Proceedings of the 29th Conference
on Decision and Control
Honolulu, Hawaii • December 1990

Nonlinear dynamic disturbance decoupling

H.J.C. Huijberts*, H. Nijmeijer* and L.L.M. van der Wegen**

* Department of Applied Mathematics
** Department of Industrial Engineering
University of Twente, P.O. Box 217, 7500 AE Enschede, The Netherlands

Abstract

In this note we introduce the Dynamic Disturbance Decoupling Problem for nonlinear systems. A local solution of the problem is given. A compensator solving the problem can be obtained by means of Singh's algorithm.

1 Problem formulation

Consider a square nonlinear multi-input-multi-output control system of the form

$$\left\{\begin{array}{rcl} \dot{x} &=& f(x)+g(x)u+p(x)q \\ y &=& h(x) \end{array}\right. \tag{1}$$

where $x \in \mathcal{X}$, an open subset of $I\!R^n$, the inputs $u \in I\!R^m$, the outputs $y \in I\!R^m$, the disturbances $q \in I\!R^r$, f and h are vector-valued analytic functions and g and p are matrix-valued analytic functions, all of appropriate dimensions. In the Disturbance Decoupling Problem (DDP) one searches for a regular static state feedback

$$u = \alpha(x) + \beta(x)v \tag{2}$$

with v a new m-dimensional control and $\beta(x)$ a nonsingular $m \times m$ matrix for all x, so that in the feedback modified dynamics

$$\dot{x} = f(x) + g(x)\alpha(x) + g(x)\beta(x)v + p(x)q \tag{3}$$

the disturbances q do not affect the outputs y. A local solution of the DDP using differential geometric tools has led to a more or less complete understanding of this problem, see e.g. [6],[7]. The nonlinear DDP forms a direct generalization of the linear DDP, cf. [9].

The purpose of this note, which summarizes the preprint [5], is to give a dynamic version of the Disturbance Decoupling Problem for a square invertible nonlinear system (1). That is, instead of a static feedback law (2) we allow for a *regular* dynamic state feedback

$$\left\{\begin{array}{rcl} \dot{z} &=& \alpha(x,z)+\beta(x,z)v \\ u &=& \gamma(x,z)+\delta(x,z)v \end{array}\right. \tag{4}$$

with z the μ-dimensional compensator state and v an m-dimensional new control, and the regularity of (4) means that the system (4) with inputs v and outputs u is invertible for all x and z. In the Dynamic Disturbance Decoupling Problem (DDDP) we require that in the modified dynamics

$$\left\{\begin{array}{rcl} \dot{x} &=& f(x)+g(x)\gamma(x,z)+g(x)\delta(x,z)v+p(x)q \\ \dot{z} &=& \alpha(x,z)+\beta(x,z)v \end{array}\right. \tag{5}$$

the disturbances q do not influence the outputs y. Clearly the static DDP forms a special case of the DDDP by assuming that $\mu = 0$.

In this note we describe a set of necessary and sufficient conditions for the local solvability of the DDDP.

2 Main result

In this section we give our main result. Instrumental in the solution of the DDDP is what we like to call a Singh compensator, which can be obtained via the so called Singh's algorithm. Singh's algorithm has been introduced in [8] for calculation of a left-inverse of a nonlinear system. It is a generalization of the algorithm from [4], which was only applicable under some restrictive assumptions. We give Singh's algorithm for the system (1) without disturbances, i.e. $q \equiv 0$, following [3]. However, our notation is slightly different from the notation employed in [3].

Step 0

Define $\tilde{y}_0 := y$, $\hat{y}_0 := \emptyset$.

Step k+1

Suppose that in Steps 0 through k, $\tilde{y}_0, \cdots, \tilde{y}_k^{(k)}, \hat{y}_k^{(k)}$ have been defined so that

$$\begin{array}{rcl} \dot{\tilde{y}}_1 &=& \tilde{a}_1(x)+\tilde{b}_1(x)u \\ &\vdots& \\ \tilde{y}_k^{(k)} &=& \tilde{a}_k(x,\{\tilde{y}_i^{(j)} \mid 1 \le i \le k-1, i \le j \le k\}) \\ && +\tilde{b}_k(x,\{\tilde{y}_i^{(j)} \mid 1 \le i \le k-1, i \le j \le k-1\})u \\ \hat{y}_k^{(k)} &=& \hat{y}_k^{(k)}(x,\{\tilde{y}_i^{(j)} \mid 1 \le i \le k, i \le j \le k\}) \end{array} \tag{6}$$

Suppose also that there exist $\tilde{y}_{i0}^{(j)}$ $(1 \le i \le k-1, i \le j \le k-1)$ such that the matrix $\tilde{B}_k := [\tilde{b}_1^T, \cdots, \tilde{b}_k^T]^T$ has full rank ρ_k on a neighborhood of $(x_0, \{\tilde{y}_{i0}^{(j)} \mid 1 \le i \le k-1, i \le j \le k-1\})$. Then calculate

$$\hat{y}_k^{(k+1)} = \frac{\partial}{\partial x}\hat{y}_k^{(k)}[f(x)+g(x)u] + \sum_{i=1}^{k}\sum_{j=i}^{k}\frac{\partial \hat{y}_k^{(k)}}{\partial \tilde{y}_i^{(j)}}\tilde{y}_i^{(j+1)} \tag{7}$$

and write it as

$$\begin{array}{rcl} \hat{y}_k^{(k+1)} &=& a_{k+1}(x,\{\tilde{y}_i^{(j)} \mid 1 \le i \le k, i \le j \le k+1\}) \\ && +b_{k+1}(x,\{\tilde{y}_i^{(j)} \mid 1 \le i \le k, i \le j \le k\})u \end{array} \tag{8}$$

Define $B_{k+1} := [\tilde{B}_k^T, b_{k+1}^T]^T$, and suppose that there exist $\tilde{y}_{i0}^{(j)}$ $(1 \le i \le k, i \le j \le k)$ such that B_{k+1} has constant rank ρ_{k+1} on a neighborhood of $(x_0, \{\tilde{y}_{i0}^{(j)} \mid 1 \le i \le k, i \le j \le k\})$. Permute, if necessary, the components of $\hat{y}_k^{(k+1)}$ so that on this neighborhood the first ρ_{k+1} rows of B_{k+1} are linearly independent. Decompose $\hat{y}_k^{(k+1)}$ as $\hat{y}_k^{(k+1)} = \left(\begin{array}{cc} \tilde{y}_{k+1}^{(k+1)^T} & \hat{y}_{k+1}^{(k+1)^T} \end{array}\right)^T$ where $\tilde{y}_{k+1}^{(k+1)}$ consists of the first $s_{k+1} := (\rho_{k+1} - \rho_k)$ rows. Since the last rows of B_{k+1} are linearly dependent on the first ρ_{k+1} rows, we can write

$$\begin{array}{rcl} \dot{\tilde{y}}_1 &=& \tilde{a}_1(x)+\tilde{b}_1(x)u \\ &\vdots& \\ \tilde{y}_{k+1}^{(k+1)} &=& \tilde{a}_{k+1}(x,\{\tilde{y}_i^{(j)} \mid 1 \le i \le k, i \le j \le k+1\}) \\ && +\tilde{b}_{k+1}(x,\{\tilde{y}_i^{(j)} \mid 1 \le i \le k, i \le j \le k\})u \\ \hat{y}_{k+1}^{(k+1)} &=& \hat{y}_{k+1}^{(k+1)}(x,\{\tilde{y}_i^{(j)} \mid 1 \le i \le k+1, i \le j \le k+1\}) \end{array} \tag{9}$$

where once again everything is rational in $\tilde{y}_i^{(j)}$. Finally, set $\tilde{B}_{k+1} := [\tilde{B}_k^T, \tilde{b}_{k+1}^T]^T$. End of Step $k+1$. ■

It should be noted that the integers $\rho_1, \cdots, \rho_k, \cdots$ defined above do not depend on the particular permutation of the rows of $\hat{y}_k^{(k+1)}$ we employ, cf. [3]. Thus, using the algorithm we obtain a uniquely defined sequence of integers $0 \leq \rho_1 \leq \cdots \leq \rho_k \leq \cdots \leq m$. We associate a notion of regularity with Singh's algorithm which in the following way.

Definition 2.1 *Let a point $x_0 \in \mathcal{X}$ be given. We call x_0 a* strongly regular point *for (1) if for each application of the algorithm the constant rank assumptions of the algorithm are satisfied.* ■

Consider an invertible system (1), i.e. $\rho_n = m$. Then we define a Singh compensator for (1) as follows. Let x_0 be a strongly regular point for (1) and apply Singh's algorithm to (1) with $q \equiv 0$. This yields at the n-th step:

$$\begin{aligned}\dot{\tilde{Y}}_n &= \tilde{A}_n(x, \{\tilde{y}_i^{(j)} \mid 1 \leq i \leq n-1, i \leq j \leq n\}) + \\ &\tilde{B}_n(x, \{\tilde{y}_i^{(j)} \mid 1 \leq i \leq n-1, i \leq j \leq n-1\})u\end{aligned} \tag{10}$$

where $\tilde{Y}_n = \left(\tilde{y}_1^T, \cdots, \tilde{y}_n^{(n-1)^T} \right)$ and where $\tilde{B}_n$ is invertible on a neighborhood of $(x_0, \{\tilde{y}_{i0}^{(j)} \mid 1 \leq i \leq n-1, i \leq j \leq n-1\})$ for some $\tilde{y}_{i0}^{(j)}$ $(1 \leq in, i \leq j \leq n)$. Then from (10) we obtain on this neighborhood:

$$u = \tilde{B}_n^{-1} \left[\dot{\tilde{Y}}_n - \tilde{A}_n \right] =: \phi(x, \{\tilde{y}_i^{(j)} \mid 1 \leq i \leq n, i \leq j \leq n\}) \tag{11}$$

Let γ_i be the lowest time-derivative and δ_i the highest time-derivative of y_i appearing in (11). Then it can be shown that ϕ is of the form

$$\begin{aligned}&\phi(x, \{y_i^{(j)} \mid 1 \leq i \leq m, \gamma_i \leq j \leq \delta_i\}) = \\ &\phi_1(x, \{y_i^{(j)} \mid 1 \leq i \leq m, \gamma_i \leq j \leq \delta_i - 1\}) + \\ &\sum_{i=1}^{m} \phi_{2i}(x, \{y_i^{(j)} \mid 1 \leq i \leq m, \gamma_i \leq j \leq \delta_i - 1\}) y_i^{(\delta_i)}\end{aligned} \tag{12}$$

Let z_i $(i = 1, \cdots, m)$ be a vector of dimension $\delta_i - \gamma_i$ and consider the system:

$$\left\{ \begin{array}{rcl} \dot{z}_i &=& A_i z_i + B_i v_i \quad (i = 1, \cdots, m) \\ u &=& \phi_1(x, z_1, \cdots, z_m) + \sum_{i=1}^{m} \phi_{2i}(x, z_1, \cdots, z_m) v_i \end{array} \right. \tag{13}$$

with inputs $v_1, \cdots, v_m$, outputs u, (A_i, B_i) $(i = 1, \cdots, m)$ in Brunovsky canonical form, and $z_i(0) = (y_{i0}^{(\gamma_i)}, \cdots, y_{i0}^{(\delta_i - 1)})$. Then (13) is called a *Singh compensator* for (1) around x_0. This compensator performs the left-inversion of the system (1), cf. [8]. Our main result can now be stated as follows.

Theorem 2.2 *Consider the square invertible system (1). Let x_0 be a strongly regular point for (1). Then the Dynamic Disturbance Decoupling Problem is locally solvable around x_0 if and only if it is solvable by means of a Singh compensator around x_0.*

Proof See [5]. ■

3 Comments

1. As noted before, the theory on the nonlinear DDP is very much based on a proper extension of the linear Disturbance Decoupling Problem. One could therefore think that similarly the nonlinear Dynamic Disturbance Decoupling Problem naturally extends the DDDP for linear systems. However one can show that for linear systems the DDDP is solvable if and only if the DDP is, see [1],[2]. Although a similar result holds for scalar output nonlinear systems, this conclusion is no longer true in the multivariable case. In other words when the number of outputs (=number of inputs) exceeds one, it may happen that the nonlinear DDDP is locally solvable whereas the nonlinear DDP is not.

2. It is straightforward to extend the result of Theorem 2.2 to nonsquare systems, i.e. systems with a different number of input and output channels, see [5].

3. Theorem 2.2 forms one -computationally direct- way of checking the local solvability of the DDDP. In [5] equivalent algebraic and differential geometric conditions for the solvability of the DDDP are given. Also one may find in the same reference an analogous treatment of the so called Dynamic Disturbance Decoupling Problem with disturbance measurements (DDDPdm) for nonlinear systems.

References

[1] G. Basile and G. Marro, *Controlled and conditioned invariant subspaces in linear system theory*, J. Optimiz. Theory Appl, 3, (1969), pp. 306-315.

[2] S.P. Bhattacharyya, *Disturbance rejection in linear systems*, Int. J. Control, 5, (1974), pp. 633-637.

[3] M.D. di Benedetto, J.W. Grizzle and C.H. Moog, *Rank invariants of nonlinear systems*, SIAM J. Control Optimiz., 27, (1989), pp. 658-672.

[4] R.M. Hirschorn, *Invertibility of Multivariable Nonlinear Control Systems*, IEEE Trans. Automat. Control, AC-24, (1979), pp. 855-865.

[5] H.J.C. Huijberts, H. Nijmeijer and L.L.M. van der Wegen, *Dynamic disturbance decoupling for nonlinear systems*, Memo.no. 835, Department of Applied Mathematics, University of Twente, (1989). Submitted for publication.

[6] A. Isidori, *Nonlinear Control Systems* (Second Edition), Springer, Berlin, (1989).

[7] H. Nijmeijer and A.J. van der Schaft, *Nonlinear Dynamical Control Systems*. Springer, New York, (1990).

[8] S.N. Singh, *A Modified Algorithm for Invertibility in Nonlinear Systems*, IEEE Trans. Automat. Control, AC-26, (1981), pp. 595-598.

[9] W.M. Wonham, *Linear Multivariable Control: a Geometric Approach* (Third Edition), Springer, New York, (1985).