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Non-Collinear Wave Mixing for a Bulk Wave Phase Velocity Measurement in an Isotropic Solid

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***Abstract*—A measurement method is presented to estimate the bulk wave phase velocity in an isotropic solid when longitudinal or shear wave velocity is known. This method is based on the non-collinear plane wave interaction theory and it does not need to estimate the phase time-of-flight and wave propagation path of ultrasonic wave in a specimen. It is necessary to measure incident angles of pump waves for estimation of the longitudinal or shear wave phase velocity. Using the proposed method, the shear wave phase velocity is measured in an aluminum specimen to be 3189 m/s ± 202 m/s and 3174 m/s ± 112 m/s at a level of confidence of 95 % depending on a selected wave mixing method.**

Keywords—non-linear ultrasonics; phase velocity; uncertainty

I. INTRODUCTION

In linear elasticity an isotropic solid with a given material density ρ is characterized by two independent stiffness constants c_{12} and c_{44} known as Lamé parameters λ and μ. Knowing all three material parameters c_{12}, c_{44} and ρ, one can estimate longitudinal and shear wave velocities in an isotropic solid from the well-known relations:

$$c_l = \sqrt{\frac{\lambda + 2\mu}{\rho}}, \quad (1)$$

$$c_s = \sqrt{\frac{\mu}{\rho}}. \quad (2)$$

However, the sound velocities or stiffness constants can be estimated from ultrasonic measurements directly, especially the longitudinal wave velocity, because transduction of the longitudinal waves does not require special techniques or special transducers as in a case of shear wave transduction [1]. Therefore the shear wave phase velocity measurements is analyzed more in detail. The shear wave velocity can be measured using a) shear wave transducers [1] or b) mode conversion when a longitudinal wave is incident to a solid at oblique angle [2, 3]. Both ways are used in research and industry widely. Alternative measurement techniques such as laser-ultrasonic techniques, ultrasonic interferometry, etc. are not discussed in this paper.

The piezoelectric shear wave transducers are used for contact measurements only. Due to that the measurement results are influenced by the contact properties between the shear wave transducers and test specimen. The measurements of the shear wave velocity employing the mode conversion can be carried out in a contactless way, e.g. using immersion ultrasonic measurement technique. In some cases electromagnetic ultrasonic transducers can be used for contactless ultrasonic transduction of the shear wave [4]. Summarizing shear wave velocity measurement methods, one can conclude that the shear wave velocity is estimated from the following generalized expression:

$$c = \frac{d}{t}, \quad (3)$$

where d is the wave propagation path in the specimen and t is the time-of-flight of the wave in the path d. In this measurements complications occur with estimation of correct time-of-flight t, because it is not easy to measure the correct phase time-of-flight, especially if the wave propagation is in a dispersive medium. Moreover, a determination of the wave propagation path d also becomes complicated when the oblique incidence of ultrasonic waves is used in the measurements. In this case d becomes a function of the wave incidence angle θ.

In this work we present a measurement method to estimate the bulk wave phase velocity in an isotropic solid when longitudinal or shear wave velocity is known. The suggested measurement method is based on the non-collinear plane wave interaction theory. Applying the method, longitudinal or shear wave phase velocity can be measured in an isotropic solid directly. It does not need to estimate the phase time-of-flight and wave propagation path of ultrasonic wave in a specimen. However, it is necessary to measure the incident angles of pump waves. Measurements of the shear wave phase velocity are analyzed in more detail, because it is more complex than the longitudinal wave phase velocity measurements.

II. BULK WAVE PHASE VELOCITY MEASUREMENT METHOD BASED ON NON-COLLINEAR WAVE INTERACTION

The synchronism conditions for two interacting phonons $\mathbf{k}_1$ and $\mathbf{k}_2$ can be written in terms of conservation laws for quasi-momentum and energy of interacting phonons [5, 6]:

$$\mathbf{k}_1 \pm \mathbf{k}_2 = \mathbf{k}_3, \quad (4)$$

$$\omega_1 \pm \omega_2 = \omega_3, \quad (5)$$

where $\mathbf{k}_i$ (i = 1,2,3) is the wave-vector of the phonon, ω_i is the angular frequency of the phonon and $\omega_i = 2\pi f_i$. f_i is the frequency. The synchronism conditions are necessary, but not

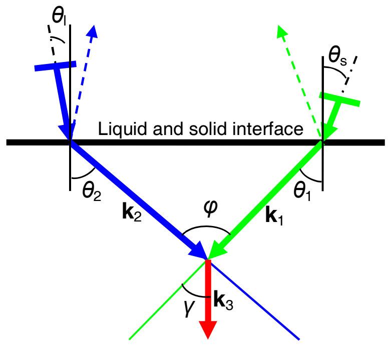

Fig. 1. Wave interaction geometry for S(ω_1)+L(ω_2)→L(ω_1+ω_2) process.

sufficient. The allowed interaction cases and their conditions are presented in literature [5, 6], and will not be discussed here. Let us analyse the wave interaction process which is denoted by S(ω_1)+L(ω_2)→L(ω_1+ω_2), where S(ω_1) and L(ω_2) are the shear and longitudinal pump waves, respectively, and L(ω_1+ω_2) is the longitudinal non-linear wave. In this case (4) is solved in the following form:

$$\left(\frac{\omega_1}{c_s}\right)^2+\left(\frac{\omega_2}{c_l}\right)^2+2\frac{\omega_1\omega_2}{c_s c_l}\cos\varphi=\left(\frac{\omega_1+\omega_2}{c_l}\right)^2, \qquad (6)$$

where φ is the angle between two interacting waves (see Fig. 1). Equation (5) contains two unknown parameters: velocity of ultrasonic pump wave (the second one is known) and interaction angle φ. Therefore it is necessary to eliminate one unknown parameter in this equation. For this, the interaction angle φ is expressed as a sum of refraction angles of the pump waves (see Fig. 1):

$$\varphi=\arcsin\left(\frac{c_s}{c_0}\sin\theta_1\right)+\arcsin\left(\frac{c_l}{c_0}\sin\theta_2\right), \qquad (7)$$

where c_0 is the known ultrasonic wave velocity in the surrounding liquid. Substituting (7) into (6), the latter transforms into the following form:

$$\left(\frac{\omega_1}{c_s}\right)^2+\left(\frac{\omega_1}{c_l}\right)^2+2\frac{\omega_1\omega_2}{c_s c_l}\times$$
$$\times\cos\left[\arcsin\left(\frac{c_s}{c_0}\sin\theta_1\right)+\arcsin\left(\frac{c_l}{c_0}\sin\theta_2\right)\right]=\left(\frac{\omega_1+\omega_2}{c_l}\right)^2. \qquad (8)$$

Equation (8) can be solved for one of unknown parameters: either for the longitudinal wave velocity c_l or for the shear wave velocity c_s. One can see that (8) is a non-linear function; therefore it is solved numerically by minimization of the objective function:

$$\min\left(f_1-f_2\right)^2, \qquad (9)$$

where f_1 and f_2 are the left and right sides of (8), respectively.

III. EXPERIMENTAL MEASUREMENTS

Experimental measurements were performed for aluminum specimen with dimensions 286 mm × 124 mm × 60 mm. Initially a pulse-echo measurement was carried out for estimation of the longitudinal wave velocity in the specimen, using a spherically focused broadband ultrasonic transducer of 10 MHz central frequency. A single pulse excitation was used. A phase-spectrum method was employed for the phase velocity data extraction from the received ultrasonic signals [7]. A very little dispersion was observed in the aluminum specimen (see Fig. 2).

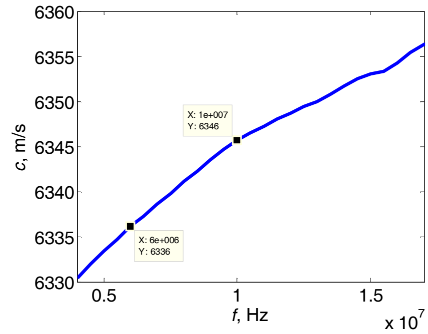

Fig. 2. Measured longitudinal wave phase velocity in the aluminum specimen.

In the non-collinear wave mixing experiment an arrangement of the test specimen and ultrasonic transducers in the pulse-echo mode is presented in Fig. 3, when the maximum amplitude of the non-linear ultrasonic wave is measured. Frequencies of the two pump waves were selected such that the non-linear wave would strike the aluminum and water interface perpendicularly. From geometry (Fig. 1) it is seen that the refraction angle of the scattered wave can be calculated from the Snell's law in the following way when the wave propagates over a solid and liquid interface:

$$\theta=\arcsin\left(\frac{c_0}{c_3}\sin\left(\theta_1-\gamma\right)\right), \qquad (10)$$

where c_0 is the ultrasonic wave velocities in the liquid, c_3 is the scattered wave's velocity. γ is the angle at which the scattered wave emerges from the point of interaction [5].

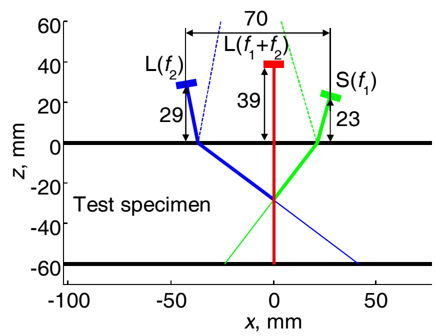

Fig. 3. Arrangement of ultrasonic transducers for the non-linear wave mixing process S(ω_1)+L(ω_2)→L(ω_1+ω_2) in the aluminum specimen (pulse-echo measurement mode).

Two broadband transducers of 5 MHz central frequency were used to generate the pump waves. The shear wave pump transducer was inclined at angle θ_s = 14° initially and driven by a burst of 30 cycles of rectangular pulses of f_1 = 4 MHz. The longitudinal wave pump transmitter was inclined at angle θ_l = 9° initially and driven by a burst of 30 cycles rectangular pulses of f_2 = 6 MHz. A fine adjustment of the angles was performed manually using rotary stages with a tuning resolution of 0.008°. The maximum amplitude of the non-linear ultrasonic wave was measured when θ_l = 10.7° and θ_s = 16.2°. The generated wave of 10 MHz was received by the spherically focused broadband receiver of 10 MHz central frequency which was perpendicular to the water and aluminum interface. Reception of ultrasonic waves was carried out from both sides of the test specimen for demonstration of the pulse-echo and through-transmission measurement capabilities. The received signals were filtered using a narrowband FIR filter with a Kaiser window and stored on a personal computer for further analysis.

Typical ultrasonic signals are presented in Figs. 4 and 5 when the pulse-echo and the through-transmission arrangement of transducers were used in the non-collinear wave mixing experiments. One can see that the through-transmission arrangement of transducers enables to receive the non-linear ultrasonic wave with a better signal-to-noise ratio, because specularly reflected pump waves do not interfere with the ultrasonic receiver.

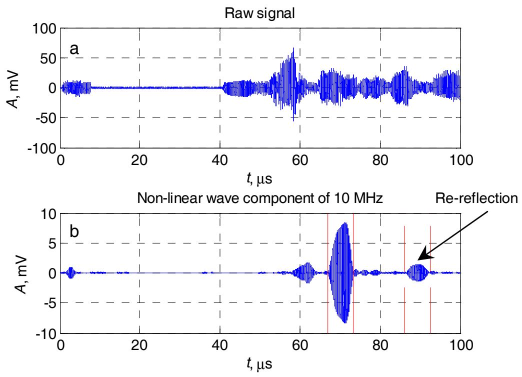

Fig. 4. $S(\omega_1)+L(\omega_2)\rightarrow L(\omega_1+\omega_2)$ wave interaction process and the pulse-echo arrangement of transducers: raw time-domain signal (a) and non-linear wave component (b). Two vertical solid lines show the first informative signal, and two vertical dashed lines show the re-reflection of the first informative signal.

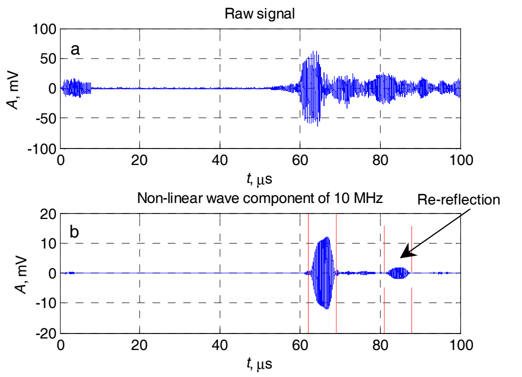

Fig. 5. $S(\omega_1)+L(\omega_2)\rightarrow L(\omega_1+\omega_2)$ wave interaction process and the through-transmission arrangement of transducers: raw time-domain signal (a) and non-linear wave component (b). Two vertical solid lines show the first informative signal, and two vertical dashed lines show the re-reflection of the first informative signal.

The measurement procedure for bulk wave phase velocity measurement contains the following steps:

1. Initial measurement of one of the ultrasonic wave phase velocities in a specimen (in our case longitudinal wave phase velocity). This step can be skipped if the velocity is known.
2. Initialization of the non-collinear wave mixing measurement assuming that the shear wave phase velocity is approximately half of the longitudinal wave velocity.
3. Iterative adjustment of the angles of the pump wave transducers until the maximum amplitude of the non-linear ultrasonic wave is measured.

A capturing of the re-reflected informative signal (see Figs. 4b and 5b) is a good stop feature for adjustment cancelation of the angles θ_l and θ_s.

Using experimental data (incident angles θ_l and θ_s, longitudinal wave velocity c_l = 6336 m/s for the pump wave of 6 MHz, and c_l = 6346 m/s for the generated non-linear wave of 10 MHz in the aluminium specimen (see Fig. 2) and wave velocity in water c_0 = 1481 m/s), the shear wave phase velocity was estimated minimizing (9). The shear wave phase velocity was found to be 3194 m/s at 4 MHz frequency. After complete disassembling and assembling of the measurement setup, the measurement was repeated. The following wave incidence angles were measured: θ_l = 10.74°and θ_s = 16.2°. In this case the estimated shear wave phase velocity was 3186 m/s at 4 MHz frequency.

The measurement results are validated employing the wave interaction process which is denoted by $S(\omega_1)+S(\omega_2)\rightarrow L(\omega_1+\omega_2)$, where $S(\omega_1)$ and $S(\omega_2)$ are the shear pump waves and $L(\omega_1+\omega_2)$ is the longitudinal non-linear wave. The through-transmission measurement mode was used in the experiment. The following pump wave incident angles were measured: θ_s = 27°and θ_s = 20.12°. The corresponding ultrasonic signal is presented in Fig. 6. The longitudinal wave velocity c_l was substituted by c_s on the left side in (8). It is important to note that the possible dispersion of the shear wave phase velocity was neglected. This assumption was used, because the shear wave phase velocity was not measured at 6 MHz frequency. Taking into account the presented assumption, the estimated shear wave phase velocity was 3174 m/s in the aluminum specimen.

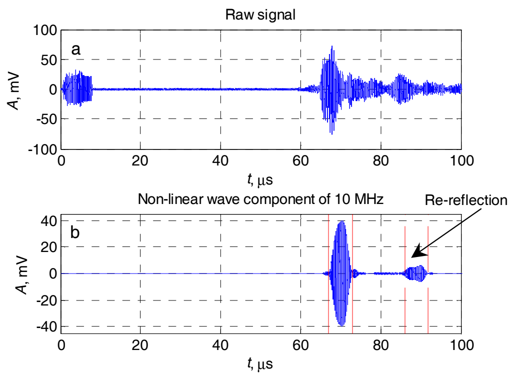

Fig. 6. $S(\omega_1)+S(\omega_2)\rightarrow L(\omega_1+\omega_2)$ wave interaction process and the through-transmission arrangement of transducers: raw time-domain signal (a) and non-linear wave component (b). Two vertical solid lines show the first informative signal and two vertical dashed lines show the re-reflection of the first informative signal.

IV. UNCERTAINTY ANALYSIS

The Monte Carlo analysis is used here for uncertainty analysis of the measured shear phase velocity in the aluminum specimen. Both used non-collinear wave mixing cases are analyzed. The following measurement models are used in the analysis:

$$\min\left[\left(\frac{\omega_1}{c_s}\right)^2+\left(\frac{\omega_2}{c_l}\right)^2-\left(\frac{\omega_1+\omega_2}{c_l'}\right)+2\frac{\omega_1\omega_2}{c_s c_l}\times \cos\left[\arcsin\left(\frac{c_s}{c_0}\sin\theta_1\right)+\arcsin\left(\frac{c_l}{c_0}\sin\theta_2\right)\right]\right]^2, \quad (11)$$

$$\min\left[\left(\frac{\omega_1}{c_s}\right)^2+\left(\frac{\omega_2}{c_s'}\right)^2-\left(\frac{\omega_1+\omega_2}{c_l'}\right)+2\frac{\omega_1\omega_2}{c_s c_s'}\times \cos\left[\arcsin\left(\frac{c_s}{c_0}\sin\theta_1\right)+\arcsin\left(\frac{c_s'}{c_0}\sin\theta_2\right)\right]\right]^2, \quad (12)$$

where (11) and (12) correspond to the wave mixing cases S(ω_1)+L(ω_2)→L(ω_1+ω_2) and S(ω_1)+S(ω_2)→L(ω_1+ω_2), respectively. c_l' and c_s' are the longitudinal and shear wave phase velocities at 10 MHz and 6 MHz frequencies, respectively. c_l and c_s are the longitudinal and shear wave phase velocities at 6 MHz and 4 MHz frequencies, respectively. Variables and their corresponding inputs used for the uncertainties analysis are listed in Table 1. It is important to note that all variables are B type and have ∞ degree of freedom. 1.5M iterations are made in the uncertainty analysis. In Table 1, the following notation is introduced: Δx_i is the deflection of the source x_i, D is the probability distribution, N and R are the normal and rectangular distributions, respectively, u_c and U are the combined standard and expanded uncertainties, respectively.

The Monte Carlo analysis shows that expanded uncertainties are 6.3% and 3.5% for the measured shear wave phase velocity in the aluminum specimen using S(ω_1)+L(ω_2)→L(ω_1+ω_2) and S(ω_1)+S(ω_2)→L(ω_1+ω_2) wave mixing cases, respectively. The level of confidence is 95.45%.

Table 1. Uncertainties budget.

Source	**S(ω_1)+L(ω_2)→L(ω_1+ω_2)**			**S(ω_1)+S(ω_2)→L(ω_1+ω_2)**		
x_i	**Value**	Δx_i	**D**	**Value**	Δx_i	**D**
θ_1, deg.	10.7	±0.5	N	27	±0.5	N
θ_2, deg.	16.2	±0.5		20.1	±0.5	
c_l, m/s	6336	±10	R			R
c_0, m/s	1481	±1		1481	±1	
c_l', m/s	6346	±10		6346	±10	
c_s', m/s				3174	±5	
u_c, m/s	±101		N	±56		N
U, m/s	±202		N	±112		N

V. DISCUSSION AND CONCLUSIONS

A bulk wave phase velocity measurement method is proposed in this work. This method is experimentally verified measuring shear wave phase velocity in an aluminum specimen. Using the Monte Carlo analysis, it is demonstrated that in the aluminum specimen the shear wave phase velocity is measured with 6.3% and 3.5% expanded uncertainties depending on a selected wave mixing method. In the uncertainty analysis it is assumed that the incident angles of two pump waves are known with a ±0.5° deflection. A lower incident angle deflection results in a lower uncertainty. For example, when the deflection is ±0.25° only, the expected uncertainty of the shear wave phase velocity becomes 1.6% for the S(ω_1)+L(ω_2)→L(ω_1+ω_2) wave mixing process. Multiple measurements of the wave velocity at different frequencies enable to establish the system of equations. Therefore it is possible to develop more advanced objective function for more accurate estimation of the phase velocity from experimental data.

The suggested bulk wave phase velocity measurement method has the following advantages: a) direct measurement of the phase velocity without the need to estimate time-of-flight of wave and wave propagation path; b) contactless measurements are possible in a through-transmission or pulse-echo modes; c) it is possible to measure ultrasonic wave phase velocity in a bonded structure without the need to know ultrasonic properties of the whole structure. It allows to steer the pump wave beams to a certain location or layer in a joined or layered structure. This advantage can be applied for quality control of embedded structures. However, synchronism conditions for the non-collinear wave interaction and employment of three transducers are to be used in the suggested measurement method.

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