

# THE EFFECT OF LINER INDUCED PHASE FLUCTUATIONS ON THE GAIN OF A CERENKOV FEL

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## Abstract

In a Cerenkov FEL (CFEL), the phase velocity of an EM wave is determined by geometry (i.e., waveguide radius and inner liner radius) and by material parameters (i.e., the dielectric constant of the liner). Changes or fluctuations in any of these parameters induce changes (or fluctuations) in the phase velocity of the EM wave and thus can degrade the gain of the CFEL. To investigate the effect of such fluctuations on the gain, a method is developed to describe the effect of a slowly varying inner liner radius on the EM wave propagation and consequently on the gain of the CFEL. As an example, results for a low gain, 800 mA Cerenkov FEL operating at a frequency of 50 GHz and a maximum beam voltage of 100 kV are presented.

## INTRODUCTION

It is well known that errors in the magnetic field of an undulator can result in (serious) degradation of the gain of an undulator based Free-Electron Laser (FEL) [1, 2]. These errors couple to the electron motion and manifest themselves as fluctuations in the phase of the electron with respect to the ponderomotive potential. As the gain is based on longitudinal phase-bunching of the electrons, these phase fluctuations will in general lower the gain of the device. The effect of the undulator errors is wavelength dependent and this leads to increasingly stringent tolerances on the undulator to avoid significant reduction in the gain at shorter wavelength [2].

In a Cerenkov FEL (CFEL), an electron beam interacts with an electromagnetic (EM) field as they co-propagate through an axial-symmetric, lined cylindrical waveguide. The longitudinal wavenumber of the EM wave is determined by geometry (waveguide radius and inner liner radius) and by material parameters (dielectric constant of the liner). Coherent amplification of EM waves takes place when the phase velocity of the waves approximately equals the electron beam velocity and the EM-wave has a longitudinal electric field component. Therefore, variations or fluctuations in the dielectric constant or in the geometry of the liner result in changes of the longitudinal wavenumber and consequently in the relative phase between the EM-wave and the co-propagating electrons. As the EM wave is the ponderomotive potential in a CFEL, it can be expected that these fluctuations will influence the gain of a CFEL as well. However, here the errors in the liner are coupled to

the radiation field whereas in the undulator based FEL the errors of the undulator are coupled to the electron motion.

To our knowledge, we are the first to investigate liner induced phase fluctuation in the ponderomotive potential and their effect on the gain of a Cerenkov FEL. This information is especially important for the design of low gain Cerenkov FEL devices (see e.g., [4]) as liners made from commercial tubes have variations in the inner radius ranging from a few hundred microns down to a few tens of microns (e.g., for precision tubes). Low gain devices are usually designed as an oscillator and have a low net-gain per pass such that any degradation of the gain seriously effects the performance of the device.

The organization of the remainder of this work is as follows. We will consider an axial-symmetric lined waveguide for which the inner radius of the liner  $r_d(z)$  varies slowly with axial distance  $z$  and is otherwise constant. We first discuss, in the limit of no electron beam, the influence of the liner fluctuations on the propagation of an EM wave in such a waveguide. Then we will use the SVAP approximation to derive the fundamental FEL equations from Maxwell's equations. We continue with applying the set of equations to a low gain CFEL that uses an 800 mA electron beam with a maximum energy of 100 kV to generate more than 1 kW in continuous wave mode, to study the effect of liner irregularities on the performance of the device. By applying a linear change of  $r_d$  with distance  $z$  the formulation can also be used to model a tapered version of the CFEL.

## WAVE PROPAGATION IN AN IRREGULAR LINED WAVEGUIDE

Consider an axial-symmetric waveguide lined with a dielectric that has a varying inner radius  $r_d(z)$  and is otherwise constant. Let  $r_g$  be the radius of the waveguide, which is also equal to the outer radius of the liner. As for the case of constant  $r_d$ , it is sufficient to solve Maxwell's equations for the longitudinal field components alone, since they completely specify the electromagnetic wave. As  $\vec{\nabla} \cdot \epsilon = 0$  within the vacuum and dielectric region separately, and assuming a  $e^{i\omega t}$  dependence for the fields, the wave equation can be written as

$$\left( \vec{\nabla}_\perp^2 + \frac{\partial^2}{\partial z^2} + \frac{\omega^2}{c^2} \right) \begin{Bmatrix} E_z(r, z) \\ B_z(r, z) \end{Bmatrix} = 0 \quad (1)$$

for each region. Here  $c = c_0$  in the vacuum region,  $c = \frac{c_0}{\sqrt{\epsilon_r}}$  in the liner region, and  $c_0$  is the speed of light in vacuum. As observed above, the longitudinal wavenumber is a function of  $z$  due to the longitudinal variation of

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$r_d(z)$ . Assuming that the longitudinal wavenumber,  $k(z)$  will be slowly varying with distance  $z$ , we will follow the WKB approximation [3] and make the ansatz that the fields have a longitudinal variation according to

$$E_z(r, z) = \frac{g(r)}{\sqrt{k(z)}} \exp\left(i \int_0^z k(z') dz'\right). \quad (2)$$

In agreement with the WKB approximation, we neglect the second order derivative in  $k(z)$ . Substituting (2) into the wave equation (1) gives

$$\left(\nabla_{\perp}^2 - k^2(z) + \frac{\omega^2}{c^2}\right) g(r) = 0, \quad (3)$$

which is Bessel's equation. However, eq. 3 still has an implicit dependence on the distance  $z$  through the longitudinal wavenumber  $k(z)$  and this would in general violate the assumption that  $g(r)$  is a function of  $r$  alone. However, we assume that the variations in  $k(z)$  are slow and small, i.e.,  $\Delta k(z) = k(z) - k_0 \ll k(z)$ , where  $k_0$  is the wavenumber corresponding to a constant  $r_d$  equal to the average liner radius  $r_{d0}$ . Thus, we find, in first approximation,  $k_z \approx k_0$  and the transverse profile of the EM-wave remains unaffected by the slowly varying inner radius. On the other hand, a small value of  $\Delta k(z)$  may give an appreciable value in the phase factor  $\exp\left(\int_0^z (k_0 + \Delta k(z')) dz'\right)$ . Thus  $\Delta k(z)$  will be retained in the phase factor. The phase  $\alpha(z, t)$  of the electromagnetic wave is given by

$$\alpha(z, t) = \int_0^z k(z') dz' - \omega t, \quad (4)$$

where the slowly varying longitudinal wavenumber is given by

$$k(z) = k_0 + \int_0^z \frac{\partial k}{\partial z'} dz' \approx k_0 + \left. \frac{\partial k}{\partial r_d} \right|_{r_{d0}} \int_0^z \frac{\partial r_d}{\partial z'} dz'. \quad (5)$$

It is assumed here that the inner radius of the liner is equal to the mean radius at  $z = 0$ . To summarize, the transverse mode pattern is not influenced by the slowly varying inner boundary of the liner which only influences the phase of the propagating EM-wave through  $k_z$ . Variations in  $r_d(z)$  are considered slow if they are slow compared to a radiation wavelength, because the SVAP approximation, used to derive the dynamical FEL equations, includes an average over one ponderomotive wavelength (i.e., a radiation wavelength).

To find the fields, within these approximations, eq. (3) is solved and the usual boundary conditions are applied. The solution consists of axial-symmetric, propagating EM-waves that can be divided into two classes. These are the well known  $TE_{0n}$  and  $TM_{0n}$  modes with respectively  $E_z = 0$  and  $B_z = 0$ . Only modes with  $E_z \neq 0$  are of interest, as these modes are responsible for the bunching of the electrons in a Cerenkov FEL. Limiting ourselves to the interesting case of a phase velocity less than  $c_0$ , the expressions for the fields, used to derive the dynamical CFEL

equations, are as follows. The transverse wavenumbers  $\kappa_n$  and  $\kappa'_n$  for the vacuum respectively liner region are given by

$$\kappa_n = \sqrt{k_{0n}^2 - \frac{\omega^2}{c_0^2}} \quad \kappa'_n = \sqrt{\epsilon_r \frac{\omega^2}{c_0^2} - k_{0n}^2}, \quad (6)$$

whereas the components for the  $TM_{0n}$  are given by

$$\vec{E}_{0n}(\vec{r}, t) = \omega \left( i I_1(\kappa_n r) \hat{e}_r - \frac{\kappa_n}{k_{0n}} I_0(\kappa_n r) \hat{e}_z \right) A_{0n}(z, t) \quad (7)$$

and

$$\vec{B}_{0n}(\vec{r}, t) = i k_{0n} \left( 1 - \frac{\kappa_n^2}{k_{0n}^2} \right) I_1(\kappa_n r) A_{0n}(z, t) \hat{e}_\theta \quad (8)$$

in the vacuum region and by

$$\begin{aligned} \vec{E}_{0n}(\vec{r}, t) = & \\ = \omega \left( i \frac{\kappa_n}{\kappa'_n} [a_n J_1(\kappa'_n r) + b_n Y_1(\kappa'_n r)] A_{0n}(z, t) \hat{e}_r - \right. & \\ \left. - \frac{\kappa_n}{k_{0n}} [a_n J_0(\kappa'_n r) + b_n Y_0(\kappa'_n r)] A_{0n}(z, t) \hat{e}_z \right) & \quad (9) \end{aligned}$$

and

$$\begin{aligned} \vec{B}_{0n}(\vec{r}, t) = & \\ = i k_{0n} \left( 1 + \frac{\kappa_n'^2}{k_{0n}^2} \right) \frac{\kappa_n}{\kappa'_n} [a_n J_1(\kappa'_n r) + & \\ + b_n Y_1(\kappa'_n r)] A_{0n}(z, t) \hat{e}_\theta & \quad (10) \end{aligned}$$

in the liner region. In these equations,

$$A_{0n}(z, t) = a_{0n} \frac{1}{\sqrt{k_n(z)}} e^{i\alpha_n(z, t)}. \quad (11)$$

## DYNAMICAL EQUATIONS FOR CFEL

In this section a non-linear formulation is given for a Cerenkov FEL with an irregular liner. As usual, we start with expressing the total field as a sum over the  $TM_{0n}$  waves (eqs. (7) to (10)) found for the irregular, axially symmetric, lined waveguide with no electron beam present [5]. The amplitude of each mode is  $z$ -dependent such that the wave can be amplified by the electron beam. This total field is substituted into Maxwell's equations with the electron beam as the driving term for the field. Using the orthogonality property of the modes, a single mode amplitude is filtered out from Maxwell's equation by multiplying it with the transverse profile of that mode and integrate Maxwell's equation over the cross-section of the waveguide. Then the slowly varying amplitude and phase (SVAP) approximation is applied to obtain the final dynamical equation that describes the amplification of the mode amplitude. The set of equations is closed by complementing it with Lorentz's equation that describes the motion of the electrons within the beam under influence of the electromagnetic fields and

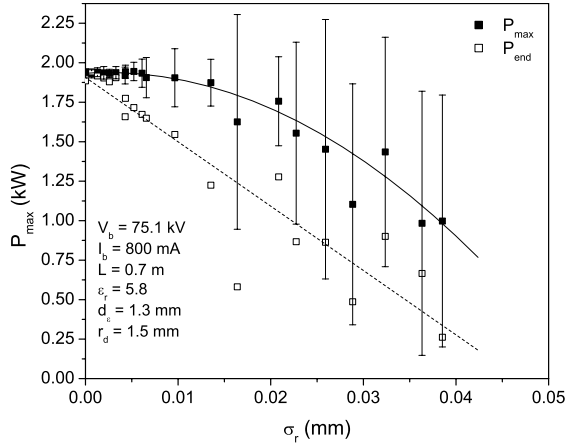


Figure 1: Calculated power as a function of the standard deviation of the liner irregularities. See the text for more details. The lines are for guidance only.

the axial magnetic guide field present. As the derivation is straightforward and very similar to [5] we only present the final result for the mode amplitude,

$$\begin{aligned}
 2\sqrt{k_n(z)}\left(1 - \frac{\kappa_n^2}{k_{0n}^2}\right)\frac{\partial a'_{0n}}{\partial z} = \\
 = -4\frac{\omega_p^2}{c_0^2}\frac{\beta_{z0}}{A_n r_{d0}^2}\int_0^{r_{d0}} dr r \left[ iI_1(\kappa_n r)\langle \frac{\beta_r}{|\beta_z|} e^{i\alpha_n} \rangle + \right. \\
 \left. + \frac{\kappa_n}{k_{0n}} I_0(\kappa_n r)\langle e^{-i\alpha_n} \rangle \right] \quad (12)
 \end{aligned}$$

where  $A_n$  is a normalization constant,  $\omega_p$  is the plasma frequency and  $a'_{0n}(z) = \frac{e}{mc} a_{0n}(z)$  with  $a_{0n}(z)$  the complex  $z$ -dependent amplitude of (11). Here,  $\langle \dots \rangle$  is an average over all electrons within one ponderomotive wavelength, i.e., one radiation wavelength. Note that with the electron beam present, the phase of the  $TM_{0n}$  mode is not solely given by  $\alpha_{0n}(z, t)$  as the dynamical CFEL equation (eq. (12)) also drives the phase of the complex amplitude  $a'_{0n}$ . Therefore the total relative phase of an electron with respect to the ponderomotive potential is given by the sum of these two phases.

## LINER IRREGULARITIES

The above given formulation is used to investigate the effect of irregularities in the inner radius of an otherwise constant liner used in a low gain CFEL operating at a nominal frequency of 50 GHz [4]. The CFEL uses a 800 mA electron beam with a radius  $r_b$  of 1mm and a maximum beam voltage  $V_b$  of 100 kV to generate an output power in excess of 1 kW. Using a liner with the following parameters,  $\epsilon_r=5.8$ ,  $r_{d0}=1.5$  mm, length  $L=0.7$  m and thickness  $d_\epsilon=1.3$  mm, the CFEL requires a beam voltage  $V_b$  of 75.1 kV to obtain maximum output power at 50 GHz. The liner

fluctuations are generated using a uniform random distribution between  $-\delta r_d$  and  $\delta r_d$  that is filtered with a low-pass spatial filter. The filter has a cut-off distance of 0.1 m and removes fast fluctuations. To estimate the influence on the maximum power, 20 different realizations of the fluctuations are generated for each maximum amplitude  $\delta r_d$ . For each realization the maximum power  $P_{max}$  or, if not saturated, the power  $P_{end}$  at the end of the liner, and the standard deviation  $\sigma_r$  of the fluctuations are calculated. The average of these values are plotted in fig. 1 for a liner length of 70 cm. The error bar in  $P_{max}$  represent the average standard deviation of  $P_{max}$  ( $P_{end}$  has a similar standard deviation). With no fluctuations, the laser has a saturated power of  $P_{max} = 1.9$  kW at a distance  $z_{sat} = 67 \pm 1$  cm. From the simulations it follows that both  $P_{max}$  and  $z_{sat}$  decrease with increasing  $\sigma_r$  and at the same time the spread in both increases, i.e.,  $z_{sat} = 55 \pm 8$  cm at  $\sigma_r = 40 \mu\text{m}$ . It follows from fig. 1 that a rms fluctuation of approx.  $40 \mu\text{m}$  is already sufficient to lower the maximum power, on average, by a factor of 2. On the other hand, fig. 1 also shows that a particular realization of the liner fluctuations can also enhance the saturated power and this will be investigated in the next section.

## TAPERED LINER

In the previous section we considered a dielectric liner with an inner radius that fluctuates slowly but randomly with distance  $z$  and found that even small fluctuations can seriously degrade the gain of a CFEL. If, on the other hand, we were to apply a linear taper starting at some distance, we expect to enhance the gain and obtain a higher output power. The latter is the result of keeping the bunch away from reaching the bottom of the ponderomotive potential well by changing the phase velocity of the EM wave. As the average electron velocity decreases, the phase velocity of the EM wave must be reduced as well to avoid saturation. Because  $dk_{0n}/dr_{d0}$  is negative, one has to reduce the inner liner radius to avoid trapping. This will reduce the gap between the electron beam and liner and will thus have limited applicability. However, it is interesting to find out if a negative taper on  $r_d(z)$  improves the performance and by how much.

We consider the same CFEL as in the previous section. The growth of the power as a function of  $z$  is shown in fig. 2 where the length of the liner has been extended to  $L = 3$  m to allow for some distance for the taper. As the CFEL has initially a gap of 0.5 mm between the electron beam and the liner, the length of the taper is limited and the maximum liner length depends on the slope of the taper. Fig. 2 shows the power as a function of  $z$  for different, negative, slopes of a taper that starts at  $z = 60$  cm. If the slope is not large enough, the laser will not reach saturation within a liner length of 3 m. The output of the laser increases with increasing slope of the taper up to the point where the liner diameter is such that the gap with the electron beam is reduced to zero and electrons start to hit the liner before the

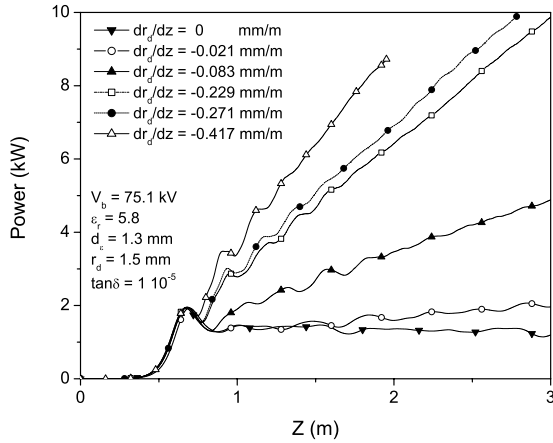


Figure 2: Calculated power as a function of distance  $z$  for different slopes of the taper. The taper starts at  $z=0.6$  m.

end is reached. At this point the output power is close to 10 kW ( $dr_d/dz = -0.229$  mm/m). A still larger slope results in a shorter distance to reach the point of zero gap and a gradual decrease of the maximum power; for a slope of  $-0.417$  mm/m the maximum power is already reduced to 8.7 kW at  $z = 1.95$  m. For a slope of  $-0.188$  mm/m we extended the liner length and found a maximum power of 10.4 kW at a distance of 3.65 m where, again, the gap was reduced to zero. It should be pointed out that although the decrease in gap as a result of the negative taper increases the coupling strength between electrons and EM wave, it is the change in phase velocity that prevents the laser from reaching saturation. To conclude, for this particular CFEL, it is found that a negative taper on  $r_{d0}$  increases the maximum attainable power with a factor of 5 before the gap is reduced to zero. Note that at that point the laser was not saturated.

Therefore, further improvements are to be expected by using different methods of tapering that avoid reducing the gap. These methods are changing the dielectric constant or the outer waveguide radius. This is currently under study.

## CONCLUSIONS

We have demonstrated that, in first approximation, the transverse mode profile of a CFEL is not influenced by a slowly varying inner radius of the liner. However the liner does effect the phase velocity of the propagating wave. We derived the dynamical CFEL equation and applied it to investigate the influence of liner fluctuations and a taper of the inner liner radius on the performance of the CFEL. It was found that an rms fluctuation that is approximately 3 % of  $r_{d0}$  is already sufficient to decrease the maximum power by a factor of 2, at least for the configuration investigated. Note that this rms fluctuation is comparable to the manufacturing accuracy of commercially available (quartz) tubes. By applying a negative taper to  $r_{d0}$  the maximum power can be increased by a factor of 5 before the gap be-

tween electron beam and liner becomes zero. At this point the laser is not yet saturated, so other methods of tapering may still further improve the performance.

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