Hydro-elasticity in flexible multibody dynamics

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Abstract

A method is presented that incorporates hydrodynamic radiation forces in the floating frame of reference method used in flexible multibody dynamics. These hydrodynamic forces are approximated such that the fully hydro-elastically coupled problem can be solved at once using standard techniques. To this end, the elastic deformation of the body that is (partially) submerged is described using Craig-Bampton modes. The generalized hydrodynamic forces caused by motion in these modes are computed using a source panel method. In a frequency domain parameter identification procedure, the solution of the remaining radiation problem is approximated by a transfer function that represents a convenient physical system. This is done such that the corresponding differential equations in the time domain describe a mechanical system. The method is illustrated for a two-dimensional partially submerged cylinder.

1 Introduction

The design and engineering of offshore operations requires analyses of very large flexible structures for many load cases. For the relevant dynamics, extensive transient simulations are necessary for which flexible multibody dynamics (FMBD) could be used. The influence of a ship's elastic hull deformations on the system dynamics can have a significant effect for a variety of state of the art operations, e.g. heavy lifting. These hull deformations may give rise to hydrodynamic forces that are not negligible. In these cases a fully hydro-elastically coupled FMBD analysis is required.

For describing the multibody dynamics, the floating frame of reference method is used. Based on linear elasticity theory, the Craig-Bampton modes of the submerged part of the ship's hull can be determined. For the hydrodynamics, a source panel method is used based on linear potential theory. This assumes small body motions, small wave heights and incompressible, inviscid and irrational flow. As a result, the transfer functions from body motion to hydrodynamic force are obtained numerically. In a parameter identification procedure, these transfer functions are approximated by the transfer functions of well-known mass-spring-damper systems. In this way, the water can be replaced by an equivalent mechanical system. This equivalent system can be used to study the dynamic behavior of the fully hydro-elastically coupled FMBD system.

2 Flexible multibody dynamics

The floating frame of reference method is covered by standard textbooks for both rigid [1] and flexible [2] multibody systems and conveniently summarized in [3]. Rigid body motion is described with respect to a global inertial frame by the position and orientation of a body's local frame, located at the center of mass of the undeformed body. Elastic deformation is described in the body's local frame by Craig-Bampton modes and is assumed small such that the local mass matrix is constant and equal to e.g. the mass matrix obtained by the finite element method, reduced to the interface points. The vector of generalized coordinates $\mathbf{q}$ is the set of rigid and flexible coordinates and uniquely defines the system's configuration.

It is assumed that the system is subjected to kinematic constraint equations of the holonomic type:

$$\boldsymbol{\Phi}(\mathbf{q}) = \mathbf{0} \tag{1}$$

The system's equations of motion are of the following general form:

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{F} \tag{2}$$

where $\mathbf{M}$, $\mathbf{C}$, $\mathbf{K}$ and $\mathbf{F}$ are the generalized mass matrix, velocity matrix, stiffness matrix and force vector, respectively. $\mathbf{M}$ and $\mathbf{C}$ are in general not constant due to geometrical nonlinearities. The system's motion is solved from the mixed system of differential-algebraic equations

$$\begin{bmatrix} \mathbf{M} & \boldsymbol{\Phi}_{\mathbf{q}}^T \\ \boldsymbol{\Phi}_{\mathbf{q}} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \mathbf{F} - \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} \\ \boldsymbol{\gamma} \end{bmatrix} \tag{3}$$

that is obtained by combining (2) and the second time derivative of (1). In this, $\boldsymbol{\Phi}_{\mathbf{q}}$ denotes the Jacobian matrix of the kinematic constraint equations, $\boldsymbol{\lambda}$ is the vector of Lagrange multipliers, $\mathbf{F}$ is the vector of generalized applied forces and $\boldsymbol{\gamma}$ is the acceleration vector of the constraint equations. For those bodies that are in contact with water, $\mathbf{F}$ also contains the generalized forces by the water on the structure.

For demonstration purposes, the fluid-structure interface is considered to be a section of a thin-walled, homogenous and prismatic cylinder with radius R. In order to determine its Craig-Bampton modes, the homogenous equilibrium equation for in-plane deformations of a thin circular beam [4] is used:

$$\frac{d^6 w}{d\theta^6} + 2\frac{d^4 w}{d\theta^4} + \frac{d^2 w}{d\theta^2} = 0 \tag{4}$$

where θ is the angular coordinate along the beam and w is the tangential displacement field. The radial displacement field u and the slope ψ of the deflection curve are related to w as:

$$u = \frac{dw}{d\theta}, \qquad \psi = \frac{1}{R}\left(\frac{d^2 w}{d\theta^2} + w\right) \tag{5}$$

The general solution of (4) is:

$$w = C_1 + C_2\theta + C_3\cos(\theta) + C_4\sin(\theta) + C_5\theta\cos(\theta) + C_6\theta\sin(\theta) \tag{6}$$

The Craig-Bampton modes can be obtained analytically by prescribing either u, w or ψ at one end of the beam a unit value, while prescribing all others a zero value and solve the relevant system of equations for the coefficients C_1 to C_6. The Craig-Bampton modes of a semi-circular beam are shown in figure 1, where the thin and thick lines indicate the undeformed and deformed shapes respectively. The analytical expressions for the coefficients C_1 to C_6 are given in table 1.

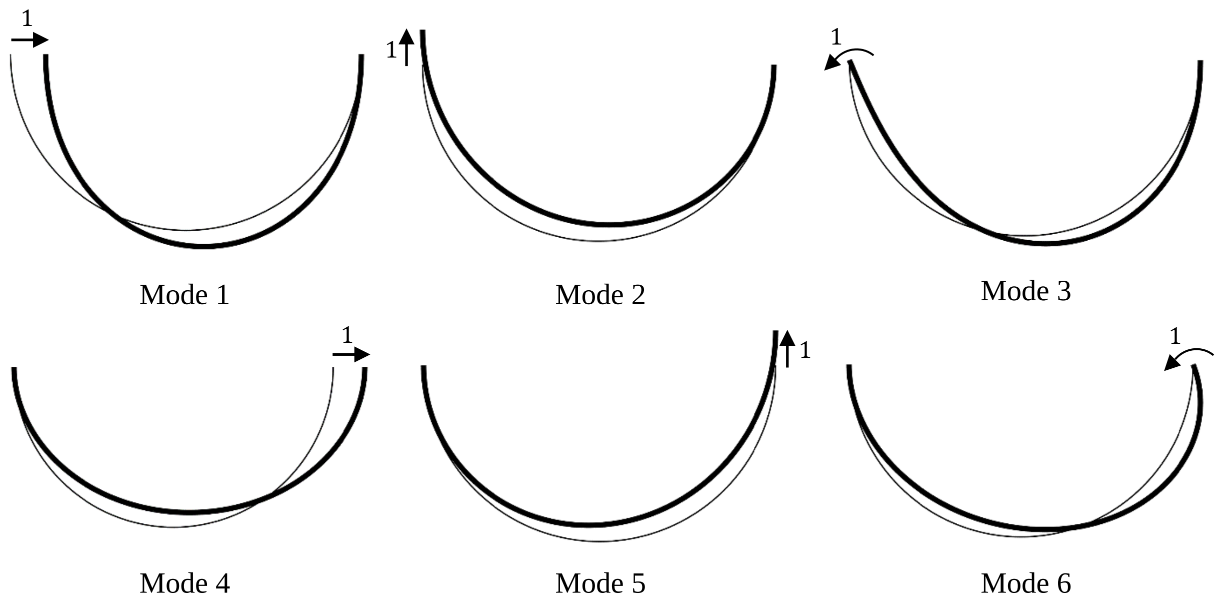

Figure 1: Craig-Bampton modes of a semi-circular beam

Mode	C_1	C_2	C_3	C_4	C_5	C_6
1	$\frac{-2\pi}{\pi^2-8}$	$\frac{-4}{\pi^2-8}$	$\frac{2\pi}{\pi^2-8}$	$\frac{4}{\pi^2-8}$	0	$\frac{\pi}{\pi^2-8}$
2	0	0	0	$\frac{1}{\pi}$	$\frac{-1}{\pi}$	0
3	$\frac{-4R}{\pi^2-8}$	$\frac{-\pi R}{\pi^2-8}$	$\frac{4R}{\pi^2-8}$	$\frac{2(\pi^2-4)R}{\pi(\pi^2-8)}$	$\frac{-R}{\pi}$	$\frac{2R}{\pi^2-8}$
4	$\frac{2\pi}{\pi^2-8}$	$\frac{4}{\pi^2-8}$	$\frac{-2\pi}{\pi^2-8}$	$\frac{-\pi^2+4}{\pi^2-8}$	0	$\frac{-\pi}{\pi^2-8}$
5	0	0	0	$\frac{-1}{\pi}$	$\frac{1}{\pi}$	0
6	$\frac{(\pi^2-4)R}{\pi^2-8}$	$\frac{\pi R}{\pi^2-8}$	$\frac{-(\pi^2-4)R}{\pi^2-8}$	$\frac{-8R}{\pi(\pi^2-8)}$	$\frac{-R}{\pi}$	$\frac{-2R}{\pi^2-8}$

Table 1: Craig-Bampton mode coefficients of a semi-circular beam

3 Linear water waves

Within the framework of linear water waves, potential flow theory is used to determine the hydrodynamic forces on the part of a ship's hull that is submerged under the free water surface. If the water is assumed to be initially undisturbed, the only contribution to the hydrodynamic force is due to radiated waves caused by the motion of the structure. The boundary value problem (BVP) for the velocity potential ϕ describing this wave radiation in deep water is governed by the Laplace equation and relevant boundary conditions [5] and is shown in figure 2. The boundary conditions are the free surface condition at $y = 0$, the sea bed condition at $y \to \infty$, the radiation conditions at $x \to \pm\infty$ and the interface condition at the submerged contour C of the body.

The linear nature of the problem allows us to write the velocity potential ϕ as the linear combination of elementary solutions:

$$\phi(\mathbf{x}, t) = \sum_j \phi_j(\mathbf{x})\, \dot{q}_j(t) \tag{7}$$

where $\phi_j(\mathbf{x})$ is the velocity potential due to a unit velocity defined by setting the time derivative of generalized coordinate q_j to 1. In this sense, q_j can be any of the generalized coordinates in the constrained equations of motion (3) that are related to the body that is (partially) submerged. Once the velocity potential is solved, the pressure can be determined from the linearized Bernoulli equation

$$p = -\rho \frac{\partial \phi}{\partial t} - \rho g y \tag{8}$$

with ρ the density of water and the latter term representing the buoyancy. Using (7) and (8), the generalized hydrodynamic force F_{ij} in direction i due to the body's motion in direction j can be computed as:

$$F_{ij} = \rho \int_C n_i \phi_j \, ds\, \ddot{q}_j = \tilde{A}_{ij} \ddot{q}_j \tag{9}$$

where s is parameterizing the submerged contour C and n_i is the generalized direction cosine in the direction of generalized coordinate q_i and $\tilde{A}_{ij}$ is the added mass in direction i due to motion in direction j.

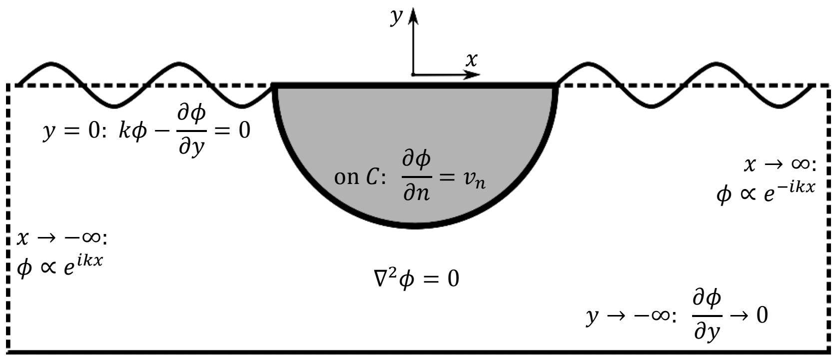

Figure 2: Boundary value problem of a partially submerged moving body

Because of the frequency dependent boundary conditions in the BVP, the added mass is complex and frequency dependent and (9) is only formally defined in the frequency domain. In order to determine the added mass, the BVP is solved for all ϕ_j in the frequency domain, assuming harmonic oscillation with unit velocity in generalized direction j. To this end, a source distribution is considered on the submerged contour:

$$\phi_j(\mathbf{x}) = \int_C \lambda(s) G(\mathbf{x}, \mathbf{x}_0)\, ds \tag{10}$$

where $\lambda(s)$ is the distributed source strength and $G(\mathbf{x}, \mathbf{x}_0)$ is the Green's function that represents the velocity potential at field point $\mathbf{x}$ due to a point source at point $\mathbf{x}_0(s)$ on C. The used form of $G(\mathbf{x}, \mathbf{x}_0)$ is in accordance with [6]:

$$G(\mathbf{x}, \mathbf{x}_0) = \frac{1}{2\pi} \log\left(\frac{r_0}{r_1}\right) - \frac{1}{\pi} \mathrm{Re}\{e^{kz} E_1(kz)\} + i e^{kz} \tag{11}$$

where k is the wave number matching deep water waves with the frequency equal to the frequency of oscillation, $E_1(z)$ the exponential integral as defined in [7] and

$$\begin{aligned} r_0 &= \sqrt{(x - x_0)^2 + (y - y_0)^2} \\ r_1 &= \sqrt{(x - x_0)^2 + (y + y_0)^2} \\ z &= (y + y_0) - i|x - x_0| \end{aligned} \tag{12}$$

Now $\lambda(s)$ is determined such that the normal velocity on the fluid-structure interface is matched. This is done numerically using a source panel method. To this end, the interface C is discretized in a sufficiently large number of panels. For every panel, integrand in (10) is replaced by its first order Taylor series about the panel's center point, which results in a solution that is second order accurate locally:

$$\phi_j(\mathbf{x}) = \sum_l \lambda_l L_l G(\mathbf{x}, \mathbf{x}_l) \tag{13}$$

where λ_l, L_l and $\mathbf{x}_l$ are respectively the source strength, length and center point of panel l. By demanding that the fluid velocity in normal direction at every center point $\mathbf{x}_l$ equals the normal velocity of the structure at that point, a system of linear equations is obtained from which the unknown source strengths are solved. With this, the added mass coefficients can be determined. Then, for any arbitrary motion of the body, the generalized hydrodynamic force F_{ij} can be expressed formally by a convolution integral:

$$F_{ij}(t) = \int_0^t A_{ij}(t - \tau) \ddot{q}_j(\tau)\, d\tau \tag{14}$$

where A_{ij} is the inverse Fourier transform of $\tilde{A}_{ij}$. In literature A_{ij} is commonly referred to as a retardation function, but this interpretation is not pursued here.

The source panel method obtained in this way, is validated by computing the added mass of a circular cylinder in rigid body motion, for which the solution is known from literature [8]. The added mass is computed for several depths H of the cylinder, where H is the distance from the free surface to the center point of the cylinder. Figure 3 shows the real and imaginary parts of the dimensionless added mass $A/\rho\pi R^2$ as a function of dimensionless frequency kR of a cylinder of which the center point is located at dimensionless depth $H/R = 0$, 1.1 and 2. Note that by this definition $H/R = 0$ corresponds to a submerged semi-cylinder and for $H/R > 1$ the cylinder is fully submerged. The curves in figure 3 are on top of the data of [8].

To demonstrate that in this way also the added mass of bodies in flexible motion can be determined, this method is applied on a circular cylinder that is given a unit velocity in the generalized direction of the Craig-Bampton modes. Figure 4 shows the real and imaginary parts of the dimensionless added mass in x-direction as a function of dimensionless frequency of a cylinder at $H/R = 0$ for all Craig-Bampton modes. It is found that the added mass of modes 1 and 4 are equal and that the added mass of modes 2, 3 and 6 are equal to each other and to minus the added mass of mode 5. This is caused by the symmetry of these modes.

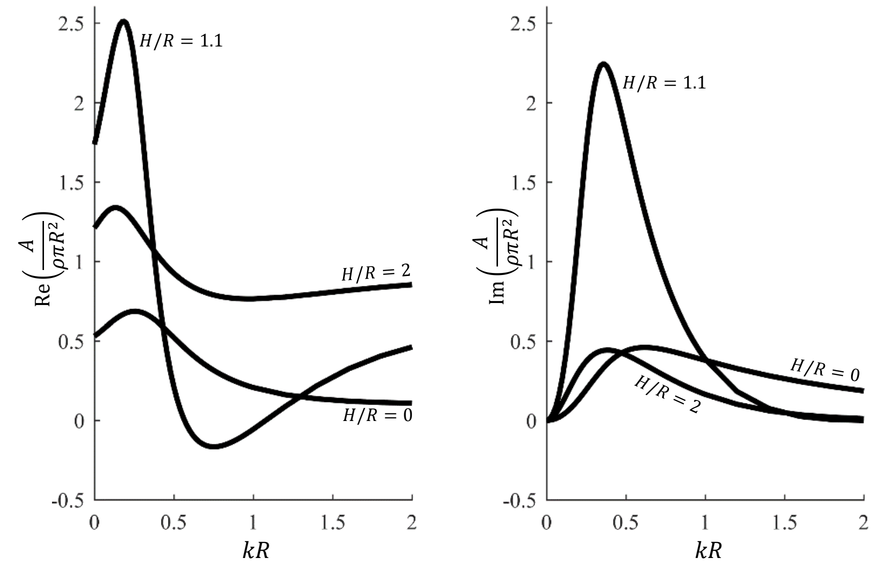

Figure 3: Added mass in x-direction of a cylinder in rigid body motion

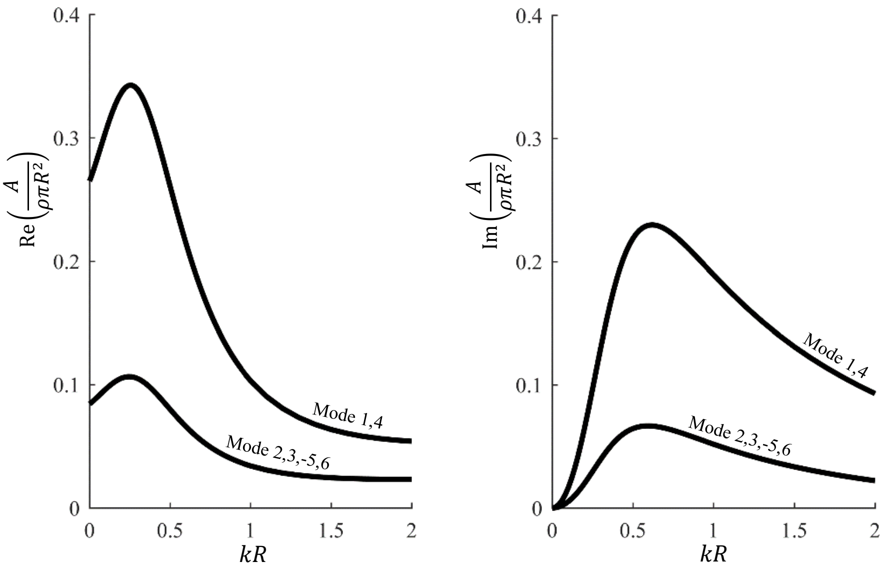

Figure 4: Added mass in x-direction of a cylinder in flexible body motion

4 Parameter identification

To include the hydro-elastic coupling in a FMBD analysis, the generalized hydrodynamic forces, as computed in (14) can be substituted in the right hand side of the constrained equations of motion (3) for those bodies that are in contact with the water. From a computational point of view, this convolution integral is very unattractive, as it needs to be evaluated at every step of the numerical time integration. Instead, a parameter identification is applied to approximate any added mass coefficient $\tilde{A}$ by a suitable transfer function $\widetilde{H}$. Many aspects of this procedure have been reported on [7,9,10] and often assume that in the Laplace domain, $\widetilde{H}$ is of the following general form:

$$\widetilde{H}(s) = K\frac{b(s)}{a(s)} = K\frac{s^n + b_{n-1}s^{n-1} + \ldots + b_1 s + b_0}{s^m + a_{m-1}s^{m-1} + \ldots + a_1 s + a_0} \tag{15}$$

Using optimization techniques, the coefficients in this expression are determined such that $\tilde{A}$ is approximated best. Based on physical considerations, a priori knowledge about the relative order of the polynomials a and b, the values of poles and zeros, the passivity of the system and so on, can be enforced in this optimization using appropriate constraint equations [11].

The common convention encountered in most references on offshore engineering is to separate the so-called infinity added mass A_∞ from (14) and interpret the real and imaginary parts of the remaining expression as added mass and added damping. The parameter identification methods commonly used are also tailored to this approach. Here, it is chosen to keep (14) as it is, because this form allows for a parameter identification with even more physical reasoning. For $\widetilde{H}$, transfer functions are used that correspond to a series of mass-spring-damper systems, as shown in figure 5.

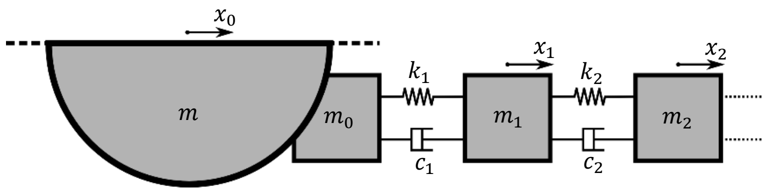

Figure 5: Identifying the hydrodynamic force by equivalent mass-spring-damper systems

The hydrodynamic force F is the interface force between m and m_0. $\widetilde{H}$ is the transfer function from acceleration $\ddot{x}_0$ to interface force F. The transfer function $\widetilde{H}_1$ of a mass-spring-damper system with one additional degree of freedom is given here:

$$\widetilde{H}_1(s) = \frac{F(s)}{s^2 x_0(s)} = m_0 \frac{s^2 + \frac{m_0 + m_1}{m_0 m_1} c_1 s + \frac{m_0 + m_1}{m_0 m_1} k_1}{s^2 + \frac{c_1}{m_1} s + \frac{k_1}{m_1}} \tag{16}$$

From (16) follows that $\widetilde{H}_1(s \to \infty) = m_0$. The value of the added mass coefficient for this limiting case is the infinity added mass A_∞, as mentioned before. This value can be determined from a related BVP [8] and is determined prior to the identification procedure. Therefore $m_0 = A_\infty$ is used directly. m_0 should be interpreted as the water mass that is in the immediate vicinity of the submerged body, as if it is rigidly connected to it.

The other system parameters are determined in the optimization procedure. To this end, s is replaced by $i\omega$ in $\widetilde{H}$ and its real and imaginary parts are determined. Here, Matlab's `fmincon` with default settings is used to minimize the squared error norm:

$$e = \left|\mathrm{Re}\left(\tilde{A}\right) - \mathrm{Re}\left(\widetilde{H}\right)\right|^2 + \left|\mathrm{Im}\left(\tilde{A}\right) - \mathrm{Im}\left(\widetilde{H}\right)\right|^2 \tag{17}$$

In order to prevent a suboptimal identification due to convergence to a local minimum, a large number of randomly generated initial values for the design variables is used. From this, the set of design variables with the lowest error is selected as the best approximation.

To illustrate this identification procedure, figure 6 shows the result of approximating the added mass in x-direction of a cylinder at $H/R = 0$ for rigid body motion. This rigid body motion can be obtained by giving Craig-Bampton modes 1 and 4 the same excitation. The black graph is the numerical approximation of the exact solution as obtained by the source panel method and the colored graphs are approximations. It can be seen that with two added degrees of freedom, the system behavior is approximated nicely. Adding more degrees of freedom will improve the estimation, though not significantly.

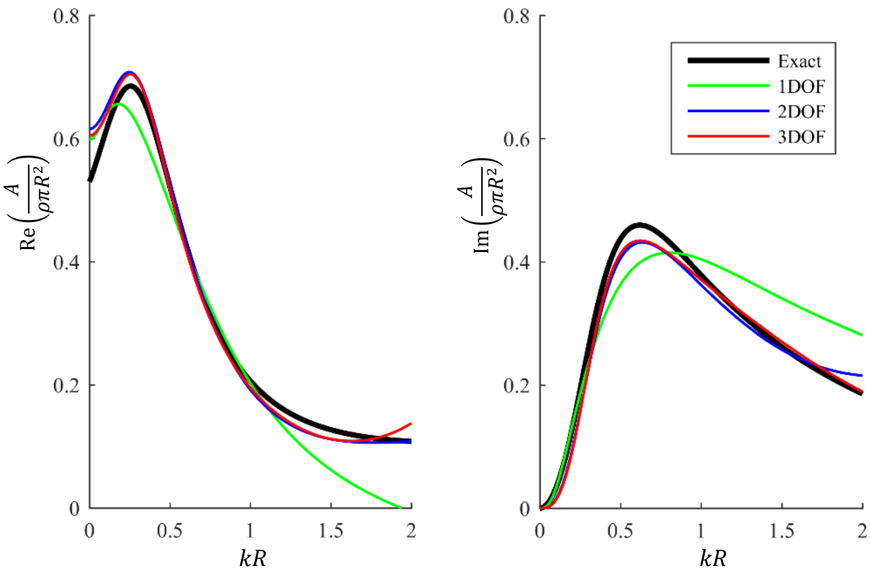

Figure 6: Added mass in horizontal direction from parameter estimation

After the identification process, it is known which equivalent mass-spring-damper system causes the same force on the submerged body as the water. With this knowledge, the constrained equations of motion of the hydro-elastically coupled flexible multibody system (3) can be modified: The generalized hydrodynamic forces are removed from $\mathbf{F}$ on the right hand side and the vector of generalized coordinates $\mathbf{q}$ is extended to include the coordinates of the equivalent mass-spring-damper systems. $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are extended accordingly. At this point, the response of a multibody system that contains a (partially) submerged body can be computed using solution procedures used in standard FMBD problems.

5 Results

For demonstration purposes, this strategy is applied on a rigidly moving cylinder with mass m at $H/R = 0$ that is constrained to move in x-direction only. The equation of motion is:

$$m\ddot{x} = F \tag{18}$$

From the identification procedure, it is known that the part of F due to hydrodynamic forces can be replaced by a 2DOF mass-spring-damper system. In this way, the equation of motion becomes:

$$\begin{bmatrix} m+m_0 & 0 & 0 \\ 0 & m_1 & 0 \\ 0 & 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} c_1 & -c_1 & 0 \\ -c_1 & c_1+c_2 & -c_2 \\ 0 & -c_2 & c_2 \end{bmatrix} \begin{Bmatrix} \dot{x} \\ \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} + \begin{bmatrix} k_1 & -k_1 & 0 \\ -k_1 & k_1+k_2 & -k_2 \\ 0 & -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} x \\ x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} F \\ 0 \\ 0 \end{Bmatrix} \tag{19}$$

This system of equations can be solved using standard techniques. The coefficients of the 2DOF mass-spring-damper system, as determined by the identification procedure, are:

$$\begin{aligned} & m_0 = 0.1153 \\ & m_1 = 0.5434, \quad k_1 = 0.2339, \quad c_1 = 0.9962 \\ & m_2 = 0.1877, \quad k_2 = 0.5522, \quad c_2 = 0.2453 \end{aligned} \tag{19}$$

Figure 7 shows the velocity response of the system when the primary body is subjected to a unit impulse excitation for a unit body mass $m = 1$. It can be seen that immediately after the initial excitation, the velocity of the primary body decreases and the velocity of the added bodies increases. With increasing time, all oscillations in the system damp out as the velocities of all bodies converge to the same constant value. This means that from this moment on, the all bodies in the system continue to move unboundedly in a rigid body mode.

The occurrence of a rigid body mode is explained by the fact that the physical systems used in the parameter identification do not connect the primary body to the fixed world, see again figure 5. From (16) it can be seen that $\widetilde{H}_1(0) = m_0 + m_1$: the total mass of the system, which is in accordance with this rigid body mode.

Note that this unit impulse response is only used here for explanatory purposes. The system is not allowed to move unboundedly, because this violates the small motion assumption made in the linear water wave theory. Moreover, in such a long range of motion, the structure's velocity would decay to zero due to drag forces. These drag forces are not accounted for by a source panel method based on linear water wave theory.

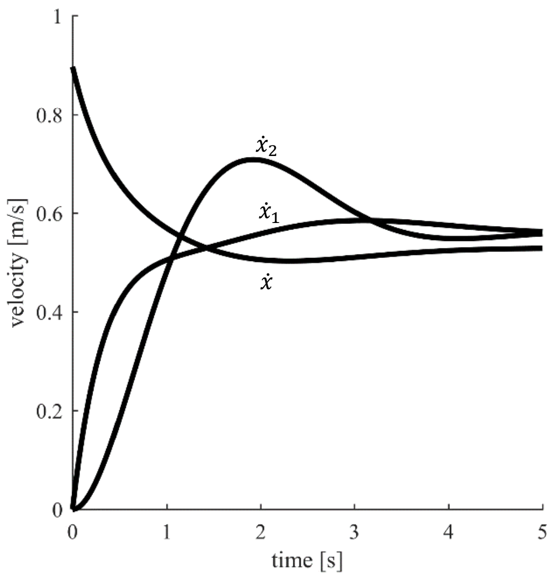

Figure 7: Unit impulse response of the equivalent mechanical system

6 Conclusions

The method presented in this work can be applied to include the hydro-elastic coupling between a ship's hull and water in a flexible multibody dynamics problem. Describing the hull's motion by Craig-Bampton modes is a convenient alternative to other elastic modes encountered in literature, because is directly related to the floating frame of reference method. In this way, the hydrodynamic forces on an arbitrary elastic body can be computed once its Craig-Bampton modes are determined using e.g. the finite element method. It is shown that the complex frequency added mass coefficients can be successfully approximated using a transfer function of an equivalent mass-spring-damper system. It is no longer necessary to interpret the results of a source panel method in terms of added mass, added damping, infinite added mass and retardation functions, as is common practice. Instead, the approximation obtained in this way can be given a clear physical interpretation, contributing to a better understanding of the overall dynamic system behavior.

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