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Regulated Output Synchronization for Heterogeneous Networks of Non-Introspective, Minimum-Phase SISO Agents Without Exchange of Controller States

Håvard Fjær Grip, Ali Saberi, and Anton A. Stoorvogel

Abstract— **In this paper we study the problem of achieving regulated output synchronization in a network of minimum-phase SISO agents. Our problem formulation is characterized by the combination of three different challenges: the network is *heterogeneous* (meaning that the agents are governed by non-identical models); the agents are *non-introspective* (meaning that they do not have access to information about their own state or output); and the agents are not allowed to exchange internal controller states via the network. To handle these challenges, we present an observer-based control methodology that combines elements of low-gain and high-gain design techniques.**

I. Introduction

In recent years, a large body of work has emerged on the topic of *synchronization*, where the goal is to secure agreement among networked agents on a common state or output trajectory. Much of this work is focused on *state synchronization* based on diffusive *state coupling*, progressing from single- and double-integrator agent dynamics (e.g., [1]–[3]) to more general agent dynamics (e.g., [4], [5]). State synchronization based on diffusive *partial-state coupling* has also been considered by several authors (e.g., [6]–[8]). In this context, Li, Duan, Chen, and Huang [9] introduced a distributed observer that makes additional use of the network by allowing the agents to exchange information with their neighbors about their internal estimates, effectively requiring another layer of communication. On the other hand, Seo, Shim, and Back [10] presented a *low-gain* control design that does not require the exchange of internal states, provided the poles of the agent dynamics are located in the closed left-half complex plane. Many of the results on the synchronization problem are rooted in the seminal work of Wu and Chua [11], [12].

A limited amount of work has also been done on *heterogeneous* networks, where the agents are governed by non-identical dynamical models. In a heterogeneous network, the agents' internal states may not be comparable to each other; thus, one often aims to achieve *output synchronization*—that is, agreement on some partial-state output.

Håvard Fjær Grip is affiliated with the Department of Engineering Cybernetics, Norwegian University of Science and Technology, 7491 Trondheim, Norway. His work was supported by the Research Council of Norway.

Ali Saberi is affiliated with the School of Electrical Engineering and Computer Science, Washington State University, Pullman, WA 99164, USA. His work was partially supported by National Science Foundation grant NSF-0901137 and NAVY grants ONR KKK777SB001 and ONR KKK760SB0012.

Anton A. Stoorvogel is affiliated with the Department of Electrical Engineering, Mathematics, and Computer Science, University of Twente, 7500 AE Enschede, The Netherlands.

Some work on heterogeneous networks has focused primarily on synchronization criteria (e.g., [13], [14]); other work has been more design-oriented [15]–[19]. Most designs for heterogeneous networks are based on either modifying the agent dynamics via local feedbacks, in order to change how the agents present themselves to the network [15]–[17]; or on synchronizing an embedded model via the network and then regulating the actual output toward the embedded model output [18], [19]. In either case, the agents are assumed to be *introspective*, meaning that they have access to information about their own state or output in addition to the information received from the network. The authors have recently considered the more challenging case of *non-introspective* agents, and developed a methodology based on a distributed high-gain observer [20]. However, like several other designs for heterogeneous networks [17], [19], it is assumed that the agents can exchange internal controller states with neighboring agents in the network.

A. Topic of This Paper

In this paper, we combine several challenges by considering output synchronization in a heterogeneous network with partial-state coupling, where the agents are non-introspective and unable to exchange controller states with neighboring agents. We focus only on SISO agent dynamics, but we note that the same principles can be applied to right-invertible MIMO agents (albeit with more complications). The only other significant restriction on the agent dynamics is that it must be minimum-phase.

Our approach will be based on a low-gain design methodology similar to that of Seo et al. [10], combined with a high-gain amplification in both the observer and controller. Unlike Seo et al. [10], we do not require the poles of the agent dynamics to be in the closed left-half complex plane, and thus our design also covers a class of homogeneous networks that, to the best of the authors' knowledge, cannot be handled by any other methods from the literature.

Our focus will be on *regulated* output synchronization, where the goal is not only agreement on some output trajectory, but convergence toward a particular trajectory specified by an autonomous exosystem. This approach can also be applied to regular synchronization without an exosystem for networks containing a directed spanning tree, by appointing the root of the spanning tree as an autonomous leader.

Zhao, Hill, and Liu [21] have previously presented a design for certain heterogeneous networks of non-introspective agents without exchange of controller states, albeit under the strict requirement of passivity.

B. Notation and Preliminaries

For a matrix A, A' denotes its transpose and A^* denotes its conjugate transpose. The Kronecker product between A and B is denoted by $A \otimes B$. We denote by $[x_1; \dots; x_n]$ the vector obtained by stacking vectors $x_1, \dots, x_n$ (similarly for matrices).

Definition 1: We say that a matrix pair (A, C) *contains* the matrix pair (S, R) if there exists a matrix Π such that $\Pi S = A\Pi$ and $C\Pi = R$.

Remark 1: Definition 1 implies that for any initial condition $\omega(0)$ of the system $\dot{\omega} = S\omega$, $y_r = R\omega$, there exists an initial condition $x(0)$ of the system $\dot{x} = Ax$, $y = Cx$, such that $y(t) = y_r(t)$ for all $t \geq 0$.[1]

II. Problem Formulation

We consider a network of N SISO agents on the form

$$\dot{x}_i = A_i x_i + B_i u_i, \quad y_i = C_i x_i, \tag{1}$$

where $x_i \in \mathbb{R}^{n_i}$, $u_i \in \mathbb{R}$, and $y_i \in \mathbb{R}$. Our goal is to achieve regulated output synchronization among the agents, meaning that $\lim_{t\to\infty}(y_i - y_r) = 0$ for all $i \in \{1, \dots, N\}$, where y_r is the output of an exosystem

$$\dot{\omega} = S\omega, \quad y_r = R\omega, \tag{2}$$

where $\omega \in \mathbb{R}^{n_r}$ and $y_r \in \mathbb{R}$. Because unobservable and asymptotically stable modes in the exosystem play no role asymptotically, we assume without loss of generality that (S, R) is observable and that the eigenvalues of S are in the closed right-half complex plane.

Assumption 1: For each $i \in \{1, \dots, N\}$, the transfer function $H_i(s) := C_i(sI - A_i)^{-1}B_i$ from u_i to y_i is minimum-phase and not identically zero.

Remark 2: Assumption 1 implies that the triple (A_i, B_i, C_i) is right-invertible, the pair (A_i, B_i) stabilizable, and the pair (A_i, C_i) detectable (see, e.g., [23, Ch. 3]).

During our design in Section III, we denote by $\bar{n}$ an upper bound on the order n_i of the agents.

A. Network Communication

The agents are in general non-introspective; hence, agent i does not have access to its own state x_i or output y_i. The information available to each agent comes from the network, in the form of a linear combination of its own output relative to that of the other agents. In particular, agent i has access to the quantity

$$\zeta_i = \sum_{j=1}^{N} a_{ij}(y_i - y_j),$$

where $a_{ij} \geq 0$.

The communication topology of the network can be described by a directed graph (digraph) $\mathscr{G}$ with nodes corresponding to the agents in the network and edges given by the coefficients a_{ij}. In particular, $a_{ij} > 0$ implies that an edge exists from agent j to i. Agent j is then called a *parent* of agent i, and agent i is called a *child* of agent j. The weight of the edge equals the magnitude of a_{ij}. We shall make use of the Laplacian matrix $G = [g_{ij}]$, where $g_{ii} = -a_{ii} + \sum_{j=1}^{N} a_{ij}$ and $g_{ij} = -a_{ij}$ for $j \neq i$, which has the property that all the row sums are zero. We can then write $\zeta_i = \sum_{j=1}^{N} g_{ij} y_j$.

In order to facilitate regulated output synchronization, we assume that a subset $\mathscr{I} \subset \{1, \dots, N\}$ of the agents have access to their own output relative to the output of the exosystem; specifically, each agent has access to the quantity

$$\psi_i = \iota_i(y_i - y_r), \quad \iota_i = \begin{cases} 1, & i \in \mathscr{I}, \\ 0, & \text{otherwise.} \end{cases}$$

Assumption 2: Every node of $\mathscr{G}$ is a member of a directed tree with its root contained in $\mathscr{I}$.

Remark 3: A directed tree is a subgraph in which every node has exactly one parent, except a single root node with no parents. Furthermore, there must exist a directed path from the root to every other agent.

We define the matrix $\bar{G} := G + \operatorname{diag}(\iota_1, \dots, \iota_N)$. It then follows from Assumption 2 and Lemma 7 of Grip et al. [20] that all the eigenvalues of $\bar{G}$ are in the open right-half complex plane. In the following sections, we shall only assume knowledge of a lower bound $\tau > 0$ on the real part of the eigenvalues of $\bar{G}$.

III. Control Design

We begin by solving the problem for a special case where the dynamics of each agent contains the exosystem dynamics, and all the agents have a common relative degree ρ. We then show that our original problem formulation can be transformed to the special case by first augmenting the agents with dynamic pre-compensators.

A. Control Design for Special Case

We consider the special case where for each $i \in \{1, \dots, N\}$, (i) the pair (A_i, C_i) contains (S, R); and (ii) the triple (A_i, B_i, C_i) is of relative degree $\rho > 0$. Then we can assume without any loss of generality that the agent model (A_i, B_i, C_i) is given in the *special coordinate basis* (SCB) [24]. This means that x_i can be partitioned as $x_i = [x_{ia}; x_{id}]$, where

$$\dot{x}_{ia} = A_{ia}x_{ia} + L_{iad}y_i, \qquad x_{ia} \in \mathbb{R}^{n_i - \rho}, \tag{3a}$$

$$\dot{x}_{id} = A_d x_{id} + B_d(u_i + E_{ida}x_{ia} + E_{idd}x_{id}), \quad x_{id} \in \mathbb{R}^{\rho}, \tag{3b}$$

$$y_i = C_d x_{id}. \tag{3c}$$

Furthermore, the eigenvalues of A_{ia} are the invariant zeros of (A_i, B_i, C_i), which are all in the open left-half complex plane due to the minimum-phase property in Assumption 1, and $A_d \in \mathbb{R}^{\rho \times \rho}$, $B_d \in \mathbb{R}^{\rho \times 1}$, and $C_d \in \mathbb{R}^{1 \times \rho}$ have the special form

$$A_d = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ 0 & 0 & \cdots & 0 \end{bmatrix}, \; B_d = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}, \; C_d = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix}.$$

[1] See [22] for a discussion of *system inclusion* and its role in network synchronization.

If an agent is not in the SCB, it can be transformed to the SCB via state and input transformations (no output transformation is required for SISO systems).

Let $\delta \in (0,1]$ and $\varepsilon \in (0,1]$ denote a low-gain and a high-gain parameter, respectively. Noting that the pair (A_d, C_d) is observable, let K be chosen such that $A_d - KC_d$ is Hurwitz, and define $K_\varepsilon = \varepsilon^{-1} S_\varepsilon^{-1} K$, where $S_\varepsilon := \mathrm{diag}(1, \ldots, \varepsilon^{\rho-1})$. Noting that (A_d, B_d) is controllable, let P_δ be the solution of the algebraic Riccati equation

$$P_\delta A_d + A_d' P_\delta - \tau P_\delta B_d B_d' P_\delta + \delta I = 0, \tag{4}$$

and define $F_{\delta\varepsilon} = -\varepsilon^{-\rho} B_d' P_\delta S_\varepsilon$. Now, for each $i \in \{1, \ldots, N\}$, define the following dynamic controller

$$\dot{\hat{x}}_{id} = A_d \hat{x}_{id} + K_\varepsilon(\zeta_i + \psi_i - C_d \hat{x}_{id}), \quad u_i = F_{\delta\varepsilon} \hat{x}_{id}. \tag{5}$$

We have the following result, which is proven in the Appendix.

Theorem 1: Suppose that for each $i \in \{1, \ldots, N\}$, the pair (A_i, C_i) contains (S, R) and the triple (A_i, B_i, C_i) is of relative degree $\rho > 0$. Let the controller for each agent be defined by (5). There exists a constant $\delta^* \in (0,1]$ such that, for each $\delta \le \delta^*$, there exists an $\varepsilon^*(\delta) \in (0,1]$ such that, for all $\varepsilon \le \varepsilon^*(\delta)$, $\lim_{t\to\infty}(y_i - y_r) = 0$ for all $i \in \{1, \ldots, N\}$.

B. Recovering the Special Case via Pre-Compensators

We now show how to recover the special case specified above, by augmenting each original agent with two dynamic pre-compensators.

Pre-Compensator 1: The purpose of the first pre-compensator is to add modes from the exosystem to agent i, so that the augmented agent dynamics contains the exosystem. Toward this end, start by constructing a state transformation $\Sigma_i \in \mathbb{R}^{n_i \times n_i}$ taking the pair (A_i, C_i) to the Kalman observable canonical form:

$$\Sigma_i^{-1} A_i \Sigma_i = \begin{bmatrix} A_{i11} & 0 \\ A_{i21} & A_{i22} \end{bmatrix}, \quad C_i \Sigma_i = \begin{bmatrix} C_{i1} & 0 \end{bmatrix},$$

where $A_{i11} \in \mathbb{R}^{\bar{n}_i \times \bar{n}_i}$ and (A_{i11}, C_{i1}) is observable. Next, let

$$O_i = \begin{bmatrix} C_{i1} & -R \\ \vdots & \vdots \\ C_{i1} A_{i11}^{\bar{n}_i + n_r - 1} & -RS^{\bar{n}_i + n_r - 1} \end{bmatrix}. \tag{6}$$

Let q_i denote the dimension of the null space of O_i, and define $r_i = n_r - q_i$. Furthermore, let $\Lambda_{iu} \in \mathbb{R}^{\bar{n}_i \times q_i}$ and $\Phi_{iu} \in \mathbb{R}^{n_r \times q_i}$ be chosen such that $O_i \begin{bmatrix} \Lambda_{iu} \\ \Phi_{iu} \end{bmatrix} = 0$ and rank $\begin{bmatrix} \Lambda_{iu} \\ \Phi_{iu} \end{bmatrix} = q_i$. The matrix Φ_{iu} has full column rank because (A_{i11}, C_{i1}) is observable (see [20, App. D]). Let therefore Φ_{io} be chosen such that $\Phi_i := [\Phi_{iu}, \Phi_{io}]$ is nonsingular. We can now state the following lemma, which is proven in the Appendix.

Lemma 1: We have that

$$\Phi_i^{-1} S \Phi_i = \begin{bmatrix} S_{i11} & S_{i12} \\ 0 & S_{i22} \end{bmatrix}, \tag{7}$$

for some matrices $S_{i11} \in \mathbb{R}^{q_i \times q_i}$, $S_{i12} \in \mathbb{R}^{q_i \times r_i}$, and $S_{i22} \in \mathbb{R}^{r_i \times r_i}$. Furthermore, there exists a nonsingular transformation $\Gamma_i \in \mathbb{R}^{r_i \times r_i}$ taking S_{i22} to the companion form

$$\Gamma_i^{-1} S_{i22} \Gamma_i = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ -s_{i1} & -s_{i2} & \cdots & -s_{ir_i} \end{bmatrix}.$$

Based on Lemma 1, let A_{ip1} denote the above companion form of S_{i22}, and define $C_{ip1} = [1, 0, \ldots, 0]$ and $B_{ip1} = [0; \ldots; 0; 1]$, so that (A_{ip1}, B_{ip1}) is controllable and (A_{ip1}, C_{ip1}) is observable. We define the following dynamic pre-compensator:

$$\dot{z}_{i1} = A_{ip1} z_{i1} + B_{ip1} v_i, \quad u_i = C_{ip1} z_{i1},$$

where $v_i \in \mathbb{R}$ is a new input.

Pre-Compensator 2: The purpose of this step is to make the relative degree of the augmented system equal to $\rho := \bar{n} + n_r$. Toward this end, let ρ_i denote the relative degree of (A_i, B_i, C_i), and define the matrices

$$A_{ip2} = \begin{bmatrix} 0 & I_{\rho - \rho_i - r_i - 1} \\ 0 & 0 \end{bmatrix}, \; B_{ip2} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}, \; C_{ip2} = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix}.$$

Define the following dynamic pre-compensator:

$$\dot{z}_{i2} = A_{ip2} z_{i2} + B_{ip2} \upsilon_i, \quad v_i = C_{ip2} z_{i2},$$

where $\upsilon_i \in \mathbb{R}$ is a new input.[2]

By stacking the original state and the state of the two pre-compensators as $\chi_i = [x_i; z_{i1}; z_{i2}]$, we obtain the following augmented agent dynamics with input υ_i:

$$\dot{\chi}_i = \mathscr{A}_i \chi_i + \mathscr{B}_i \upsilon_i, \quad y_i = \mathscr{C}_i \chi_i, \tag{8}$$

where

$$\mathscr{A}_i = \begin{bmatrix} A_i & B_i C_{ip1} & 0 \\ 0 & A_{ip1} & B_{ip1} C_{ip2} \\ 0 & 0 & A_{ip2} \end{bmatrix}, \mathscr{B}_i = \begin{bmatrix} 0 \\ 0 \\ B_{ip2} \end{bmatrix}, \mathscr{C}_i = \begin{bmatrix} C_i & 0 & 0 \end{bmatrix}.$$

We can now state the following result, which recovers the result of Theorem 1 for general systems satisfying Assumption 1. The proof can be found in the Appendix.

Theorem 2: The augmented agent dynamics (8) satisfies Assumption 1, and moreover (i) the pair $(\mathscr{A}_i, \mathscr{C}_i)$ contains (S, R); and (ii) the triple $(\mathscr{A}_i, \mathscr{B}_i, \mathscr{C}_i)$ is of relative degree $\rho > 0$.

IV. Example

We illustrate the results on a network of ten agents. Agents 1–5 are standard double integrators, whereas agents 6–10 are second-order oscillators:

$$A_i = \begin{bmatrix} 0 & 0.01 \\ -0.01 & 0 \end{bmatrix}, \quad B_i = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C_i = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

The exosystem is also a second-order oscillator:

$$S = \begin{bmatrix} 0 & 0.1 \\ -0.1 & 0 \end{bmatrix}, \qquad R = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

[2]Note that it is possible to have $\rho - \rho_i - r_i = 0$. In this case, the pre-compensator should be defined simply as $v_i = \upsilon_i$.

The network topology is described by the adjacency matrix

$$A = \begin{bmatrix} 0 & 1 & 0.2 & 0 & 0 & 0.1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0.1 \\ 0.2 & 0 & 0 & 0.9 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.7 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.2 & 0 \end{bmatrix}.$$

The relative output of the exosystem is available only to agent 10 (i.e., $\mathscr{I} = \{10\}$), which satisfies Assumption 2. A lower bound on the real part of the eigenvalues of $\bar{G}$ is $\tau = 0.5$. An upper bound on n_i is $\bar{n} = 2$, and we therefore operate with $\rho = 2 + n_r = 4$.

The agent models do not satisfy the special case in Section III-A, and hence the first step is to add pre-compensators to the agents. We illustrate this process for the double-integrator dynamics. First note that, because (A_i, C_i) is observable, the Kalman observable canonical form is the same as the model itself with $A_{i11} = A_i$ and $C_{i1} = C_i$. After calculating O_i, we find that $q_i = 0 \implies r_i = 2$. Hence, Λ_{iu} and Φ_{iu} are empty, and we can choose $\Phi_{io} = I$. It follows that $\Phi_i^{-1} S \Phi_i = S$, meaning that $S_{22} = S$, which can be taken to the companion form via $\Gamma_i = \mathrm{diag}(1, 10)$. Note that $\rho_i = 2$ and $r_i = 2$, and hence the second pre-compensator is simply a direct feedthrough according to footnote 2. The resulting augmented agent dynamics is given by the matrices

$$\mathscr{A}_i = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -0.01 & 0 \end{bmatrix}, \quad \mathscr{B}_i = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad \mathscr{C}_i = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}',$$

which are already in the SCB. We follow a similar procedure for the oscillator dynamics of agents 6–10.

After application of the pre-compensators, we get dynamics that satisfy the special case in Section III-A. We therefore proceed by selecting $K \approx [3.08; 4.24; 3.08; 1.00]$, such that $A_d - KC_d$ is Hurwitz. Next, we solve the algebraic Riccati equation (4) with $\delta = 10^{-12}$, which yields $B_d' P_\delta \approx [1.41 \cdot 10^{-6}, 1.27 \cdot 10^{-4}, 5.74 \cdot 10^{-3}, 0.15]$. Finally, defining $K_\varepsilon = \varepsilon^{-1} S_\varepsilon^{-1} K$ and $F_{\delta\varepsilon} = -\varepsilon^{-4} B_d' P_\delta S_\varepsilon$, we find that stability is achieved for for $\varepsilon = 0.5$, which yields $K_\varepsilon \approx [6.15; 16.94; 24.62; 16.00]$ and $F_{\delta\varepsilon} \approx [-2.26 \cdot 10^{-5}, -2.02 \cdot 10^{-3}, -2.30 \cdot 10^{-2}, -0.30]$. Fig. 1 shows the simulated outputs together with the output of the exosystem.

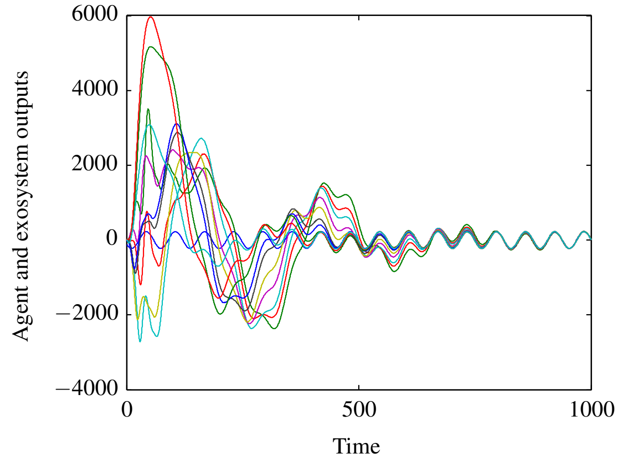

Fig. 1. Agent and exosystem outputs for simulation example

Appendix

Proof of Theorem 1

For each $i \in \{1, \ldots, N\}$, let $\bar{x}_i = x_i - \Pi_i \omega$, where Π_i is such that $\Pi_i S = A_i \Pi_i$, $C_i \Pi_i = R$ in accordance with Definition 1. Then $\dot{\bar{x}}_i = A_i x_i - \Pi_i S \omega + B_i u_i = A_i x_i - A_i \Pi_i \omega + B_i u_i = A_i \bar{x}_i + B_i u_i$. Furthermore, the synchronization error $e_i = y_i - y_r$ is given by $e_i = C_i x_i - R\omega = C_i x_i - C_i \Pi_i \omega = C_i \bar{x}_i$. Since the dynamics of the $\bar{x}_i$ system with output e_i is governed by the same triple (A_i, B_i, C_i) as the dynamics of agent i, we can decompose it in the same way as in (3), by writing $\bar{x}_i = [\bar{x}_{ia}; \bar{x}_{id}]$, where

$$\begin{aligned} \dot{\bar{x}}_{ia} &= A_{ia} \bar{x}_{ia} + L_{iad} e_i, \\ \dot{\bar{x}}_{id} &= A_d \bar{x}_{id} + B_d (u_i + E_{ida} \bar{x}_{ia} + E_{idd} \bar{x}_{id}), \end{aligned}$$

and $e_i = C_d \bar{x}_{id}$. Define $\xi_i = S_\varepsilon \bar{x}_{id}$ and $\hat{\xi}_i = S_\varepsilon \hat{x}_{id}$. Then it is easy to confirm that we can write

$$\begin{aligned} \dot{\bar{x}}_{ia} &= A_{ia} \bar{x}_{ia} + L_{iad} C_d \xi_i, \\ \varepsilon \dot{\xi}_i &= A_d \xi_i - B_d B_d' P_\delta \hat{\xi}_i + \varepsilon^\rho B_d (E_{ida} \bar{x}_{ia} + E_{idd} S_\varepsilon^{-1} \xi_i), \end{aligned}$$

with $e_i = C_d \xi_i$. Furthermore, noting that $\sum_{j=1}^N g_{ij} = 0$, we can write $\zeta_i + \psi_i = \sum_{j=1}^N g_{ij} y_j + \iota_i (y_i - y_r) = \sum_{j=1}^N g_{ij} (y_j - y_r) + \iota_i (y_i - y_r) = \sum_{j=1}^N \bar{g}_{ij} e_j$, where $\bar{g}_{ij}$ represents the coefficients of $\bar{G}$. We therefore have $\varepsilon \dot{\hat{\xi}}_i = A_d \hat{\xi}_i + K \sum_{j=1}^N \bar{g}_{ij} C_d \xi_j - K C_d \hat{\xi}_i$. Let $\xi = [\xi_1; \ldots; \xi_N]$, $\hat{\xi} = [\hat{\xi}_1; \ldots; \hat{\xi}_N]$, and $\bar{x}_a = [\bar{x}_{1a}; \ldots; \bar{x}_{Na}]$. Then

$$\begin{aligned} \dot{\bar{x}}_a &= A_a \bar{x}_a + L_{ad} (I_N \otimes C_d) \xi, \\ \varepsilon \dot{\xi} &= (I_N \otimes A_d) \xi - (I_N \otimes B_d B_d' P_\delta) \hat{\xi} \\ &\quad + \varepsilon^\rho (I_N \otimes B_d)(E_{da} \bar{x}_a + E_{dd} (I_N \otimes S_\varepsilon^{-1}) \xi), \\ \varepsilon \dot{\hat{\xi}} &= (I_N \otimes A_d) \hat{\xi} + (\bar{G} \otimes K C_d) \xi - (I_N \otimes K C_d) \hat{\xi}, \end{aligned}$$

where $A_a = \mathrm{diag}(A_{ia}, \ldots, A_{N1})$, $L_{ad} = \mathrm{diag}(L_{1ad}, \ldots, L_{Nad})$, $E_{da} = \mathrm{diag}(E_{1da}, \ldots, E_{Nda})$, and $E_{dd} = \mathrm{diag}(E_{1dd}, \ldots, E_{Ndd})$.

Let U be defined such that $U^{-1} \bar{G} U = J$, where J is the Jordan form of the matrix $\bar{G}$, and define $\nu = (J U^{-1} \otimes I) \xi$ and $\hat{\nu} = (U^{-1} \otimes I) \hat{\xi}$. Note that the eigenvalues of $\bar{G}$ along the diagonal of J are all in the open right-half complex plane, as explained in Section II-A. We have

$$\begin{aligned} \dot{\bar{x}}_a &= A_a \bar{x}_a + L_a (U J^{-1} \otimes C_d) \nu, \\ \varepsilon \dot{\nu} &= (I_N \otimes A_d) \nu - (J \otimes B_d B_d' P_\delta) \hat{\nu} \\ &\quad + \varepsilon^\rho (J U^{-1} \otimes B_d)(E_{da} \bar{x}_a + E_{dd} (U J^{-1} \otimes S_\varepsilon^{-1}) \nu), \\ \varepsilon \dot{\hat{\nu}} &= (I_N \otimes A_d) \hat{\nu} + (I_N \otimes K C_d) \nu - (I_N \otimes K C_d) \hat{\nu}. \end{aligned}$$

Next, let $\tilde{\nu} = \nu - \hat{\nu}$. We then have

$$\begin{aligned}
\dot{\bar{x}}_a &= A_a\bar{x}_a + L_a(UJ^{-1}\otimes C_d)\nu,\\
\varepsilon\dot{\nu} &= (I_N\otimes A_d)\nu - (J\otimes B_dB_d'P_\delta)\nu + (J\otimes B_dB_d'P_\delta)\tilde{\nu}\\
&\quad + \varepsilon^\rho(JU^{-1}\otimes B_d)(E_{da}\bar{x}_a + E_{dd}(UJ^{-1}\otimes S_\varepsilon^{-1})\nu),\\
\varepsilon\dot{\tilde{\nu}} &= (I_N\otimes(A_d-KC_d))\tilde{\nu} - (J\otimes B_dB_d'P_\delta)\nu + (J\otimes B_dB_d'P_\delta)\tilde{\nu}\\
&\quad + \varepsilon^\rho(JU^{-1}\otimes B_d)(E_{da}\bar{x}_a + E_{dd}(UJ^{-1}\otimes S_\varepsilon^{-1})\nu).
\end{aligned}$$

Finally, let $\eta = [\nu_1;\tilde{\nu}_1;\dots;\nu_N;\tilde{\nu}_N]$. Then

$$\begin{aligned}
\dot{\bar{x}}_a &= A_a\bar{x}_a + L_a(UJ^{-1}\otimes C_d)\nu,\\
\varepsilon\dot{\eta} &= \bar{A}\eta + \varepsilon^\rho M(E_{da}\bar{x}_a + E_{dd}(UJ^{-1}\otimes S_\varepsilon^{-1})\nu),
\end{aligned}$$

where

$$\bar{A} = I_N\otimes\begin{bmatrix}A_d & 0\\ 0 & A_d-KC_d\end{bmatrix} + J\otimes\begin{bmatrix}-B_dB_d'P_\delta & B_dB_d'P_\delta\\ -B_dB_d'P_\delta & B_dB_d'P_\delta\end{bmatrix}$$

and

$$M = \left(I_N\otimes\begin{bmatrix}I_\rho\\ I_\rho\end{bmatrix}\right)(JU^{-1}\otimes B_d).$$

Due to the upper block-triangular structure of $\bar{A}$, we know that its eigenvalues are the eigenvalues of the matrices

$$\bar{A}_i := \begin{bmatrix}A_d-\lambda_iB_dB_d'P_\delta & \lambda_iB_dB_d'P_\delta\\ -\lambda_iB_dB_d'P_\delta & A_d-KC_d+\lambda_iB_dB_d'P_\delta\end{bmatrix},$$

where λ_i is the i'th eigenvalue of $\bar{G}$ along the diagonal of J. Noting that A_d has all its poles in the closed left-half complex plane, the matrix $\bar{A}_i$ corresponds to the system matrix from Seo et al. [10, Eq. (19)] except for the appearance of λ_i instead of $\lambda_i - 1$ in the second row. The proof of [10, Theorem 4] can now be followed to prove that $\bar{A}_i$ is Hurwitz for all δ less than some sufficiently small $\delta^* > 0$. Note that δ^* is independent of the high-gain parameter ε.

Next, let $\tilde{P} = \tilde{P}' > 0$ be the solution of the Lyapunov equation $\tilde{P}\bar{A} + \bar{A}^*\tilde{P} = -I$. Furthermore, let $P_a = P_a' > 0$ be the solution of the Lyapunov equation $P_aA_a + A_a'P_a = -I$, which exists because A_a is block-diagonal with elements $A_{1a},\dots,A_{Na}$, each of which are Hurwitz. Define the Lyapunov function $V = \varepsilon\eta^*\tilde{P}\eta + \varepsilon^\rho\bar{x}_a'P_a\bar{x}_a$. Then

$$\begin{aligned}
\dot{V} &= -\|\eta\|^2 + 2\varepsilon^\rho\mathrm{Re}(\eta^*\tilde{P}M(E_{da}\bar{x}_a + E_{dd}(UJ^{-1}\otimes S_\varepsilon^{-1})\nu)\\
&\quad - \varepsilon^\rho\|\bar{x}_a\|^2 + 2\varepsilon^\rho\mathrm{Re}(\bar{x}_a'P_aL_a(UJ^{-1}\otimes C_d)\nu).
\end{aligned}$$

Clearly, we have that $\varepsilon^\rho\|\mathrm{Re}(\eta^*\tilde{P}ME_{da}\bar{x}_a)\| \le \varepsilon^\rho m_1\|\eta\|\|\bar{x}_a\|$ for some $m_1 > 0$. Also, noting that S_ε contains no powers of ε higher than $\rho - 1$, we have $2\varepsilon^\rho\|\mathrm{Re}(\eta^*\tilde{P}ME_{dd}(UJ^{-1}\otimes S_\varepsilon^{-1})\nu)\| \le \varepsilon m_2\|\eta\|\|\nu\| \le \varepsilon m_2\|\eta\|^2$ for some $m_2 > 0$. Finally, $\varepsilon^\rho\|\mathrm{Re}(\bar{x}_a'P_aL_a(UJ^{-1}\otimes C_d)\nu)\| \le \varepsilon^\rho m_3\|\bar{x}_a\|\|\nu\| \le \varepsilon^\rho m_3\|\bar{x}_a\|\|\eta\|$ for some $m_3 > 0$. Hence, we have

$$\dot{V} \le -(1-m_2\varepsilon)\|\eta\|^2 - \varepsilon^\rho\|\bar{x}_a\|^2 + 2\varepsilon^\rho(m_1+m_3)\|\eta\|\|\bar{x}_a\|,$$

where m_1, m_2, and m_3 are independent of ε. By inspecting the principal minors of the corresponding quadratic form, we find that $\dot{V}$ is negative definite for all sufficiently small ε. Hence, $\lim_{t\to\infty}\eta = 0$ and $\lim_{t\to\infty}\bar{x}_a = 0$. This implies $\lim_{t\to\infty}\xi_i = 0$ for all $i\in\{1,\dots,N\}$, which in turn implies $\lim_{t\to\infty}e_i = 0$. ■

Proof of Lemma 1

The columns of $[\Lambda_{iu};\Phi_{iu}]$ span the unobservable subspace of the pair $(\mathrm{diag}(A_{i11},S),[C_{i1},-R])$, which is $\mathrm{diag}(A_{i11},S)$-invariant, and hence

$$\begin{bmatrix}A_{i11} & 0\\ 0 & S\end{bmatrix}\begin{bmatrix}\Lambda_{iu}\\ \Phi_{iu}\end{bmatrix} = \begin{bmatrix}\Lambda_{iu}\\ \Phi_{iu}\end{bmatrix}U_i, \tag{9a}$$

$$\begin{bmatrix}C_{i1} & -R\end{bmatrix}\begin{bmatrix}\Lambda_{iu}\\ \Phi_{iu}\end{bmatrix} = 0. \tag{9b}$$

for some $U_i\in\mathbb{R}^{q_i\times q_i}$. It follows that $S\Phi_{iu} = \Phi_{iu}U_i$, which means that

$$S\begin{bmatrix}\Phi_{iu} & \Phi_{io}\end{bmatrix} = \begin{bmatrix}\Phi_{iu} & \Phi_{io}\end{bmatrix}\begin{bmatrix}U_i & S_{i12}\\ 0 & S_{i22}\end{bmatrix},$$

for some matrices S_{i12} and S_{i22}. This, in turn, implies (7) with $S_{i11} = U_i$.

Next, note that, because (S,R) is observable, we have $\mathrm{rank}\begin{bmatrix}S-\lambda I\\ R\end{bmatrix} = n$ for all eigenvalues λ of S, which implies that $\mathrm{rank}(S-\lambda I) = n-1$ for all eigenvalues of S. Due to the triangular form obtained via the similarity transform in (7), we therefore have $\mathrm{rank}(S_{i22}-\lambda I) = r_i - 1$ for all eigenvalues λ of S_{i22}; that is, the geometric multiplicity of each eigenvalue is 1. It follows from this that S_{i22} is a *non-derogatory* matrix that can be transformed to the companion form [25, Section 7.4.6]. ■

Proof of Theorem 2

Since the pre-compensators are zero-free and have their poles in the right-half complex plane, no pole-zero cancellations occur in the augmented system (which is not identically zero), and hence it has the same invariant zeros as the original system and satisfies Assumption 1. The relative degree of the two pre-compensators are r_i and $\rho - \rho_i - r_i$. The relative degree of augmented dynamics (8) is therefore $\rho_i + r_i + \rho - \rho_i - r_i = \rho$.

Next, to show that $(\mathscr{A}_i,\mathscr{C}_i)$ contains (S,R), we start by showing that there exists a Π_i such that $\Pi_iS = \mathscr{A}_{i1}\Pi_i$, $\mathscr{C}_{i1}\Pi_i = R$, where

$$\mathscr{A}_{i1} = \begin{bmatrix}A_i & B_iC_{ip1}\\ 0 & A_{ip1}\end{bmatrix},\quad \mathscr{C}_{i1} = \begin{bmatrix}C_i & 0\end{bmatrix}.$$

Post-multiplying by Φ_i and defining $\bar{\Pi}_i := \Pi_i\Phi_i$, it can be seen from the proof of Lemma 1 that we get the equivalent expression

$$\begin{aligned}
\begin{bmatrix}\bar{\Pi}_{i11} & \bar{\Pi}_{i12}\\ \bar{\Pi}_{i21} & \bar{\Pi}_{i22}\end{bmatrix}\begin{bmatrix}U_i & S_{i12}\\ 0 & S_{i22}\end{bmatrix} &= \begin{bmatrix}A_i & B_iC_{ip1}\\ 0 & A_{ip1}\end{bmatrix}\begin{bmatrix}\bar{\Pi}_{i11} & \bar{\Pi}_{i12}\\ \bar{\Pi}_{i21} & \bar{\Pi}_{i22}\end{bmatrix},\\
\begin{bmatrix}C_i & 0\end{bmatrix}\begin{bmatrix}\bar{\Pi}_{i11} & \bar{\Pi}_{i12}\\ \bar{\Pi}_{i21} & \bar{\Pi}_{i22}\end{bmatrix} &= \begin{bmatrix}R\Phi_{iu} & R\Phi_{io}\end{bmatrix}.
\end{aligned}$$

From (9) we have $A_{i11}\Lambda_{iu} = \Lambda_{iu}U_i$. By Remark 2, the pair (A_i,C_i) is detectable, and hence the eigenvalues of the matrix A_{i22} are in the open left-half complex plane. Since the eigenvalues of U_i are in the closed right-half complex plane, we can therefore find a solution X_i of the Sylvester equation $X_iU_i = A_{i22}X_i + A_{i21}\Lambda_{iu}$ (see, e.g., [26, App. 2.A]). It follows that

$$\begin{bmatrix}\Lambda_{iu}\\ X_i\end{bmatrix}U_i = \begin{bmatrix}A_{i11} & 0\\ A_{i21} & A_{i22}\end{bmatrix}\begin{bmatrix}\Lambda_{iu}\\ X_i\end{bmatrix}.$$

Letting $\bar{\Pi}_{i11} = \Sigma_i[\Lambda_{iu}; X_i]$, we therefore have $\bar{\Pi}_{i11} U_i = A_i \bar{\Pi}_{i11}$. Furthermore, using the identity $C_{i1}\Lambda_{iu} = R\Phi_{iu}$ from (9), we have $C_i\bar{\Pi}_{i11} = [C_{i1}, 0][\Lambda_{iu}; X_i] = C_{i1}\Lambda_{iu} = R\Phi_{iu}$.

Let $\bar{\Pi}_{i21} = 0$. Next, consider the equations $\bar{\Pi}_{i11}S_{i12} + \bar{\Pi}_{i12}S_{i22} = A_i\bar{\Pi}_{i12} + B_i\Xi_i$, $C_i\bar{\Pi}_{i12} = R\Phi_{io}$ with unknowns $\bar{\Pi}_{i12}$ and Ξ_i. This set of *regulator equations* is solvable if the Rosenbrock system matrix $\begin{bmatrix} A_i-\lambda I & B_i \\ C_i & 0 \end{bmatrix}$ has rank $n_i + 1$ for each λ that is an eigenvalue of S_{i22} [26, Corollary 2.5.1]. The normal rank of this matrix is $n_i + 1$, because the system is right-invertible [27, Proposition 3.1.6]. The matrix retains its normal rank for each λ that is an eigenvalue of S_{i22}, since these are all in the closed right-half complex plane while the invariant zeros of (A_i, B_i, C_i) are all in the open left-half complex plane. Finally, consider the equations $\bar{\Pi}_{i22}S_{i22} = A_{ip1}\bar{\Pi}_{i22}$, $C_{ip1}\bar{\Pi}_{i22} = \Xi_i$ with unknown $\bar{\Pi}_{i22}$. To see that these can be solved, note we can equivalently write $\tilde{\Pi}_{i22}S_{i22} = S_{i22}\tilde{\Pi}_{i22}$, $C_{ip1}\Gamma_i^{-1}\tilde{\Pi}_{i22} = \Xi_i$, where $\tilde{\Pi}_{i22} = \Gamma_i\bar{\Pi}_{i22}$. Letting $\bar{O}_i$ denote the observability matrix of the pair $(\mathrm{diag}(S_{i22}, S_{i22}), [C_{ip1}\Gamma_i^{-1}, -\Xi_i])$, it follows from the Cayley-Hamilton theorem that

$$\operatorname{rank}\bar{O}_i = \operatorname{rank}\begin{bmatrix} C_{ip1}\Gamma_i^{-1} & -\Xi \\ \vdots & \vdots \\ C_{ip1}\Gamma_i^{-1}S_{i22}^{r_i-1} & -\Xi_i S_{i22}^{r_i-1} \end{bmatrix} \le r_i.$$

The first r_i columns of the above matrix constitute the observability matrix of the observable pair $(S_{i22}, C_{ip1}\Gamma_i^{-1})$, and it follows that $\tilde{\Pi}_{i22}$ can be chosen such that $\bar{O}_i[\tilde{\Pi}_{i22}; I] = 0$; that is, $[\tilde{\Pi}_{i22}; I]$ spans the unobservable subspace of $(\mathrm{diag}(S_{i22}, S_{i22}), [C_{ip1}\Gamma_i^{-1}, -\Xi_i])$. Then $C_{ip1}\Gamma_i^{-1}\tilde{\Pi}_{i22} = \Xi_i$ and

$$\begin{bmatrix} S_{i22} & 0 \\ 0 & S_{i22} \end{bmatrix} \begin{bmatrix} \tilde{\Pi}_{i22} \\ I \end{bmatrix} = \begin{bmatrix} \tilde{\Pi}_{i22} \\ I \end{bmatrix} \begin{bmatrix} S_{i22} & 0 \\ 0 & S_{i22} \end{bmatrix},$$

which implies $S_{i22}\tilde{\Pi}_{i22} = \tilde{\Pi}_{i22}S_{i22}$.

Combining the above expressions, we have

$$\begin{bmatrix} \bar{\Pi}_{i11} & \bar{\Pi}_{i12} \\ \bar{\Pi}_{i21} & \bar{\Pi}_{i22} \end{bmatrix} \begin{bmatrix} U_i & S_{i12} \\ 0 & S_{i22} \end{bmatrix} = \begin{bmatrix} \bar{\Pi}_{i11}U_i & \bar{\Pi}_{i11}S_{i12} + \bar{\Pi}_{i12}S_{i22} \\ 0 & \bar{\Pi}_{i22}S_{i22} \end{bmatrix}$$
$$= \begin{bmatrix} A_i\bar{\Pi}_{i11} & A_i\bar{\Pi}_{i12} + B_i\Xi_i \\ 0 & A_{ip1}\bar{\Pi}_{i22} \end{bmatrix} = \begin{bmatrix} A_i & B_iC_{ip1} \\ 0 & A_{ip1} \end{bmatrix} \begin{bmatrix} \bar{\Pi}_{i11} & \bar{\Pi}_{i12} \\ \bar{\Pi}_{i21} & \bar{\Pi}_{i22} \end{bmatrix}$$

and

$$\begin{bmatrix} C_i & 0 \end{bmatrix} \begin{bmatrix} \bar{\Pi}_{i11} & \bar{\Pi}_{i12} \\ \bar{\Pi}_{i21} & \bar{\Pi}_{i22} \end{bmatrix} = \begin{bmatrix} C_i\bar{\Pi}_{i11} & C_i\bar{\Pi}_{i12} \end{bmatrix} = \begin{bmatrix} R\Phi_{iu} & R\Phi_{io} \end{bmatrix}.$$

Defining $\mathscr{B}_{i1} = [0; B_{ip1}]$, we can write

$$\mathscr{A}_i = \begin{bmatrix} \mathscr{A}_{i1} & \mathscr{B}_{i1}C_{ip2} \\ 0 & A_{ip2} \end{bmatrix}, \quad \mathscr{C}_i = \begin{bmatrix} \mathscr{C}_{i1} & 0 \end{bmatrix}.$$

It is now straightforward to see that the matrix $\Pi_i^* := [\Pi_i; 0]$ verifies that the pair $(\mathscr{A}_i, \mathscr{C}_i)$ contains (S, R). ∎

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