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A FLEXIBLE ROTOR ON FLEXIBLE SUPPORTS: MODELING & EXPERIMENTS

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ABSTRACT

In this study, a flexible rotor with variable support stiffness has been analyzed. Simple support models consisting of mass, spring systems are extracted from modal analysis of the isolated support and by applying static loads to the finite element model of the supports. The derived equivalent models of the supports are then implemented in the finite element based structural model which predicts the dynamic behavior of the rotor. Finally experimental modal analysis of the rotor is performed with different support stiffnesses. The experimental and theoretical results have been compared and different support modeling approaches have been examined.

NOMENCLATURE

B Compliance matrix
c Equivalent support damping coefficient
F Force vector
f Applied force in x and y direction
I Inertia of the support disc
K Stiffness matrix
k Equivalent support stiffness
k_b Translational stiffness of the bearing
k_s Translational stiffness of the support
k_{sr} Rotational stiffness of the support
k_r Rotational stiffness of the bearing
k_i i^{th} element of the support stiffness matrix
M Applied moment in x and y direction

m Mass of the support disc
q Vector of nodal displacements
x Translational deformation
θ Rotational deformation
$\Delta\omega$ Frequency difference between half power points
ω_n Natural frequency
ζ Damping ratio

INTRODUCTION

Flexible mechanical systems are gaining increasing significance. The trend for the design of modern rotating equipment is to build machines with flexible rotor-bearing systems, higher speeds and lower weight. This can be accomplished by predicting rotor dynamic behavior properly. Since the flexible supports have an important role on the dynamics of the structure, they should be taken into account in the rotordynamic structural model [1–9]. However including a detailed model of the support requires much effort and increases the computational time drastically. Therefore equivalent support models are deducted from experimental modal analysis [1–3, 5, 10, 11], from static loading tests [9] and by reduction of finite element models. In this study, an experimental setup which consists of a flexible rotor and flexible supports has been analyzed. The flexible supports are composed of three beams and a support disc. Modeling approaches have been developed in order to extract equivalent simple models which consists of masses, dampers and springs. Experimental modal analysis of the isolated supports and finite element model of the supports have been used to determine the parameters of the

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equivalent support models. Then these support models are implemented into the structural finite element model in MATLAB based on Timoshenko beam theory. Experimental modal analysis of the complete setup has been performed and compared with theoretical results using different support modeling approaches.

EXPERIMENTAL SETUP

The analyzed experimental set-up is shown in Fig. 1. The experimental setup consists of a stepped rotor (4), flexible coupling (7), flexible supports (1,2,3,6,10,11), high speed angular contact ball bearings (3,6) and the motor (9). The dimensions of the stepped rotor are given in Fig. 2 . The material properties of the rotor, the disc and beams of the support structure are provided in Tab.1

the support is changed. A detailed explanation of the experimental setup can be found in previous studies [12,13].

Table 1. MATERIAL PROPERTIES

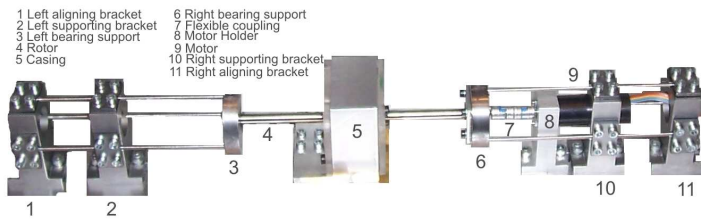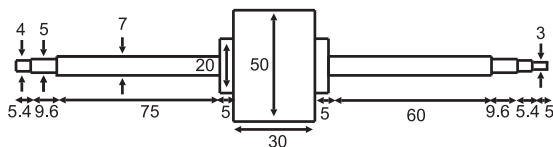|		Density	7730 kg/m^3
Rotor	Modulus of Elasticity	205 GPa
	Total Mass	522 g
Support Disc	Total Mass	162 g
Beams	Density	7850 kg/m^3
	Modulus of Elasticity	200 GPa

Figure 1. THE EXPERIMENTAL SETUP

Figure 2. THE DIMENSIONS OF THE STEPPED ROTOR (mm)

The rotor is suspended on high speed angular ball bearings mounted on the flexible supports. These bearings are selected since they are available in the market, easily mountable, and capable of supporting both axial and radial loads. Bearings are mounted on the discs of the flexible supports. These supports enable to investigate the dynamic behavior of the rotor with different support stiffness. The supports are designed to be axially rigid and radially flexible. Each support is composed of a disc in which the bearing is mounted, three beams to determine the stiffness, two brackets- one for the alignment and the other for the adjustment of the support stiffness. The support stiffness is determined by changing the position of the supporting bracket. In this way the length of the beams is altered and the stiffness of

EXPERIMENTAL MODAL ANALYSIS OF THE ISOLATED SUPPORT

Experimental modal analysis of the isolated support has been performed in order to examine the dynamic behavior of the support with different support beam lengths and to extract equivalent support models. A shaker is used for excitation and measurements are obtained by a laser doppler vibrometer and orthogonally placed accelerometers (Fig. 3).
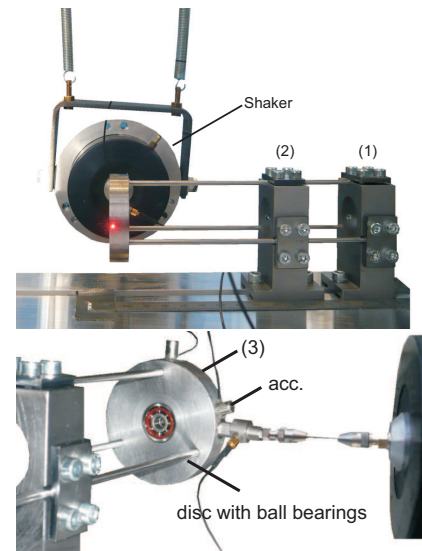

Figure 3. EXPERIMENTAL MODAL ANALYSIS OF THE ISOLATED SUPPORT

The frequency response functions (FRF) of the support disc at different support beam lengths are presented in Fig. 4. It is

observed that the beam length has a significant effect on the support dynamics. Extraction of equivalent models from FRFs for different support beam lengths is discussed in the following sections.
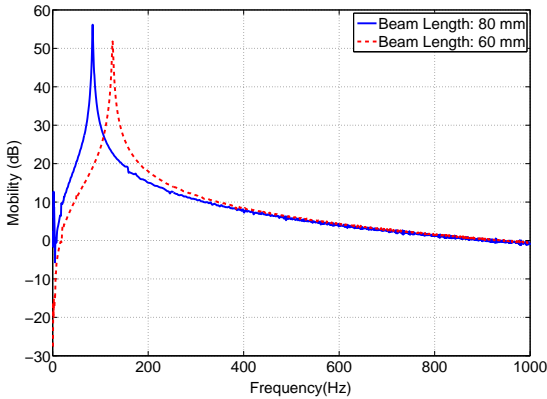

Figure 4. SUPPORT FRF WITH DIFFERENT BEAM LENGTHS

FINITE ELEMENT MODEL OF THE SUPPORT

A finite element model for the supports has been generated to extract the equivalent stiffness matrix for the supports using the commercial software package ANSYS. The supports are modeled by structural solid element:Solid92. Static loading simulations are required to determine the parameters of the support model. However in order to examine the capability of the model in representing the dynamic behavior of the support, at first a modal analysis of the model has been performed and compared with the experimental results. Fig. 5 illustrates the FE model and Tab. 2 presents the results of the modal analysis simulations and experiments for different beam lengths.
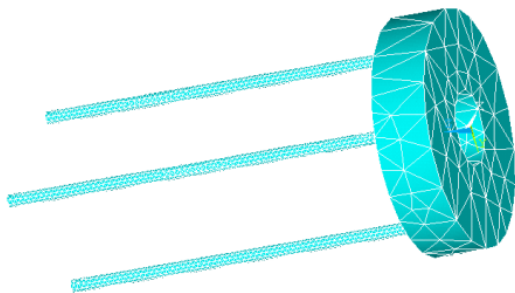

Figure 5. ANSYS MODEL OF THE SUPPORT

Table 2. NATURAL FREQUENCIES OF THE SUPPORT

	Simulation (Ansys)		
Beam Length	70 mm	80 mm	90 mm
Natural frequency	108.8 Hz	89.5 Hz	75.4 Hz
	Experiments		
Beam Length	70 mm	80 mm	90 mm
Natural frequency	101.3 Hz	83.8 Hz	73.8 Hz

The theoretical results are in fair agreement with the experimental ones. So this Ansys model can be used to extract the simplified equivalent stiffness matrix with coupled terms for rotation and translation. The determination of the equivalent support stiffness matrix will be explained in the upcoming sections.

EXPERIMENTAL MODAL ANALYSIS OF THE TOTAL SYSTEM

The modal analysis of the complete system with different beam lengths has been performed at standstill to obtain the dynamic behavior of the complete system. Random excitation has been provided by the shaker mounted on the support. The response is measured by the accelerometers mounted on the supports and the LDV directed towards the rotor surface. At first the motor is disassembled and the supporting bracket moved to the support disc making the supports rigid. In this way only the flexibility of the ball bearings are effective and they can be determined. The related FRF is shown in Fig. 6.
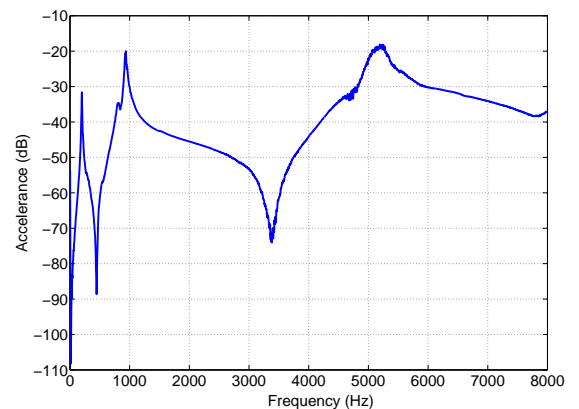

Figure 6. FRF OF THE ROTOR WITH RIGID SUPPORTS

Afterwards the motor is mounted and the experiments are repeated for support beam lengths of 72 mm, 80 mm and 90 mm

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at standstill respectively. The FRF results for these three configurations are given in Fig. 7. The natural frequencies of the rotor at different support beam length are summarized in Tab. 3. Changing the beam length results in a significant change of the dynamics of the structure. Not only rigid body mode natural frequencies change but also the natural frequency belonging to the first flexural mode changes by altering the beam length. In the next sections, the experimental results will be compared with the theoretical results obtained from different support models.
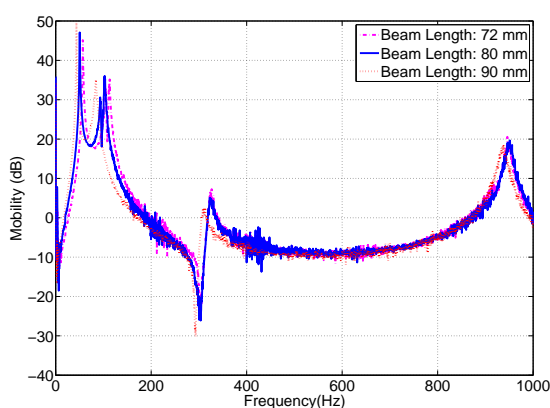

Figure 7. FRFs AT DIFFERENT SUPPORT BEAM LENGTH

Table 3. NATURAL FREQUENCIES-DIFFERENT CONFIGURATIONS

Beam Length:	72 mm	80 mm	90 mm
1st Rigid body	57.2 Hz	50.6 Hz	43.7 Hz
2nd Rigid body	113.4 Hz	102.8 Hz	84.0 Hz
1st Flexural	327.2 Hz	320.1 Hz	311.6 Hz

EQUIVALENT SUPPORT MODELS
Support: Mass-Spring-Damper

In the first model the support has been modeled as a mass, spring and damper and only translational degrees of freedom have been taken into account. Bearing stiffness has been ignored since the bearing stiffness is much higher than the support stiffness. The first support model and the rotor are illustrated in Fig.8. The equivalent stiffness and equivalent damping coefficient for the support are extracted from experimental modal analysis of the isolated support as shown in Fig. 4. An equivalent
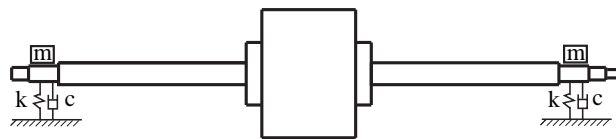

Figure 8. FIRST SUPPORT MODEL

spring-damper system can be constructed from the modal analysis data of the isolated support by using the well known peak amplitude method. The details of this method are explained by Ewins [14]. For each beam length, the stiffness is calculated by $k = \omega_n^2 m$ where m is the mass of the support disc. The damping ratio is $\zeta = \Delta\omega/\omega_n$ from which the support damping coefficient is found. Then these mass, stiffness and damping values representing the supports are implemented in the structural model for the theoretical dynamic analysis of the total system. The theoretical and experimental results are compared in Tab. 4.

Table 4. NATURAL FREQUENCIES

Experimental	Theoretical
50.6 Hz	51.4 Hz
102.8 Hz	80.1 Hz
320.1 Hz	263 Hz
962.1 Hz	877 Hz

There is a significant difference between the theoretical and experimental results. Therefore the support model has to be enhanced.

Support & Bearing with Rotational DOF

The support model consisting of a translational spring and damper and the mass directly added to the connection node has been observed not to be adequate to predict the dynamic behavior of the system. Therefore an additional degree of freedom has been added and the bearing rotational and translational stiffness have been taken into account. In addition the rotational stiffness has also been included in the support model. The bearing translational stiffness has been estimated from the supplier manual and in order to determine the rotational stiffness of the bearing the simulations for rigid supports (support beam length: 0 mm) are performed with different bearing stiffnesses and compared with the experimental results. In this way the support stiffness has been eliminated and the bearing stiffness can be independently examined. Then, appropriate rotational stiffness for the bearing has been determined. After determining the bearing stiffness the rotational and translational stiffness of the supports should be de-

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termined. The previously explained ANSYS model of the supports (Fig. 5) is used to extract the rotational and translational stiffness of the supports statically. Force and moment has been applied in the bearing positions, corresponding displacement and rotation has been obtained. The translational and rotational stiffness values are calculated by dividing the force and moment by displacement and rotation ($k_s = f/x$, $k_{sr} = M/\theta$). The corresponding model is shown in Fig.9.
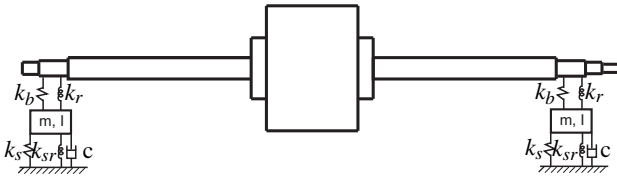

Figure 9.   SUPPORT MODEL WITH ROTATIONAL DOF

After adding the rotational DOFs to the model the results have been improved. Tab. 5 illustrates the new results with this modeling approach.   Remarkable improvement has been

Table 5.   NATURAL FREQUENCIES

Experimental	Theoretical
50.6 Hz	53.8 Hz
102.8 Hz	108.1 Hz
320.1 Hz	313.5 Hz
962.1 Hz	896.1 Hz

achived by taking the rotational DOFs of the supports into account. The increase of the natural frequencies are resulted from the disc mass which is connected to the rotor with springs resembling the bearing. In the previous modeling approach this mass was directly added to the rotor model. The first two natural frequencies are in fair agreement for both experiments and theory. However the estimation for the flexural mode natural frequency could be improved.

Support & Bearing with Coupled Rotational DOF

Finally the cross coupling between rotational and translational stiffness terms is included in the support stiffness matrix. This support model and the rotor are shown in Fig.10.

The same stiffness values are used for the bearings as in the previous model. However the support stiffness has been calculated with a different approach. Static loads (force and moment) are applied on the support disc and deformations (translational
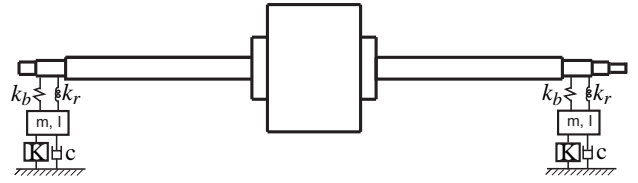

Figure 10.   SUPPORT MODEL WITH COUPLED STIFFNESS TERMS

and angular displacement) are obtained in order to calculate the equivalent stiffness matrix for the support. The stiffness matrix with cross coupled terms for the support is calculated by:

$$\mathbf{BF} = \mathbf{q} \qquad (1)$$

where $\mathbf{B}$ is the compliance matrix, $\mathbf{q}$ is the displacement vector, $\mathbf{F}$ is the force vector and $\mathbf{K}$ is the stiffness matrix given as:

$$\mathbf{q} = \begin{bmatrix} x \\ \theta \end{bmatrix}, \quad \mathbf{F} = \begin{bmatrix} f \\ M \end{bmatrix}, \quad \mathbf{K} = \mathbf{B}^{-1} = \begin{bmatrix} k_1 & k_2 \\ k_3 & k_4 \end{bmatrix}$$

The simulations with this support model are performed and compared with the experimental results. The experimental, theoretical results and the difference for a support beam length of 80 mm are presented in Tab. 6.

Table 6.   NATURAL FREQUENCIES

Experimental	Theoretical
50.6 Hz	54.1 Hz (3.6 %)
102.8 Hz	108.2 Hz (5.4 %)
320.1 Hz	325.8 Hz (1.8 %)
962.1 Hz	934.9 Hz (2.8 %)

The final support model yields some improvement in the prediction of dynamic behavior of the rotor. The better estimations for the third and fourth natural frequency are achieved. Therefore this support modeling approach with rotational DOF and cross coupled terms are capable of representing the dynamic behavior of the flexible supports.

CONCLUSION

A flexible rotor on flexible supports has been studied. Each flexible support includes three beams, two brackets and a bearing housing. Different approaches for the modeling of the flexible supports have been examined. Equivalent simple models have

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been extracted and applied to a structural model. Then the theoretical results have been compared with the experiments. At first simple models consisting of mass-spring and damper have been extracted. These models could not predict the second and third natural frequency properly. Then the rotational DOF, rotational stiffness and inertia been included in the extracted models. The experimental results and theoretical results are in fair agreement. Finally the cross coupled terms of the support stiffness matrix have been included and additional improvement in theoretical results has been achieved. The parameters used for different support models have been provided in App. A.

ACKNOWLEDGMENT

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Appendix A : Parameters Used in the Support Models

1) Mass-Spring-Damper
c= 8.39 Ns/m
m=162 g
k=4.76×10^4 N/m

2) Support & Bearing with Rotational DOF
I=2.767×10^{-5} kgm^2
k_s=5.16×10^4 N/m
k_{sr}=1.03×10^4 N/rad
k_b=1×10^7 N/m
k_r=500 N/rad

3) Support & Bearing with Coupled Rotational DOF
k_1= 5.20×10^4 N/m
k_2= -0.21×10^4 N/rad
k_3= -0.21×10^4 N/m
k_4= 1.03×10^4 N/rad