# The splitting value: analyzing the value of buying in separate lots 

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#### Abstract

Summary We consider the situation in which a buyer has the option to buy a whole project or to buy a number of separate lots from different suppliers that together constitute the project. Buying a whole project reduces the coordination and management costs. First we show that buying a whole project (the bundle) might increase the project costs. Then we investigate this possible increase in project costs by analytical means. This analysis proves the increase in costs. Finally we visualize the size of the cost increase by simulation techniques. In the simulated cases, buying in separate lots turns out to save up to 10 percent of the total purchasing price. This leads to the conclusion that there is a positive effect associated with splitting into lots, we call it the splitting value.


## Keywords

Bundling, lots, splitting value, purchasing decisions

## Introduction

In many cases buying organizations have a choice whether to buy a project in one whole deal or to buy the project in a number of smaller packages, called lots. Consider the example of buying a new building as a whole from one main contractor or to contract your own architect, excavation, electrical installation, elevator, plumbing, landscaping etc. A contract for the project as a whole should reduce your efforts in selecting, contracting and managing the subcontractors, which might be worth something (or a great deal) to you. On the other hand if you contract for the whole project, you might expect your contractor not to do everything by his own company and to contract a number of specialized subcontractors. Now he has to select, contract and manage these subcontractors, for which he presumably will include a mark-up in the total contract amount. An additional issue is that a main contractor usually works with a fixed list of pre-selected subcontractors or uses mixed ways to select their subcontractors. So the main contractor A might select plumber X, whereas plumber Y may be better or less expensive. In other words the selection of subcontractors is limited. It is this last effect, limited subcontractor selection, which we study here.

In this paper we show that buying in separate lots instead of bundled packages creates a clear splitting value that results in lower contract costs. Current purchasing research, however, offers little guidance on this value. In this paper we (1) describe the splitting value of buying
in separate lots and (2) use analytical and simulation techniques to analyze the occurrence and the size of the value and the influence of number of lots and number of bidders. This research is just one building block of a more extensive research project that also includes empirical research of bundling and splitting decisions in various industries and analytical investigations of the cost of splitting.

## The splitting effect

Assume a buyer is searching for the lowest price, so per lot he will select the bidder with the lowest price. If he buys in separate lots he will search for the lowest price on each individual lot. The total price he pays will be the sum of these individual lowest prices. When buying the combination of these lots in one package (a bundle), he will compare prices per bundle and search for the lowest bundle price. This might lead to different results.

In an example we show how. Suppose a buyer has to buy 3 lots of works on a building, namely plumbing, carpenting and painting. Each of the lots is offered at prices in a certain range. When the buyer chooses to buy the lots separately, the suppliers will place a bid for each separate lot. The buyer first selects the least expensive supplier for each lot and then combines the winners of both lots. When the buyer chooses to buy the lots in one bundle, the suppliers have to form a consortium and combine their separate bids in one combined bid for the bundle. We assume that consortia are formed in a random way. So the resulting consortium is a random combination of suppliers for the various lots. After the consortium bids are in, the buyer selects the least expensive combination. First, we present this example in table I where suppliers are asked to bid for each lot.

| Supplier | Bids for <br> plumbing | Supplier | Bids for <br> carpenting | Supplier | Bids for <br> painting |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 50 | 7 | 45 | 10 | 105 |
| 2 | 48 | 8 | 39 | 11 | 110 |
| 3 | 47.5 | 9 | 44 | 12 | 95 |
| 4 | 51 |  |  | 13 | 91 |
| 5 | 50.5 |  | 14 | 99 |  |
| 6 | 49.5 |  |  |  |  |
|  |  |  |  |  |  |


| Winning <br> supplier | Best bid for <br> plumbing | Winning <br> supplier | Best bid for <br> carpenting | Winning <br> supplier | Best bid for <br> painting | Total price <br> for the job |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | 47.5 | 8 | 39 | 13 | 91 | 177.5 |

Table I: an example for buying in separate lots
We see in table I that the combination of the best bids on the separate lots will be the sum of the bids of supplier 3, 8 and 13. In the situation that the buyer chooses to buy in one bundle, the suppliers form consortia. The consortia can be formed in many ways. The classical way would be for a lead bidder to select other consortium members based on lowest price. But then it is not clear who acts as a lead bidder in this case, it could either be the plumber, carpenter or the painter. Sometimes the consortium is formed by suppliers who have formed a long lasting relationship. But it could also be formed ad hoc, to minimize the effort of forming the consortium. The mixture of various ways to form the consortium can lead to numerous possible consortia formats. In this light it seems reasonable to assume that consortia are
formed randomly in this example. In table II we expand the example with bids from 4 consortia.

| Consortium | Consortium <br> members | Bids for the bundle, <br> sum of all bids | Winning <br> consortium | Best consortium <br> bid |
| :---: | :---: | :---: | :---: | :---: |
| A | $1,7,10$ | 200 | B | 185 |
| B | $4,8,12$ | 185 |  |  |
| C | $3,9,11$ | 201.5 |  |  |
| D | $6,8,14$ | 187.5 |  |  |
|  |  |  |  |  |

Table II: an example of buying in a bundle
From table II we see that the minimum bid for the bundle (minimum of the sum) exceeds the sum of the minimum bids for the separate lots by $185-77.5=7.5$, some 4.1 percent of the total offer. This shows that the bundled bids could be more expensive than separate bids. In the sequel of this paper we will prove that the total contract price for buying in separate lots is always (for any number of lots and any number of bidders) at most equal to the best contract price for the bundle. For the remainder of this paper we refer to the difference between the combined best prices of the separate lots and the best price of the bundle as the 'splitting value'.

## Literature review

The importance of transaction costs when buying has been stressed by Williamson (1979). In transactions the asymmetry of information plays an important role in the extent the buyer can extract the maximum benefits from the trade (Adams and Yellen, 1976). In the case of buying a bundle, the buyer does not know the exact valuation of the separate lots. The buyer only knows the combined bids for the bundle. Because of this lack of information he does not have the information to form the optimal combination of bids for the bundle, and realise all possible gain from the trade. When buying in separate lots the buyer can obtain the information about the lowest bids for each lot, and therefore the information about the optimal combination of bids. However, transaction economics literature does not further investigate what the specific consequences of buying in bundles or separate lots could be.

Another relevant research area in this topic is the bidding process itself. An overview of bidding models and auction theory is given by Klemperer (1999) and Rothkopf and Harstad (1994), who present an extensive list of auctions, bidding structures and underlying theories. However, the concept of the splitting value from a purchasing perspective seems to be forgotten in this research area.

In purchasing literature, authors do describe effects of bundling purchasing volumes. The fact is mentioned that bundling leads to economies of scale due to increased volume and lower administrative costs (Looman et al., 2002). Also increased buying power is mentioned (Ramsey, 2001) as well as the fact that, bundling can lead to poor purchasing performance when the process of creating the bundle is poor (Mabert and Schoenherr, 2001). The study of Beall et al. (2003) confirmed this significance and described two commonly used bundling strategies, market basket bundling and individual bundling. Schoenherr and Mabert (2006, 2007) also mention the existence of degrees of freedom in bidding requirements. The bundle composition can be generated by the buyer, encouraged by the buyer or generated by the supplier. Although all of these contributions stress the importance for the determination of the
appropriate bundle and describe the environment in which a bundle is to be formed, nothing like the splitting value has been mentioned so far. Furthermore the studies mentioned are of a descriptive and conceptual nature and no attempts have been made to estimate or determine the size of the effects by analytical means. This lack of interest could be the result of irrelevance of the topic to buying practitioners. However, in marketing literature, the strategic importance of bundling has been proven (e.g. Stremersch and Tellis, 2002). Also the optimality of bundling from the seller's point of view has been studied by many (e.g. Bakos et al., 1999, 2000, or Eppen et al., 1991). Therefore, if bundling and specially the optimality of bundling are considered to be important from the sellers' perspective, it should be analyzed from the buyers' perspective as well.

This study contributes to the limited body of research on bundling strategies from a buyer's perspective. Adding to recent qualitative and descriptive research on the effects of bundling strategies in buying situations, we focus on the splitting value in buying bundles in an analytical way.

## Analytical formulation

In the numerical example we saw a positive splitting value when a buyer selects the lowest bid from each individual lot instead of the lowest bid for the combined lots. If we take $P$ as the price we pay for all $n$ lots, and $b_{i, j}$ as the bid of supplier $j$ on lot $i$, we can formulate the two different formulas for the price paid when buying in separate lots or in one bundle ( $P_{\text {bundle }}$ and $P_{\text {split }}$. Buying in separate lots means that we take the sum of the lowest bids on each lot:

$$
\begin{equation*}
P_{\text {split }}=\sum_{i=1}^{n} \min _{j}\left(b_{i, j}\right) \tag{1}
\end{equation*}
$$

When buying the whole deal at once, we take the lowest sum of the bids of one supplier:

$$
\begin{equation*}
P_{\text {bundle }}=\min _{j} \sum_{i=1}^{n}\left(b_{i, j}\right) \tag{2}
\end{equation*}
$$

The existence of the splitting value can be described as the fact that the $P_{\text {bundle }}$ is always the same size or larger than $P_{\text {split }}$. This is rather straightforward to prove. For each lot $i$, every arbitrary bid from all $j$ bids (one from each supplier) is always equal or larger than the lowest bid for that lot:
$b_{i, j} \geq \min _{j}\left(b_{i, j}\right)$
Since (3) holds for all $i$ every arbitrary combination of bids for lot $i$ is also equal to or larger than the summation of all lowest bids for all lots $i$ :

$$
\begin{equation*}
\sum_{i=1}^{n}\left(b_{i, j}\right) \geq \sum_{i=1}^{n} \min _{j}\left(b_{i, j}\right) \tag{4}
\end{equation*}
$$

That $j$ for which the left handside of (4) is smallest, is still larger or equal to the summation of the lowest bids.

$$
\begin{equation*}
\min _{j} \sum_{i=1}^{n}\left(b_{i, j}\right) \geq \sum_{i=1}^{n} \min _{j}\left(b_{i, j}\right) \tag{5}
\end{equation*}
$$

And so we see that $P_{\text {bundle }}$ is always the same size or larger than $P_{\text {split }}$.

$$
\begin{equation*}
P_{\text {bundle }} \geq P_{\text {split }} \tag{6}
\end{equation*}
$$

In the next section we analyze the size of the splitting value $P_{\text {bundle }}-P_{\text {split }}$.

## Formal analysis

For further analysis, and for the sake of simplicity, we assume that all suppliers place an independent bid. So we assume there is no collusion on the market. Also we assume that the suppliers place a bid from the same probability function $f(x)$ with cumulative function $F(x)$. Each bid on a lot is a random pick from the distribution. This means, for example, that if the bids are uniformly distributed between 90 and 110, the suppliers will place bids randomly between these boundaries. Logically we are looking in this case for the lowest bid placed for a particular lot. This is the field of order statistics. The expected value of the lowest bid ( $X_{m i n}$ ) with $n$ bidders can be described as:

$$
\begin{equation*}
E\left[X_{\min }\right]=\int_{0}^{\infty}(1-F(x))^{n} d x \tag{7}
\end{equation*}
$$

The calculation of this expected value is not very straightforward and requires some simplification. To clarify our presentation we make some assumptions. We assume that there are two similar lots, named $l$ and 2 , of the same size, the same goods or services. Also the bids of the suppliers on these two lots are randomly placed out of a $[0,1]$ uniform distribution. The distribution functions for these two lots are then:

$$
f_{1}(x)=f_{2}(x)= \begin{cases}1 & \text { if } 0 \leq x \leq 1  \tag{8}\\ 0 & \text { otherwise }\end{cases}
$$

The sum of the lowest bids for two lots ( $P_{\text {split }}$ ) would make (with $n$ the number of bidders):

$$
\begin{equation*}
P_{\text {split }}=2 \cdot E\left[X_{\min }\right]=2 \cdot \frac{1}{n+1}=\frac{2}{n+1} \tag{9}
\end{equation*}
$$

If we want to know what the distribution of combined bids is we need the sum of the two uniform distributions. The combined distribution function $f_{c}(z)$ can be described as:

$$
\begin{equation*}
f_{c}(z)=\int_{-\infty}^{+\infty} f_{1}(z-y) f_{2}(y) d y \tag{10}
\end{equation*}
$$

$f_{2}(y)=1$ if $y$ is between 0 and $1:$
$f_{c}(z)=\int_{0}^{1} f_{1}(z-y) d y$
With some steps omitted:

$$
f_{c}(z)=\left\{\begin{array}{cl}
z & \text { if } 0 \leq z \leq 1  \tag{12}\\
2-z & \text { if } 1 \prec z \leq 2 \\
0 & \text { otherwise }
\end{array}\right.
$$

The cumulative function is then:

$$
F_{c}(z)=\left\{\begin{array}{cl}
1 / 2 z^{2} & \text { if } 0 \leq z \leq 1  \tag{13}\\
2 z-1 / 2 z^{2}-1 & \text { if } 1 \prec z \leq 2 \\
1 & \text { otherwise }
\end{array}\right.
$$

If we combine that with the formula for the expected bid (7), we get the lowest expected bid out of the sum of two distributions ( $E\left[X_{\text {min, sum }}\right]$ ), which equals $P_{\text {bundle }}$ in this situation::

$$
\begin{equation*}
P_{\text {bundle }}=E\left[X_{\text {min, sun }}\right]=\int_{0}^{\infty}\left(1-F_{c}(z)\right)^{n} d z=\int_{0}^{1}\left(1-1 / 2 z^{2}\right)^{n} d z+\int_{1}^{2}\left(1-\left(2 z-1 / 2 z^{2}-1\right)\right)^{n} d z(1 \tag{14}
\end{equation*}
$$

This lowest expected bid $E\left[X_{\text {min ,sum }}\right]$ is the $P_{\text {bundle }}$ which can be compared with the outcome of $P_{\text {split }}$ calculated earlier (9). However, the sophisticated integrals in combination with the variable $n$ in the formula for $P_{\text {bundle }}$ (14) are difficult to solve. Even in this case where for the sake of simplicity some assumptions have been made, it shows that a pure analytical solution would take too much effort to be of practical relevance. This also justifies the next step: the use of simulations to analyze the splitting value further. In the next section we continue the paper with a simulation approach.

## Simulation

To analyze the splitting value when buying in lots or bundles we conducted simulations of purchasing situations. In these simulations we focused on the difference between the lowest separate bids on lots and bids on bundles of lots. First we have to decide what variables are important to investigate. In the analytical formulas (see (1) and (2)) we see three variables which can influence the outcome of this splitting value:

- Number of bidders $(j)$ : variation in number of suppliers competing for the lot
- Bids placed (i): variation in bid distribution and/or bid spread
- Number of lots ( $n$ ): variation in the number of lots

We realize in practice more variables may be of influence. But for the sake of simplicity we choose to focus on the variables provided by the analytical analysis. When we look into the
number of bidders we expect that with a higher value of this variable, lower bids will be placed. Logically with a high number of bidders there is a greater probability of a low bid, than with a small number of bidders. First we simulate $P_{\text {bundle }}$ and $P_{\text {split }}$ for situations with two lots and with 1 to 20 suppliers, who bid according to a uniform distribution of [90,110], hence place a bid between 90 and 110 euros. The results are presented in figure I.

## [Insert figure I here]

As we can see in figure I the splitting value adds to a lower price paid when buying in two separate lots. The space between $P_{\text {bundle }}$ en $P_{\text {split }}$ is determined as the difference between the two, and a proof of the existence of the splitting value. This value reaches its top around 6 or 7 bidders, when the gap is not widening any more. Although we conclude that a healthy number of suppliers is certainly necessary for the value to be present, adding more suppliers will not add to the value.

To investigate the effect of difference in bids, we will vary the bidspreads to see what impact they have on the splitting value. We will vary the intervals of the uniform distribution in the next simulation. We used intervals of [80, 120], [90, 110] and [95, 105]. In this way we can also vary the bid spread in the bids with 40,20 and 10 percent respectively. This time we visualize the splitting value as a percentage of savings when buying in separate lots, compared to buying in a bundle. The results are presented in figure II.

## [Insert figure II here]

The splitting value depends on the order of first selecting the best suppliers for the lot and then summing the different lots versus vice versa. It is clear that when the value is based on an earlier selection of the best bids, the bidspread has a large impact on the size of the splitting value. In other words; if the prices of the best bids differ more, it pays off to select your suppliers earlier, before summing the lots. In this perspective the bidspread counts as a sort of multiplier for the splitting value. With the number of bidders and bidspread investigated, we still have to investigate the impact of the splitting itself in the splitting value, e.g. the variation of the number of lots. In the next simulation we vary the number of lots between 1 and 20, and use the same intervals [80, 120], [90, 110] and $[95,105]$ to create various bidspreads. The results are presented in figure III.

## [Insert figure III here]

As we see in the figure the splitting value reaches savings as high as ten percent with a bidspread of 40 percent, and more than 12 lots. Again we see that the savings are clearly generated by the effect of splitting in lots. Another observation is the effect of adding (or splitting into) another lot is generating additional savings in a decreasing manner. Eventually adding a lot will not generate additional savings compared to buying in a bundle. An interesting phenomenon might be the marginal splitting value (MSV). This MSV could give insight in the decision, whether to add (or split into) another lot or not.

To summarize these simulations have shown that in a market with a healthy number of suppliers, the splitting value exists where the addition to the value decreases with the number of lots. The spread in the bids works as a multiplier in the splitting value. To make the value even more visible for practitioners a tool has been created to calculate the splitting value for
every instance in number of lots, suppliers and bidspread. The tool can be downloaded at www.utips.eu.

## Conclusions

In this paper we have analyzed the topic of bundling compositions and the effects of bundling from a buyer's perspective. This study describes a new effect in bidding on volumes, splitted in lots, called the splitting value. Not only did we describe a case to visualize the value, we also conducted formal analysis and simulations to investigate this value. In both formal and simulation studies we proved that the splitting value exists and is substantial.

An analytical calculation of the splitting value seems to be impractical, because it would take too much effort to be of practical relevance. Instead we chose to use simulations which showed us that the splitting value is dependent on a healthy amount of competition in the bidding process. But adding more bidders in the process after reaching this healthy amount, in the simulation around six, adds little to the splitting value. Further simulations revealed that the bidspread acts as a multiplier for the splitting value. The higher the bidspread, the higher the splitting value will be. Finally we investigated the effect of the number of lots on the splitting value. Adding a lot to the bundle will generate additional splitting value but decreasingly so. We conclude that the existence of a marginal splitting value (MSV) could be of importance for bundling composition decision making.

Of course the splitting value is not the only effect which occurs when buying in bundles or not. Other effects, which have now been left out of scope, could have opposite effects on the savings, generated by the splitting value. But with savings up to ten percent in comparison with bundled buying the splitting value has shown its potential for further research.

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Figures


Figure I: price paid when buying two lots bundled or splitted with various numbers of bidders


Figure II: savings compared to combined bids by various bidspreads and number of bidders and two lots


Figure III: savings compared to combined bids by various bidspreads and number of lots and five bidders

