Time of Arrival Estimation in Pulsar-Based Navigation Systems

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Abstract **—This paper focuses on the Time of Arrival (TOA) estimation problem related to new application of pulsar signals for airplane-based navigation. The aim of the paper is to propose and evaluate a possible algorithm for TOA estimation that consists of epoch folding, filtering, CFAR detection, cross-correlation and TOA calculation. The TOA estimation algorithm proposed is verified using real experimental data obtained from the Westerbork radio observatory in The Netherlands. The performance of the proposed TOA algorithm is evaluated in terms of SNR at the cross-correlator input and the TOA accuracy.**

Index Terms—**Pulsar signal detection, estimation, navigation**

I. INTRODUCTION

Pulsars are rotating neutron stars that periodically emit broadband electromagnetic pulses. The emission period is the same as the rotation period. Although individual pulses vary in strength and shape, the average pulse shape is stable and characterizes each pulsar. In [1-4, 6], the authors discuss the possibility of an autonomous system for navigation of airplanes that is based on pulsar timing data. The pulsar periodic signals have timing stabilities comparable to atomic clocks and provide characteristic temporal signatures that can be used as natural navigation beacons, quite similar to the use of GPS satellites for navigation on Earth. By comparing pulse arrival times measured on-board the spacecraft with predicted pulse arrivals at some reference location, the spacecraft position can be determined autonomously with accuracies on the order of 5 kilometres. Therefore this new technology is an alternative to standard navigation based on radio tracking by ground stations, without the disadvantages of uncertainty increasing with distance from Earth and the dependence on ground control. The practical realization of the idea of pulsar-based navigation in space is difficult, firstly, the insufficient number of known pulsars, and secondly - sophisticated technology to detect them. But in recent years the situation has changed significantly. Since the discovery of the first pulsar in 1967, approximately 1800 pulsars have been found.

The key problem in pulsar navigation systems is the estimation of the time of arrival (TOA) of the pulsar pulses [1]. If the pulsar pulses have a relatively stable width, then the TOA refers to some fiducial point of the pulsar profile. There are many algorithms to estimate the TOA of pulsar signals relatively to the pulsar template [6]. The cross-correlation (CC) method is basic solution of the TOA problem [6]. According to [1], the CC method correlates the signal after epoch folding with the pulsar template. The time argument that corresponds to the maximum peak in the output of the cross-correlation function is considered as an estimate of the time delay of the epoch folded signal. According to [1], the standard algorithm for TOA estimation of pulsar signals, implemented in most of radio observatories, usually includes epoch-folding (signal integration during a lot of pulse repetition periods) and correlation. The conceptual scheme of TOA estimation of pulsar signals is presented in Fig.1.

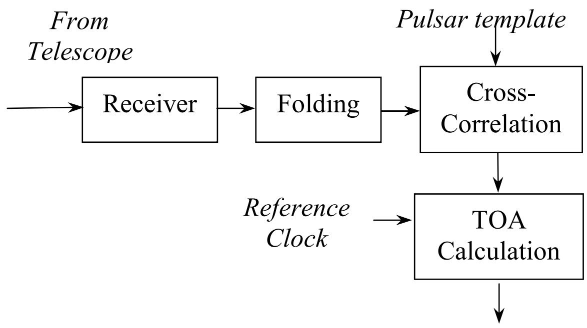

Figure 1. The basic concept of TOA estimation of pulsar signals

The key constraints towards the accuracy of TOA estimation are the very low signal-to-noise ratio (SNR) of the received pulsar signals, corrupted by impulsive interference, broadband interference and sometimes periodic terrestrial radiation, and the time-consuming processing of pulsar signals [3, 4]. As shown in [1], the accuracy of TOA estimation is inversely proportional to the level of SNR at the correlator input. To improve the correlation peak detection and time delay estimation, various filters can be used before the cross correlation. This technique is called generalized cross-correlation (GCC) [7]. The GCC method, proposed by Knapp and Carter, is the very popular technique due to their accuracy and computational complexity. The role of the filter in GCC method is to ensure a large sharp peak in the obtained cross-correlation thus ensuring a high time delay resolution.

In this paper we propose a possible algorithm to estimate TOA of pulsar signals that can be used in a pulsar-based navigation system mounted on the airplane board. The algorithm includes the following stages: epoch folding, filtering, automatic detection, time delay estimation and, finally, TOA calculation. After epoch folding and filtering, the filter output is processed by a CFAR detection algorithm. If the signal at the filter output is detected, then the time delay of the filtered signal relatively to the pulsar template can be estimated. The time argument that corresponds to the maximum peak in the output of the cross-correlation function is considered as an estimate of the time delay of the filtered signal. The TOA estimate is calculated on the base of the time delay estimate, reference clock and the filter group delay. In this algorithm the role of the filter is to improve SNR at the cross-correlator input and the role of the CFAR detector is to control false alarms. The TOA estimation accuracy of the proposed algorithm is evaluated using the experimental data obtained from the Westerbork radio observatory in The Netherlands. The experimental data contain the noisy signal received from pulsar B0329+54. The impact of each of the three filters, Moving Averaging Filter (MAF), MAF with Jumping Window (MAFJW) and the Matched Filter (MF), is evaluated in terms of SNR estimated at the filter output, and the corresponding accuracy of TOA estimation.

II. Signal Processing

The block-scheme of the proposed algorithm for TOA estimation of pulsar signals is shown in Fig.2.

A. Epoch Folding

The integration of the received signal during K sequential repetition periods of the input data (usually called as epoch folding) is done as non-coherent. It is the standard way that is implemented in most of radio observatories. After epoch-folding, the output signal y at time discrete n is formed as:

$$x[n] = \frac{1}{K}\sum_{k}[I_{s,k} + N_{1,k}]^2 + \frac{1}{K}\sum_{k}[Q_{s,k} + N_{2,k}]^2 \quad (1)$$

In (1), $I_{s,k}$ and $Q_{s,k}$ are the quadrature components of the received pulsar signal in the k-th repetition period, $N_{1,k}$ and $N_{2,k}$ are quadrature components of zero mean Gaussian noise with variance σ^2 in the k-th period. As follows from (1), the output signal $x[n]$ is distributed according the non-central chi-square law with $2K$ degrees of freedom. It can be seen that for K=1, the chi-square distribution becomes the exponential distribution.

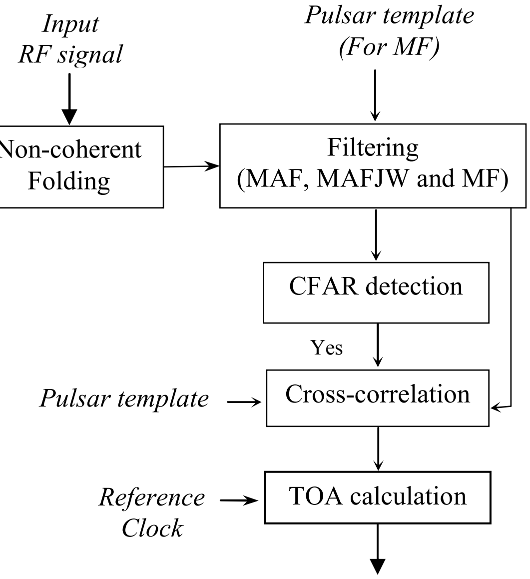

Figure 2. Proposed algorithm for TOA estimation

B. Moving Average Filter

The Moving Average Filter (MAF) is a simple low-pass FIR filter, which is often used for smoothing the sampled noisy signal. At a discrete time n, it takes $(2N+1)$ samples from the moving window of the input data (x_{n-N}, $x_{n-N-1},\ldots,x_{n-1}$, x_n, $x_{n+1},\ldots,x_{n+N}$), calculates the average of those samples and produces an output sample y_n. The MAF output is formed as:

$$y[n] = \frac{1}{2N+1}\sum_{k=-N}^{N} x[n-k] \quad (2)$$

The filter impulse response in the time domain is 1/ (2N+1), where (2N+1) is the window length of the filter. The key problem is the optimal choice of the window length, i.e N. When choosing the window length of the MAF we must take into account not only the level of suppression of the noise variance, but the degree of distortion of the useful signal immersed in noise. If f_{cut} is the highest frequency of the signal at the MAF output, then the improvement in SNR at the MAF output can be expressed as:

$$H(N) = \frac{1}{2N+1}(1 + 2\sum_{k=1}^{N}\cos(\frac{2\pi k f_{cut}}{f_s}))^2 \quad (3)$$

From (3) follows that the optimal window size, i.e. $(2N_{opt}+1)$ must be determined according to the following optimization criterion:

$$N_{opt} = \max\{H(N)\}_N \quad (4)$$

C. Moving Average Filter with Jumping Window

The Moving Average Filter with a Jumping Window (MAFJW) is a modified version of the MAF. It divides the period with N samples into L non-overlapping intervals (windows) of size M and calculates the average of samples in each interval. Therefore, the MAFJW acts not only as a low-pass filter but a decimator as well. When the signal processing is carried out in the time domain, the use of the MAFJW can be very useful in the sense of reducing the processing time.

D. Matched Filter

According to the theory of matched filtering (MF), the filter impulse response is calculated as:

$$h[n] = p[N-n+1] / \| p[N-n+1] \| \quad (5)$$

where $p[n]$ is the pulsar template . The performance of the matched filter in the frequency domain can be expressed mathematically as:

$$y[n] = IFFT\{H(f) \cdot X(f)\} \quad (6)$$

H(f) is the frequency impulse response of the matched filter, and *X(f)* is the signal spectrum at the matched filter input.

E. CFAR Detection

The CFAR detection approach is based on the criterion of Neyman – Pearson. According to this criterion, the following algorithm can be used for testing a simple hypothesis H_1 (pulsar signal is present) against a simple alternative H_0 (pulsar signal is absent):

$$\begin{aligned} H_1 &: if \max\{y(n)\} \geq T_{fa} \cdot \sum_{l=1}^{L} y'(l) \\ H_0 &: \qquad\qquad otherwise \end{aligned} \quad (7)$$

In the decision rule (7), *y[n]* is the signal power at the CFAR detector input, *L* is the size of a reference window for estimating the noise power and *y'[l]* are the signal power samples located in the noise zone, where *l* varies inside the interval *[1, L]*. In order to define the noise zone all input power samples *y[n]* are sorted in the ascended order and the first *L* sorted samples are defined as a noise zone. The detection constant T_{fa} is determined in accordance with the probability of false alarm, which is maintained by the detection algorithm. We assume that the hypothesis H_1 is verified only in one single sample. Using the search strategy "Maximum" the probability of false alarm is defined as:

$$P_{FA} = 1 - [1 - P_{fa}]^N \quad (8)$$

In (8), *N* is the total number of samples, and P_{fa} is the probability of false alarm in a single sample. In case of mean zero Gaussian noise at the receiver input, the probability of false alarm to be maintained in a single sample is:

$$P_{fa} = 1/(1+T_{fa})^L \quad (9)$$

The probability P_{fa} is defined as a solution of the equation (8) for a given probability of false alarm P_{FA}. The solution of (9) gives the detection constant:

$$T_{fa} = P_{fa}^{-1/L} - 1 \quad (10)$$

F. TOA Estimation

The Time of Arrival of pulsar signal is determined as:

$$TOA = t_{ref} + t_{dif} - t_{group} \quad (11)$$

In (11), t_{ref} is a reference clock, t_{dif} is the time delay of the filtered signal relatively to the pulsar template, and t_{group} is the group delay of the filter. The time difference t_{dif} is the lag of the cross-correlation peak. According to [1], the rough estimate of the accuracy of TOA estimation can be roughly estimated as:

$$\sigma_{TOA} = \Delta \cdot P / SNR_{cor} / N \quad (12)$$

In (12), Δ is the width of the normalized pulsar profile determined at the level of 0.7 (number of samples), *P* is the repetition period of pulsar signals (seconds), SNR_{cor} is the value of SNR determined at the cross-correlator input, and *N* is the total number of samples within a repetition period.

G. SNR estimation

The value of SNR at the correlator input is calculated as the following ratio:

$$SNR_{cor} = [y_{s,\max} - mean(y_n)] / \sigma(y_n) \quad (13)$$

In the ratio (13) y_S is the signal taken from the interval where the pulsar signal is present, and y_n is the signal taken from the interval where only noise is present.

III. Numerical Results

The aim of the study is to verify and evaluate the performance of the proposed algorithm for TOA estimation (shown in Fig.2) using the experimental records of the signals received from pulsar B0329+54 at the Westerbork radio observatory. The template of pulsar B0329+54 taken from the EPN database is sampled at the sampling rate of 1.4493 KHz and it contains 1024 samples. The experimental data are sampled at a frequency of 40 MHz, and the number of samples of the input signal within a repetition period is 28582316. In order to perform the matched filtering and the cross-correlation at the sampling frequency of 40 MHz, we approximated the pulsar template as a sum of three Gaussian pulses (Fig.3).

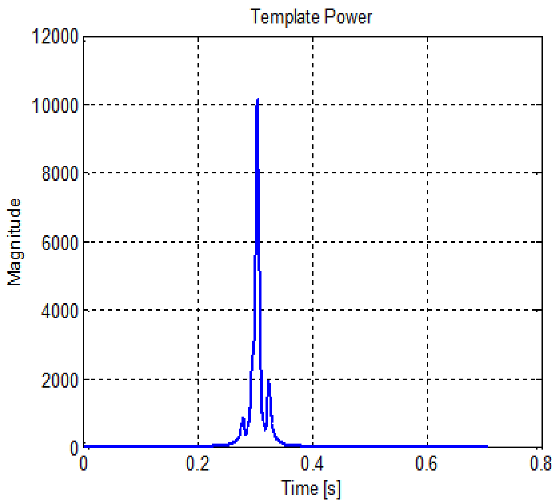

Figure 3.Template of the pulsar B0329+54

The corresponding amplitude spectrum of the pulsar template is shown in Fig. 4. It can be seen that the frequency bandwidth of the pulsar template is approximatly 200 Hz. In that case the cutoff frequency of 100 Hz can be used for optimization of the window length of the MAF filter using the optimization criterion (4).

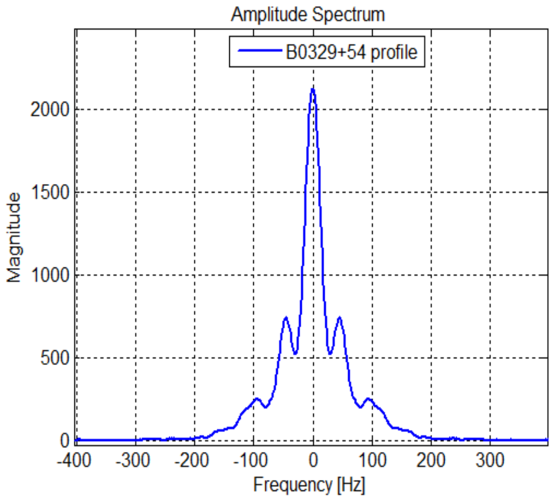

Figure 4. Amplitude spectrum of the pulsar template

The SNR values calculated at the output of four types of signal processing are summarized in Table I. The first column in the table contains different numbers of integrated periods during the epoch-folding. The second column contains the SNR values estimated in case when before correlation only the epoch-folding procedure is performed. The next three columns contain the SNR values estimated in case when before correlation the signal processing includes two stages – epoch-folding and filtering. The optimal window length of the MAF is calculated using the optimization criterion (4) for f_{cut}=100 Hz and f_S=40 MHz.. The calculated MAF window is 148403.

TABLE I

Number of Integrated Periods	SNR [dB] Only Folding	SNR [dB] *Folding & MAFJW*		
		Folding & MAFJW	*Folding & MAF*	*Folding & MF*
1	-	10.63	13.17	15.0
13	10.46	21.12	23.20	25.2
27	10.67	22.83	25.26	27.13
41	11.35	23.35	25.66	27.61
55	11.47	24.07	26.60	28.38
69	12.0	24.64	27.18	28.82
83	12.2	24.86	27.30	29.15
97	12.48	25.42	28.08	30.07
111	12.83	25.46	27.99	29.99
125	13.03	25.75	28.52	30.58

It can be seen that when the number of integrated periods is greater than 13, the SNR values at the output of the three filters are high enough in order to guarantee the high probability of signal detection at the CFAR detector output. The numerical results also show that the most values of SNR are achieved at the MF output. For comparison, the signals after epoch folding are shown in Fig.5 for 13 integrated periods and in Fig. 6 for 125 integrated periods. Comparing Fig. 5 and Fig.6 we can see that even 125 integrated periods are not sufficient for visible separation of the signal from noise.

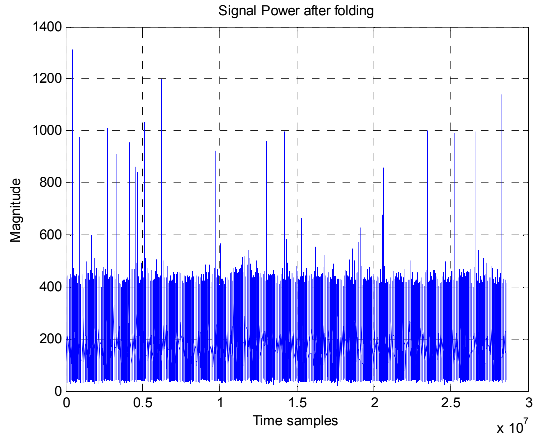

Figure 5. Signal after epoch folding (13 periods)

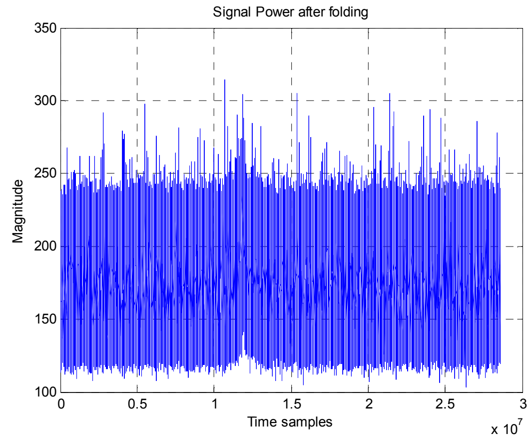

Figure 6. Signal after epoch folding (125 periods)

The cross-correlation coefficients between the epoch folded signal and the pulsar template are plotted in Fig.7 for 13 integrated periods and in Fig.8 for 125 integrated periods. It can be seen that in both case the cross-correlation coefficient does not exceed the level of 0.132. It means that the maximal value of the cross-correlation coefficient remains very low regardless of the increase in the number of periods. This explains the lower accuracy of TOA estimation accordinng to the standard approach, which is insufficient for reliable TOA estimation.

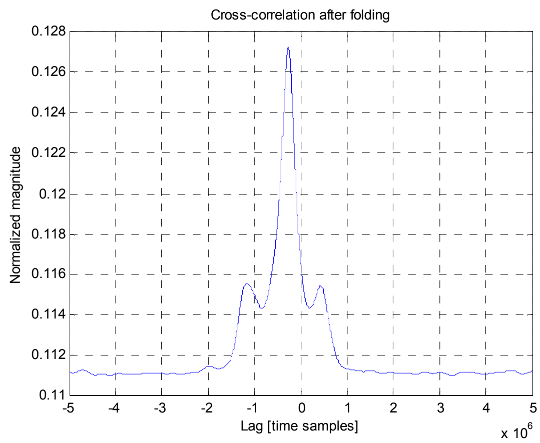

Figure 7. Cross-correlation coefficient (13periods, max=0.127)

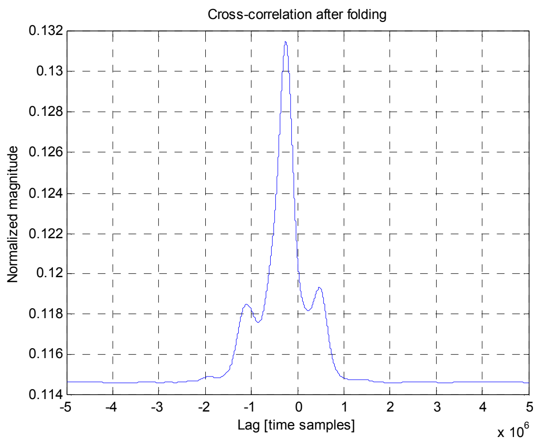

Figure 8. Cross-correlation coefficient (125 periods, max=0.131)

The output of the matched filter together with the CFAR threshold is shown in Fig.9. In contrary to Fig. 7 and Fig. 8, the cross-correlation coefficient between the MF output and the pulsar template is high and equals 0.81 (Fig.10).

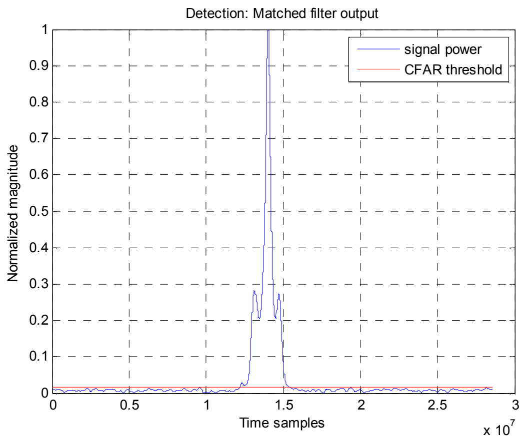

Figure 9.The output of the MF (13 periods)

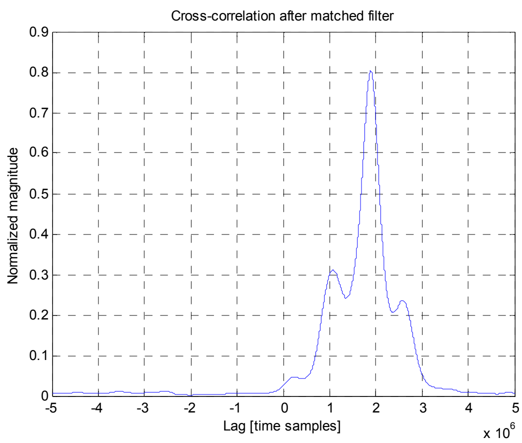

Figure 10. Cross-correlation coefficient (13 periods, max=0.81)

The high value of the cross-correlation coefficient allows us to obtain the reliable estimate of the time delay of the filtered signal. Figure 10 also shows that the group delay of the matched filter must be taken into account in TOA calculation.

Table II contains the rough estimates of the accuracy of TOA estimation (σ_{TOA}), which are calculated according to (12).

TABLE II

Number of integrated Periods	TOA accuracy [s]		
	Folding & MAFJW	*Folding & MAF*	*Folding & MF*
1	$3.0995\cdot10^{-4}$	$1.7270\cdot10^{-4}$	$1.1331\cdot10^{-4}$
13	$2.7688\cdot10^{-5}$	$1.7151\cdot10^{-5}$	$1.0821\cdot10^{-5}$
27	$2.4748\cdot10^{-5}$	$1.0673\cdot10^{-5}$	$6.9388\cdot10^{-6}$
41	$1.6569\cdot10^{-5}$	$9.7339\cdot10^{-6}$	$6.2128\cdot10^{-6}$
55	$1.4037\cdot10^{-5}$	$7.8395\cdot10^{-6}$	$5.2034\cdot10^{-6}$
69	$1.2311\cdot10^{-5}$	$6.8594\cdot10^{-6}$	$4.7020\cdot10^{-6}$
83	$1.1703\cdot10^{-5}$	$6.6725\cdot10^{-6}$	$4.3580\cdot10^{-6}$
97	$1.0287\cdot10^{-5}$	$5.5575\cdot10^{-6}$	$3.5260\cdot10^{-6}$
111	$1.0193\cdot10^{-5}$	$5.6923\cdot10^{-6}$	$3.6668\cdot10^{-6}$
125	$9.5342\cdot10^{-6}$	$5.0383\cdot10^{-6}$	$3.1354\cdot10^{-6}$

It can be seen that the accuracy of TOA estimation varies from $3\cdot10^{-4}$ s to $3\cdot10^{-6}$ s depending on the type of the used filter and the number of integration periods.

IV. Conclusions

A possible algorithm for TOA estimation of weak pulsar signals is proposed and evaluated using the experimental data of pulsar B0329+54 provided by the Westerbork radio observatory. The results obtained show that the presented algorithm can be successfully used for processing of pulsar signals.

Acknowledgment

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