A generalization of a Coherence Multiplexing System

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Abstract

Coherence Multiplexing is a relatively unknown form of a CDMA-system, especially used in access networks for optical communication systems. Usually a Coherence Multiplex System uses delay-filters at the transmitter and at the receiver to perform the code. In this paper an extension to other filter types is described. The performance, in terms of the signal-to-noise ratio, can be calculated. The result is a simple expression in wich only the bit-time, the coherence-time of the source and the number of simultaneous users is involved. With the use of a continuous source the signal to noise ratio will be proportional to the inverse of the square of the number of simultaneous users. Such a system is in fact a spectral code system. A further extension will be made with the use of a pulsed source and by replacing the filters in the previous case by a bank of filters. Each element of that filterbank also has a delay, different for each element, included. It will be shown that in that case the signal-to-noise ratio can be made proportional to the inverse of the number of users instead of the square of the number of users, which means an important improvement with respect to the number of users that can be handled. This system can be named a spectro-temporal code system.

1. Introduction

Coherence Multiplexing is a relatively unknown form of Code Division Multiple Access (CDMA). It is particularly suitable in access networks for optical communication systems since it imposes less severe constraints on transmitter and receiver components than for instance WDM, which requires very stable lasers and receiver filters in order to avoid crosstalk between adjacent channels. Coherence Multiplexing is a technique that utilises the random phase jitter of a broadband laser or LED, by transmitting two versions of a source signal and correlating these two in the receiver. Coherence Multiplexing in its conventional form is extensively discussed in [1] and [2] and is illustrated in figure 1.

The symbols in figure 1 correspond to the following quantities:

- $\mathbf{x}(t)$: Electric field of the input lightwave;
- $m(t)$: Information carrying datasignal (rectangular, either +1 or -1);
- $\mathbf{z}(t)$: Electric field of the transmitted lightwave;
- $\mathbf{w}_a(t)$: Electric field of the output lightwave of the upper receiver output branch;
- $\mathbf{w}_b(t)$: Electric field of the output lightwave of the lower receiver output branch;
- $\mathbf{I}_a(t)$: Current through the upper receiver photodiode;
- $\mathbf{I}_b(t)$: Current through the lower receiver photodiode;
- $\mathbf{I}(t)$: Residual output current of the receiver;
- T_{tr} : Difference in phase delay between upper and lower transmitter branches;
- T_{re} : Difference in phase delay between upper and lower receiver branches;

where boldface characters denote random processes.

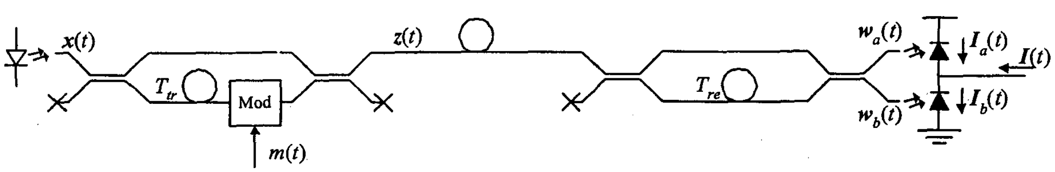

Figure 1: *A conventional Coherence Multiplexing System*

All electrical fields are circular complex gaussian distributed bandpass signals and the currents are real baseband signals.

The left part of figure 1 denotes a transmitter that converts the input signal $x(t)$ into a signal $z(t)$, consisting of two versions of the source signal $x(t)$. The second version is shifted in time with respect to the first version by a time T_{tr}, and modulated by $m(t)$.

In the receiver, $z(t)$ is divided in two versions that are shifted in time with respect to one another by a time T_{re}. One can prove that the phase shifts in the right coupler cause the difference of the two photodiode currents to be proportional to the product of these two versions of $z(t)$, provided that all couplers are lossless and perfectly balanced. The average value of the output current $I(t)$ is thus proportional to the crosscorrelation of these two versions for zero timeshift.

Since the source signal $x(t)$ suffers from phase jitter, two signals can only have a non-zero crosscorrelation function for zero timeshift when they are nearly synchronized. Consequently, $I(t)$ will only have a non-zero average if we choose T_{re} to be equal to T_{tr}. Choosing the T_{tr}'s sufficiently apart for different transmitters is thus a way to enable the receiver to select a desired transmitter.

The performance of such a system is mainly limited by three types of noise:

- Beat noise, which is a consequence of the random character of the interfering signal terms;
- Shot noise, which is caused by the fact that the photon-electron conversion in the photodiodes is a discrete process;
- Thermal amplifier noise.

One can show, however, that the latter two noise components can be neglected when the transmitted light power is large enough (see [2]). Beat noise thus forms a so-called noise-floor in the receiver output current. By calculating the autocorrelation function of the output current $I(t)$ and its corresponding spectral density, one can show that the signal-to-beat noise ratio in a Coherence Multiplexing System with M active transmitters is proportional to $\frac{1}{M^2}$, independent of the transmitted light power (see [2]). Particularly for large M, this greatly limits the overall signal-to-noise ratio.

It is thus desirable to find a way to reduce the beat noise in the output current. In this paper, an alternative form of a Coherence Multiplexing System is proposed, in which the signal-to-beat noise ratio is proportional to $\frac{1}{M}$ instead of $\frac{1}{M^2}$.

2. A generalized Coherence Multiplexing System

A more generalized form of a Coherence Multiplexing System can be obtained by replacing the delay lines in both the transmitters and receivers by (other) linear filters. This is illustrated in figure 2. The symbols in the filter boxes denote the impulse responses of the corresponding filters.

We can now express both the average output current and the beat noise spectral density as a function of the impulse response of the filters in figure 2, to be able to impose demands on these impulse responses for minimizing both crosstalk and noise.

Therefore, we have to distinguish M transmitters and M receivers. Each transmitter i has its own transmitter filters $h_{tr,a,i}$ and $h_{tr,b,i}$ and information datasignal $m_i(t)$, and each receiver r has its own receiver filters $h_{re,a,r}$ and $h_{re,b,r}$. All transmitters have identically distributed source signals $x_i(t)$ (same wavelength, same power, same spectrum) that are mutually independent.

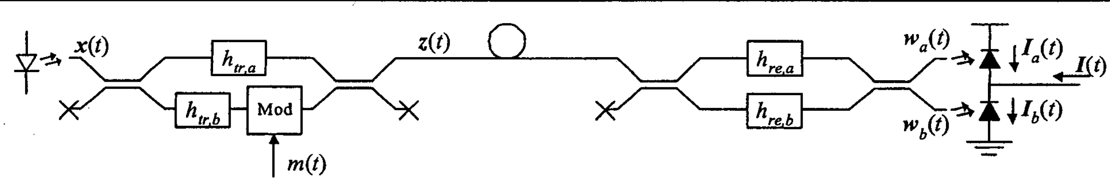

Figure 2: A generalized Coherence Multiplexing System

$$\langle \boldsymbol{I}_r(t)\rangle = \frac{R_{pd}}{16\cdot M}\cdot\left(\begin{array}{l} \displaystyle\sum_{i=1}^{M}\int_{-\infty}^{\infty}\left(\begin{array}{l}\left[H_{re,a,r}(f)\cdot H_{re,b,r}{}^*(f)+H_{re,a,r}{}^*(f)\cdot H_{re,b,r}(f)\right]\cdot\\ \left[\left|H_{tr,a,i}(f)\right|^2+\left|H_{tr,b,i}(f)\right|^2\right]\end{array}\right)\cdot S_{xx^*}(f)\cdot\mathrm{d}f \\ \displaystyle -\sum_{\substack{i=1\\ i\neq r}}^{M} m_i\cdot\int_{-\infty}^{\infty}\left(\begin{array}{l}\left[H_{re,a,r}(f)\cdot H_{re,b,r}{}^*(f)+H_{re,a,r}{}^*(f)\cdot H_{re,b,r}(f)\right]\cdot\\ \left[H_{tr,a,i}(f)\cdot H_{tr,b,i}{}^*(f)+H_{tr,a,i}{}^*(f)\cdot H_{tr,b,i}(f)\right]\end{array}\right)\cdot S_{xx^*}(f)\cdot\mathrm{d}f \\ \displaystyle -m_r\cdot\int_{-\infty}^{\infty}\left(\begin{array}{l}\left[H_{re,a,r}(f)\cdot H_{re,b,r}{}^*(f)+H_{re,a,r}{}^*(f)\cdot H_{re,b,r}(f)\right]\cdot\\ \left[H_{tr,a,r}(f)\cdot H_{tr,b,r}{}^*(f)+H_{tr,a,r}{}^*(f)\cdot H_{tr,b,r}(f)\right]\end{array}\right)\cdot S_{xx^*}(f)\cdot\mathrm{d}f \end{array}\right) \qquad (1)$$

Average output current

The instantaneous output current of receiver r can be found by observing one bit period and assuming that $m(t)$ is constant during that period. We then write both $w_{a,r}(t)$ and $w_{b,r}(t)$ as a sum of four convolutions, which corresponds to the four possible paths that a lightwave can travel from the source to either of the photodiodes, thereby being filtered and/or multiplied by m and phase-shifted by 90° when a coupler is 'crossed' (see [3]). The output current is then equal to the difference of the average powers of $w_{a,r}(t)$ and $w_{b,r}(t)$ times the responsivity R_{pd} of the photodiodes.

The average output current can be found by taking the expected value of the resulting expression. The result is given in (1), in which $S_{xx^*}(f)$ is defined as the power spectral density function of the source signal $\boldsymbol{x}(t)$, and uppercase H's are the transfer functions corresponding to the lowercase h's.

The average output current consists of the following terms:

- M bias terms;
- M–1 crosstalk terms;
- One information term.

Ideally, the filters are chosen such that all terms are cancelled except the latter one, which is proportional to the desired information datasignal and which should thus be maximized.

This can be done by choosing the transfer functions of all transmitter and receiver filters to be orthogonal to each other in the non-zero part of the source spectrum, except for the filter pairs of corresponding transmitters and receivers, which should be either equal or complex conjugates. The mathematical notation is given in (2).

$$H_{tr,a,i}\perp H_{tr,b,j}\ \forall\ i,j \qquad (2a)$$
$$H_{tr,a,i}\perp H_{tr,a,j}\ \forall\ i\neq j \qquad (2b)$$
$$H_{tr,b,i}\perp H_{tr,b,j}\ \forall\ i\neq j \qquad (2c)$$
$$H_{tr,a,i}\perp H_{re,a,j}\ \forall\ i\neq j \qquad (2d)$$
$$H_{tr,b,i}\perp H_{re,b,j}\ \forall\ i\neq j \qquad (2e)$$

$$(H_{re,a,I}=H_{tr,a,i}\wedge H_{re,b,I}=H_{tr,b,i}\ \forall\ i)\vee$$
$$(H_{re,a,i}=H_{tr,a,i}{}^*\wedge H_{re,b,i}=H_{tr,b,i}{}^*\ \forall\ i) \qquad (2f)$$

The receiver filters should thus either be equal or matched to the corresponding transmitter filters. Both options result in the same average receiver output current, which is proportional to $\frac{1}{M^2}$. In total, a set of $2{\cdot}M$ different (orthogonal) filters is needed.

Beat noise spectral density

The beat noise spectral density can be found by Fourier transforming the autocorrelation function of the instantaneous receiver output current $\boldsymbol{I}_r(t)$ (see section 2.1), which results in a DC-term (which corresponds to the information-carrying average receiver current) and a broadband noise term. If, however, we define the beat noise spectral density $S_{I_rI_r^*}(f)$ of $\boldsymbol{I}_r(t)$ as the Fourier transform of the covariance function of $\boldsymbol{I}_r(t)$, the DC-term is cancelled and only the noise spectrum remains. For computing the performance of the system, only the lower part of this spectrum is interesting, since the information-carrying signal part is confined to this part of the spectrum. One can prove that this can be written as in (3), provided that the receiver filters are either equal or matched to the corresponding transmitter filters, as suggested in section 2.1. Both options result in the same beat noise spectral density.

$$S_{I_rI_r^*}(0)=\frac{R_{pd}^2}{256\cdot M^2}\cdot\sum_{i=1}^{M}\sum_{j=1}^{M}\int_{-\infty}^{\infty}\left(\begin{array}{l}\left[H_{tr,a,r}{}^*(f)\cdot H_{tr,b,r}(f)+H_{tr,a,r}(f)\cdot H_{tr,b,r}{}^*(f)\right]^2\cdot\\ \left|H_{tr,a,i}(f)-m_i\cdot H_{tr,b,i}(f)\right|^2\cdot\left|H_{tr,a,j}(f)-m_j\cdot H_{tr,b,j}(f)\right|^2\cdot S_{xx^*}^2(f)\end{array}\right)\cdot\mathrm{d}f \qquad (3)$$

Expanding this equation results in $64\cdot M^2$ integrals of products of 8 transfer functions and the square of the source spectrum. We assume that the filters are chosen such that these integrals are non-zero only when these 8 transfer functions form 4 pairs of complex conjugated transfer functions. In that case only $8\cdot M^2+4\cdot M+2$ of these integrals remain. Provided that the filters are chosen such that the remaining integrals all account for a same noise contribution, the beat noise spectral density $S_{I_r I_r^*}(0)$ is given by (4).

$$S_{I_r I_r^*}(0) \propto \frac{8\cdot M^2+4\cdot M+2}{M^2} \qquad (4)$$

Signal-to-beat noise ratio

The received signal power can be found by squaring the average receiver output current.

The beat noise power can be minimized by low-pass filtering the output current of the receiver. The bandwith of this filter should be $1/T_b$, in order to be able to detect the information bits. The resulting beat noise power can be approximated by multiplying the power spectral density for zero frequency by the bandwith $2/T_b$ of the information carrying signal $m(t)$.

By dividing these two we find a signal-to-beat noise ratio SNR_b that is given by the proportionality in (5).

$$SNR_b \propto \frac{1}{4\cdot M^2+2\cdot M+1} \qquad (5)$$

Obviously, the generalized Coherence Multiplexing System in figure 2 does not satisfy our goal as far as the signal-to-beat noise ratio is concerned; a more advanced structure is needed for that.

3. A slotted generalized Coherence Multiplexing System

In this section, an extension to the generalized Coherence Multiplexing System in figure 2 is proposed and analyzed. We will show that, using this alternative, we can have a signal-to-noise ratio that is proportional to $\frac{1}{M-1}$.

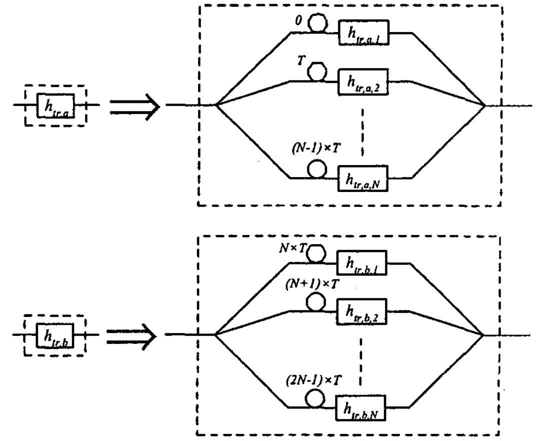

Figure 3: Substitution of transmitter filters

System description

This is accomplished by dividing each bit-time of the transmitted signal into $2\cdot N$ timeslots of length $T = T_b/(2\cdot N)$, where T_b is the length of a bit. We can do this by replacing the continuous light source in figure 2 by a pulsed light source with a duty cycle of $\frac{1}{2\cdot N}$, and replacing the transmitter filters by a filterbank of N filters, each delayed by a specific multiple of T, as shown in figure 3. Consequently, we have:

$$h_{tr,a}(t) = \frac{1}{\sqrt{N}}\cdot\sum_{k=1}^{N} h_{tr,a,k}\left(t-(k-1)\cdot T\right) \qquad (6)$$

and

$$h_{tr,b}(t) = \frac{1}{\sqrt{N}}\cdot\sum_{k=1}^{N} h_{tr,b,k}\left(t-(N+k-1)\cdot T\right) \qquad (7)$$

Using this structure, the transmitted signal will be slotted in time, each slot being filtered by a different filter, as illustrated in figure 4.

Note that only the last N slots are proportional to the information bit m.

$(x \otimes h_{tr,a,1})(t)$	$(x \otimes h_{tr,a,2})(t-T)$	…	$(x \otimes h_{tr,a,N})(t-(N-1)\cdot T)$	$-m \cdot (x \otimes h_{tr,b,1})(t-N\cdot T)$	$-m \cdot (x \otimes h_{tr,b,2})(t-(N+1)\cdot T)$	…	$-m \cdot (x \otimes h_{tr,b,N})(t-(2\cdot N-1)\cdot T)$
slot 1	slot 2		Slot N	slot N+1	slot N+2		slot 2·N

Figure 4: One bit-time of the transmitted signal

In the receiver, a similar substitution is made for the receiver filters, so we have:

$$h_{re,a}(t) = \frac{1}{\sqrt{N}} \cdot \sum_{k=1}^{N} h_{re,a,k}\left(t-(k-1)\cdot T\right) \qquad (8)$$

and

$$h_{re,b}(t) = \frac{1}{\sqrt{N}} \cdot \sum_{k=1}^{N} h_{re,b,k}\left(t-(N+k-1)\cdot T\right) \qquad (9)$$

Performance for one transmitter and one receiver

We can now compute the performance of the new system. For convenience, we will first observe the situation in which only one transmitter and one receiver are involved. Every subbranch in the receiver introduces a different delay and a different filtering. The outputs of the two filterbanks are then multiplied to form the output current of the receiver. This output current can be constructed using table I.

Each column in table I represents a timeslot in the output current. Each row represents the output of one receiver filter branch. The sum of the upper N rows thus represents the output of the upper receiver filter bank, and the sum of the lower N rows represents the output of the lower receiver filter bank. The output current is proportional to the the product of these two sums. This results in N^2 current components per timeslot. These components only have a non-zero average value if they are constructed by multiplying two contributions that correspond to the same source pulse, since two contributions that do not correspond to the same source pulse are not correlated.

Consequently, for example all components in timeslot N have zero average, since the contributions in the lower rows do not correspond to the same bit as the contributions in the upper rows. All components that do not disappear in this way have an average value that can be written as in (10).

$$\int_{-\infty}^{\infty} H_{tr,a\,\mathrm{or}\,b,k} \cdot H_{re,a,k'} \cdot H_{tr,a\,\mathrm{or}\,b,l}{}^{*} \cdot H_{re,b,l'}{}^{*} \cdot S_{xx^*} \cdot \mathrm{d}f \qquad (10)$$

If we (again) assume that such a term is only non-zero if the four transfer function form two pairs of complex conjugated functions, and that all transmitter filters are different and all receiver filters are different, we can conclude that there are no bias terms (all remaining terms are proportional to the message bit m).

Two choices for the relation between transmitter and receiver filters seem interesting:

- Equal filter banks:

$H_{re,a,k}=H_{tr,a,k}$ and $H_{re,b,k}=H_{tr,b,k}$ $\forall\ k$; (11)

- Matched filter banks:

$H_{re,a,k}=H_{tr,b,N+1-k}{}^{*}$ and $H_{re,b,k}=H_{tr,a,N+1-k}{}^{*}$ $\forall\ k$; (12)

In the case of equal filter banks, we have N^2 non-zero average output current components, non-uniformly divided over 2·N–1 timeslots. In the case of matched filters, we also have N^2 non-zero average output current components, but this time the average output current is concentrated in timeslot 2·N. Intuitively, this seems a more favourable situation, since we want to improve the overall signal-to-noise ratio. The average receiver current in slot 2·N can be written as in (13).

One can show that the beat noise spectral density in slot 2·N has 4·N^4 terms and can be written as in (14).

	$\frown$	$\frown$	$\frown$	$\frown$	$\frown$	$\frown$	$\frown$	$\frown$	$\frown$	$\frown$	$\frown$	$\frown$	$\frown$	$\frown$	$\frown$	$\frown$
$a,1$	$h_{tr,a,1}$ $h_{re,a,}$	$h_{tr,a,2}$ $h_{re,a,}$	…	$h_{tr,a,N}$ $h_{re,a,}$	m $h_{tr,b,1}$ $h_{re,a,}$	m $h_{tr,b,2}$ $h_{re,a,}$	…	m $h_{tr,b,N}$ $h_{re,a,}$	…	…	…	…	…	…	…	…
	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
$a,2$	…	$h_{tr,a,1}$ $h_{re,a,}$	…	$h_{tr,a,N}$ -1 h	$h_{tr,a,N}$ $h_{re,a,}$	m $h_{tr,b,1}$ $h_{re,a,}$	…	m $h_{tr,b,N}$ -1 h	m $h_{tr,b,N}$ $h_{re,a,}$	…	…	…	…	…	…	…
	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…
	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
a,N	…	…	…	$h_{tr,a,1}$ $h_{re,a,}$	$h_{tr,a,2}$ $h_{re,a,}$	$h_{tr,a,3}$ $h_{re,a,}$	…	m $h_{tr,b,1}$ $h_{re,a,}$	m $h_{tr,b,2}$ $h_{re,a,}$	m $h_{tr,b,3}$ $h_{re,a,}$	…	…	…	…	…	…
	$\smile$	$\smile$	$\smile$	$\smile$	$\smile$	$\smile$	$\smile$	$\smile$	$\smile$	$\smile$	$\smile$	$\smile$	$\smile$	$\smile$	$\smile$	$\smile$
	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×
	$\frown$	$\frown$	$\frown$	$\frown$	$\frown$	$\frown$	$\frown$	$\frown$	$\frown$	$\frown$	$\frown$	$\frown$	$\frown$	$\frown$	$\frown$	$\frown$
$b,1$	…	…	…	…	$h_{tr,a,1}$ $h_{re,b,}$	$h_{tr,a,2}$ $h_{re,b,}$	…	$h_{tr,a,N}$ $h_{re,b,}$	m $h_{tr,b,1}$ $h_{re,b,}$	m $h_{tr,b,2}$ $h_{re,b,}$	…	m $h_{tr,b,N}$ $h_{re,b,}$	…	…	…	…
	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
$b,2$	…	…	…	…	…	$h_{tr,a,1}$ $h_{re,b,}$	…	$h_{tr,a,N}$ -1 h	$h_{tr,a,N}$ $h_{re,b,}$	m $h_{tr,b,1}$ $h_{re,b,}$	…	m $h_{tr,b,N}$ -1 h	m $h_{tr,b,N}$ $h_{re,b,}$	…	…	…
	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…
	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
b,N	…	…	…	…	…	…	…	$h_{tr,a,1}$ $h_{re,b,}$	$h_{tr,a,2}$ $h_{re,b,}$	$h_{tr,a,3}$ $h_{re,b,}$	…	m $h_{tr,b,1}$ $h_{re,b,}$	m $h_{tr,b,2}$ $h_{re,b,}$	m $h_{tr,b,3}$ $h_{re,b,}$	…	…
	$\smile$	$\smile$	$\smile$	$\smile$	$\smile$	$\smile$	$\smile$	$\smile$	$\smile$	$\smile$	$\smile$	$\smile$	$\smile$	$\smile$	$\smile$	$\smile$
Slot	1	2	…	N	N+1	N+2	…	$2{\cdot}N$	1	2	…	N	N+1	N+2	…	$2{\cdot}N$
Bit	1								2							

Table I: Construction of the output current

$$\langle I(t)\rangle = -\frac{R_{pd}}{8\cdot N^2}\cdot m\cdot\sum_{k=1}^{N}\sum_{l=1}^{N}\int_{\infty}^{\infty}\left|H_{tr,a,k}(f)\right|^2\cdot\left|H_{tr,b,l}(f)\right|^2\cdot S_{xx^*}(f)\cdot \mathrm{d}f \qquad (13)$$

$$S_{II^*}(0) = \frac{R_{pd}^2}{64\cdot N^4}\cdot\sum_{k=1}^{N}\sum_{k'=1}^{N}\sum_{l=1}^{N}\sum_{l'=1}^{N}\int_{-\infty}^{\infty}\left|H_{tr,a,k}(f)\right|^2\cdot\left|H_{tr,b,k'}(f)\right|^2\cdot\left|H_{tr,a,l}(f)\right|^2\cdot\left|H_{tr,b,l'}(f)\right|^2\cdot S_{xx^*}^2(f)\cdot \mathrm{d}f \qquad (14)$$

Performance for *M* transmitters and *M* receivers

When we have a system with M transmitters and M receivers, each transmitter i has N filters $h_{tr,a,i,k}$ and N filters $h_{tr,b,i,k}$ and each receiver j has N filters $h_{re,a,j,k}$ and N filters $h_{r,b,j,k}$ with $1 \le k \le N$. When we consider one receiver r, we have to make M tables like Table I, each containing different transmitter filters.

The output current of receiver r consists of M^2 components. Each component arises as an interaction between signals from transmitter i and from transmitter j, where $1 \le i,j \le N$. Since these signals are not correlated for $i \ne j$, the average receiver output current consists of only M components (one for each transmitter). Consequently, we have one desired signal component and $M-1$ crosstalk components. Since the transmitters are generally not synchronized in time, these components are not synchronized either. For example, if transmitter i is lagging exactly one slottime compared to transmitter r, the crosstalk component from transmitter i in slot $2\cdot N$ of receiver r can be computed by observing slot $2\cdot N-1$ in the table corresponding to transmitter i. Consequently, all transmitter filters of all transmitters have to be orthogonal in order to avoid crosstalk for any timing situation between the transmitters. If we then choose the receiver filters to be matched to the transmitter filters as we did in section 3.2, the resulting average output current of receiver r is equal to (15).

Each of the M^2 combinations of two transmitters i and j contributes to the beat noise in slot $2\cdot N$ of the output current of receiver r. Each component can be constructed by combining the upper N rows in the correct column of the table corresponding to transmitter i and the lower N rows in the correct column of the table corresponding to transmitter j. Which columns to choose depends on the timing of transmitter i and j with respect to transmitter r. By counting the number of beat noise terms for every timing situation, one can show that the number of beat noise terms in timeslot $2\cdot N$ is:

- $2\cdot N^2$ if neither i nor j is equal to r;
- $2\cdot N^3$ if either i or j is equal to r;
- $4\cdot N^4$ if both i and j are equal to r (as we saw in section 3.2),

irrespective of the mutual timing of the interfering transmitters. This is illustrated in table II.

$\downarrow j$ \ $\rightarrow i$	1	2	...	r	...	M
1	$2\cdot N^2$	$2\cdot N^2$	...	$2\cdot N^3$	...	$2\cdot N^2$
2	$2\cdot N^2$	$2\cdot N^2$	...	$2\cdot N^3$	...	$2\cdot N^2$
...	...	...	...	...	...	...
r	$2\cdot N^3$	$2\cdot N^3$	...	$4\cdot N^4$	...	$2\cdot N^3$
...	...	...	...	...	...	...
M	$2\cdot N^2$	$2\cdot N^2$	...	$2\cdot N^3$	...	$2\cdot N^2$

Table II: *Number of beat noise components for each combination of interfering transmitters i and j*

$$\langle I_r(t)\rangle = -\frac{R_{pd}}{8\cdot N^2\cdot M}\cdot m_r\cdot\sum_{k=1}^{N}\sum_{l=1}^{N}\int_{\infty}^{\infty}\left|H_{tr,a,r,k}(f)\right|^2\cdot\left|H_{tr,b,r,l}(f)\right|^2\cdot S_{xx^*}(f)\cdot \mathrm{d}f \qquad (15)$$

Using this table, we can figure out that the total number of beat noise terms in the output current of receiver r is equal to:

$$2 \cdot (M-1)^2 \cdot N^2 + 4 \cdot (M-1) \cdot N^3 + 4 \cdot N^4 \quad (16)$$

Before we compute the signal-to-beat noise ratio, we have to note that the received bit-time is shortened when N is increased, which increases the required bandwith of the output low-pass filter and thereby decreases the signal-to-beat noise ratio by an extra factor $1/N$.

The received signal-to-beat noise ratio is thus given by (17).

$$SNR_b \propto \frac{N}{(M-1)^2 + 2 \cdot (M-1) \cdot N + 2 \cdot N^2} \quad (17)$$

We can optimize this equation with respect to N, which results in:

$$N_{opt} = \frac{M-1}{2} \quad (18)$$

$$SNR_{b,\max} \propto \frac{1}{M-1} \quad (19)$$

A total set of $M(M-1)$ orthogonal filters is needed in order to attain this result.

4. Conclusions

When the delay lines in a conventional Coherence Multiplexing System are replaced by filters, we see that a transmitter-receiver pair is matched when their filters are either equal or matched (where equal delays were demanded in the conventional approach). Both choices result in the same received signal power and noise power. The received signal-to-noise ratio is still proportional to the inverse of the square of the number of users, however, so the performance of the system is not significantly improved with respect to the conventional approach. When a pulsed light source is used and the filters are replaced by banks of filters, each filter element having a delay, the results for equal and matched filter banks are not the same. Only the matched filter banks situation is considered. In that case, the signal energy of a received bit is confined to only a small part of the bit-time, and signal-to-beat noise ratio can be made proportional to the inverse of the number of users instead of the inverse of the square of the number of users.

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