A correlating receiver for OFDM at low SNR

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***Abstract*—By extending OFDM symbols, acceptable BER performance can be achieved at low SNRs. Two alternative differential receiver architectures are presented, a receiver based on a FX correlator (Fourier transformation before correlation) and based on an XF correlator (correlation before Fourier transformation). To reduce the complexity and hence the power consumption of both the ADC and the first digital processing stage single- or two bit quantization is used. The receiver based on the XF correlator is more suited to exploit such coarse quantization.**

Two basic effects are visible if coarse quantization is used. First, the BER performance is reduced due to the introduction of quantization errors. Second, beyond certain SNR levels, the BER performance does not increase due to the correlation between quantization errors. Furthermore, oversampling increases BER performance considerably. For single bit quantization with oversampling, acceptable BERs ($< 10^{-3}$) can be achieved for a limited SNR range for symbol extension factors of 32 and 64. In case of two bit quantization without oversampling, the results are comparable with single bit quantization with two times oversampling. For two bit quantization in combination with two times oversampling, acceptable BERs are achieved for symbol extension factors 8, 16, 31 and 64.

***Index Terms*—Correlation, Differential phase shift keying, Fourier transforms, Frequency division multiplexing, Modulation**

I. Introduction

Traditionally, the electromagnetic spectrum has been segmented into relatively narrow bands where different bands are used for different services. Access is either free (e.g. ISM bands) or regulated by means of licenses. Because of this segmentation, spectrum efficiency has become the major design parameter for communication systems. This has lead to a widespread use of high order modulation schemes where carriers are modulated in amplitude and/or phase. One of the main characteristics of higher order modulation schemes is that a linear increase of communication capacity (bits per second per Herz) is accompanied with an exponential increase in power [1]. Especially for mobile applications, limited power is available. For that reason, the segmented spectrum in practice offers limited communication capacity to the different services. Besides the technical challenges concerning the realization of narrow-band, high bit-rate, high power transmitters and receivers, there are other issues as well. Because of the worry with respect to global warming, minimizing power consumption has become a key issue. Furthermore, since the discussion on the effects of prolonged exposure to electromagnetic radiation is still ongoing, it is wise to avert this discussion by emitting as less power as possible. Concluding, because of technical and societal reasons, the current policy of segmenting the spectrum has become obsolete. To counteract the segmentation of spectrum, two major research areas have evolved: ultra wide-band (UWD, [2]) and Cognitive Radio (CR, [3]). In UWB, a relatively large frequency span has been made available with tight restrictions on the average emitted power levels per unit frequency. Cognitive Radio is based on the observation that, although the segmented spectrum is completely allocated to different services, large parts of the spectrum are hardly used. CR aims at exploiting the unused part of the spectrum as an unlicensed, secondary user.

Anticipating on the results of these two recent developments, our paper is based on the premis that, in the near future, large frequency bands will be accessible for communication purposes. Consequently, spectrum efficiency will no longer be the decisive design parameter, but power efficiency (power consumed by transmitter and receiver) and low emission (energy per bit) will be.

The communication system presented in this paper is based on the availability of a large frequency band and a low required spectral capacity (less than 2 bits per second per Hertz). This prevents us from using power inefficient, high order modulation schemes. The main question that then remains is how to design a power efficient, large bandwidth system. Key issue is to prevent the use of high-frequency high-resolution Analog to Digital converters in the receiver to reduce power consumption (see [4]). We propose to use extended symbols, constructed via repetition of low-power symbols in the time domain. Note that extending symbols results in lower data rates. To prevent vulnerability to frequency offsets because of extended symbol lengths, differential modulation will be used. The receiver architecture is based on coarse quantization and low resolution cross correlation to reduce power consumption. In this paper we first introduce OFDM based on extended symbols (ES-OFDM). Both the transmitter and receiver are addressed. For the receiver, two different architectures are presented. First, an architecture based on the FX correlator (see e.g. [5]), second, an architecture based on an XF correlator (see e.g. [6]). A comparison between these architectures can be found in [7]. The performance of ES-OFDM is analyzed using differential modulation disregarding quantization effects. In section III we analyze the effects of coarse quantization in combination with oversampling.

II. Differential ES-OFDM

Differential ES-OFDM is based on the assumption that the receiver is synchronized to the transmitter in time and

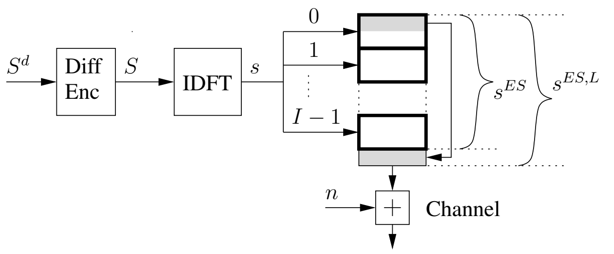

Fig. 1. Base-band equivalent model of the differential ES-OFDM transmitter

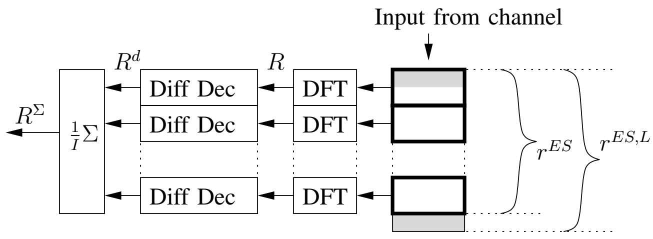

Fig. 2. Base-band equivalent model of the differential ES-OFDM FX receiver

frequency but not necessarily in phase. Phase offsets might be caused by (relatively small) frequency offsets. Using differential modulation will make ES-OFDM more robust against these small frequency offsets. However this comes at the expense of SNR performance. Both frequency- and time differential demodulation detection (FDDD and TDDD respectively) can be used in an OFDM system [8], [9]. In this paper, we will use FDDD modulation.

A. *Transmitter*

A base-band equivalent model of the relevant parts of the ES-OFDM transmitter for differential modulation is presented in figure 1.

At the transmitter, a modulator (not shown in Figure 1) produces S^d which consists of N complex values (indicated as S^d_f, $f = 0, 1, ..., N-1$), where each value is a constellation point from a chosen modulation scheme. In this paper we restrict ourselves to BPSK. The carriers are differentially encoded.

$$S_f = S_{f-1} \cdot S^d_f, \quad f = 1, 2, ..., N-1 \tag{1}$$

$$S_0 = 1 \tag{2}$$

S is transformed into the time domain through the IDFT giving s.

$$s_t = \frac{1}{\sqrt{N}} \sum_{f=0}^{N-1} S_f e^{j\frac{2\pi f t}{N}}, \quad t = 0, 1, ..., N-1 \tag{3}$$

I copies of s are concatenated giving s^{ES}.

$$s^{ES}_t = s_{\mathrm{MOD}(t,N)}, \quad t = 0, 1, ..., IN-1 \tag{4}$$

where $\mathrm{MOD}(, N)$ indicates the modulo N operator. Consequently, the data rate is reduced with a factor I. The last L samples of s ($L \leq N$) act as a cyclic prefix completing $s^{ES,L}$. The values of this extended symbol are shifted out serially and transmitted through the channel. Note that the word 'symbol' is used for representations in both the time and frequency domain. We assume an additive white Gaussian noise (AWGN) channel adding n to $s^{ES,L}$.

Extending a symbol with a factor I implies that the rate at which IDFTs are executed is reduced with the same factor I. Note that extended symbols can also be generated by increasing the IDFT size with a factor I and only loading each Ith carrier with information. However, this is computationally inefficient compared to extending symbols as described above.

B. *FX receiver*

The first receiver that is presented is based on the more traditional OFDM receiver architecture. In this paper, we refer to this receiver as the FX receiver. A base-band equivalent model of the relevant parts of the ES-OFDM receiver for differential modulation is presented in figure 2.

The first step is to remove the cyclic prefix. The resulting extended symbol is r^{ES}.

$$r^{ES}_t = s^{ES}_t + n_t, \quad t = 0, 1, ..., IN-1 \tag{5}$$

At the receiver, we identify I blocks, where each block contains N samples

$$r_{t,i} = r^{ES}_{\mathrm{MOD}(t+iN,IN)}, \quad t = 0, 1, ..., N-1 \quad \text{and} \quad i = 0, 1, ..., I-1 \tag{6}$$

Each individual block is converted into the frequency domain. Since the DFT is a linear operation, the signal and noise parts can be separated.

$$R_{f,i} = \mathrm{DFT}(r_{t,i}) = S_f + N_{f,i}, \quad f = 0, 1, ..., N-1 \ \text{and} \ i = 0, 1, ..., I-1 \tag{7}$$

Each individual block is differentially decoded by the 'Diff Dec' block. This operation is defined as

$$R^d_{f,i} = R_{f,i} \cdot R^*_{f-1,i}, \quad f = 1, 2, ..., N-1 \tag{8}$$

where * indicates the complex conjugate. From the I blocks of data, corresponding values are averaged:

$$R^{\Sigma}_f = \frac{1}{I} \sum_{i=0}^{I-1} R^d_{f,i} \tag{9}$$

Neglecting the noise contributions and using expressions 1 to 9, we see that the output of the receiver equals

$$R^{\Sigma}_f = |S_{f-1}|^2 S^d_f \tag{10}$$

which is a scaled version of the transmitter input S^d_f.

At the receiver, the number of DFTs and differential decoding operations per symbol is proportional to I. Consequently, the number of operations per bit is increased proportional to I. One of the options to reduce the computational complexity is to reduce the resolution of digital signals. Another advantage of this approach is that the resolution of the Analog to Digital

Converter (ADC) can be reduced as well leading to an additional power reduction. However, using the approach described above, the reduced resolution cannot be fully exploited since the DFT is generating higher resolution signals internally and also the final result R will have increased resolution requirements. For that reason we propose to use an alternative architecture which can exploit limited resolution ADCs (or coarse quantization) to the full.

The differential decoding operation in expression 8 can be considered as a correlation operation (indicated as 'X'). This operation is preceded by a DFT operation (indicated as 'F'). The architecture presented above is therefore referred to as an FX receiver; the Fourier transform followed by a correlation. In our second receiver architecture, the order is reversed and is called the XF receiver.

C. XF receiver

To describe the differential ES-OFDM XF-receiver, we will rearrange the expressions for the FX-receiver. Using the shift theorem (see [10]) and expression 8, expression 9 can be rewritten as

$$
\begin{aligned}
R^{\Sigma}_{f} = \;& \frac{1}{I}\sum_{i=0}^{I-1} R_{f,i}\cdot R^{*}_{f-1,i} \;=\; \frac{1}{I}\sum_{i=0}^{I-1} R^{*}_{f-1,i}\cdot R_{f,i} \\
= \;& \frac{1}{I}\sum_{i=0}^{I-1}\left(\frac{1}{\sqrt{N}}\sum_{t_1=0}^{N-1}\left(r_{t_1,i}e^{j\frac{2\pi t_1}{N}}\right)^{*}e^{j\frac{2\pi f t_1}{N}}\right) \\
& \cdot\left(\frac{1}{\sqrt{N}}\sum_{t_2=0}^{N-1} r_{t_2,i}e^{-j\frac{2\pi f t_2}{N}}\right)
\end{aligned}
\tag{11}
$$

The summation over t_2 can start at any index within a symbol. We choose to add t_1 to the index.

$$
\begin{aligned}
R^{\Sigma}_{f} = \;& \frac{1}{IN}\sum_{i=0}^{I-1}\sum_{t_1=0}^{N-1}\left(r_{t_1,i}e^{j\frac{2\pi t_1}{N}}\right)^{*}e^{j\frac{2\pi f t_1}{N}} \\
& \cdot\left(\sum_{t_2=0}^{N-1} r_{(t_1+t_2),i}e^{-j\frac{2\pi f(t_1+t_2)}{N}}\right)
\end{aligned}
\tag{12}
$$

If we define $t_1 + iN = t$, $t_2 = \tau$ and use expression 6, expression 12 becomes

$$
\begin{aligned}
R^{\Sigma}_{f} = \;& \frac{1}{IN}\sum_{t=0}^{IN-1}\left(r^{ES}_{t}e^{j\frac{2\pi t}{N}}\right)^{*}e^{j\frac{2\pi f t}{N}} \\
& \cdot\left(\sum_{\tau=0}^{N-1} r^{ES}_{\mathrm{MOD}(t+\tau,IN)}e^{-j\frac{2\pi f(t+\tau)}{N}}\right)
\end{aligned}
\tag{13}
$$

The exponential parts dependent on t can be removed, and by exchanging the summation order, expression 13 becomes

$$
\begin{aligned}
R^{\Sigma}_{f} = \;& \sum_{\tau=0}^{N-1}\left(\frac{1}{IN}\sum_{t=0}^{IN-1}\left(r^{ES}_{t}e^{j\frac{2\pi t}{N}}\right)^{*}\right. \\
& \left.\cdot\, r^{ES}_{\mathrm{MOD}(t+\tau,IN)}\right)e^{-j\frac{2\pi f\tau}{N}} \\
\triangleq \;& \sqrt{N}\;\cdot\;\mathrm{DFT}(z_\tau)
\end{aligned}
\tag{14}
$$

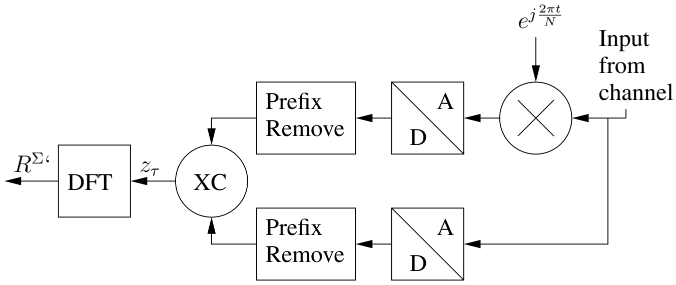

Fig. 3. Base-band equivalent model of the differential ES-OFDM XF receiver

We see that R^{Σ}_{f} is proportional to the DFT of a cross-correlation result z_τ. To produce z_τ, the received extended symbol r^{ES} is correlated with a frequency shifted version of the same symbol. The frequency shift corresponds to one carrier spacing. The resulting architecture is presented in figure 3 where the ADCs are also depicted.

After frequency conversion, both the frequency shifted and original signals are quantized and sampled and the cyclic prefix is removed. The resulting signals are (cross-)correlated by the XC function producing z_τ. The cross correlation result is converted into the frequency domain. The final result R^{Σ} can then be demodulated. In the XF receiver, in contrast with the FX receiver, the rate at which the DFT is executed is reduced with a factor I. However, the (cross-)correlator XC is an additional element and its optimization from a computational point of view is discussed in section III.

D. Bit Error Rates for differential ES-OFDM

To determine the relation between the SNR of the DFT output R^{Σ} and the BER for differential ES-OFDM, we use results presented in [11]. In that paper, a continuous time domain analysis of a DPSK demodulator is presented where a bandpass filtered signal is correlated with a delayed version of the same signal. In section II-C, we have shown that, for each f, R^{Σ}_{f} is the result of the (cross-)correlation of two modulated carriers. Because of the orthogonality of the carriers, a differential ES-OFDM receiver can be considered as $N-1$ parallel DPSK demodulators.

In the expression relating SNR to BER, presented in [11], two parameters are important. First, the energy per bit over noise spectral density ratio ($\frac{E}{N_0}$) and second, the product of the bandwidth of the bandpass filter and the integration time (FT). For differential ES-OFDM using BPSK, the energy per bit over noise spectral density ratio equals $\mathrm{SNR}\cdot I$ and the bandwidth-integration time product equals $\Delta f \cdot \frac{NI}{N\Delta f} = I$, where Δf is the inter-carrier spacing. The final result of [11], for differential ES-OFDM using BPSK in case of an AWGN channel becomes

$$
\mathrm{BER} = \frac{1}{2^{I}}e^{-(I\cdot\mathrm{SNR})}\sum_{i=0}^{I-1}\frac{(I\cdot\mathrm{SNR})^{i}}{i!}\sum_{j=i}^{I-1}\frac{1}{2^{j}}C^{j+I-1}_{j-i}
\tag{15}
$$

$$
C^{n}_{k} = \frac{n!}{k!(n-k)!}
\tag{16}
$$

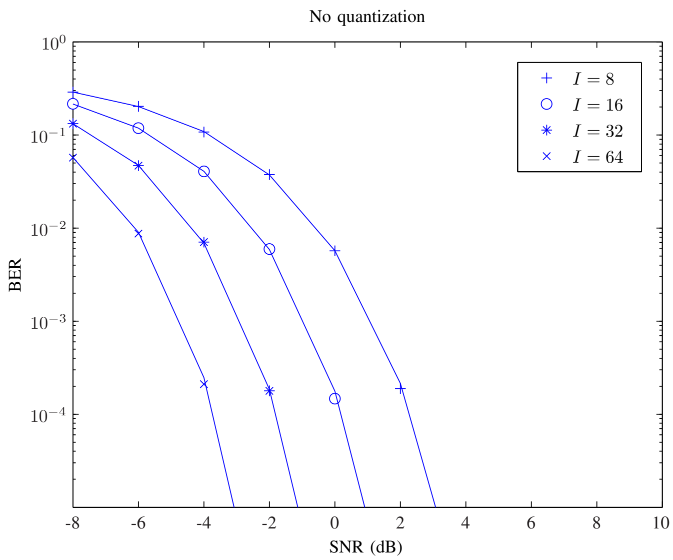

Fig. 4. BERs for differential ES-OFDM for an AWGN channel.

This expression has been used to determine the theoretical BER for differential ES-OFDM. A simulation model has been implemented for a system with 64 carriers ($N = 64$), a cyclic prefix length of 8 ($L = 8$), $I = 8, 16, 32$ and 64 and double precision floating point data representation. In Figure 4, theoretical (curves) and simulated BERs (markers) are presented for an AWGN channel.

We see that doubling the extended symbol length improves the SNR performance. From Figure 4 it can be seen that each doubling of I increases the SNR performance with approximately 2.1 dB. This way, differential ES-OFDM is able to operate a very low signal levels.

III. Quantization Effects

Because of the segmented spectrum and high order modulation schemes, relatively low frequency sampling and high resolution quantization have been used in communication receivers. The applicability of coarse quantization in communication receivers has therefore not really been an issue. However, since ES-OFDM is based on relatively large bandwidths and low power spectral densities, coarse quantization might be suitable for realizing low-power receivers. As mentioned in section II-B, the XF receiver is more suited to exploit coarse quantization than the FX receiver (see also [7]). In expression 14, the frequency shifted and original signals are multiplied. If the signals are coarsely quantized, a low resolution multiplier can be used, reducing the complexity considerably. In the extreme case, single bit quantization, the multiplier simply is an exclusive NOR. Furthermore, in this extreme case, the summation operation can be realized by a simple up-down counter. Even for two bit quantization, very simple implementations exist already for quite some time [12]. Besides the simplicity of the digital processing, the ADC becomes very simple as well. Because of their low resolution, the ADC and the cross-correlator can operate at high frequency realizing a broadband receiver. The first processing stage requiring high resolution, the DFT, is executed only once every IN samples

TABLE I
Degradation d due to coarse quantization

n (bits)	η	d
1	1	1.57
	2	1.35
2	1	1.14
	2	1.06
3	1	1.05
∞(analog)	-	1.0

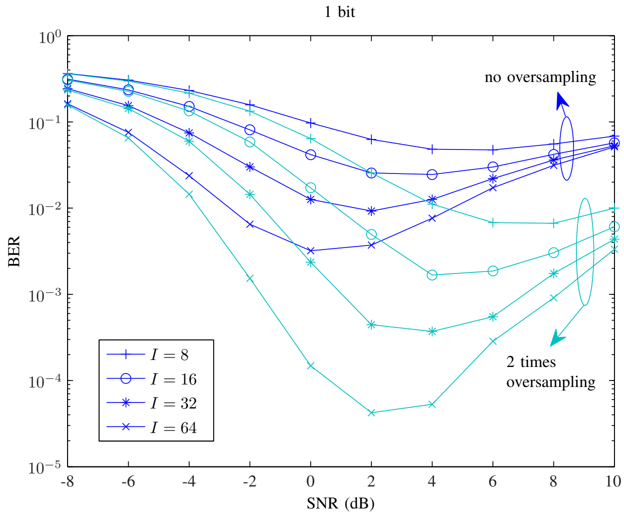

Fig. 5. BERs for differential ES-OFDM for an AWGN channel using single bit quantization.

leading to a small computational load. However, the relatively simple digital implementation comes at the cost of increased quantization noise levels.

Quantization effects within correlators have been studied intensively in the context of Radio Astronomy, see e.g. [12], [13] and [14]. Generally, performance of correlators using coarse quantization is expressed in a degradation factor d which is defined as the ratio of the (rms-)SNR of an analog correlator output and the (rms-)SNR of a digital correlator output. The degradation factors for different oversampling factors η and optimum decision levels are presented in Table I are from [15].

Based on this table we conclude that the interesting cases are the single- and two-bit cases. Going from two- to three bit will result in a limited performance increase.

Although the effects of coarse quantization are known, it is interesting to investigate them in relation to ES-OFDM. To evaluate the effects, an executable model of a correlator using single bit or two bit quantization has been realized. The BER performance when using single bit quantization is presented in Figure 5

The effects of using single bit ADCs in both paths of the XF receiver are shown. Both the results for a critically sampled system and a system with two times oversampling are given. Below -2 dB, the BER results follow the performance trends

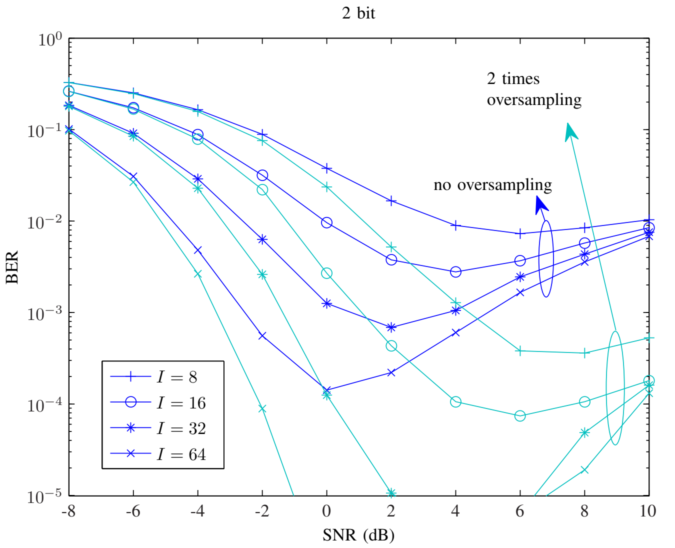

Fig. 6. BERs for differential ES-OFDM for an AWGN channel using two bit quantization.

presented in Figure 4. However, because of the introduction of quantization, the performance is degraded with roughly the factors presented in Table I. Above -2 dB, the correlation between the quantization errors introduced in the two paths, starts to dominate the BER performance.

Furthermore, the effect of oversampling is clearly visible. Two times oversampling decorrelates the quantization errors, improves BER performance considerably and leads to acceptable BERs ($< 10^{-3}$) for $I = 32$ and $I = 64$ for limited SNR ranges.

The simulation results in case of two bit quantization are shown in Figure 6.

Going from a single bit quantizer to a two bit quantizer without oversampling has a similar effect as introducing two times oversampling in the single bit case. Oversampling a two bit quantizer improves performance considerably. For two bit quantization, acceptable BER performance is achieved for $I = 32$ and $I = 64$ and limited SNR ranges. Two times oversampling leads to acceptable BER performance for all four values of I.

IV. Conclusion

In this paper we introduced differential ES-OFDM. By means of extending symbols, acceptable BER performance can be achieved at low SNRs against lower data rates. The required computational capacity of a differential ES-OFDM transmitter is reduced when extending symbols, the complexity of the differential receiver is not. For that reason, the complexity of only the receiver was analyzed in more detail. An option to reduce the complexity and hence the power consumption of the ADC and the first digital processing stage, is to use single- or two bit quantization. The receiver based on the XF correlator (correlation before Fourier transformation) is more suited to exploit such coarse quantization the the FX correlator (Fourier transformation before correlation).

The effects of coarse quantization in combination with oversampling have been analyzed by means of simulation for symbol extension factors $I = 8$, 16, 32 and 64. Two basic effects are visible. First, the BER performance is reduced due to the introduction of quantization errors. Second, beyond certain SNR levels, the BER performance does not increase if transmit power is increased. This is due to the correlation between the quantization errors that are introduced in the two paths of the differential XF receiver.

Oversampling increases BER performance considerably. Single bit quantization without oversampling will not lead to acceptable BERs ($< 10^{-3}$) for the extension factors mentioned. If oversampling is used, acceptable BERs can be achieved. In case of two bit quantization and no oversampling the results are comparable with single bit quantization with two times oversampling. Two bit quantization in combination with two times oversampling improves the performance even further

The effect of coarse quantization has been studied for AWGN channels. Future work will be to use more realistic channel models and ways to mitigate the effects of the channel. Furthermore, scenarios will be developed for application of coarsely quantized ES-OFDM for Cognitive Radio, UWB and also sensor networks.

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