1972. Wave climate at Gold Coast, Qld, 'iv. Tech. Papers, 13(5):1-17.

1992. Frequency dependent cross-shore A barred shoreface. Marine Geol., 106.

Model for the formation of longshore f. J. Geophys. Res. (to be submitted)

The siting of beach nourishment place- 7-24.

Global climate change and the rising e Coastal Zone Management Subgroup, ks and Water Management, Directorate herlands.

Morphodynamic modelling of shoreface bars

Suzanne J.M.H. Hulscher *

Abstract

In this paper the behaviour of near-shore large-scale bars is studied using a morphological model. The model describes the interaction between the sandy sea bed and propagating weakly-dispersive, weakly-nonlinear free low-frequency waves, which are incident at the offshore boundary. A linear stability analysis is performed to study the formation and migration of bed patterns. The fastest growing sea-bed wave in this model has a wavelength close to the recurrence length; it moves slowly shorewards. These results are compared with the ridges on the North American, Atlantic shelf and show good agreement. The model shows that the bars grow due to wave-skewness differences between uphill and downhill moving surface waves and the bars migrate due to differences in the shape (peakedness and skewness) between the surface waves straight above the top and the ones straight above the trough. The bar pattern in the present analytical model agrees with the results of related numerical studies; however, in the present study the bars migrate, whereas this was impossible in the numerical studies because the boundary conditions were fixed.

Introduction

The sandy bed of many shelf seas shows a variety of submarine rhythmic sea-bed patterns. This paper focuses on a specific model for shoreface bars, a pattern having multiple ridges whose crests are oriented (nearly) parallel to the shoreline.

The fields of sand ridges in the North Atlantic shelf [*Swift et al., 1972a*] occur at a typical mean depth ($H \sim 15$ m) on a very gentle mean slope (order 0.0005) towards the coast. The mean crest line spacing in all these systems shows a variation between 1.4 km and 6.1 km. The mean ridge length is typically 10 times the ridge spacing. The height of these ridges is of the order of 4 m.

*University of Twente, Civil Engineering & Management, P.O. box 217, 7500 AE Enschede, The Netherlands.

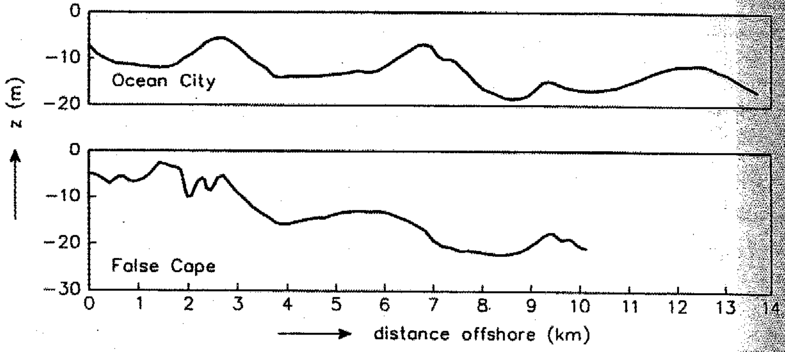

Figure 1: *Cross-section of shore-face ridge systems of False Cape, near the Virginia inner shelf region, and Ocean City area. After Duane et al. [1972]*

On the North Atlantic shelf tidal currents are medium to low. At these locations far more energy is supplied by wind waves, storm currents and the associated wave trains [*Duane et al., 1972, Swift et al, 1972b*], which are expected to play a leading role in the morphodynamical processes. Two specific fields of shoreface ridges on the North Atlantic shelf are shortly discussed now: those located near False Cape and Ocean City; representative cross-sections are presented in figure 1.

Comparisons of the depth surveys in the False Cape area [*Swift et al., 1972a*] of 1922 and 1969 indicate landward migration of 140 m for the offshore ridges system, although the authors themselves have serious doubts about the accuracy of the earliest observation. Given an average ridge spacing of 5 km, it will take 1.5 millennia to move up one complete bar. The bars near Ocean City have shown an offshore migration of 150-450 m over 80 years [*Duane et al., 1972*], indicating a period of 1 to 3 millennia for the migration of one complete bar offshore.

The numerical nonlinear morphological model of *Boczar-Karakiewicz & Bona* [1986] simulates the nonlinear evolution of multiple bar systems due to the harmonic wave-wave interaction. For low-frequency waves this model simulates successfully the bottom topography near False Cape and Ocean City. This shows that the low-frequency waves are indeed capable of forming such ridge systems, but the formation mechanism is still not understood in detail. Yet, there are some points of further discussion:

- The model does not predict bar migration, which does occur in nature
- The influence of initial and boundary conditions on the model results is unclear
- It is not clear how the bed-wave interaction process works which underlies the formation and migration of the bed waves

T(s)	H(m)	A(m)	$\lambda_w(m)$	ϵ	μ
47	12.5	0.15	518	0.02	0.15
80	15	0.15	969	0.01	0.097
80	20	0.2	$1.10\,10^3$	0.01	0.11

Table 1: *Model parameters for free low-frequency waves at various depths, for the North-Atlantic, North American shelf.*

The model here is studied using linear stability analysis, which has been found successful in the study of bar formation due to tide-topography interactions [*Hulscher*, 1996]. Such an analytical technique yields the initial behaviour (growth and migration) of small-amplitude bed forms. In this way the initially fastest-growing bottom pattern is found. As this investigation method explicitly yields the initial feedback between water waves due to the bottom perturbation, it will contribute to a better understanding of the underlying bar processes.

Model description

The morphological model, derived in [*Hulscher*, 1996], describes the interaction between the sandy sea bed and propagating weakly-dispersive, weakly-nonlinear free low-frequency waves, which are incident at the offshore boundary. The model consist out of

- The Boussinesq flow model in one horizontal co-ordinate x, including weak friction and weakly forcing of low-frequency waves due to the modulating character of wind waves.
- Bedload sediment transport model. The sand is transported due to the near bed streaming of the nonlinear low-frequency waves. The transport is stirred by wind waves, a slope effect in included.
- Conservation of sand on the model domain

By using the Boussinesq model is required that the dispersion, measured by $\mu = k^*H$, and the nonlinear effects of the free surface elevation, measured by $\epsilon = A/H$ are such that $\epsilon = \mathcal{O}(\mu^2)$. Herein k^* is the wave number of the low-frequency waves, A is the amplitude of the surface waves. Table 1 shows examples of the values of the small parameters ϵ and μ.

In this study the reference values ϵ=0.01, μ=0.1 and H = 15 m are chosen. As this study focuses on a length scale larger that the wavelength of the low-frequency waves, a new spatial variable χ for the morphological length scale is introduced, the ratio between these scales is chosen here as ϵ_L=0.01. Furthermore a friction parameter r is involved, the value of the Jonsson friction coefficient f_w = 0.24 leads to the value r = 1.0. Sediment transport values are taken as usual, see *Hulscher* [1996].

Averaging over the period of the low frequency waves leads here to the morphological time scale yields

$$T_m = 930 \pm 10 \ \ \text{yr}.$$

Stability Analysis

This paper examines whether the presence of near-shore multiple bar systems (crests parallel to the shoreline) can be associated with instabilities in the morphological system. Due to the system the nondimensional velocity of the low-frequency waves is described by its first and second harmonic component as follows:

$$u(x,\chi,t) = C_{\beta 1}\beta_1(\chi)\cos\varphi + C_{\beta 2}\left[-B_s(\chi)\sin(2\varphi) + B_c(\chi)\cos(2\varphi)\right], \quad (1)$$

in which β_1 gives the rescaled amplitude of the first harmonic component and B_s, B_c the 90° shifted and the in phase component of the second harmonic component of the water wave. The constants $C_{\beta 1,\beta 2}$ are scaling constants which are necessary to express the mathematical problem in terms of equal order. The phases are expressed in

$$\varphi(x,\chi,t) \ = \ kx - t + \phi(\chi), \quad (2)$$

in which ϕ is the (real, not continuous) phase of the first harmonic component. These surface waves are asymmetric in two ways: firstly, they can have more peaked tops than troughs (or vice versa), and, secondly, they have slightly steeper fronts than backs (or vice versa). The first effect depends on the ratio $\breve{B}_c/\breve{\beta}_1$ and is indicated as peakedness; larger positive values mean more peaked wave tops than troughs. The second effect depends on the ratio $-\breve{B}_s/\breve{\beta}_1$ and in this paper is indicated as skewness. The skewness increases for larger differences in the steepness of the front and back slopes of the water waves.

The basic state The stability analysis can be carried out by first defining a basic state which describes water waves over a sea floor without a rhythmic topography. The flow is characterised by $\psi = (\beta_1, B_s, B_c)$, the basic flow is indicated as $\breve{\psi}$. Here the basic flow corresponds to an asymmetrical (positive peaked and positive skewed), permanent surface wave which propagates over a flat sea bed, see for details *Hulscher* [1996].

Bed perturbations The next step is to introduce sine-like bed perturbations h with arbitrary wavenumbers κ in the horizontal direction χ, shorter than the wavenumber k of the incoming water waves. In the case of a small bed perturbation the solution is expanded as follows

$$(\psi, h) = (\breve{\psi}, 0) + \int \left(\tilde{\psi}, \tilde{h}\right) e^{-i\kappa\chi} \mathrm{d}\kappa + c.c., \quad (3)$$

which means that harmonic boundary conditions have been taken. This type of boundary condition has the advantage that the free behaviour in the system can be investigated.

Growth rate Subsequently, one studies the initial interaction between bed forms and waves and determines the growth rate ω which shows whether the bed perturbations are amplified or reduced. A basic state is said to be stable if arbitrary bed form disturbances with nonzero wavenumbers decay exponentially in time. This implies that all bed perturbations κ have negative growth rates. Conversely, if there is at least one bed perturbation κ with a positive growth rate ω, the basic state is unstable. After some algebra, see *Hulscher* [1996] the growth rate ω can be written as

$$\omega(\kappa) \ = \ i\kappa \frac{(i * D_1\kappa + D_2)}{(\lambda_1 + i\kappa)(\lambda_2 + i\kappa)(\lambda_3 + i\kappa)} - \kappa^2\hat{\lambda}, \quad (4)$$

in which D_i are real coefficients. The λ_i are eigenvalues of the hydrodynamic-stability problem; λ_1 is real and negative, the remaining two are complex conjugates $\lambda_2 = \bar{\lambda}_3$. These eigenvalues are connected with the so-called reccurence length $\kappa = |Im(\lambda_2)|$.

The first term on the right-hand side of (4) describes the complex growth (amplification and migration) of the sea-bed perturbation κ due to the feedback given by the fluid. The last term of (4) is always real and negative; it decreases quadratically in κ. This term hinders the development of small-wavenumber bed patterns.

Results

Growth rate of bed perturbations The resulting real and imaginary parts of the growth rate $\omega(\kappa)$ as a function of the wavenumber κ are shown in figure 2.

Due to the scaling and rescaling as shown in *Hulscher* [1996] the bed mode κ is related to the dimensional sea-bed wavelength λ_m^* following:

$$\lambda_m^* = \frac{\lambda_w}{2\pi\kappa S_{12}\sqrt{2S_{22}\epsilon}} = \frac{H}{\mu\kappa S_{12}\sqrt{2S_{22}\epsilon}}, \quad (5)$$

herein, the factor $S_{12}\sqrt{2S_{22}} \simeq 2.7$ [*Hulscher*, 1996].

The imaginary part of the growth rate is related to the migration of a particular bed perturbation κ. The imaginary part of the growth rate $Im(\omega)$ denotes the number of bed waves which pass over on a certain position in the morphological time scale. This means that the sea-bed wave migration period yields $T_{migr} = T_m/Im(\omega)$.

In order to discuss the results dimensionally, the reference values as discussed in section model description have been chosen. The dimensional bed wavelength

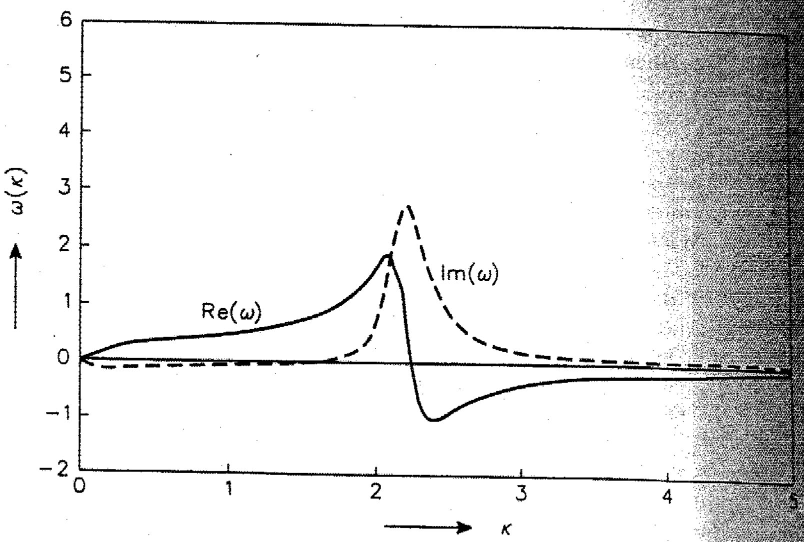

Figure 2: *Growth rate Re(ω) (solid) and Im(ω) (dashed) as a function of the wavenumber κ. Parameters are μ=0.10, ϵ = ϵ$_L$ = 0.01 and r=1.0. For more information, see text.*

then becomes $\lambda_m = 5.5\,\mathrm{km}/\kappa$. This means that sea-bed patterns which have wavelengths down to 2.4 km ($\kappa \geq 2.25$) initially grow. The most unstable mode has a wavelength of $\lambda_m = 2.6\,\mathrm{km}$ ($\kappa \simeq 2.1$). The growth rate factor is related to the morphological time scale, which means that the typical time scale for morphological changes of the fastest growing bed forms is 520 years, and even longer for larger bed forms.

Bed waves which have crest-spacing of 3.3 km or longer ($\kappa < 1.65$) show small negative values, $-0.2 \leq Im(\omega) \leq 0$, which corresponds to a migration in the seaward direction of one bed wave in a period of at least 4.5 millennia. An initially shoreward movement is found for sea-bed waves shorter than 3.3 km ($\kappa \geq 1.65$). However, note that wavelengths shorter than 2.4 km are not amplified but decay exponentially in time. The most unstable mode gives a positive value for the imaginary part, $Im(\omega) = 1.4$, which means a shoreward migration period of 670 years. Note that the mode having the smallest migration period is marginally stable: it neither grows nor decays. Note also that the fastest growing mode migrates shoreward.

The spatial and temporal characteristics of the bed forms in this model are comparable to the ridge systems on the North-American North-Atlantic shelf, as reported in *Swift et al.* [1972a]. The model shows slow migration of the bed features in either direction, which is in agreement with the observations. However, the fastest growing mode always migrates shorewards, one order of magnitude faster than the estimates derived from the observations. Since these estimates themselves give rise to doubts, this aspect deserves further study.

Flow-topography feedback Here the shape of the surface waves is investigated, under conditions which lead to growth and migration of the perturbed bottom.

The sea-bed perturbation $\epsilon\hat{h}\cos(\kappa\chi)$ leads to spatially periodic changes in the ratio of the harmonic components, given by the complex values of $\tilde{\beta}_1, \tilde{B}_s, \tilde{B}_c$. As a function of the bed wavenumber κ figure 3 shows the real and the imaginary part of the perturbed harmonic components $\tilde{\beta}_1/\epsilon, \tilde{B}_s/\epsilon, \tilde{B}_c/\epsilon$. The rescaled form is chosen here as an order ϵ change in topography, it balances order 1 changes in the harmonic components, which subsequently leads to order ϵ changes in the residual velocity near the sea bed. The perturbations in the basic shape of the surface waves which are caused by the bed-topography perturbation are thus known exactly here.

Figures 2 and 3 imply that growing modes correspond to $Im(\tilde{B}_s) > 0$, whereas decaying modes show $Im(\tilde{B}_s) < 0$. Physically, $Im(\tilde{B}_s)$ corresponds to spatial differences in surface-wave skewness at the sides of the bar, such that $Im(\tilde{B}_s) > 0$ gives steeper fronts for uphill moving waves than for the front which move downhill. So in this model a growing bar has steeper surface wave-fronts on the seaside than on the shore side. If these surface-wave differences produces bars in the real world will, this will indicate whether the presently studied mechanism works and is therefore an interesting topic to investigate.

From the previous considerations it is clear that the real part of the flow perturbations is generated indirectly due to friction. Figures 2 and 3 imply that in the case of seaward migration $Re(\tilde{\beta}_1) < 0$ and $Re(\tilde{B}_c) > 0$, which means more peaked waves over the crests than in the troughs. In the case of shoreward migration of the bars, this effect is dominated by $Re(\tilde{B}_s) > 0$, which means steeper fronts at the crests than in the troughs. In this model therefore the migration orientation of the bars is determined by differences in surface-wave peakedness (seaward) or skewness (shoreward) between top and trough. Again it still has to be verified, whether these results agree with morphological behaviour in reality.

Comparison of results with related models In order to compare the results with those of the numerical evolution model of *Boczar-Karakiewicz & Bona* [1986] special choices have to be made. The model in the present paper is limited to small bed amplitudes and thus it is only comparable to the earliest stage of the evolution of the finite amplitude bars in the numerical model. Furthermore, the small parameters in the present model $\epsilon, \epsilon_L, \mu$ have to be used differently, in order to create similar circumstances here, ϵ in equation (5) has to be replaced by $\mu/(2\pi)$ [*Hulscher*, 1996]. Under these conditions equation (5) gives the following

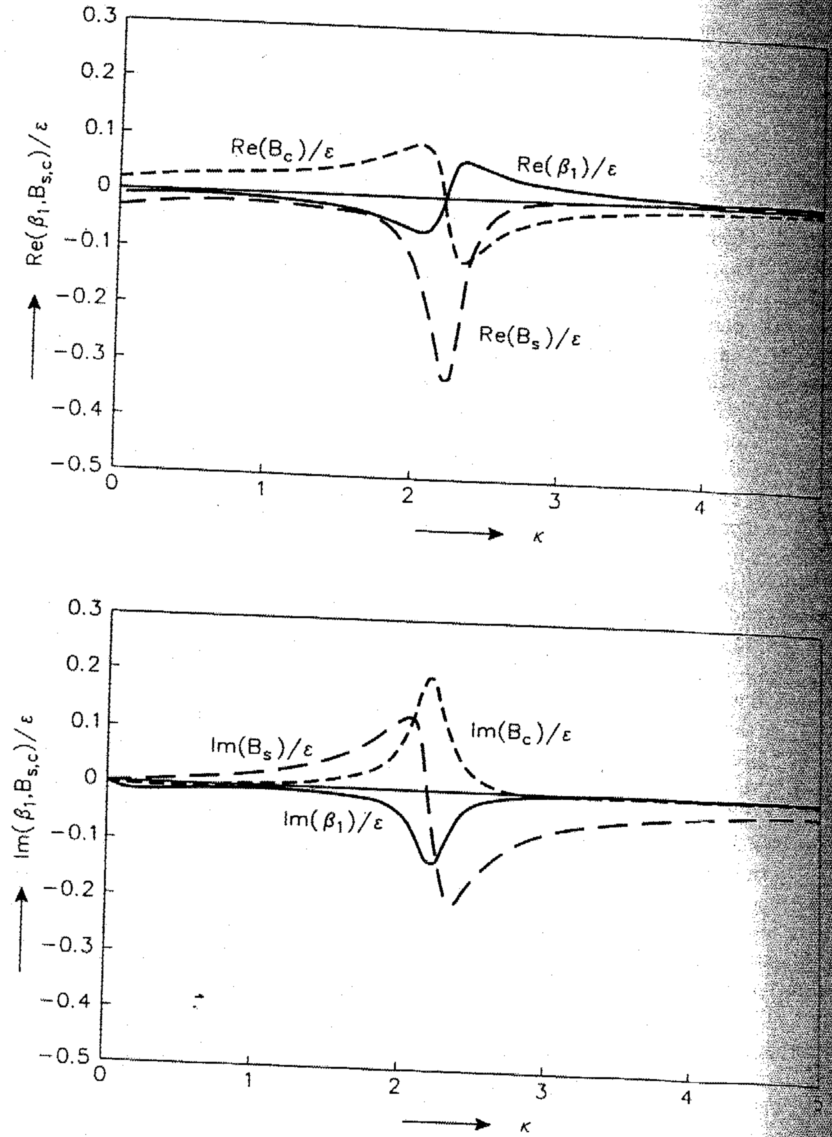

Figure 3: *The (a) real and (b) imaginary part of the wave components* $\tilde{\beta}_1/\epsilon, \tilde{B}_s/\epsilon, \tilde{B}_c/\epsilon$ *as function of the bed wave number* κ. *For parameters see text.*

formula to estimate the dimensional wavelength of the most unstable bar κ_f beyond critical conditions

$$\lambda_m^* \simeq \frac{\lambda_w}{\kappa_f S_{12}\sqrt{2S_{22}}\mu} = \frac{2\pi H}{\kappa_f S_{12}\sqrt{2S_{22}}\mu^2}. \qquad (6)$$

From the previous results it is estimated that $\kappa_f \simeq 2.1 \pm 0.1$.

Boczar-Karakiewicz & Bona [1986] estimated the relative wave amplitudes being $\epsilon = 0.05$; the typical wavelength of the incoming low-frequency waves at False Cape and Ocean City are λ_w^*=2.5 km; which mean periods of some T= 80 s. Substitution of these values into equation (6), taking realistic depth-values, that is between H=30 m and H=15 m, yields bars with wavelengths between 1.7 and 1.3 km. This agrees with the model results as presented in *Boczar-Karakiewicz & Bona* [1986], in which the recurrence length of the surface waves is of the order of magnitude of the bar spacing.

However, some critical remarks are called for here. The previous derivation shows the sensitivity of the bar spacing to the wavelength of the incoming waves: for every sea-bed wavelength a suitable low-frequency wave can be selected which is able to form it. A model result which is so dependent on the (variable) extrinsic conditions is not satisfactory.

A second comment concerns the boundary conditions. The present analysis has shown that migration of bars, either seawards or shorewards, occurs naturally in the present morphological model. As the sea bed at the seaward side in the *Boczar-Karakiewics*-model is fixed, the migration of the bed features is simply not able to occur. As the migration and the growth of sea-bed features are both due to divergences in the sediment transport and are therefore closely connected processes, a physically realistic simulation of bars requires that movement is permitted.

Discussion/Conclusion

This study confirms the conclusion of *Boczar-Karakiewicz & Bona* [1986] that the ridge systems on the South Eastern North American Atlantic Shelf [*Swift et al.*, 1972a] may be formed by weakly dispersive and weakly nonlinear low-frequency waves. The most preferred bed wavelength equals the recurrence length of the surface wave amplitudes. The present study quantifies the strong relation between the sea-bed wavelength and the wavelength of the incoming surface waves. The present analysis shows that both onshore as offshore movement of bars under conditions of an overall onshore oriented sediment transport are intrinsic processes in this system. Because the solution technique is analytical, the model enables detailed investigations to be made in order to discover the underlying processes.

According to this model the topographically induced growth of sand bars is connected to wave-skewness differences at the sides of the bar, in such a way that the propagating surface waves show steeper fronts when going uphill than when going downhill. Bar migration is connected to the differences in wave shapes between the surface waves at top and trough. In this model more peaked waves at the crests cause bars to move seawards; steeper fronts at the crests coincide shoreward migration. It would be interesting to find out whether such wave-shape differences actually occur in nature. It may be that the validity of the processes established here even extends to different types of waves (wind- or storm-waves).

The various papers on the numerical evolution model, the basis of the morphological model in this paper [*Boczar-Karakiewicz et al, 1995* and references herein] show striking agreement between numerical simulations and observations of barred profiles. However, the strong relation between the resulting sea-bed wavelength and the low-frequency wavelength tends to undermine evidence for the validity of the model. Moreover, a fixed sea bed at the seaward boundary keeps the bars from moving and so the inherent sea-bed behaviour in this model cannot develop.

Acknowledgements This paper is based on work in the PACE-project, in the framework of the EU-sponsored Marine Science and Technology Programme (MAST-III), under contract no. MAS3-CT95-0002. It was co-sponsored jointly by Delft Hydraulics and RIKZ, within the framework of the Netherlands Centre for Coastal Research. The work was started during the author's stay at the INRS-Océanologie Rimouski, Canada. The author thanks Prof. B. Boczar-Karakiewicz for introducing her to her morphological evolution model. The model has been a continuous source of inspiration for the study which has led to this paper.

References

Boczar-Karakiewicz, B. and Bona, J.L., Wave-dominated shelves: a model of sand ridge formation by progressive infragravity waves, in *Shelf sands and sandstones*, edited by R.J. Knight and J.R. McLean, pp 163-179, Can. Soc. Petr. Geol. Memoir II, 1986.

Boczar-Karakiewicz, B., Forbes D.L. and Drapeau, G., Nearshore bar development in southern gulf of St. Lawrence, *J. of Waterw., Port, Coast. and Oc. Engineering*, 121, 1, 49-60, 1995.

Duane, D.B., Field, M.E., Meisburger, E.P., Swift, D.J.P. and Williams, S.J., Linear shoals on the Atlantic inner continental shelf, Florida to Long Island, in *Shelf sediment transport: process and pattern*, edited by D.J.P. Swift, D.B. Duane and O.H. Pilkey, pp 447-498, Dowden, Hutchinson & Ross, Inc., 1972.

Hulscher, S.J.M.H., Formation an migration of large-scale rhythmic sea-bed patterns, PhD-Thesis IMAU, Utrecht University, 1996.

Swift, D.J.P., Kofoed, J.W., Saulsbury, F.P., Sears, P., Holocene evolution of the shelf surface, central and Southern Atlantic Shelf of North America, in *Shelf sediment transport: process and pattern*, edited by D.J.P. Swift, D.B. Duane and O.H. Pilkey, pp 499-574, Dowden, Hutchinson & Ross, Inc., 1972.