are considered as complex numbers that are squared. This has the effect that strong orientations have a higher vote in the average orientation than weaker orientations and this approach results in the cleanest expressions.

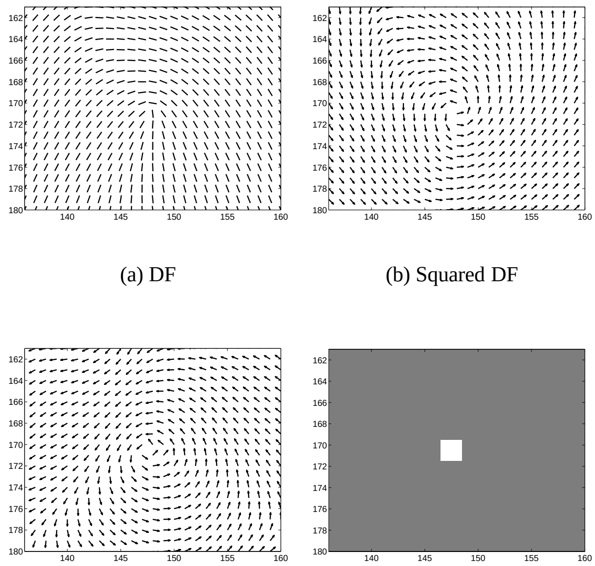
Another difference with the methods of other researchers is that we do not estimate the average DF for a number of blocks in the image. Instead, the DF is estimated for each pixel in the image using a Gaussian window $W$ for averaging. Using $G_{xx} = \sum_W G_x^2$, $G_{yy} = \sum_W G_y^2$ and $G_{xy} = \sum_W G_x G_y$, the average gradient direction $\theta$, with $-\frac{1}{2}\pi < \theta \leq \frac{1}{2}\pi$, is given by:

$$\theta = \frac{1}{2} \angle \left( G_{xx} - G_{yy}, 2G_{xy} \right) \qquad (2)$$

## 3. Singular point extraction

*Singular points* (SPs) are the discontinuities in the DF. They are clearly visible in a fingerprint image, as can be seen from Figure 1, which shows examples of the two different types of SPs that are called *core* and *delta*. SPs can for instance be used for the registration (alignment) of two fingerprints of the same finger.

Many different algorithms for SP extraction are known from literature. Examples of these algorithms are based on



(a) DF (b) Squared DF

(c) Gradient (d) Rotation

**Figure 2: Processing steps in the extraction of a core.**

sliding neural networks [4], local energy of the DF [5], ratio of the sine of the DF in two adjacent regions [6], and the Poincaré index [7]. However, these algorithms provide somewhat unsatisfactory results, since they provide continuous measures that indicate how much the local DF resembles a SP. Postprocessing steps are necessary to interpret the outputs of the algorithms and to make the final decisions, resulting in missed and false SPs.

The method that is presented here, is based on the Poincaré index [7]. It can be explained using Figure 2(a). Following a counter-clockwise closed contour in the DF around a core results in a cumulative change of $\pi$ in the orientation, while carrying out this procedure around a delta results in $-\pi$. On the other hand, when applied to a location that does not contain a SP, the cumulative orientation change will be zero.

Although the Poincaré index provides means for consistent detection of SPs, the question arises how to calculate this measure. Apart from the problem of how to calculate cumulative orientation changes over contours efficiently, a choice has to be made on the optimal size and shape of the contour. A possible implementation is described in [8], which claims that a square curve with a length of 25 pixels is optimal. A smaller curve results in spurious detections, while a larger curve may ignore core-delta pairs which are close to each other. Postprocessing steps interpret the results.

In the implementation that is proposed in this paper, such choices don't have to be made. The method generates an exact binary output that indicates whether a SP is present or not. It doesn't need postprocessing steps and the cumulative orientation changes over contours are implemented very efficiently in standard 2-dimensional filters. Furthermore, the method is capable of detecting SPs that are located only a few pixels apart.

In the rest of the text, all calculations are made for the case of a core. It is left to the reader to adapt them to the case of a delta. First, the squared DF is taken. This eliminates the transition of $\pi$ which is encountered in the DF between the orientations $\theta = \frac{1}{2}\pi$ and $\theta = -\frac{1}{2}\pi$. As a result, the Poincaré index is doubled. The orientation of the squared DF is depicted in Figure 2(b) for the area around the core example.

Instead of summing the changes in orientation, it is possible to sum the gradients of the squared orientation as well. The gradient vector **J** can be efficiently precalculated for the entire image by:

$$\begin{bmatrix} J_x(x,y) \\ J_y(x,y) \end{bmatrix} = \nabla 2\theta(x,y) = \begin{bmatrix} \frac{\partial 2\theta(x,y)}{\partial x} \\ \frac{\partial 2\theta(x,y)}{\partial y} \end{bmatrix} \quad (3)$$

In the calculation of the discrete version of this gradient, both components of $J$ should be calculated 'modulo $2\pi$', such that they are always between $-\pi$ and $\pi$. This makes the transition from $2\theta = -\pi$ to $2\theta = \pi$ continuous or, in other words, the orientation is considered to be cyclic. The gradient vectors of the squared orientation around the core is shown in Figure 2(c).

The next step is the application of Green's Theorem, which states that instead of calculating a closed line-integral over a vector field, the surface integral over the rotation of this vector field can be calculated. This theorem is applied to the summation of the gradients of the squared orientation over the contour:

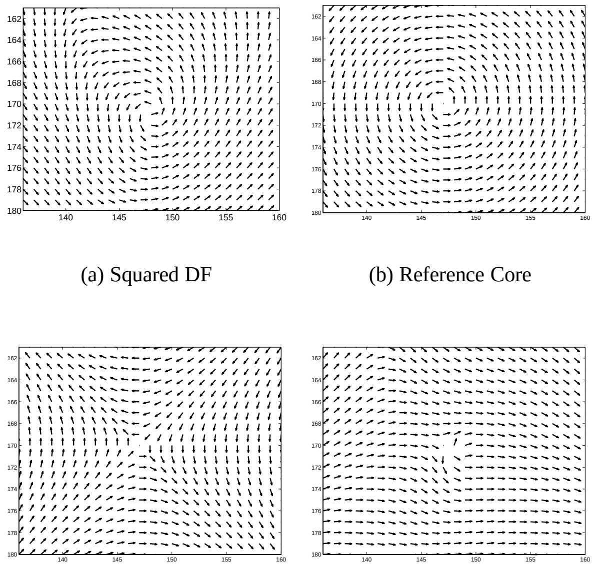
$$\sum_{\partial A} J_x + J_y = \sum_{A} \mathrm{rot}[J_x \ J_y]^T = \sum_{A} \left( \frac{\partial J_y}{\partial x} - \frac{\partial J_x}{\partial y} \right) \quad (4)$$

where $A$ is the area and $\partial A$ is the contour.

Since the DF is assumed to be smooth, $A$ can be taken as small as a square of $1 \times 1$ pixel. This results in a very efficient method for computation of the Poincaré index. Application of the proposed method will indeed lead to the desired SP locations. Unlike all other SP extraction methods, a core results in a Poincaré index of $2\pi$, a delta in $-2\pi$ while the index for the rest of the image is exactly equal to 0. This is illustrated in Figure 2(d) for the core example and the area around it.

## 4. Orientation of singular points

The last subject of this paper is the estimation of the *orientations* $\varphi$ of the extracted SPs. The method that is described here, makes use of the squared gradient vectors in the neighborhood of an SP, both for the image to be analyzed and for the reference SP. The averaged squared gradients of the core, repeated in Figure 3(a), ideally look like the reference model in Figure 3(b). For a core at $(x, y) = (0, 0)$, this reference model is given by:



(a) Squared DF (b) Reference Core

(c) Conjugated (d) Orientation

**Figure 3: Processing steps in the calculation of the orientation of a core.**

$$Core_{\mathrm{Ref}} = \frac{(y, -x)}{\sqrt{x^2 + y^2}} \quad (5)$$

The usefulness of the squared gradients is caused by the fact that, when the gray-scale image rotates around the core, in effect all components of the averaged squared gradient matrix rotate over the same angle. Therefore, the model of a core that has rotated over an angle $\varphi$ is given by a reference model with all its components multiplied by $e^{j\varphi}$.

$$Core_{\varphi} = Core_{\mathrm{Ref}} \cdot e^{j\varphi} \quad (6)$$

This property is used for the estimation of the orientation of the core. The orientation of the core, with respect to the reference model, is found by taking the inner product of the estimated squared gradient data and the complex conjugated of the reference model, depicted in Figure 3(c). Then, it is divided by the number matrix elements $N$, and the angle is taken.

$$\hat{\varphi} = \angle \frac{1}{N} \sum_{x,y} Core^*_{\mathrm{Ref}}(x,y) \cdot Core_{\mathrm{Obs}}(x,y) \quad (7)$$

The averaging operator provides an accurate estimate for the orientation $\varphi$, while, if the observed core is exactly a rotated version of the reference core, this estimate gives:

$$\begin{aligned} \hat{\varphi} &= \angle \frac{1}{N} \sum_{x,y} Core^{*}_{\mathrm{Ref}}(x, y) \cdot Core_{\mathrm{Ref}}(x, y) \cdot e^{j\varphi} \\ &= \angle \frac{1}{N} \sum_{x,y} |Core_{\mathrm{Ref}}(x, y)|^2 \cdot e^{j\varphi} \\ &= \angle e^{j\varphi} = \varphi \end{aligned} \quad (8)$$

For a delta, a related reference model can be made. Because of the symmetries in a delta, its relative orientation with respect to the reference model is given by one third of the angle resulting from Equation 7, which was used for a core.

## 5. Conclusions

In this paper, methods have been presented for the estimation of a high resolution directional field from fingerprints. It is shown how, from the directional field, very accurate detection of the singular points and the orientations of those points can be obtained. These estimates can for instance be used for accurate registration (alignment) of two fingerprints in a fingerprint verification system.

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