STRIBECK CURVE FOR STARVED CONCENTRATED CONTACTS

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Abstract This paper discusses a mixed lubrication model in order to predict the Stribeck curve for starved lubricated line contacts. The model is an extension of the mixed lubrication model of Gelinck and Schipper [1]. In order to build the starved Stribeck curve model, the contact model of Greenwood and Williamson [2] and the EHL film thickness for starved line contacts making use of the assumption of Johnson et al. [3] is combined. The starved solution to be implemented in the EHL component is obtained by fitting numerical data of Wolveridge et al. [4] who computed the starved film thickness for smooth line contacts. Calculations are presented for different oil supply layer thickness over roughness values. For values of oil layer thickness over roughness ratio (h_{oil}/σ_s) larger than approximately 6, the Stribeck curve and separation do not change. If the oil layer thickness over roughness ratio is in the range of 6 to 0.7 friction starts to increase and the film thickness decreases. When the oil layer thickness over roughness ratio is less than approximately 0.7 the Stribeck curve tends to transform into a straight line and separation stays on the same value as in the BL regime.

Keywords: Stribeck curve, starved lubrication, mixed lubrication, film thickness, oil layer thickness over roughness ratio.

1. INTRODUCTION

One of the developments in design is to reduce the size of the components in constructions while transmitting the same or even higher loads, resulting in severe operational contact conditions. This means that the contact between the different components do not operate anymore under (elasto-) hydrodynamic lubrication (EHL) conditions but mixed lubrication (ML) or even boundary lubrication (BL) is more likely to occur. Therefore, the Stribeck curve is an important tool to determine in which lubrication regime a contact operates. In Gelinck and Schipper [1] a model is presented in order to predict the Stribeck curve for line contacts. This model is based on the combination of the Greenwood and Williamson [2] contact model and the full film theory using the

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mixed lubrication model of Johnson et al. [3]. With this model one is able to predict friction and determine the transitions between the different lubrication regimes i.e. (Elasto-) Hydrodynamic Lubrication (EHL), Mixed Lubrication (ML) and Boundary Lubrication (BL). This model is based on the assumption that enough lubricant is supplied to the contact, e.g. fully flooded conditions. However, in many applications the amount of lubricant available is not sufficient and the contacts operate under starved lubrication conditions. For a designer it is very important to know the friction and the transitions between the different lubrication regimes (EHL, ML and BL) under starved lubrication conditions.

In the literature much attention is paid to starvation, however, the investigations were conducted for the full-film situation (no contact takes place between the opposing surfaces) with the emphasis on the inlet boundary conditions and cavitation. Most of these investigations were applied to the circular contact situation. For the starved line contact situation, a few were published, for instance Wolveridge et al. [4].

In the work of Wolveridge et al. [4] a correction on the film thickness formula for line contacts due to starvation is presented. Combining this modified film thickness relation for starved line contacts with the model of Gelinck and Schipper [1] will result in a mixed lubrication model for starved line contacts.

In this article the consequences of starvation on friction depicted in the generalised Stribeck curve due to change in film formation are discussed.

2. MIXED LUBRICATION MODEL

The total normal load F_T is shared by the hydrodynamic action and the interacting asperities:

$$F_T = F_C + F_H, \tag{1}$$

where F_C is the load carried by the asperities and F_H the load carried by the hydrodynamic component. Therefore, friction in mixed lubrication is a summation of the friction due to asperity interaction (boundary lubrication) and friction due to shearing the lubricant.

2.1 Load Carried by Asperity Interaction

Greenwood and Williamson [2] showed that the load carried by the asperities is given by:

$$p_a(x) = \frac{2}{3} n \beta \sigma_s \sqrt{\frac{\sigma_s}{\beta}} E' F_{3/2}\left(\frac{h(x)}{\sigma_s}\right) \tag{2}$$

with h the separation between two surfaces, n the density of the asperities, β the average radius of the asperities, and σ_s the standard deviation of the height distribution of the asperities. $F_{3/2}(h)$ is defined as:

$$F_{3/2}(h) = \int_h^\infty (s - h)^{3/2}\, \phi(s)\, \mathrm{d}s, \tag{3}$$

where $\phi(s)$ is the height distribution of the asperity summits. In this paper a Gaussian height distribution of the asperities has been assumed:

$$\phi(s) = \frac{1}{\sqrt{2\pi}}\, \mathrm{e}^{-1/2s^2}. \tag{4}$$

The elasticity modulus used in Equation (2) is defined as:

$$\frac{2}{E'} = \frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2} \tag{5}$$

with E_i the elasticity modulus of the surfaces and ν_i the Poisson's ratio.

In Gelinck and Schipper [1] the pressure distribution was calculated on the basis of Equation (2), making the nominal pressure $p[h(x)]$ due to the deformed asperities consistent with the gap $h[p(x)]$ resulting from the bulk deformation. A relation for the central (maximum) pressure p_C compared to its smooth value as characterising the complete pressure distribution is derived:

$$\frac{p_C}{p_{\mathrm{Hertz}}} = \left[1 + \left(a_1 n'^{\,a_2} \sigma_{S'}{}^{a_3} W^{a_2 - a_3}\right)^{a/4}\right]^{1/a_4} \tag{6}$$

in which p_{Hertz} is the maximum Hertzian pressure (for the other parameters the reader is referred to the Appendix).

2.2 EHL Component

Based on EHL calculations Moes [7] derived a fit in dimensionless form for the central film thickness in an EHL line contact:

$$H_C = \left[\left(H_{RI}^{7/3} + H_{EI}^{7/3}\right)^{3/7s} + \left(H_{RP}^{7/2} + H_{EP}^{7/2}\right)^{2/7s}\right]^{s^{-1}} \tag{7}$$

with

$$s = \frac{1}{5}\left(7 + 8\exp\left[-2\frac{H_{EI}}{H_{RI}}\right]\right). \tag{8}$$

H_{RI}, H_{EI}, H_{RP} and H_{EP} are the dimensionless film thickness asymptotes for respectively the Rigid-Isoviscous, Elastic-Isoviscous, Rigid-Piezoviscous and Elastic-Piezoviscous situation as described in the Moes-diagram. The reader is referred to the Appendix, Gelinck and Schipper [1] or to Moes [7] for further details of the Moes diagram and the asymptotes.

2.3 The Model

According to Johnson et al. [3] the total pressure in an ML contact can be divided into an EHL component and a BL component.

Two coefficients γ_1 and γ_2 according to Johnson et al. [3] are defined:

$$\gamma_1 = \frac{F_T}{F_H}, \quad \gamma_2 = \frac{F_T}{F_C}. \tag{9}$$

It is shown by Johnson et al. [3] and Gelinck and Schipper [1] that the results of the EHL and BL calculations can be combined in the ML regime, if E' is replaced by E'/γ_1, F_T by F_T/γ_1 and n by $n \cdot \gamma_2$. For the EHL component this results in:

$$H_C = \left[\gamma_1^S \left(H_{RI}^{7/3} + \gamma_1^{14/15} H_{EI}^{7/3}\right)^{3/7S} + \left(H_{RP}^{7/2} + H_{EP}^{7/2}\right)^{2/7S}\right]^{1/S} \tag{10}$$

with

$$s = \frac{1}{5}\left(7 + 8\exp\left[-2\frac{H_{EI}}{H_{RI}} \cdot \gamma_1^{-2/5}\right]\right) \tag{11}$$

and for the BL component is:

$$\frac{p_C}{p_{\mathrm{Herz}}} = \left[1 + (a_1 n'^{\,a_2}\, \sigma_{s'}{}^{a_3} W^{a_2-a_3} \gamma_2^{a_2})^{a_4}\right]^{1/a_4} \frac{1}{\gamma_2}. \tag{12}$$

As the load increases or the surface becomes smoother, p_C approaches the Hertzian pressure corresponding to the fraction of load carried by the asperities.

With the equations given so far the fractions of load of the BL component and the EHL component can be calculated.

2.4 Friction Force

The total friction force F_f for a certain velocity is the sum of the balanced friction forces of the interacting asperities and the shear force of the hydrodynamic component:

$$F_f = \sum_{i=1}^{N} \iint_{A_{C_i}} \tau c_i \, \mathrm{d}A_{C_i} + \iint_{A_H} \tau_H \, \mathrm{d}A_H \tag{13}$$

with N the number of asperities in contact, A_{Ci} the area of contact of a single asperity, i; τC_i the shear stress at the asperity contact i; A_H the contact area of the hydrodynamic component; and τ_H the shear stress of the hydrodynamic component.

The coefficient of friction f_{C_i} of a single asperity can be written as:

$$f_{C_i} = \frac{\tau C_i}{p_{C_i}} \tag{14}$$

with p_{C_i} the normal pressure of a single asperity. According to Briscoe et al. [8] the ratio of the shear strength and the local contact pressure is nearly constant and therefore the coefficient of friction is approximately constant for all asperity contacts. Thus the first term in Equation (13) can also be written as:

$$\sum_{i=1}^{N} \iint\limits_{A_{C_i}} \tau C_i \, \mathrm{d}A_{C_i} = f_C F_C, \tag{15}$$

where F_C is the total normal load carried by the asperities. The value of f_C is determined from experiments.

For the shear force in the lubricant film the Eyring model is used:

$$\tau_H(\dot{\gamma}) = \tau_0 \cdot \operatorname{arcsinh}\left(\frac{\eta\dot{\gamma}}{\tau_0}\right), \tag{16}$$

where η is calculated according to the Roelands equation, the pressure used to calculate the viscosity is the average pressure of the hydrodynamic component, $\dot{\gamma}$ is the shear rate (ν^{dif}/h, ν^{dif} is the difference in velocity of the two surfaces or sliding velocity) and τ_0 is the Eyring shear stress.

The coefficient of friction can now be written as:

$$f = \frac{F_f}{F_T} = \frac{f_C F_C + \iint\limits_{A_H} \tau_H(\dot{\gamma}) \, \mathrm{d}A_H}{F_T}. \tag{17}$$

Combining this result with Equation (16) results in the coefficient of friction for Stribeck curves:

$$f = \frac{F_C F_C + \tau_0 \, A_H \operatorname{arcsinh}\left(\frac{\eta \nu^{\mathrm{dif}}}{h\tau_0}\right)}{F_N}. \tag{18}$$

3. THE STARVED MODEL

In Figure 1 a two dimensional representation of the starved contact situation is presented in which h_{oil} is the supplied oil layer thickness and h'_{oil} is the oil layer thickness when the surface tension is taken into account; x_i and x'_i are the lubricant inlet lengths belonging to the aforementioned lubricant layer thicknesses; h^*, b and V are the starved film thickness, half contact width and the velocity respectively.

The surface tensions depend on the properties of the lubricant and the surface, Tian and Bhushan [10]. In this paper h_{oil} and h'_{oil} are taken equal. According to Crook [11] the oil layer thickness in the converging entry is given

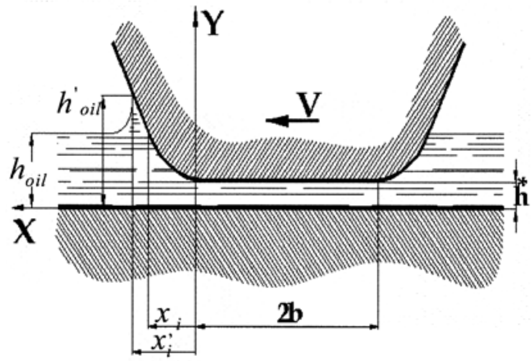


Figure 1. A schematic representation of starved situation.

with good approximation by:

$$h_{\mathrm{oil}} = h^* \left[1 + \frac{4\sqrt{2}}{3} \phi^{3/2} \right], \tag{19}$$

where ϕ is a nondimensionless coordinate:

$$\phi = b^{1/3} x_i \big/ (2Rh^*)^{2/3} \tag{20}$$

in which R is the contact radius.

The ratio β between the film thickness for the starved (h^*) and fully flooded condition (h) is derived by Wolveridge et al. [4], based on computational solutions of Orcutt and Cheng [9], as a function of the dimensionless lubricant inlet length ψ_i:

$$\frac{h^*}{h} = \frac{H_C^*}{H_C} = \beta = f(\psi_i), \tag{21}$$

where

$$\psi(i) = b^1 3x_i \big/ (2Rh)^{2/3}. \tag{22}$$

In order to implement the numerical solution of Wolveridge et al. in Gelinck and Schipper's model an analitical solution is needed.

The numerical solutions are well fitted by the following equation:

$$\beta = \frac{h^*}{h} = \frac{2}{\pi} \arctan\,(2.7 \cdot \psi_i) \tag{23}$$

as may be seen in Figure 2. If in the EHL component of the mixed lubrication model, the film thickness formula is replaced by the fitted starved film thickness formula (23), the starved mixed lubrication model is completed.

In the next section the results of the presented starved Stribeck model for two different sets of roughness parameters are presented.

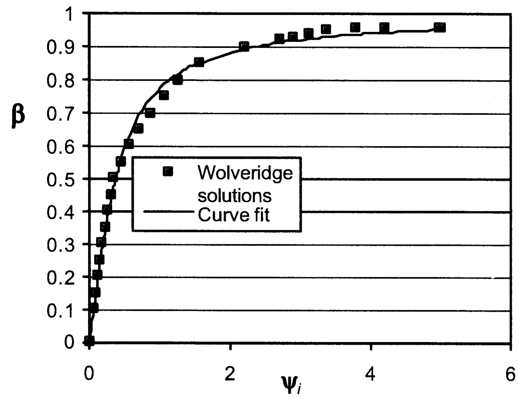


Figure 2. The approximation of numerical solutions of Wolveridge et al. [4].

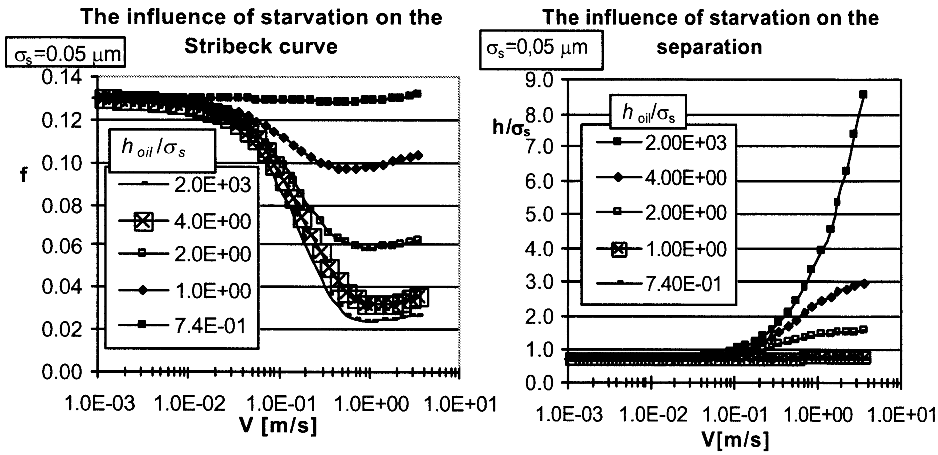
Table 1. Parameters used for calculations.

Property	Value Reference Case 1	Value Reference Case 2
n	$1 \cdot 10^{11}$ m^{-2}	$13.0 \cdot 10^{9}$ m^{-2}
β	$1 \cdot 10^{-5}$ m	$2.6 \cdot 10^{-6}$ m
σ_s	$0.05 \cdot 10^{-6}$ m	$0.487 \cdot 10^{-6}$ m
R_a	$0.0565 \cdot 10^{-6}$ m	$0.55 \cdot 10^{-6}$ m
B	$10.0 \cdot 10^{-3}$ m	$12.7 \cdot 10^{-3}$ m
E'	$231 \cdot 10^{9}$ Pa	$231 \cdot 10^{9}$ Pa
R	$20.0 \cdot 10^{-3}$ m	$19.0 \cdot 10^{-3}$ m
η_0	0.0202 Pa·s	0.0374 Pa·s
α	$2.0 \cdot 10^{-8}$ Pa^{-1}	$2.0 \cdot 10^{-8}$ Pa^{-1}
τ_0	$2.5 \cdot 10^{6}$ Pa	$2.5 \cdot 10^{6}$ Pa
f_c	0.13	0.13
F_N	1000 N	1000 N

4. RESULTS OF STARVED STRIBECK CURVE CALCULATIONS

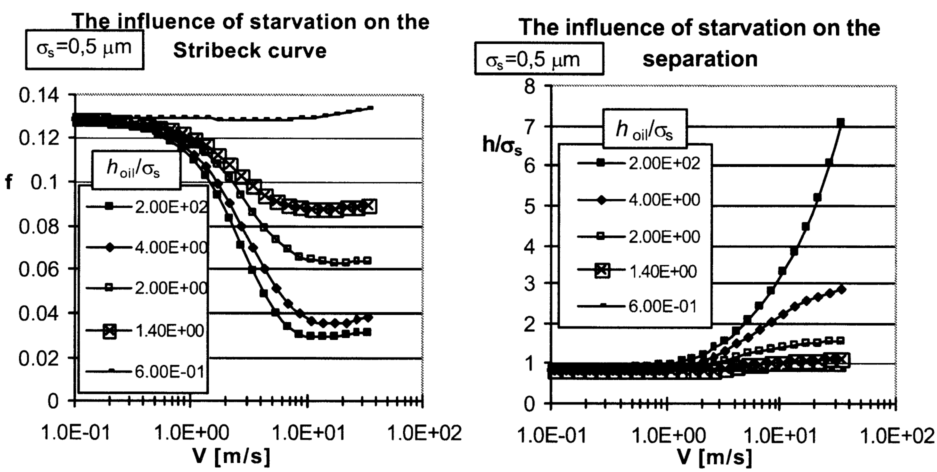
The Stribeck curve for starved lubrication conditions can now be calculated by varying the velocity. Two sets of surface parameters in combination with the supplied oil layer thickness are used to show their influence on friction under starved conditions. The parameters used are presented in Table 1.

The surface parameters of reference case 1 are of a spur gear transmission and the parameters of reference case 2 are are taken from Johnson and Spence [12].



(a) (b)

Figure 3. Stribeck curve (a) and separation (b) for reference case 1.



(a) (b)

Figure 4. Stribeck curve (a) and separation (b) for reference case 2.

The results of the calculation of the Stribeck curve and the separation for the two reference cases performed for different oil layer thickness over rougness ratios, h_{oil}/σ_s are given in Figures 3 and 4.

The tendency of the starved Stribeck curve and the corresponding separation as a function of oil layer thickness ratio h_{oil}/σ_s can be described as follows:

- when the oil layer thickness (h_{oil}) is larger than, say, 6 times the standard deviation of summits heights (σ_s) the Stribeck curve and separation do not change compared to the fully flooded condition.

- if the oil layer thickness ratio (h_{oil}/σ_s) is between 6 and, say, 0.7 the friction level in the EHL and ML regimes starts to increase and the separation decreases, Figures 3b and 4b. The mixed lubrication regime becomes less steep in this range of oil layer thickness over roughness ratio. The transition from BL to ML and from ML to EHL respectively occurs approximately at the same transition velocity only the friction level changes (Figures 3a and 4a). The Stribeck curve for starved lubricated contact "pivots" at the transition ML/BL as the ratio of h_{oil}/σ_s decreases.
- for values of oil layer thickness over roughness ratio's (h_{oil}/σ_s) less than $\sim$ 0.7 the Stribeck curve tends to transform into a straight line, Figures 3a and 4a and the separation stays on the same level as in the BL regime, Figures 3b and 4b.

Similar results have been found for point contacts by Liu [5].

5. CONCLUSIONS

A model has been developed in order to calculate the starved Stribeck curve and separations in the mixed lubrication regime. The model is a combination of the Gelinck and Schipper mixed lubrication model and numerical calculations for starved smooth lubricated line contacts of Wolveridge et al. [4]. The surface tensions which give a thicker film and heat development which give lower film are neglected, however in real contacts these two main effects may compensate each other.

The calculations have been performed for two sets of roughness parameters. In terms of oil layer thickness over roughness ratio (h_{oil}/σ_s) the results show the same trend for the two reference cases. For values of oil layer thickness over roughness ratio (h_{oil}/σ_s) larger than approximately 6 the Stribeck curve and separation do not change. If oil layer thickness over roughness ratio is in the range of 6 to 0.7 the friction starts to increase and the film thickness decreases. When the oil layer thickness over roughness ratio is less than approximately 0.7 the Stribeck curve tends to transform into a straight line (constant friction level) and separation stays at the same value as in BL regime.

APPENDIX

In Equation (6) the dimensionless numbers n', σ_s' and W, as well as the values of the constants a_i were used. These numbers are defined as:

$$n' = \frac{342}{\pi} n R \sqrt{\beta R}, \quad \sigma_s' = \frac{\pi}{8}\frac{\sigma_s}{R} \quad \text{and} \quad W = \frac{F_T}{BE'R}.$$

The values of the parameters a_i in eq. (6) are $a_1 = 0.953$, $a_2 = 0.0337$, $a_3 = -0.442$ and $a_4 = -1.70$, respectively.

In Equation (7) the function fit of the Moes-diagram is given. The asymptotes in the Moes-diagram are given as:

$$
\begin{aligned}
H_{RI} &= 3M^{-1}, \quad H_{RP} = 1.287L^{2/3} \\
H_{EI} &= 2.621M^{-1/5} \quad \text{and} \quad H_{EP} = 1.31M^{-1/8}\,L^{-3/4}
\end{aligned}
$$

in which

$$
\begin{aligned}
H_C &= \frac{h}{R}\left(\frac{E'R}{\eta_0 v^+}\right)^{1/2}, \quad M = \frac{F_T}{BE'R}\left(\frac{E'R}{\eta_0 v^+}\right)^{1/2} \\
L &= \alpha E'\left(\frac{E'R}{\eta_0 v^+}\right)^{-1/4},
\end{aligned}
$$

where h is the film thickness and v^+ is the sum velocity.

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