A DIANA-ELEMENT FOR THE DYNAMIC ANALYSIS OF LAMINATE PLATES

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Abstract. This paper describes the main characteristics of a new DIANA element to treat the dynamic behaviour of structures made of three-layer laminate plate, including the effect of damping. This element is not only suited for laminates with two metal skins and a viscoelastic layer, but also for composite materials like ARALL. Sample calculations are presented showing the effect of a viscoelastic layer on the dynamic characteristics. In addition attention is paid to the interaction of the vibrating structure with a surrounding fluid.

1. Introduction

Laminated plates offer benefits such as lighter structures or more energy dissipation (higher damping values) compared to the more traditional materials. For this reason many publications appeared in this field. For a good overview on the publications in this field reference is made to papers by Bert [2] and by Noor and Burton [8].

There are different ways in modelling a laminate plate. The first one is to treat it as a single-layer plate with an equivalent stiffness, in which the displacements across the thickness are described with a single expression (see figure 1).

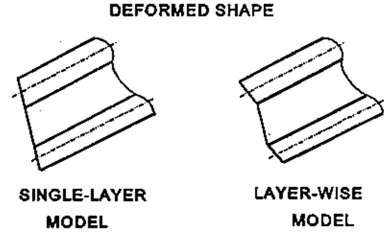

Fig 1: Single-layer and layer-wise model

A second way to model a laminate plate is a layer-wise description, in which the displacements of every layer are described by separate functions, continuous through the thickness, including the layer interfaces (see figure 1).

A combination of the single-layer and layer-wise models is also a possibility. In that case adjacent layers are grouped together into so-called sublaminates. In each sublaminate the displacement field is continuous through the thickness. On the interfaces of the different sublaminates, the displacement fields are taken continuously.

Various combinations can be used for the polynomial order of the displacement fields. For the inplane displacements in many cases a linear function over the thickness is used and the transverse displacement is supposed to be constant through the thickness. These are the first-order theories.

The Kirchhoff theory, also called classical plate theory (CPT), is an example of such an approach. The transverse shear strains are put zero, so this theory cannot represent shear deformations and it is only valid for thin plates. Another first-order theory is the Mindlin theory where the shear deformations are assumed to be constant through the thickness.

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For this reason this theory is also known as the first-order shear deformation theory. The Mindlin theory results in constant transverse shear strains through the element thickness and thus provides constant transverse shear stresses.

2. Model Description

For the present purpose a layer-wise model is combined with a first-order plate theory. Reasons for not using a higher order theory are the lower number of degrees of freedom involved and the fact that the dynamic behaviour of a laminate plate is well described by a first-order theory. The dynamic behaviour is a global phenomenon and local differences will be cancelled out by taking the integral over the domain. The distribution of the transverse shear stress is not accurately represented, because this is a local phenomenon. The first-order theory used here is the Reissner-Mindlin theory, because it can represent shear deformations.

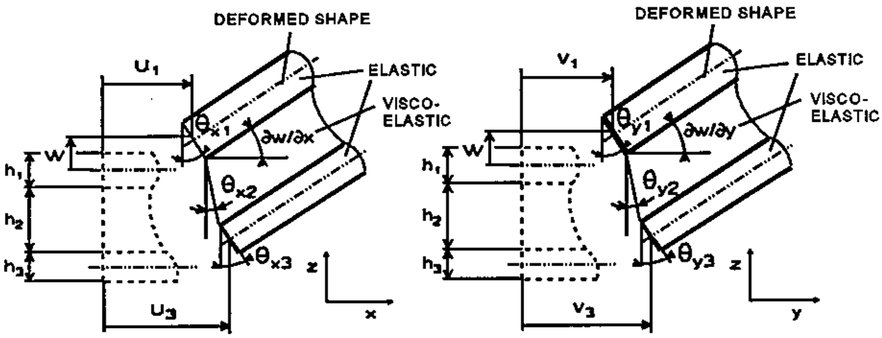


Fig 2: Model degrees of freedom

The finite element treated in this paper consists of three layers. The displacement fields are:

$$\bar{u}_i(x,y,z) = u_i(x,y) - \theta_{xi}(x,y)\,z_i$$
$$\bar{v}_i(x,y,z) = v_i(x,y) - \theta_{yi}(x,y)\,z_i \qquad (-\tfrac{1}{2}h_i \le z_i \le -\tfrac{1}{2}h_i) \quad i = 1,2,3 \tag{1}$$
$$\bar{w}_i(x,y,z) = w_i(x,y)$$

Where i is the layer-index. The displacements of a point on the reference plane of the i^{th} laminate layer are given by u_i, v_i en w_i, while θ_{xi} and θ_{yi} denote the rotations about the y and x axis and z_i the coordinate in thickness direction, which is zero on the reference plane of the corresponding layer (see figure 2).
The normal strains become:

$$\varepsilon_{xx,i} = \frac{\partial u_i}{\partial x} - z_i \frac{\partial \theta_{xi}}{\partial x} \quad ; \quad \varepsilon_{yy,i} = \frac{\partial v_i}{\partial y} - z_i \frac{\partial \theta_{yi}}{\partial y} \tag{2}$$

The shear strains become:

$$\gamma_{xy,i} = \frac{\partial u_i}{\partial y} + \frac{\partial v_i}{\partial x} - z_i\left(\frac{\partial \theta_{xi}}{\partial y} + \frac{\partial \theta_{yi}}{\partial x}\right) \qquad \text{(in plane)} \tag{3}$$

$$\gamma_{xz,i} = \frac{\partial w}{\partial x} - \theta_{xi} \quad ; \quad \gamma_{yz,i} = \frac{\partial w}{\partial y} - \theta_{yi} \qquad \text{(transverse)} \tag{4}$$

Further the following assumptions are made:

1 The normal stresses in thickness direction are neglected as is done in all plate theories, so: $\sigma_{zz}=0$. This means that the thickness of the plate is supposed to be small compared to its length.
2 The strain in thickness direction (ε_{zz}) is zero, because of the constant transverse displacement field. This is only valid when the layer material is sufficiently stiff in thickness direction.
3 The model is supposed to be linear, which means small displacements and strains.
4 There is no slip between the layers. This assumption results in four interface conditions for the two interfaces.

In the model an arbitrarely situated point in the (x-y)-plane contains 15 degrees of freedom: 5 per layer. The four interface relations combined with the condition of a constant transverse displacement reduces this number to 9.

The degrees of freedom chosen are (see figure 2):
- the inplane displacements of both skin-layers (u_1, v_1, u_3, v_3),
- the transverse displacement (w) and
- the two rotations around the inplane axis of the two skin-layers (θ_{x1}, θ_{y1}, θ_{x3}, θ_{y3}).

The five degrees of freedom of the midlayer can be expressed in those of the skin-layers.

Next the assumption is made that the rotations of the two skin-plates are equal, so:

$$\theta_{x1} = \theta_{x3} = \theta_x \quad ; \quad \theta_{y1} = \theta_{y3} = \theta_y \tag{5}$$

This assumption is justified for laminated plates, which are symmetric in-thickness direction or which have thin skin plates. For strongly asymmetric laminates, this assumption will introduce only a small error. After the introduction of (5) the final number of degrees of freedom in a point of the laminate plate is reduced from 9 to 7.

The strain energy Π_{in} of a three-layer laminate element (area A) can be split up into three parts, i.e. the energies due to membrane (m), bending (b) and shear (s) deformations:

$$\Pi_{in} = \frac{1}{2}\sum_{i=1}^{3}\left(h_i\int_A \{\varepsilon_{mi}\}^T\,[E_i]\,\{\varepsilon_{mi}\}\,dA + \frac{1}{12}h_i^3\int_A \{\varepsilon_{bi}\}^T\,[E_i]\,\{\varepsilon_{bi}\}\,dA + h_i\int_A \{\varepsilon_{si}\}^T\,[G_i]\,\{\varepsilon_{si}\}\,dA\right) \tag{6}$$

where:

$$\{\varepsilon_{mi}\} = \begin{Bmatrix} \dfrac{\partial u_i}{\partial x} \\ \dfrac{\partial v_i}{\partial y} \\ \dfrac{\partial u_i}{\partial y} + \dfrac{\partial v_i}{\partial x} \end{Bmatrix} \; ; \; \{\varepsilon_{bi}\} = \begin{Bmatrix} \dfrac{\partial \theta_{xi}}{\partial x} \\ \dfrac{\partial \theta_{yi}}{\partial y} \\ \dfrac{\partial \theta_{xi}}{\partial y} + \dfrac{\partial \theta_{yi}}{\partial x} \end{Bmatrix} \; ; \; \{\varepsilon_{si}\} = \begin{Bmatrix} \dfrac{\partial w}{\partial x} - \theta_{xi} \\ \dfrac{\partial w}{\partial y} - \theta_{yi} \end{Bmatrix} \tag{7}$$

and in case of an orthotropic material behaviour:

$$[E_i] = \begin{bmatrix} \dfrac{E_{xx}}{1-\nu_{xy}\nu_{yx}} & \dfrac{E_{yy}\nu_{xy}}{1-\nu_{xy}\nu_{yx}} & 0 \\ \dfrac{E_{xx}\nu_{yx}}{1-\nu_{xy}\nu_{yx}} & \dfrac{E_{yy}}{1-\nu_{xy}\nu_{yx}} & 0 \\ 0 & 0 & G_{xy} \end{bmatrix}_i \; ; \; [G_i] = \begin{bmatrix} \dfrac{1}{k_x} G_{xz} & 0 \\ 0 & \dfrac{1}{k_y} G_{yz} \end{bmatrix}_i \tag{8}$$

in which k_x and k_y are the shear factors for the x-direction and y-direction, respectivily.

The kinetic energy T of the three-layer laminate element equals:

$$T = \frac{1}{2}\sum_{i=1}^{3} \left[\rho_i h_i \int_A (\dot{u}_i^2 + \dot{v}_i^2 + \dot{w}^2)\, dA + \frac{\rho_i h_i^3}{12} \int_A (\dot{\theta}_{xi}^2 + \dot{\theta}_{yi}^2)\, dA \right] \tag{9}$$

The first term of the kinetic energy is due to longitudinal and transverse inertia and the second due to rotatory inertia.

At this stage of the formulation it becomes important to choose the type of element. In this study an isoparametric Mindlin-type of element is selected. This kind of element behaves very well compared to other types of elements, e.g. discrete-Kirchhoff elements and the formulation is relatively simple. The only disadvantage of these elements is their sensitivity for shear-locking. This can be avoided by using a reduced integration rule for the shear deformation part of equation (6). This may introduce so-called zero-energy modes (also called mechanisms). These are modes with a non-zero deformation, for which the elastic energy becomes zero, leading to a singular stiffness matrix.

Triangular elements do not behave well if a reduced integration is applied. Cook, Malkus and Plesha [4] showed that from the rectangular elements, the 4-node element (bilinear element) and the 9-node element (Lagrange element) are very susceptible to zero-energy modes, especially when also for the bending deformation a reduced integration rule is used. The 8-node element (Serendipity element) is free of zero-energy modes under the condition that only the shear deformation part is integrated with a reduced rule and the bending deformation part with a full integration rule (selective integration). Another

variant is the Heterosis element, which also consists of nine nodes, but where the mid-node only has rotations as degrees of freedom. This element tries to inherit the good properties of both the Lagrange and the Serendipity element, which explains the name Heterosis. In contrast to the Serendipity element the Lagrange element behaves well for very thin plates.
Both the Serendipity and Heterosis element have been implemented in DIANA, but only the Serendipity element will be discussed here, because both elements have the same behaviour in dynamic analyses.

The discretization of the displacements and rotations for the Serendipity element gives:

$$u_i=\sum_{k=1}^{8} N_k u_{ik}\,;\, v_i=\sum_{k=1}^{8} N_k v_{ik}\,;\, w=\sum_{k=1}^{8} N_k w_k\,;\, \theta_{xi}=\sum_{k=1}^{8} N_k \theta_{xik}\,;\, \theta_{yi}=\sum_{k=1}^{8} N_k \theta_{yik} \qquad (10)$$

Here $N_k = N_k(x,y)$ represents the k^{th} shape or interpolation function. These expressions substituted in (7), (6) and (9) and applying Lagrange's principle gives, after summation over the elements, the set of equations of motion of the structure:

$$[M]\{\ddot{u}\} + [K]\{u\} = \{F\} \qquad (11)$$

in which [M], [K] and {F} respectively are the assembled mass matrix, the stiffness matrix and the force vector. The vector {u} contains the structural degrees of freedom. The eigenvalues of the system can be determined from the homogeneous form of this matrix-vector equation.

3. Numerical Tests: Static Analysis

For a first validation of the static behaviour of the new element (denoted as CQ56L and in the latest version of DIANA as MLAYSH), we consider a uniformly loaded (q) laminated square plate, which is simply supported at all its edges and which is modelled with 10 by 10 CQ56L-elements. The three layers are of equal thickness and have the same material properties. The material is isotropic.

$E = 7.\ 10^4\ N/mm^2$
$\nu = 0.34$
$k = 1.2$
$q = 0.1\ N/mm^2$

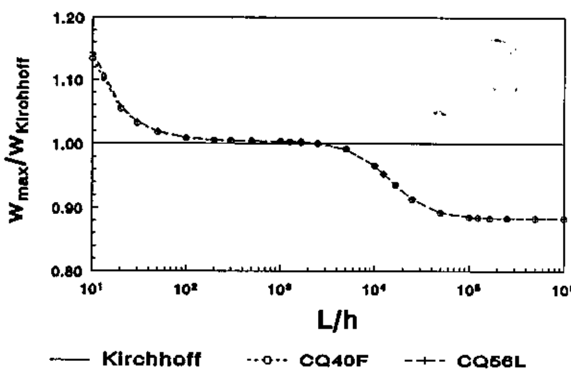


Fig 3: Relative deflection against length to thickness ratio

The results are depicted in figure 3, showing the normalized maximum deflection of the plate against the ratio of its length (L) and thickness (h). The deflection is normalized with the corresponding value obtained with the Kirchhoff theory. The normalized deflection should converge to unity for large values of L/h (thin plate), because then the shear deformation is negligible, which is a basic assumption in the Kirchhoff theory. For L/h values larger than 3.10^3 there is no longer convergence to unity caused by the shear-locking phenomenon. An improvement is possible by introducing lower shearfactors k_x and k_y. This factors may be chosen such, that they are a function of the ratio of the element magnitude and the element thickness as mentioned in ref. [1,9].
For small L/h values the solution deviates from unity, because the Kirchhoff theory is no longer valid. The same behaviour is observed in the literature on this element. The results for the laminate element, CQ56L, are also compared with the results of the non-laminate shell element in DIANA, CQ40F (see also figure 3). From this figure it can be concluded that for static problems the laminate plate element shows the same behaviour as other shell elements do, as long as the element length to thickness ratio is not too large.

4. Numerical Tests: Dynamic Analysis

The dynamic behaviour of the laminate element has been validated against numerical values published by others for a 'thick' isotropic plate, for a 'thick' orthotropic plate and for a cross-ply laminate plate.

4.1 Thick Plate of Isotropic, respectively Orthotropic Material

The plate considered here is a square plate, which is simply supported at all its edges. The length to thickness ratio L/h is 10. The plate which in fact consists of only one type of material is modelled with the laminate element by taking the layers equally thick with the same material properties. The plate is discretized again with 10 by 10 elements. The results are presented as the natural frequencies Ω, which are made dimensionless according to:

$$\Omega = \omega h \sqrt{\frac{\rho}{C}} \tag{12}$$

Where C is a relevant material stiffness property. In the presented example of orthotropic material the relationship between the two elasticity moduli is $E_{xx} \approx 2.E_{yy}$. For further details see ref. [6].
The shear factor is taken 1.1 for both cases. This is somewhat lower than usually applied in the Mindlin theory (1.2). It can be shown, see ref. [6], that the factor 1.1 provides a better approximation of the shear energy for layered models.
A selection of the natural frequencies of the flexural (I) and extensional modes (II, III), from [6], is presented in table 1 for the isotropic case and in table 2 for the orthotropic case. For extensional mode II the deformations in x- and y-direction do have the same sign (both tension or both compression), for extensional mode III the opposite takes place. The frequencies are compared with the exact values derived with the 3D-elasticity

theory and with numerical values calculated with a higher-order theory presented by Cho, Bert and Striz [3]. Cho e.a. used third-order polynomials for the inplane displacement fields and a quadratic polynomial for the out-of-plane displacement field, combined with a layer-wise model. This model should deliver the most accurate results. The results calculated with the model of Cho e.a. are also presented in table 1. The plate is treated as a single-layer (N=1) and as a four-layer laminate (N=4), respectivily.
From the table it can be concluded that the results obtained with the laminate element show good agreement with the exact values and with the results of the higher order theory.

Theory	Isotropic				Orthotropic			
	Mode	I	II	III	Mode	I	II	III
Exact	1 1	0.0931	0.7498	0.4443	1 1	0.0474	0.3941	0.2170
CQ56L		0.0930	0.7510	0.4443		0.0474	0.3940	0.2170
Cho (N=1)		0.0930	0.7516	0.4443		0.0474	0.3941	0.2170
Cho (N=4)		0.0931	0.7499	0.4443		0.0474	0.3941	0.2170
Exact	1 2	0.2226	1.1827	0.7025	1 2	0.1033	0.5624	0.3450
CQ56L		0.2219	1.1875	0.7025		0.1032	0.5626	0.3452
Cho (N=1)		0.2219	1.1897	0.7025		0.1033	0.5626	0.3451
Cho (N=4)		0.2224	1.1832	0.7025		0.1033	0.5624	0.3450
Exact	1 3	0.4171	1.6654	0.9935	1 3	0.1888	0.7600	0.4953
CQ56L		0.4152	1.6800	0.9939		0.1885	0.7611	0.4957
Cho (N=1)		0.4147	1.6856	0.9935		0.1885	0.7610	0.4955
Cho (N=4)		0.4164	1.6668	0.9934		0.1887	0.7601	0.4953

***Table 1:** Comparison of the dimensionless natural frequencies Ω of a homogeneous plate of isotropic, respectively orthotropic material*

4.2 Cross-ply Laminate

In table 2 the natural frequency of the first flexural mode of a three-layer (0°/90°/0°) and a two-layer (0°/90°) cross-ply laminate are presented for various length to thickness (L/h) ratios. The laminate plate is again simply supported at all its edges. The material properties are:

$$E_{yy}/E_{xx} = 4 \;\; ; \;\; G_{xy}/E_{yy} = 0.6 \;\; ; \;\; \nu_{xy} = 0.25 \;\; ; \;\; G_{xz}/E_{yy} = 0.6 \;\; ; \;\; G_{yz}/E_{yy} = 0.5 \;\; ; \;\; k = 1.1$$

In the present example the dimensionless frequency is defined by:

$$\Omega = \omega \frac{L^2}{h} \sqrt{\frac{\rho}{E_{yy}}} \tag{13}$$

The numerical results obtained with the laminate element for the three-layer laminate (CQ56L) agree very well with the results presented by Cho e.a. for $L/h \geq 10$. In this region the difference is less than 1 percent. This difference increases for smaller L/h ratios (very thick plates). This is due to the fact that the assumptions $\sigma_{zz}=0$ and $\varepsilon_{zz}=0$ (plate theory) are no longer valid here. Cho e.a. did not make this assumption (resulting in substantially more degrees of freedom), so his results will be more accurate. However, to the authors' opinion one should not use plate elements to model these situations, but volume elements.

L/h	three-layer (0°/90°/0°)				two-layer (0°/90°)			
	CQ56L	% Error to Cho	Cho e.a.	CPT	CQ56L	% Error to Cho	Cho e.a.	CPT
2	5.242	13.0	5.923	15.830	5.216	8.44	4.810	8.499
5	10.565	1.02	10.673	18.215	8.819	5.14	8.388	10.584
10	14.965	0.67	15.066	18.652	10.467	1.92	10.270	11.011
20	17.589	0.31	17.535	18.767	11.077	0.55	11.016	11.125
25	18.023	0.17	18.054	18.780	11.158	0.36	11.118	11.139
50	18.661	0.05	18.670	18.799	11.270	0.09	11.260	11.158
100	18.833	0.01	18.835	18.804	11.299	0.03	11.296	11.163

Table 2: *Comparison of the dimensionless natural frequencies , of a cross-ply three-layer (0 °/90 °/0 °) and a two-layer (0 °/90 °) square laminate plate*

The results for the two-layer laminate show only slightly larger deviations, even though the laminate is asymmetric in material properties and modelling (the 90° layer is modelled by two layers of the element). So also in this case it is justified to assume equal rotations of the two skin plates.
Table 2 confirms the expectation that the classical plate theory (CPT) is not able to represent the real dynamic behaviour of the cross-ply laminate.

4.3 Laminate Plate with Viscoelastic Midlayer

One of the aims of the present study was to include the damping effect of a viscoelastic midlayer. For this purpose complex elasticity constants are introduced:

$$E_2^* = E_2 (1 + i\,\beta) \quad \text{and} \quad G_2^* = G_2 (1 + i\,\beta)$$

with E_2, G_2 and β being the elasticity modulus, the shear modulus and the so-called loss factor, respectively.
As a first application the dynamic response of a square laminate plate was considered, excited by a uniformly distributed, harmonic load. The plate was simply supported at all edges. The dimensions were chosen fully arbitrarely, to obtain convenient natural frequencies. The amplitude of the load was 10^5 N/m^2 and the frequency of oscillation was varied between 3 and 4.2 Hz with steps of 0.02 Hz. The amplitude of the centre of the

plate as a function of the frequency, presented in figure 4, clearly reveals not only the presence of a resonance frequency around 3.6 Hz but also the effect of the loss factor β on the amplitude.
It can be observed that, in contrast with the expectations, an increase in loss factor is coupled with an increase in resonance frequency. This phenomenon has been observed earlier by Durocher and Solecki [5] and a physical interpretation has been attempted by Locht [7] with the help of an equivalent two-degrees-of-freedom system.

$L = 100$ m
$h_i = 1, 8, 1$ m (i=1,2,3)

skin plates:
$E_1 = E_3 = 69$ GPa
$\rho_1 = \rho_3 = 2700$ kg/m^3
$\nu_1 = \nu_3 = 0.3333$

midlayer:
$E_2 = 2.4$ GPa
$\rho_2 = 1022$ kg/m^3
$\nu_2 = 0.3333$

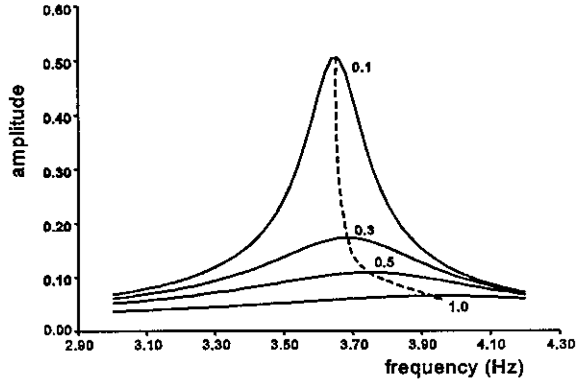


Fig 4: Response of a simply supported laminate plate with viscoelastic midlayer for β = 0.1, 0.3, 0.5 and 1.0

4.4 Acoustic Coupled Laminate Plate with Viscoelastic Midlayer

The final example is meant to illustrate the combined effect of the presence of visco-elastic damping and the acoustic coupling with the surrounding air on the dynamic behaviour of a square laminate plate. For details on the way in which this coupling is achieved in DIANA reference is made to [1].
The example taken from [7] concerns a closed cavity, at one side covered with a flexible laminate plate. All other walls are assumed to be rigid (see figure 5).
The material properties of the laminate and the dimensions of the cavity are chosen such that the natural frequencies of the plate without the effect of the air ('uncoupled') are not too far apart from the natural frequencies of the 'uncoupled' acoustic modes (i.e. the acoustic modes in the cavity with all walls rigid).
Since it was the intention to consider only the lowest double-symmetric modes of the plate, it suffices to model one quarter of the box (see figure 5) . The plate was modelled with 4 8-node laminate plate elements (MLAYSH-Serendipity-version), the air with 5 x 16 acoustic elements (HX8HT) and the interface plate-air with 4 interface elements (BQ24S9). For the determination of the resonance frequencies of the coupled system, the plate was excited in its centre by means of a harmonic normal force with a frequency ranging from 12 to 20 Hz.

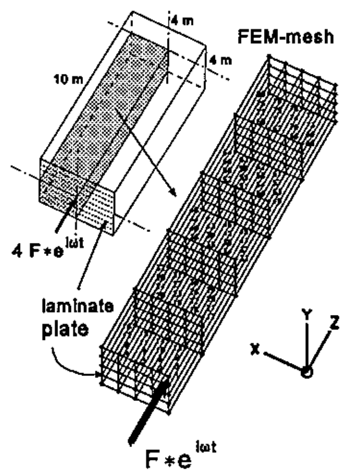


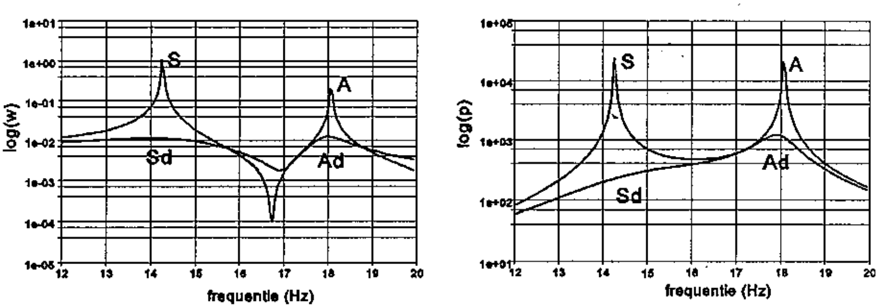
Fig 5: Example of acousto-elastically coupled laminate plate

Material properties laminate:
$E_1 = 210$ GPa ; $\rho_1 = 2400$ kg/m^3 ; $\nu_1 = 0.3$
$E_2 = 0.024$ GPa ; $\rho_2 = 1100$ kg/m^3 ; $\nu_2 = 0.3$; $\beta = 0.5$
$E_3 = 210$ GPa ; $\rho_3 = 2400$ kg/m^3 ; $\nu_3 = 0.3$
air: velocity of sound: 340 m/s ; $\rho = 1.125$ kg/m^3
dimensions laminate: $h_1 = h_3 = 0.2$ mm ; $h_2 = 0.6$ mm

The results of the computations of the response of the system are collected in table 3 and examples of the frequency response functions in terms of displacements and pressure perturbation in the centre of the plate versus frequency are depicted in figure 6. Table 3 reveals that the uncoupled resonance frequency of the plate increases from 14.97 to 15.11 Hz when the loss factor β is increased from 0 to 0.5. The coefficient of (structural) damping, η, of the overall system becomes 0.291.
Due to the presence of the air in the cavity the resonance frequency of the plate decreases from 14.97 to 14.25 for β=0 and from 15.11 to 14.60 for β=0.5. In the latter case the coefficient of damping η decreases from 0.291 to 0.240.

β	UNCOUPLED MODES			COUPLED MODES			COUPLED MODES light		
		FREQ	η	TYPE	FREQ	η	TYPE	FREQ	η
0.0	PLATE	14.97	-	S	14.25	-	S	12.05	-
	AIR	17.28	-	A	18.05	-	A	20.80	-
0.5	plate	15.11	0.291	Sd	14.60	0.240	Sd	12.22	0.182
	air	17.28	-	Ad	17.93	0.043	Ad	20.85	0.058

Table 3: Natural frequencies of uncoupled air and laminate plate, and the coupled system



S = structure dominated A = acoustic dominated d = damped

Fig 6: Frequency response functions of the acousto-elastic coupled system

The frequency of the uncoupled acoustic mode increases from 17.28 to 18.05 Hz without viscoelastic damping (β=0) and from 17.28 to 17.93 Hz for β=0.5. In the last mentioned situation the acoustic mode becomes also damped (η=0.043). Evidently, in this example, the plate experiences the air enclosed in the cavity as an additional mass. Moreover, the coupled acoustic mode becomes damped because of the accompanying motion of the plate.
The influence of the air enclosed in the cavity on the dynamic behaviour of the laminate plate becomes more pronounced for a structure which has less mass. This is demonstrated by reducing both the stiffness coefficients and the densities of the laminate plate with a factor 10. This has no effect on the 'uncoupled' resonance frequency but seriously affects the interaction between the vibrating plate and the air. The results are given in table 3 under the header 'COUPLED MODES light'.

5. Conclusions

The results of the new three-layer laminate element (CQ56L - MLAYSH), based on a first-order shear deformation theory for the inplane displacement fields and a constant out-of-plane displacement field, in combination with a layer-wise model, show a very good agreement with the higher order-model used by Cho e.a.. The element is suited to determine the dynamic behaviour of thin and thick isotropic and orthotropic plates as well as cross-ply and angle-ply laminates, including the effect of viscoelastic damping.
With the capability to use this element in combination/interaction with acoustic elements a new 'tool' has been added to DIANA which enables the study of problems related with noise and noise abatement.

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