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Efficient Characterization of Labeling Uncertainty in Closely-spaced Targets Tracking

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***Abstract*—In this paper we propose a novel solution to the labeled multi-target tracking problem. The method presented is specially effective in scenarios where the targets have once moved in close proximity. When this is the case, disregarding the labeling uncertainty present in a solution (after the targets split) may lead to a wrong decision by the end user. We take a closer look at the main cause of the labeling problem. By modeling the possible crosses between the targets, we define some relevant labeled point estimates. We extend the concept of *crossing objects*, which is obvious in one dimension, to scenarios where the objects move in multiple dimensions. Moreover, we provide a measure of uncertainty associated to the proposed solution to tackle the labeling problem. We develop a novel, scalable and modular framework in line with it. The proposed method is applied and analyzed on the basis of one-dimensional objects and two-dimensional objects simulation experiments.**

I. Introduction

Multiple Target Tracking (MTT) refers to the problem of estimating the state of the objects in the scene. This general definition encapsulates the subproblem of deciding which estimates belong to which object. This subproblem is generally referred to as the track labeling problem and it is well-known that it becomes particularly challenging when considering the tracking of objects that become closely-spaced every now and then [1].

Many methods face the MTT problem incorporating labels. Conventional plot-based examples are the Joint Probabilistic Data Association (JPDA) filter and the Multiple Hypothesis Tracking (MHT) filter [2] [3]. These methods work quite satisfactory as long as the objects remain well-separated, but tend to produce errors when exposed to closely-spaced object scenarios. Standard JPDA leads to *track coalescence*, while MHT might lead to *track swaps*, moreover both errors might get unnoticed without special built-in mechanisms. In addition, when objects move very close to each other, the measurements will get unresolved.

Track-before-detect (TBD) algorithms work with unthresholded measurements. So TBD approaches could in general tackle challenging scenarios which may lead to unresolved or merged measurements with a plot-based approach. However, similar problems as with plot-based methods will arise when extracting point estimates from the posterior density: MMSE estimates can lead to *track coalescence* and MAP estimates can lead to *track swaps* [4]. The reason for such unacceptable performance (both in plot-based and TBD algorithms) is that the labeling ambiguity is overlooked.

In some tracking problems, the track labels are irrelevant. Not surprisingly, explicitly ignoring track labels, results in better state estimation [5]. For instance, a modified JPDA filter designed to estimate a set of (unordered) state partitions, known as the Set JPDA filter, avoids the *track coalescence* problem of JPDA [6].

In many tracking problems however, the generation of track labels that are consistently pointing to the same objects over time is relevant. At this point it needs to be pointed out that this problem cannot be solved by solely adopting a labeled Random Finite Set (RFS) solution to the tracking problem. Although such a labeled-RFS tracking algorithm is very powerful and even desired, it might still produce erroneous estimates under closely-spaced object situations.

In this paper we argue that although the particular problem is often being referred to as a track labeling problem, the root of the problem however is the joint multi-target position uncertainty. Therefore, the solution to the problem lies in the efficient characterization of this uncertainty. Existing methods for characterization of the uncertainty in the labeling problem [7] [8] [9] [10] are either inefficient, not extendable to scenarios with more than two objects or unsuited for TBD.

We envisage the solution to the (labeled) MTT problem to be a finite number of relevant labeled point estimates (RLPEs), $t!$ to be exact (where t is the number of targets), along with their certainty measures. We call this method *cross modeling* since it is very intuitive (in the low dimensional case) that two RLPEs represent a cross between the objects. The solution proposed is scalable and of low numerical complexity.

The paper is organized as follows, in section II we discuss about the existent methods that consider labeling uncertainty characterization and their drawbacks. In section III we revisit the root cause of the uncertainty in the labeling problem and give the motivation to our proposed solution. In section IV we present our approach based on *modeling crosses* between objects. Then, we provide a possible scheme for extraction of labeled point estimates and the derivation of the associated certainty measures. Finally, we present the actual algorithm and simulation results for a toy scenario. In section V we present the modularity and scalability of our approach. To

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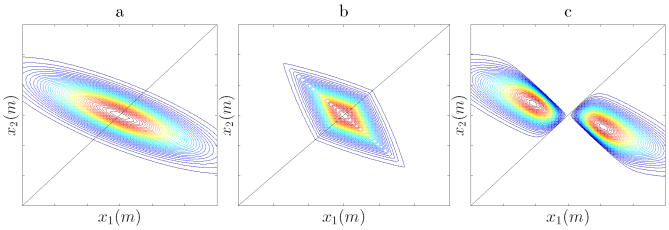

Fig. 1. a: posterior density, b: invariant part of the posterior density, c: strictly variant part of the posterior density.

illustrate this, a TBD application for a two two-dimensional targets scenario is provided. We conclude in section VI.

II. Related work

Blom and Bloem [11] introduced a decomposition of the exact Bayes posterior density into the weighted sum of permutation invariant and permutation strictly variant components. In Remarks 1 and 2 of the same work, practical consequences of having permutation invariant and permutation strictly variant posterior densities are explained. If the joint posterior density is permutation invariant and the tracker is required to make a hard decision to assign labels to tracks, it will frequently make wrong choices (about half of the time). On the other hand, when it is permutation strictly variant, there is no uncertainty at all about track identity. As noticed by García-Fernández in [12] Section 5.3.2, the last condition is only necessary but not sufficient.

One may extend the same reasoning from these two extreme cases to all the rest where the posterior density is a weighted composition of the two parts (see Figure 1). By these means, the amount of *permutation invariancy* in the posterior density may be used as a measure of uncertainty in the labeling problem.

Some particle filter algorithms relying on this idea have been presented in [9] and [10]. In [9], Blom and Bloem derive a filter that accounts for the uncertainty in the labeling problem and allows *track coalescence* avoidance of the extracted point estimates. The proposed point estimates extraction scheme is based on the so-called Track Continuity Assignment method. The associated uncertainty measures are given by the calculation of the so-called Track Swap probabilities. However, the solution is not general in the sense that it does not apply to general measurement models.

In [10], García-Fernández at al. developed a particle filter that is able to maintain the inherent multi-modality that appears in the posterior density when two objects separate after being in close proximity. By these means, the (undesired) *self-resolving* phenomenon of the multi-target sequential Monte Carlo (M-SMC) particle filter [13] is avoided. The filter allows to extract labeling information relying on the unique decomposition and point estimates that provide lower optimal subpattern assignment (OSPA) errors than MMSE or MAP estimators. However, the complexity of the algorithm is high ($O(N_p^2)$ where N_p is the number of particles). Moreover, mentioned methods based on the unique decomposition have no apparent extension to handle scenarios with more than 2 objects.

Aoki et al [7] provided their solution based on the following two questions: How can one optimally assign labels to two tracks? what is the probability that the assignment is incorrect, i.e. that track swap has occurred? As suggested by the first question, this method leads to hard labeling decisions (based on some optimal ordering of the unlabeled estimates). However, uncertainty in the labeling problem is taken into account by tackling the second question. That is to say, for a given labeling decision, this method estimates a measure of its certainty, resulting in situational awareness enhancement when tracking closely spaced objects. The work contains a formulation of the so called labeling probabilities and shows how to approximate them for two MTT algorithms: the M-SMC filter and the MHT filter. It suggests a MMOSPA-based (minimum mean Optimal Subpattern Assignment) point estimates extraction scheme as well, although any other method to obtain an unlabeled set of point estimates can be applied. In another contribution of the same authors [16], simulations are presented which show appealing results. In this case, they present a new MTT algorithm called Rao-Blackwellized marginal M-SMC filter that successfully avoids the (undesired) *self-resolving* phenomenon suffered by the M-SMC filter [13]. A complete overview of this approach is contained in [14] and [15], where the so called Labeling Uncertainty Aware particle filter (LUA-PF) is presented. The main drawback is that this method leads to algorithms with computational complexity $O(N_p^2 t!^2)$ (where N_p is the number of particles and t is the number of targets). Such a computational bottle neck is prohibitive for tracking scenarios with more than two objects.

In [8], Svensson et al. argue that target tracking is fundamentally about computing the posterior probabilities of targets' trajectories. By this means, labels are not even needed because the support of the filtering density contains all possible hypothesis of (already linked) sets of tracks. Their work presents a new state variable along with the development of all theoretical tools to use it in a filtering framework and a closed-form recursion for a conjugate family of distributions. However, as in MHT, the number of hypothesis grows exponentially and approximations involving pruning and merging techniques are required in practice. Furthermore, it is not apparent that this method allows direct extensions to TBD problems.

III. Root cause of the uncertainty in the labeling problem

Let us consider for a moment that Figure 2a shows the JMPD at a certain time instant k. Clearly, it is most likely that the object labeled as blue is further away from the sensor than the object labeled as red. However, a non-neglectable probability mass is given to some regions of the joint multi-target position space (JMPS) where the object labeled as red would be the farthest. This reflects that the first cause of the labeling problem is the dispersion of the JMPD. In fact, the labeling problem is not due to uncertainty in the labels.

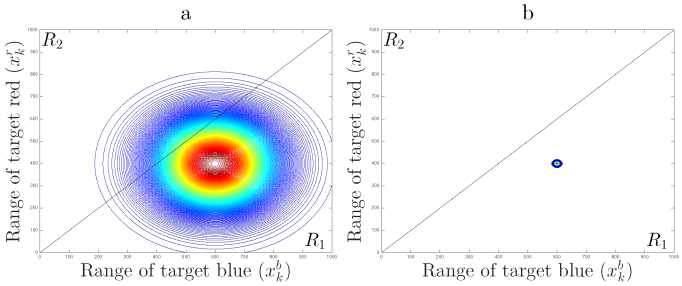

Fig. 2. The density in Figure a) represents the common noisy knowledge about the information of interest. The density in Figure b) represents an almost ideal noise-free knowledge about the information of interest.

After all, the set of labels is created by the tracker (blue and red) and therefore are completely certain.

If the JMPD was sharper (see Figure 2b) the probability that the object labeled as red is further away would become negligible. In this case, the labeling decision is "easy" and one would not consider it to be a "problem" at all. Thus, from a given JMPD two relevant probabilities can be calculated: the probability that the object labeled as blue is further away than the object labeled as red (see Equation 1), and its complement (see Equation 2). We define $f_X(x_k^b, x_k^r)$ to be the JMPD of the random variables x_k^b and x_k^r, which represent the range of the objects labeled as blue and red at time instant k respectively. Since, in general, the integrals in Equation 1 and 2 will not have closed-form solution we have to resort to numerical approximations.

$$p(x_k^b > x_k^r | Z_k) = \int\int_{R_1} f_X(x_k^b, x_k^r) dx_k^b dx_k^r \tag{1}$$

$$p(x_k^b \leq x_k^r | Z_k) = \int\int_{R_2} f_X(x_k^b, x_k^r) dx_k^b dx_k^r \tag{2}$$

In our view, the first point to tackle a (labeled) MTT problem is to define what is considered a "solution" to the problem. Moreover, it seems reasonable that the definition of "the solution" depends on the end user of the application. The JMPD is the richest representation of the information of interest. It contains labeled information for any arbitrarily selected region of the JMPS together with a measure of its certainty, namely the probability mass concentrated in this particular region. The JMPD is then, per se, a solution to the (labeled) MTT problem. However, is this solution compact enough for, say, a radar operator? Indeed, the information should be summarized to display a more concise situational awareness to the end user. In this summarizing process, namely point estimates extraction, information about the dispersion of the JMPD gets lost. To illustrate the idea, the two JMPDs in Figures 2a and 2b lead to the same point estimate (no matter if the estimator is based on MMSE or MAP). However, the dispersion is quite different. Therefore, it is in this process of extracting point estimates where the labeling problem comes into place.

In this article we propose that the solution to the labeling problem should be composed of a finite number of relevant

Fig. 3. Representation of targets' positions at discretized time instants. Trajectories considering that objects have (physically) crossed an even number of times fall within the representation on top. Trajectories considering that objects have (physically) crossed an odd number of times fall within the representation at the bottom.

labeled point estimates (RLPEs) along with their certainty measures. Our problem statement is then formulated by the following question: what amount of certainty can be extracted from the JMPD to designate to all the RLPEs? Self-evidently, this question requires a formal definition of what we mean by relevant. To do so we exploit the idea of *modeling crosses* between the objects. This is because when the objects are (or have been) in close proximity, the problem of labeling boils down to decide whether or not the objects have crossed with respect to a previous initialization stage.

Figure 3 illustrates the idea of *cross modeling* in the most simplistic scenario where the labeling problem appears: two one-dimensional objects approach each other, stay closely-spaced for a while and finally split. As commonly done in the literature, we consider that at some initial point in time $k = 0$, labeling can be done free of uncertainty. In practice, this means that at $k = 0$, the targets are not closely spaced and therefore, the tracker can unambiguously place a label to each of them. Let us say blue to the target that is farther away and red for the other. Thus, effectively, only one RLPE is needed to summarize the entire JMPD (at $k = 0$). However, at a later time ($k = k'$), when the objects are separated again, after moving closely for a while, two RLPEs are worth considering. One where the farthest object is blue and the nearest one red (example trajectories on top of the figure) and the other with opposite colors, positions being the same (example trajectories at the bottom of the figure). In the latter case, the objects are in a crossed position compared to that at $k = 0$.

It is important to note that each of these two RLPEs represent a single point in the JMPS. A measure of (un)certainty still needs to be associated to each of these RLPEs, so that the dispersion of the JMPD is well represented. We consider the mentioned solution (composed of two RLPEs together with associated certainty measures) to be informative enough as well as compact. In fact, this solution is richer than considering a single point estimate and not as overwhelming as providing the JMPD approximation.

IV. Cross modeling in one dimension

Modern algorithms approach the (labeled) MTT problem using two different state variables: a vector composed of ordered partitions and a set of partitions where labels are

included. Practically speaking, the difference between both approaches is that scenarios with unknown and variable number of targets can be tackled more efficiently with sets. In this work we will be considering scenarios where the number of targets is fixed and known and thus a vector-based implementation suffices. When using a particle filter, the uncertainty in the labeling problem translates to a situation where different labeling w.r.t. target identity are proposed by different particles. This situation, where the available information allows more than one labeling possibility is referred as "mixed labeling" by Boers and Driessen [13].

This work aims at overcoming the drawbacks of current methods. Therefore, it does not extend any of the ideas in the literature. In this section, the simplest set-up (with two targets moving along a line as in Figure 3) will be considered to convey the intuition behind our method. Later, it will become apparent that it allows direct extensions to more targets moving in higher number of spatial dimensions. It is not intended to improve the estimation of the JMPD, so no modeling involving additional knowledge is used to this end.

In the method by Aoki [7] and García-Fernández [10], labels are used explicitly whereas in the method by Svensson they are not due to their very different problem statement. In this sense, our method is closer to the firsts ones, so one objective is to avoid the computational complexity demanded there.

A. Notion of order

To describe our solution we introduce a notion of *order* in the filtering process. It is important to remark that this order is introduced at a higher level than the order that is inherent in a vector-based implementation. In fact, it is independent from the underlying state variable considered. This higher level notion of *order* is represented by a discrete variable with sample space cardinality equal to $t!$, where t represents the number of targets.

Suppose that the state variable is a vector where partitions are stacked: $s_k = [x_k^b \ \dot{x}_k^b \ x_k^r \ \dot{x}_k^r]^T$. Positions of each partition are denoted as x_k and velocities as $\dot{x}_k$. b and r are the labels for the first and second partition respectively (blue and red), k denotes a generic discrete time instant. The variable *order* at time step k is defined in Equation 3:

$$o_k = d(s_k) = \begin{cases} 1 & \text{if } x_k^b > x_k^r \\ 2 & \text{otherwise} \end{cases} \tag{3}$$

The definition of *order* will in turn define all $t!$ RLPEs that compose a solution at time instant k. The JMPS is divided in $(t!)$ subspaces. $o = 1$ determines R_1 and $o = 2$ determines R_2 (see Figure 2).

We should remark here that in the context of Figure 3, where at $k = 0$, the target further away is assigned label blue and the other label red, the RLPE extracted from $o = 1$ should be 100% certain. Later on, if the targets get close in position but not in velocity then they will cross each other without remaining closely-spaced at all. In this case as well, there would not be any uncertainty in the labeling decision and the RLPE extracted from $o = 2$ will have the full certainty. It is only when the two objects come close both in the position and velocity dimensions that we will need to consider both RLPEs along with their certainty measures.

B. RLPEs and associated certainty measures

In this section we define the certainty measures associated with the RLPEs as well as one possible way to select these RLPEs. When using particle-based approximations, Equation 3 divides the particles in two clusters depending on whether they fall in the region R1 or R2 (see Figure 2). To motivate the choice of the certainty measure it is convenient to consider the probabilistic graphical model representation of our method. This is an extension of the traditional Hidden Markov Model (see Figure 4). The arrows marked with d in Figure 4 represent the probabilistic influence of the state (s_k) on the *order* (o_k). Due to the definition of d in Equation 3, the influence is not probabilistic, but deterministic.

Functions f and h represent the general nonlinear dynamic system and observation models respectively (see Equations 4 and 5). n_k, z_k and v_k represent the process noise of the transition model, the measurement and the observation noise respectively at time k.

$$s_{k+1} = f(s_k, n_k) \quad (4) \qquad z_k = h(s_k, v_k), \ k \in \mathbb{N} \quad (5)$$

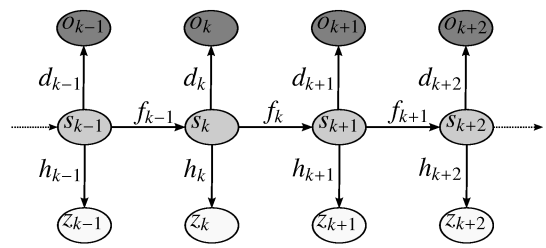

Fig. 4. Bayes network representation of our approach.

Equation 3 defines a bijection between the sample space of the variable *order* and the different clusters of the particle cloud. We define the certainty measures associated with the RLPEs as the posterior probability mass function of the order given the measurements $p(o_k|Z_k)$. By these means, the integrals in Equations 1 and 2 may be approximated by $p(o_k = 1|Z_k)$ and $p(o_k = 2|Z_k)$ respectively.

When the JMPD is estimated by a particle filter, the RLPEs and the associated certainty measures can be obtained as follows. Suppose the particle filter representation of the JMPD is given by the weighted particles $\{s_k^i, w_k^i\}_{i=1}^{N_p}$ and the definition of *order* is applied per particle. That is to say, $o_k^i := d(s_k^i)$ is the *order* of the i-th particle at time k. Moreover, w_k^{i*} and o_k^{i*} are used to denote the weight after resampling and the order after resampling of particle i at time instant k respectively. The notation $x_k^{i,c}$ refers to the position of target labeled as c by the i-th particle at time k.

The suggested point estimates extraction method to calculate the two RLPEs is the sample mean of each cluster:

$$\begin{gathered} \text{RLPE}_k(o = m) = [\bar{x}_k^b(o = m), \bar{x}_k^r(o = m)]^T \\ \bar{x}_k^c(o = m) = \frac{\sum_{i:o_k^{i*}=m} x_k^{i,c}}{\#\{o_k^{i*} = m\}}, \quad m = \{1, 2\}, \ c = \{b, r\} \end{gathered} \tag{6}$$

The associated certainty measures using the particle-based approximation are derived as follows. The calculation holds if resampling is performed after weights update $\forall k$:

$$p(o_k = 1|Z_k) = p(d(s_k) = 1|Z_k) = \int_{\{d(s_k)=1\}} f(s_k|Z_k) ds_k \approx$$

$$\underset{\substack{\textit{particle} \\ \textit{aproximation} \\ f(s_k|Z_k)}}{\approx} \sum_{i=1}^{N_p} \mathbb{1}_{\{d(s_k^i)=1\}} w_k^{i*} = \sum_{i:x_k^{i,b} > x_k^{i,r}} w_k^{i*} = \frac{\#\{o_k^{i*} = 1\}}{N_p} \quad (7)$$

$$p(o_k = 2|Z_k) \approx \frac{\#\{o_k^{i*} = 2\}}{N_p} \quad (8)$$

The proposed solution is of low complexity ($O(N_p)$). Interestingly, existent methods rely in different estimators to extract (unlabeled) point estimates achieving low MOSPA errors. Based on that, they have to evaluate all possible permutations to find out the labeling probabilities and provide an optimal labeled point estimate. Instead, in our method, the clustering produced by the definition of *order* allows simple extraction based on MMSE estimation (so already labeled point estimates) and straightforward certainty estimation (see Equations 6, 7 and 8).

C. Algorithm and simulation results

We have applied our proposed solution (see Algorithm 1) in a similar scenario to the one in Figure 3. The simulation results are shown in Figures 5 and 6. The RLPE($o = 1$) supports the hypothesis that the maneuvers of the two labeled objects lead to non-crossed trajectories with respect to the initialization $\forall k$. The certainty measure associated ($p(0 = 1|Z_k)$ curve) is extracted from the particle approximation of the JMPD in step 11 of Algorithm 1. Over the first 18 seconds of the simulation, the RLPE($o = 1$) holds full certainty. No single particle belongs to the cluster $o = 2$ and therefore, no representative can be extracted from there (see Figure 6). As expected, $p(o_k = 1|Z_k)$ drops down to around 0.5 as soon as the objects are closely-spaced. This means that labeling information have been lost. Labeling information cannot be recovered after the split as suggested by the measure of certainty. Indeed, the likelihood is permutation invariant because it is assumed that the measurements do not carry any information of identity. The RLPE($o = 2$) supports the hypothesis that the maneuvers of the two labeled objects lead to crossed trajectories with respect to the initialization $\forall k$. In this particular case, where only two targets are considered, the certainty measure of RLPE($o = 2$) can be directly extracted from the one of RLPE($o = 1$). However, the presented method will provide certainty measures for every RLPE ($t!$ in total) when considering a larger number of targets as well.

V. Extendability of the method

This section is devoted to justify the scalability and efficiency of the method in more complex scenarios. Also, we conduct an experiment, simulating the benchmark scenario involving two two-dimensional targets. The straightforward

1 $k = 0$
2 Draw N_p samples s_k^i from $p(s_k)$
3 Draw N_p samples n_k^i from $p(n_k)$
4 $k = k + 1$
5 $s_k^i = f(s_{k-1}^i, n_{k-1}^i)$
6 Calculate o_k^i according to Equation 3
7 Given z_k, obtain $\tilde{w}_k^i = p(z_k|s_k^i)$
8 Normalize weights $w_k^i = \tilde{w}_k^i / \sum_{j=1}^{N_p} \tilde{w}_k^j$
9 Resample from $\hat{p}(s_k|Z_k) = \sum_{i=1}^{N_p} w_k^i \delta(s_k - s_k^i)$:
10 Extract RLPEs according to Equations 6
11 Obtain certainty measures of RLPEs according to Equations 7 and 8
go to 3

Algorithm 1: Pseudo-code of the Extended M-SMC particle filter algorithm. Extensions of the plain vanilla M-SMC particle filter are highlighted in blue color.

extension from our toy example to higher number of one-dimensional targets is possible by simply extending the definition of *order*. For instance, Equation 9 gives the definition of *order* in a set-up of three objects moving along a line. The new label is considered to be green, so the state vector with ordered partitions will become: $s_k = [x_k^b \ \dot{x}_k^b \ x_k^r \ \dot{x}_k^r \ x_k^g \ \dot{x}_k^g]^T$. Same way as in Equation 3, two RLPEs extracted from any couple of clusters represent a cross between, at least, one couple of objects.

$$o_k = d(s_k) = \begin{cases} 1 & \text{if } x_k^b > x_k^r, x_k^r > x_k^g \\ 2 & \text{if } x_k^b > x_k^g, x_k^g \geq x_k^r \\ 3 & \text{if } x_k^r \geq x_k^b, x_k^b > x_k^g \\ 4 & \text{if } x_k^r > x_k^g, x_k^g \geq x_k^b \\ 5 & \text{if } x_k^g \geq x_k^b, x_k^b > x_k^r \\ 6 & \text{if } x_k^g \geq x_k^r, x_k^r \geq x_k^b \end{cases} \quad (9)$$

In general, the proposed method provides a modular solution where one needs only to design the proper definition of *order* in the context of the problem. Also, extensions to scenarios where the targets move in an arbitrary n-dimensional space can be considered as long as the definition of order is consistent. If this is the case, Equations 6, 7 and 8 can be extended to the $t!$ possible values of *order* right away. Figure 7 gives the graphical idea. Therefore, the computational complexity remains linear in the number of particles regardless of how many objects and dimensions are considered.

A. Two-dimensional objects scenario: definition of order

The extension of the definition of order from one to two dimensions is non-trivial. However, the same general principle applied in Equation 3 holds. First, since only two objects are considered to be present, the entire four-dimensional JMPS should be divided into two (2!) subspaces. Second, one of the RLPEs should represent a "cross" between the two objects as compared to the time instant $k = 0$. The problem is that the exact meaning of what is a cross between two objects in two dimensions is not as clear as it was with one-dimensional targets. Therefore, before we define o_k we have to define a cross in a two-dimensional space.

Note that Equation 3 can be re-written using a reference point c, as indicated by Equation 10. The reference point is the central position between the positions of the two partitions and

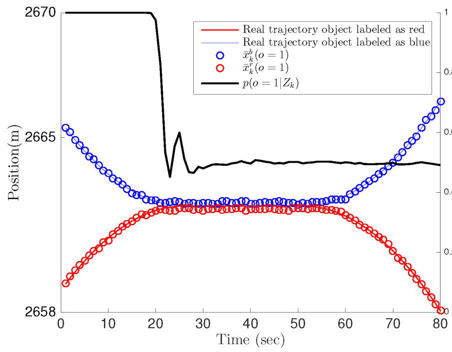

Fig. 5. RLPE(o = 1) and associated certainty measure.

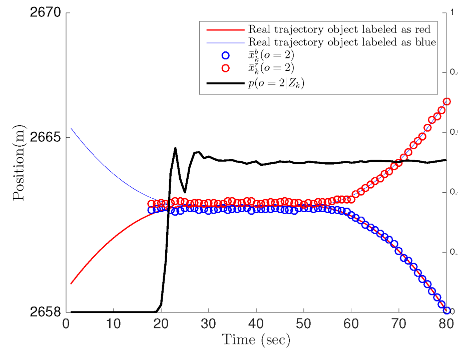

Fig. 6. RLPE(o = 2) and associated certainty measure.

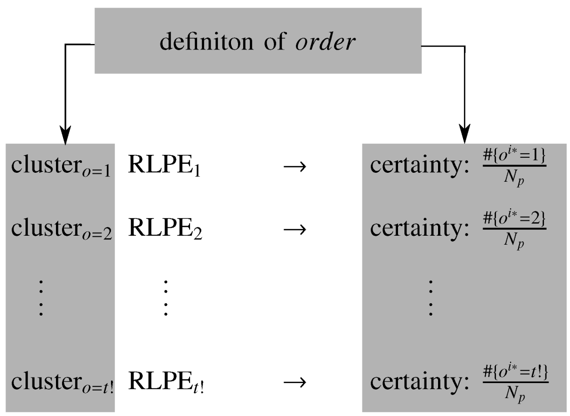

Fig. 7. The proposed method is modular. Extensions only require to design the back box containing the definition of *order*.

can be used to determine if the position of the blue object is on the left-hand-side (*order* = 1) or if it is on the right-hand-size (*order* = 2). It seems intuitive then that one particle ordered as 1 is "crossed" with respect to another particle that is ordered as 2.

$$o_k = d(s_k) = \begin{cases} 1 & \text{if } x_k^b > x_k^r \\ 2 & \text{if } x_k^b \leq x_k^r \end{cases} = \begin{cases} 1 & \text{if } x_k^b > c > x_k^r \\ 2 & \text{if } x_k^b \leq c \leq x_k^r \end{cases} \tag{10}$$

where $c = \frac{x_k^b + x_k^r}{2}$

Where a point is used to subdivide the real line in the one-dimensional targets case, for the two-dimensional targets case, a straight line appropriately subdivides the plane. Similarly, this line should be defined in an equivalent manner as the central point was defined in Equation 10. Also, what left and right (with respect to this line) means need to be defined carefully.

The straight line considered is the perpendicular bisector between two points, ${}_{(x_k^b, y_k^b)} \perp_{(x_k^r, y_k^r)}$. In Equation 11 an informal definition of *order* is provided, where $\hookleftarrow$ and $\hookrightarrow$ can be read as "to the left of" and "to the right of" respectively.

$$o_k = d(s_k) = \begin{cases} 1 & \text{if } (x_k^b, y_k^b) \hookleftarrow {}_{(x_k^b, y_k^b)}\perp_{(x_k^r, y_k^r)} \hookrightarrow (x_k^r, y_k^r) \\ 2 & \text{if } (x_k^r, y_k^r) \underset{=}{\hookleftarrow} {}_{(x_k^b, y_k^b)}\perp_{(x_k^r, y_k^r)} \underset{=}{\hookrightarrow} (x_k^b, y_k^b) \end{cases} \tag{11}$$

According to Equation 11 particles belonging to the cluster $o = 1$ have the blue object on the left-hand-side of the perpendicular bisector and particles belonging to the cluster $o = 2$ have the blue object on the right-hand-side of the perpendicular bisector. Although one can always draw the perpendicular bisector between two points, what is left and right with respect to this line depends on the sense of motion. Therefore, one extra degree of freedom should be tackled. Luckily, velocity dimensions of the objects are estimated as well, so this information can be included to construct the definition of *order*:

$$o_k = d(s_k) = \begin{cases} 1 & \text{if } m_\perp x_k^b + b_\perp > y_k^b \text{ and } cvp > 0 \text{ or} \\ & \text{if } m_\perp x_k^b + b_\perp < y_k^b \text{ and } cvp < 0 \\ 2 & \text{if } m_\perp x_k^b + b_\perp \leq y_k^b \text{ and } cvp > 0 \text{ or} \\ & \text{if } m_\perp x_k^b + b_\perp \geq y_k^b \text{ and } cvp < 0 \end{cases} \tag{12}$$

where $cvp = \frac{\dot{x}_k{}^b + \dot{x}_k{}^r}{2}$, $m_\perp$ is the slope and $b_\perp$ is the y-intercept of the perpendicular bisector respectively.

B. Track-before-detect model

For the numerical experiment we consider a TBD model as in [17]. The partition of target t is denoted by $s_k^t = [x_k^t \ \dot{x}_k^t \ y_k^t \ \dot{y}_k^t]^T$. The state variable is composed of ordered partitions, to accommodate the scenario composed of two two-dimensional objects with labels b and r the state variable is: $s_k = [x_k^b \ \dot{x}_k^b \ y_k^b \ \dot{y}_k^b \ x_k^r \ \dot{x}_k^r \ y_k^r \ \dot{y}_k^r]^T$. The dynamic model for each targets is the commonly used near-constant velocity, defined in cartesian coordinates as:

$$s_{k+1}^t = f(T, s_k^t) + g(T, s_k^t) w_k, \ \ k \in \mathbb{N} \tag{13}$$

$$f(T, s_k^t) = \begin{pmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{pmatrix} s_k^t$$

Where T is the revisit time or data rate. In order to capture object accelerations the process noise $g(T, s_k^t) w_k$ has to be considered, w_k is assumed to be standard white Gaussian. The process noise input model for each target is given by:

$$g(T, s_k^t) = \begin{pmatrix} \frac{1}{2}(\frac{1}{3} a_{x,max}) T^2 & 0 \\ \frac{1}{3}(a_{x,max}) T & 0 \\ 0 & \frac{1}{2}(\frac{1}{3} a_{y,max}) T^2 \\ 0 & \frac{1}{3}(a_{y,max}) T \end{pmatrix}$$

where $a_{x,max}$ and $a_{y,max}$ are the maximum accelerations that the objects can perform.

The measurement model used defines the power reflections of the two objects for each cell. One measurement z_k is composed of $N_r \times N_b$ power measurements z_k^{ij}, where $k \in \mathbb{N}$ and N_r and N_b are the number of range and bearing cells.

$$z_k^{ij} = |z_{A,k}^{ij}|^2 = |A_k^{(1)} h_A^{(1)ij} + A_k^{(2)} h_A^{(2)ij} + n_I(t_k) + in_Q(t_k)|^2 \quad (14)$$

where A_k is the complex amplitude of the target.

$$A_k = \tilde{A}_k^{(t)} e^{i\phi_k}, \ \phi_k \in (0, 2\pi) \quad (15)$$

$$h_A^{(t)ij}(s_k^t) = e^{\frac{(r_i - r_k^t)^2}{2R} - \frac{(b_j - b_k^t)^2}{2B}}, \ i = 1, ..., N_r, \ j = 1, ..., N_b \quad (16)$$

r_k^t and b_k^t are the range and bearing of the target t and R and B are constants related to the size of the cells.

$$r_k^t = \sqrt{(x_k^t)^2 + (y_k^t)^2} \text{ and } b_k^t = arctan(\frac{y_k^t}{x_k^t}) \quad (17)$$

The noise in Equation 14 is complex Gaussian, where $n_I(t_k)$ and $n_Q(t_k)$ are independent, zero-mean white Gaussian with variance σ_n^2. These measurements, conditioned on the state are assumed to be exponentially distributed [18],

$$p(z^{ij}|s_k) = \frac{1}{\mu_0^{ij}} e^{-\frac{1}{\mu_0^{ij}} z_k^{ij}} \quad (18)$$

where

$$\mu_0^{ij} = (\tilde{A}_k^{(1)})^2 (h_A^{(1)ij}(s_k, t_k))^2 + (\tilde{A}_k^{(2)})^2 (h_A^{(2)ij}(s_k, t_k))^2 + 2\sigma_n^2 \quad (19)$$

Assuming that the noise is independent from cell to cell and that the reflections of the two targets are independent, the likelihood function to apply in Step 7 of Algorithm 1 is:

$$p(z_k|s_k) = \prod_{ij} p(z^{ij}|s_k) \quad (20)$$

C. Numerical results

We consider a specific scenario which has been analyzed recurrently over the last years as a test case for different approaches that tackle the labeling problem. This scenario is composed of two separated targets describing trajectories approaching each other and splitting after some time moving closely-spaced.

In order to run the experiments, the system set-up defined in the previous section is encapsulated in Algorithm 1. Three simulations are carried out:

1) In and out closely-spaced formation of objects ($N_p = 3 \cdot 10^5$)
2) In and out overlapped formation of objects ($N_p = 3 \cdot 10^5$)
3) In and out overlapped formation of objects ($N_p = 10^5$)

The RLPEs and the corresponding certainty measures are presented in Figures 8, 9 and 10. The outcome of each simulation is shown by columns. The figures show some remarkable (common) aspects.

- Labeling is not compromised until the objects reach 420m in x axis. Therefore, in the first part of the simulation, any current labeled approach (even without accounting for labeling characterization) would perform equivalently.
- Labeling uncertainty decreases when the objects get close enough to each other. In particular, information about identities is lost almost completely after a few seconds in the simulations with overlapped objects. Here is where uncertainty characterization enhances situational awareness. If labeling uncertainty is not taken into account, any hard decision regarding the association between labels and positions will be considered (wrongfully) as the correct one.
- Once the information about identities is lost partially or completely it does not seem to be recovered again.
- The point estimates that result from our suggested track extraction method are not biased towards the centroid of the real objects locations. Interestingly, to avoid *track coalescence* there is no need to calculate any MOSPA minimizer. Instead, straightforward track extraction is possible by simply averaging the positions of the particles in the same *order* by partitions.

The analysis of the differences between the three simulations suggests relevant conclusions as well.

- Identity information degenerates faster for the cases where the objects overlap.
- The third simulation presents accentuated instability in the estimation of the labeling certainty after objects separation (compared to the second scenario). This is due to the *self-resolving* property inherent in a particle filter implementation, which is more visible when the number of particles used is scarce. This property refers to the incapacity of the filter to approximate multi-modal densities for an arbitrary long time. However, since the proposed method leads to a linear complexity algorithm, we can increase the number of particles to mitigate the effect.

VI. Conclusions

In this work we have presented a new way of defining solutions to the (labeled) MTT problem. We have proposed a solution to be a few relevant (labeled) position estimates with their associated certainty measures. The new method avoids the drawbacks present in the only few existing methods that explicitly deal with a measure of (un)certainty in a solution to the (labeled) MTT problem. The proposed method provides a modular framework to arrive at the solution where the notion of *order* plays a key role in describing the "crosses" between objects. In the proposed method the definition of *order* is isolated in the sense that it will change depending on the problem at hand but once properly designed, the method is extendable to an arbitrary number of objects and dimensionality. More importantly, the method is computationally efficient, with a linear computational complexity in the number of particles irrespective of the dimensionality.

As for future work, it will be interesting to see if this concept can be used in a scenario where the number of targets is not known. The notion of *order* is independent of the way in which labels are included. This suggests that the formulation can be extended from ordered vectors to labeled random finite sets in order to do cardinality estimation as well. Another focus for future work will be on the design of new *order* definitions and integration of our method in MTT algorithms that avoid the *self-resolving* phenomenon.

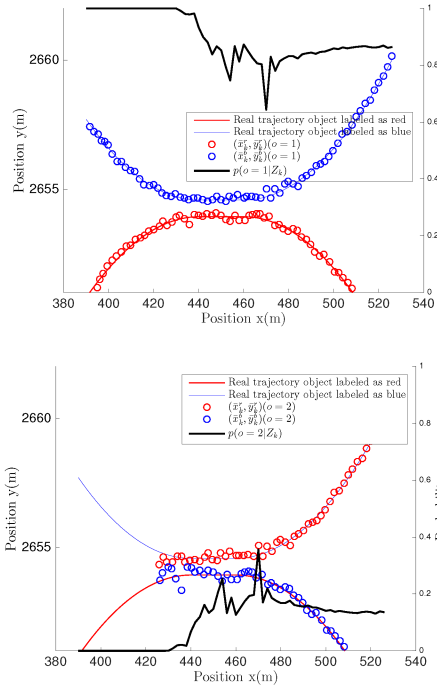

Fig. 8. In and out closely-spaced objects formation ($N_p = 3 \cdot 10^5$). RLPE($o = 1$) and its certainty measure (Fig. on top). RLPE($o = 2$) and its certainty measure (Fig. at the bottom).

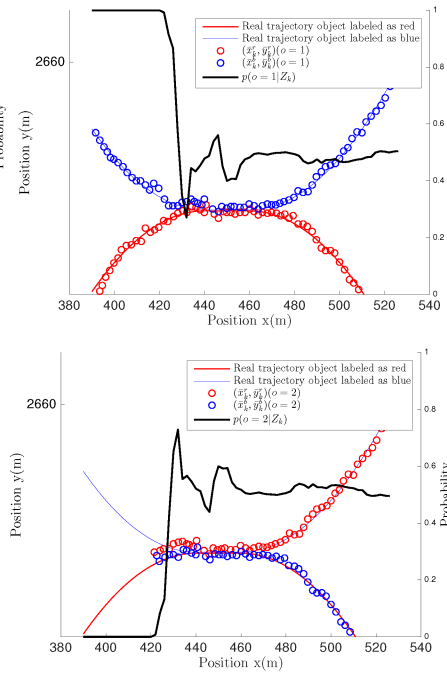

Fig. 9. In and out overlapped objects formation ($N_p = 3 \cdot 10^5$). RLPE($o = 1$) and its certainty measure (Fig. on top). RLPE($o = 2$) and its certainty measure (Fig. at the bottom).

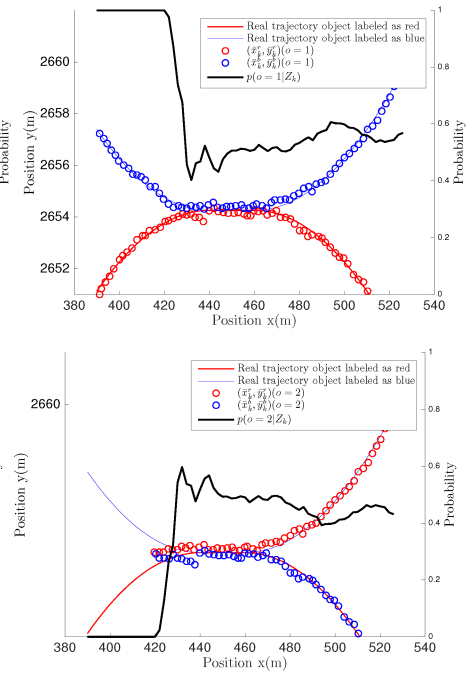

Fig. 10. In and out overlapped objects formation ($N_p = 10^5$). RLPE($o = 1$) and its certainty measure (Fig. on top). RLPE($o = 2$) and its certainty measure (Fig. at the bottom).

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References

[1] S. Blackman and R. Popoli, "Design and analysis of Modern Tracking Systems," Norwood, MA: Artech House, 1999

[2] T. Fortmann, Y. Bar-Shalom and M. Scheffe, "Sonar tracking of multiple targets using joint probabilistic data association," in *IEEE Journal of Oceanic Engineering*, vol. 8, no. 3, pp. 173-184, Jul 1983.

[3] D. Reid, "An algorithm for tracking multiple targets," in *IEEE Transactions on Automatic Control*, vol. 24, no. 6, pp. 843-854, Dec 1979

[4] H. Blom, E. Bloem, Y. Boers and H. Driessen, "Tracking closely spaced targets: Bayes outperformed by an approximation?," in *Information Fusion, 2008 11th International Conference on*, Cologne, 2008, pp. 1-8.

[5] M. Guerriero, L. Svensson, D. Svensson and P. Willett, "Shooting two birds with two bullets: How to find Minimum Mean OSPA estimates," in *Information Fusion, 2010 13th Conference on*, Edinburgh, 2010, pp. 1-8

[6] L. Svensson, D. Svensson and P. Willett,"Set JPDA algorithm for tracking unordered sets of targets," in *Information Fusion, 2009 12th International Conference on*, Seattle, WA, 2009, pp. 1187-1194

[7] E. H. Aoki, Y. Boers, L. Svensson, P. Mandal and A. Bagchi,"A Bayesian look at the optimal track labelling problem," in *Data Fusion & Target Tracking Conference (DF&TT 2012)*: Algorithms & Applications, 9th IET, London, 2012, pp. 1-6.

[8] L. Svensson and M. Morelande, "Target tracking based on estimation of sets of trajectories," in Information Fusion, 2014 17th International Conference on, Salamanca, 2014, pp. 1-8

[9] H. Blom and E. Bloem, "Decomposed particle filtering and track swap estimation in tracking two closely spaced targets," in *Information Fusion, 2011 14th International Conference on*, Chicago, IL, 2011, pp. 1-8.

[10] A. García-Fernández, M. Morelande and J. Grajal, "Particle filter for extracting target label information when targets move in close proximity," in *Information Fusion, 2011 14th International Conference on*, Chicago, IL, 2011, pp. 1-8

[11] H. Blom and E. Bloem, "Permutation invariance in Bayesian estimation of two targets that maneuver in and out formation flight," in *Information Fusion, 2009 12th International Conference on*, Seattle, WA, 2009, pp. 1296-1303

[12] A. F. García-Fernández, Detection and tracking of multiple targets using wireless sensor networks Ph.D. dissertation, Universidad Politcnica de Madrid, Madrid, Spain, 2011 [Online]. Available: http://oa.upm.es/9823/

[13] Y. Boers, E. Sviestins and H. Driessen, "Mixed Labelling in Multitarget Particle Filtering," in *IEEE Transactions on Aerospace and Electronic Systems*, vol. 46, no. 2, pp. 792-802, April 2010.

[14] E. H. Aoki, Y. Boers, L. Svensson, P. K. Mandal, and A. Bagchi, "A Bayesian solution to multi-target tracking problems with mixed labelling" in Memorandum 2036 (July 2014). ISSN 18744850. Available from: http://www.math.utwente.nl/publications Department of Applied Mathematics, University of Twente, Enschede, The Netherlands

[15] E. H. Aoki, "Characterization of uncertainty in Bayesian estimation using Sequential Monte Carlo methods", Ph.D. dissertation, Department of Applied Mathematics, University of Twente, Enschede, The Netherlands

[16] E. H. Aoki, Y. Boers, L. Svensson, P. K. Mandai and A. Bagchi, "The Rao-Blackwellized marginal M-SMC filter for Bayesian multi-target tracking and labelling," in *Information Fusion, 2012 15th International Conference on*, Singapore, 2012, pp. 90-97

[17] Y. Boers and J. N. Driessen, "Multitarget particle filter track before detect application," in *IEE Proceedings - Radar, Sonar and Navigation*, vol. 151, no. 6, pp. 351-357, 10 Dec. 2004

[18] M. I.Skolnik, "Introduction to Radar Systems" McGraw-Hill, Inc, Singapore, 1981, 2nd. edn.