Spectral Weighting Functions for Single-symbol Phase-noise Specifications in OFDM Systems

Fokke Hoeksema, Roel Schiphorst, Kees Slump
University of Twente,
Department of Electrical Engineering, Mathematics and Computer Science (EEMCS)
Signals and Systems Group,
P.O. box 217 – 7500 AE Enschede – The Netherlands
E-mail: {f.w.hoeksema,r.schiphorst,c.h.slump}@utwente.nl

***Abstract*— For the specification of phase-noise requirements for the front-end of a HiperLAN/2 system we investigated available literature on the subject. Literature differed in several aspects. One aspect is in the type of phase-noise used (Wiener phase-noise or small-angle phase noise). A Wiener phase-noise based analysis leads to contradictions with the type of analysis normally used in the solid state oscillator literature. However, a phase-noise spectrum with a Wiener phase-noise shape can be used provided that the small-angle approximation is satisfied.**

An other aspect is whether a Fourier Series or DFT based approach is used. The approaches use weighting functions to relate phase-noise power spectral densities to phase-noise power. The two types of analysis are presented in a unified fashion that allows easy comparison of the weighting functions involved. It can be shown that for practical purposes results are identical.

Finally phase-noise specifications for the HiperLAN/2 case are presented.

***Index Terms*— Hiperlan/2, Phase-noise.**

I. Introduction

In our *Software Defined Radio* (SDR) project we aim at combining two different types of standards, Bluetooth and HiperLAN/2 on one common platform. We focus on the radio front-end of a receiver, so from antenna (RF signal) till and including demodulator (raw bits). Our demonstrator under construction consists of a flexible wide-band analog front-end, a PCI-board with ADCs and Sample Rate Conversion and a GPP software implementation (P4, 2.4GHz) of the demodulator. The system is described in more detail in [1] and [2].

Two questions need to be answered: 1/. 'What are the phase-noise requirements for the analog quadrature mixing system used?' and 2/. 'Is a symbol synchronization system (e.g. based on the one described in [3]) necessary given the burst-duration in HiperLAN/2 and given the phase-noise requirements under 1/.?' In this paper we focus on the first question.

To be able to specify the phase-noise requirements for the analog quadrature mixing system we turned to the available literature on phase-noise and OFDM. The articles investigated differ in several aspects (see section III). Most characteristically, some use a Fourier-Series (FS) analysis approach and some use a Discrete Fourier Transform (DFT) analysis. Moreover, we would like to use the specification parameter commonly used by analog synthesizer designers $\mathcal{L}(f_{m0})$ (see section II) to relate to the OFDM performance metrics.

In this paper we first describe phase-noise and define the specification parameter used by analog front-end designers (solid state oscillator literature).

In section IV both the DFT and FS analysis are presented. Basically, weighting functions are defined that need to be applied to the phase-noise spectrum in order to asses its consequences for an OFDM system.

In section V we relate the $\mathcal{L}(f_{m0})$ parameter to OFDM performance metrics for the HiperLAN/2 case. Finally, conclusions are drawn.

II. Phase Noise

An ideal harmonic oscillator provides a signal $x(t) = A\cos(\omega_0 t + \varphi_0)$. On a spectrum analyzer this produces a spectral line with power $P_x = A^2/2$. In reality however, one observes sidebands that may be attributed both to amplitude and phase variations in the oscillator. Due to the presence of limiting mechanisms in the oscillator (or a limiter before the spectrum analyzer) one generally assumes that the amplitude variations can be neglected and that the sidebands are entirely due to variations in phase. These so-called phase-noise sidebands can be attributed to noise in the oscillator circuit (device noise, substrate noise, supply noise) and are modelled by a signal

$$x(t) = A\cos(\omega_0 t + \varphi_0 + \varphi(t)) \tag{1}$$

with $\varphi(t)$ a (zero-mean) stochastic process [4], [5].

While the (double sided) power spectral density (PSD) of the phase-noise $S_{\varphi\varphi}(f)$ *cannot* be directly measured, the single sideband (SSB) PSD $S_x(f)$ of the oscillator signal $x(t)$ *can* be measured using a spectrum analyzer.

Phase noise is characterized by a measurement on $x(t)$ in which the SSB phase-noise-to-carrier ratio $\mathcal{L}(f_m)$ at an offset frequency f_m from the carrier frequency f_0 is determined. With $P_{SSB}(f_0 + f_m; 1\,\mathrm{Hz})$ the SSB power in a band of 1 Hz width, we have

$$\mathcal{L}(f_m) \triangleq \frac{P_{SSB}(f_0 + f_m; 1\,\mathrm{Hz})}{P_x} \approxeq \frac{S_x(f_0 + f_m)}{A/2}. \tag{2}$$

which is measured in [dBc/Hz]. This refers to the power in a 1 Hz band being measured relative to the carrier power.

The specification (2) above enables measurement of $\mathcal{L}(f_m)$ over a range of offset frequencies f_m, resulting in a so-called spectral mask. This spectral mask has a characteristic shape, with close to the carrier a region with a f^{-3} slope (-30 dB/decade), then a large f^{-2} region (-20 dB/decade) and finally a white-noise floor region [4], [5]. For unambiguous specification of phase-noise the complete spectral mask has to be prescribed. In the analog design-world however, often only a *single number* $\mathcal{L}(f_m)|_{f_m=f_{m0}}$ is specified (for a particular offset frequency f_{m0}), assuming a f^{-2} sideband slope [4, p.68]. Below, we will relate this number to performance metrics used to asses the consequences of phase noise in a HiperLAN/2 based OFDM system.

Remains the issue of relating the SSB PSD $S_x(f)$ observed by the spectrum analyzer (or $\mathcal{L}(f_m)$ using some phase-noise measurement equipment) to the (double sided) phase-noise PSD $S_{\varphi\varphi}(f)$. For this, one evokes "FM-theory" and the "small-angle approximation (SMAP)" to relate these PSDs to one another (e.g. [4], [5], [6]).

A. Small-Angle Phase Noise

The analysis of phase noise in solid state oscillator literature (e.g. [5], [4]) is based on the so-called small-angle approximation. In general, the complex envelope of the bandpass signal in (1) is given by $\tilde{x}(t) = A\,e^{j\,\varphi(t)}$. In the small-angle approximation this complex envelope is assumed to be equal to $\tilde{x}(t) = A\,(1 + j\,\varphi(t))$. Moreover, the phase-noise process is assumed to be stationary. The autocorrelation function of the complex envelope is then given by

$$R_{\tilde{x}\tilde{x}}(\tau) \triangleq E[\tilde{x}(t+\tau)\tilde{x}^*(t)] = A^2\,(1+R_{\varphi\varphi}(\tau)). \tag{3}$$

Fourier transformation of this autocorrelation function gives $S_{\tilde{x}\tilde{x}}(f) = A^2\delta(f) + A^2 S_{\varphi\varphi}(f)$. Applying the standard relations for bandpass signals (e.g. see [7]) we find for the SSB PSD

$$S_x(f) = \frac{A^2}{2}\,\delta(f - f_0) + \frac{A^2}{2}\,S_{\varphi\varphi}(f - f_0). \tag{4}$$

The signal power P_x is

$$\begin{aligned} P_x &= \int_0^\infty S_x(f)\,df = \frac{A^2}{2} + \frac{A^2}{2}\int_0^\infty S_{\varphi\varphi}(f - f_0)\,df = \\ &\approxeq \frac{A^2}{2} + \frac{A^2}{2}\int_{-\infty}^{\infty} S_{\varphi\varphi}(f_m)\,df_m \triangleq \frac{A^2}{2} + \frac{A^2}{2}\,\sigma_\varphi^2 \end{aligned} \tag{5}$$

in which we substituted $f_m = f - f_0$ and assumed that some "phase noise bandwidth" is far smaller than the carrier frequency. Moreover we defined the total phase noise power σ_φ^2. As can be seen, indeed, for $P_x \approxeq \frac{A^2}{2}$ to hold, the total phase-noise power should be less than, say 0.1 rad^2. Finally, the SSB phase-noise-to-carrier ratio $\mathcal{L}(f_m)$ can be derived by substituting (4) into (2) as

$$\begin{aligned} \mathcal{L}(f_m) &= \frac{S_x(f_0 + f_m)}{P_x} \approxeq \frac{A^2/2\,S_{\varphi\varphi}(f_m)}{A^2/2} = \\ &= S_{\varphi\varphi}(f_m) = S_\varphi(f_m)/2. \end{aligned} \tag{6}$$

Observe that $\mathcal{L}(f_m)$ can be measured while $S_{\varphi\varphi}(f_m)$ is a construct from "FM theory" and SMAP.

B. Wiener Phase Noise

In [8] the effects of so-called *Wiener phase noise* on an OFDM system are addressed. The Wiener phase-noise model is used to relate the line width of *lasers* to the white noise process that causes this width [9]. As such, it has nothing to do with solid-state oscillators.

The SSB PSD of the bandpass signal is given by

$$S_x(f) = \frac{2/\sigma_w^2}{1 + \left(\frac{f - f_0}{f_n}\right)^2} \tag{7}$$

in which σ_w^2 is the variance of the white noise in the Wiener process [10, pp. 321,344]). One recognizes the magnitude-response of a (first order) Butterworth type of filter with cutoff frequency f_n. As we may assume that $f_0 \gg f_n$ it can be shown that the signal power $P_x = 1/2$, as is to be expected. The so-called "laser line-width" is defined as $B_\varphi \triangleq 2\, f_n$; we will call B_φ the phase-noise bandwidth. So for the jitter variance we have $\sigma^2_{\Delta\varphi(\tau)} = 2\,\pi\, B_\varphi\, |\tau| = 4\,\pi\, f_n\, |\tau|$, which is the starting assumption in [8].

It can be shown that one *cannot* simply use the Wiener phase-noise case in the "FM-theory"/SMAP approach that is found in the solid-state oscillator literature. What one can do however is use the Wiener phase-noise *shape* as a phase-noise PSD. This results in[1]

$$S_{\varphi\varphi,a}(f_m) = \frac{\sigma^2_{\varphi,a}}{\pi\, f_n}\, \frac{1}{1+\left(\frac{f_m}{f_n}\right)^2} \tag{8}$$

as an approximate phase-noise mask and one can make sure that $\sigma^2_{\varphi,a} \leq 0.1\,\text{rad}^2$.

C. Single-number phase-noise specification

Using (8) a single number $\mathcal{L}(f_{m0})$ can be used for phase-noise specification. In the region for which $f_m >> f_n$ we have that

$$S_{\varphi\varphi,a}(f_m) \simeq \frac{\sigma^2_{\varphi,a}}{\pi}\, \frac{f_n}{f_m^2} \triangleq \hat{\mathcal{L}}(f_m) \tag{9}$$

Specifying the phase-noise at an offset frequency of 1 MHz from the carrier with $\hat{\mathcal{L}}(1\,\text{MHz}) \triangleq \mathcal{L}_{dB,1}$ we have

$$\sigma^2_{\varphi,a} \cdot f_n = 10^{\mathcal{L}_{dB,1}/10 + {}^{10}\log(\pi)+12} \tag{10}$$

One observes the ambiguity in the single-number specification here: for a chosen and fixed phase-noise variance, multiple cutoff-frequencies satisfy the specification; for a chosen and fixed cutoff-frequency multiple phase-noise variances do.

III. Phase-Noise in OFDM Literature

There exists an extensive amount of literature regarding phase-noise and OFDM. In our work we used [8], [11], [12], [13] and [14], so we are by no means comprehensive let alone complete. In the studies often both results of analysis and simulations are presented. Here, we are interested in the results of the analysis.

In all papers, the effects of phase-noise on the reception of a *single* OFDM symbol are addressed. The reported results differ in type of phase-noise considered (e.g. Wiener phase-noise; Small-angle phase-noise), in type of OFDM system models used (DFT-based analysis; FS-based analysis), in type of channel models used (small phase-noise; with or without phase-offset), in type of phase-noise specification-parameter used (e.g. one-sided spectral "line width" (so f_n); phase-noise bandwidth (so B_φ); complete two-sided analog phase-noise PSD (so, the spectral mask)) and in type of the used performance metrics (SNR Degradation w.r.t. the no phase-noise situation (D_{SNR}); phase-noise power in relation to received carrier-symbol power (SNR_{ICI}) and Symbol Error Rate (SER)).

Garcia [12] relates the SNR Degradation w.r.t. the no phase-noise situation to the symbol-energy to noise-density ratio E_s/N_0:

$$D_{SNR} = 10\,{}^{10}\log(1+\sigma^2_{\varphi,a}\,\frac{E_s}{N_0}) \quad \text{in [dB].} \tag{11}$$

The formula holds when the small-angle approximation holds. Stott [13] uses the total ICI phase-noise power in relation to received carrier-symbol power SNR_{ICI} that can be defined as

$$SNR_{ICI} = 10\,{}^{10}\log(\frac{1}{\sigma^2_{\varphi_\Sigma,r}}) \quad \text{in [dB]} \tag{12}$$

in which $\sigma^2_{\varphi_\Sigma,r}$ is the total ICI power for the rth carrier symbol, see (19).

In table I an overview is presented.

Author	PN Type	OFDM Model	PN spec. param.	Perf. Metric
Pollet [8]	Wiener	FS	f_n	D_{SNR}
Garcia [11], [12]	Small angle	DFT	B_φ	D_{SNR}
Stott [13]	Small angle	FS	$S_{\varphi\varphi,a}(f)$	SNR_{ICI}
El-Tannay [14]	Small angle	DFT	$S_{\varphi\varphi,a}(f)$	SER

TABLE I
LITERATURE ON PHASE-NOISE (PN) IN OFDM.

Stott [13] and El-Tannay [14] use a weighting-function based approach that enable to establish the phase-noise power in relation to received carrier-symbol power. El-Tannay proceeds from there to arrive at the SER. Their approaches differ in OFDM model used, their weighting functions also. In section IV a (brief) derivation is presented that starts with ideas from Pollet [8] and allows the weighting functions to be derived and presented in a unified way thus allowing easy comparison.

IV. Phase Noise Analysis

A. Fourier Series Analysis

Consider the transmission of a single OFDM symbol of which the useful part starts at $t = 0$.

[1]The subscript ",a" refers to the analog domain (continuous-time), in section IV-B, it is contrasted to a subscript ",d" referring to the digital domain (discrete-time).

Neglecting the cyclic prefix of the OFDM symbol, the complex envelope of the transmitted signal is

$$\tilde{s}(t) = \begin{cases} \sum_{k=0}^{N-1} C_k \,.\, e^{\jmath \frac{2\pi}{T_u} kt} & \text{if} \quad 0 \le t \le T_u, \\ 0 & \text{elsewhere} \end{cases} \tag{13}$$

in which T_u is the useful-part duration, $C_k \in \mathbb{C}$ the complex symbol for the k-th subcarrier and N the number of sub-carriers. The r-th received complex symbol B_r is given by

$$B_r = \frac{1}{T_u} \int_{t=0}^{T_u} \tilde{r}(t) \,.\, e^{-\jmath \frac{2\pi}{T_u} rt} \, dt, \tag{14}$$

in which $\tilde{r}(t)$ is the received complex envelope.

For a flat channel without noise, the relation between transmitted an received complex envelope is given by

$$\tilde{r}(t) = \tilde{s}(t) \,.\, e^{-\jmath \, \varphi(t)} = \tilde{s}(t) \,.\, (1 - \jmath \, \varphi(t)) \tag{15}$$

in which $\tilde{s}(t)$ is the transmitted complex envelope and $\varphi(t)$ represents the (possibly large) phase offset and the (small angle) phase-noise of both transmitter and receiver. The phase offset is corrected by the receiver so the small-angle phase-noise approximation results in the final step above ($\varphi(0) = 0$).

Following [8] and using the small-angle approximation (like in [13]) it can be shown that

$$\hat{B}_r = C_r - \jmath \, C_r \,.\, \varphi_0 - \jmath \sum_{k=0,\, k \neq r}^{N-1} C_k \,.\, \varphi_{r-k} \tag{16}$$

$$\varphi_0 = \frac{1}{T_u} \int_{t=0}^{T_u} \varphi(t) \, dt \tag{17}$$

$$\varphi_l = \frac{1}{T_u} \int_{t=0}^{T_u} \varphi(t) \,.\, e^{-\jmath \frac{2\pi}{T_u} lt} \, dt \tag{18}$$

In (16) we recognize the wanted r-th carrier symbol C_r; an error component that is equal for all symbols C_r, the so-called *Common Phase Error* (CPE) component and an error component that shows the interference of other carrier symbols ($k = 0, \ldots, N-1,\ k \neq 0$) into the wanted carrier symbol C_r, the so-called *Inter-Carrier Interference* (ICI) component.

Due to the presence of pilot tones in the system, the Common Phase Error can be estimated and (partly) corrected [11], [12], [14]. The ICI component of the error has a (Gaussian) noise character and can be tackled in case an equalizer is used. In this paper we deem the ICI component the most harmful.

A power-based analysis [13] of the error components relates the power of the received (zero mean) complex symbol $\sigma_b^2 = E[|\hat{B}_r|^2]$ to the power of the transmitted (zero mean) complex symbol $E[|C_r|^2]$ and the error power. We assume that *all* carriers are *identically* loaded[2]. Moreover we assume that they are loaded with identically distributed zero-mean symbols, so $\forall\, k \;\; E[|C_k|^2] = \sigma_c^2$. The carriers are assumed to be loaded with interleaved data, so $E[C_m C_k^*] = \sigma_c^2 \, \delta_{mk}$ (Kronecker delta). The last assumption is that the phase noise and the carrier symbols are statistically independent, $E[\varphi_l C_m] = 0$. Using (16) we find:

$$\begin{aligned} E[|\hat{B}_r|^2] &= E[|C_r|^2] \,.\, E[|\hat{I}_0|^2] + \\ &\quad + \sum_{k=0,\, k \neq r}^{N-1} E[|C_k|^2] \,.\, E[|\hat{I}_{r-k}|^2] = \\ &= \sigma_c^2 + \sigma_c^2 \,.\, \sigma_{\varphi_0}^2 + \sigma_c^2 \,.\, \sum_{k=0,\, k \neq r}^{N-1} \sigma_{\varphi_{r-k}}^2 \\ &\triangleq \sigma_c^2 + \sigma_c^2 \,.\, \sigma_{\varphi_0}^2 + \sigma_c^2 \,.\, \sigma_{\varphi_\Sigma, r}^2 \end{aligned} \tag{19}$$

in which σ_c^2 is the (transmitted) carrier symbol power, $\sigma_{\varphi_0}^2$ is the Common Phase Error (CPE) power, $\sigma_{\varphi_l}^2$ the variance of (filtered) modulated phase noise and $\sigma_{\varphi_\Sigma, r}^2$ the total Inter-Carrier Interference (ICI) power for the rth carrier symbol.

The variance of the phase noise $\sigma_{\varphi_0}^2$ at the end of the useful period T_u can be found observing that φ_0 results from subsequent filtering and sampling (at time instant $t = T_u$) of $\varphi(t)$. The used filter, a so-called boxcar filter, has an impulse response $h(t) = 1/T_u \, (u(t) - u(t - T_u))$ (with u(t) the unit step function). It follows that:

$$(17) \Rightarrow \sigma_{\varphi_0}^2 = \int_{-\infty}^{\infty} \text{sinc}^2(f.\, T_u) \,.\, S_{\varphi\varphi,\, a}(f) \, df \tag{20}$$

In which $S_{\varphi\varphi,\, a}(f)$ is the power spectral density (PSD) of the (continuous-time phase-noise). For the filtered modulated phase-noise we find

$$\begin{aligned} &(18) \Rightarrow \\ &\sigma_{\varphi_l}^2 = \int_{-\infty}^{\infty} \text{sinc}^2(f.\, T_u) \,.\, S_{\varphi\varphi,\, a}(f + l \,.\, f_u) \, df = \\ &= \int_{-\infty}^{\infty} \text{sinc}^2(f.\, T_u - l) \,.\, S_{\varphi\varphi,\, a}(f) \, df, \;\; f_u = 1/T_u. \end{aligned} \tag{21}$$

By substituting (21) into the summation factor of the

[2]In HiperLAN/2 this is not the case: apart from data carrier-symbols there are pilot carrier-symbols and zero carrier-symbols inserted.

third term of (19) we find:

$$\sigma^2_{\varphi_\Sigma,r} = \sum_{k=0,\,k\neq r}^{N-1} \sigma^2_{\varphi_{r-k}} =$$
$$= \sum_{k=0,\,k\neq r}^{N-1} \int_{-\infty}^{\infty} \mathrm{sinc}^2(f.T_u - (r-k))\,.\,S_{\varphi\varphi,\,a}(f)\,df =$$
$$\triangleq \int_{-\infty}^{\infty} W^{(r)}_{ICI,\,a}(f)\,.\,S_{\varphi\varphi,\,a}(f)\,df \quad \text{in which}$$
$$W^{(r)}_{ICI,\,a}(f) = \sum_{k=0,\,k\neq r}^{N-1} \mathrm{sinc}^2(f.T_u - (r-k)) \tag{22}$$

The equation above shows that the ICI power is determined by a weighting function $W^{(r)}_{ICI,\,a}(f)$ of the phase-noise PSD. Also the CPE power in (20) can be re-formulated in this way:

$$\begin{aligned}\sigma^2_{\varphi_0} &= \int_{-\infty}^{\infty} \mathrm{sinc}^2(f.T_u)\,.\,S_{\varphi\varphi,\,a}(f)\,df = \\ &\triangleq \int_{-\infty}^{\infty} W_{CPE,\,a}(f)\,.\,S_{\varphi\varphi,\,a}(f)\,df\end{aligned} \tag{23}$$

So, using these two weighting functions, the phase-noise PSD can be related to the CPE and ICI error components. The difficulty is that $W^{(r)}_{ICI,\,a}(f)$ depends on the actual carrier r we are considering, see (22). It would come in handy to have only two weighting functions, one for the CPE component of the error and one (in stead of the N/2 different ones) for the ICI component. This can be done straightforwardly by observing $\sigma^2_{\varphi_0} + \sigma^2_{\varphi_\Sigma,r} \leq \sigma^2_{\varphi,a}$. Then, one can define a weighting function $W_{ICI,\,a}(f)$ that is independent of the carrier number r as

$$W_{ICI,\,a}(f) \triangleq 1 - W_{CPE,\,a}(f) \tag{24}$$

and use it to assess the phase-noise spectrum in an approximate fashion.

B. Discrete Fourier Transform Analysis

As OFDM systems operate on discrete signals, the Fourier series analysis above is less according the nature of the case than the Discrete Fourier Transform (DFT) analysis presented in this section (see also [14]).

In the digital case the complex envelope of the transmitted signal and the received symbol for the r-th carrier $\tilde{B}_r$ are given by (compare with (13) and (15))[3]:

$$\tilde{s}[\,i\,] = \sum_{k=0}^{N-1} C_k\,.\,e^{\jmath\,\frac{2\pi}{N}\,ki}\,,\; \tilde{B}_r = \frac{1}{N}\sum_{i=0}^{N-1} \tilde{r}[\,i\,]\,.\,e^{-\jmath\,\frac{2\pi}{N}\,ri} \tag{25}$$

The channel is given by the discrete-time version of (15). Now, an approach can be followed that basically leads to an expression similar to (19). However, the expressions for the Common Phase Error (CPE) power $\sigma^2_{\tilde{\varphi}_0}$ and total Inter-Carrier Interference (ICI) power for the rth carrier symbol $\sigma^2_{\tilde{\varphi}_\Sigma,r}$ differ from the ones in the Fourier Series case.

The variance of the phase-noise $\sigma^2_{\tilde{\varphi}_0}$ is:

$$\begin{aligned}\tilde{\varphi}_0 &= \frac{1}{N}\sum_{i=0}^{N-1} \varphi[\,i\,] \Rightarrow \\ \sigma^2_{\tilde{\varphi}_0} &= \frac{1}{2\pi}\int_{-\pi}^{\pi} S_{\tilde{\varphi}_0\tilde{\varphi}_0,d}(e^{\jmath\,\Omega})\,d\Omega\end{aligned} \tag{26}$$

with $S_{\tilde{\varphi}_0\tilde{\varphi}_0,d}(e^{\jmath\,\Omega})$ the PSD of a boxcar-filtered discrete-time phase-noise signal and Ω the angular frequency for discrete-time signals. The relation between the discrete-time phase-noise PSD $S_{\varphi\varphi,d}(e^{\jmath\,\Omega})$ and the boxcar-filtered version is given by:

$$\begin{aligned}&S_{\tilde{\varphi}_0\tilde{\varphi}_0,d}(e^{\jmath\,\Omega}) = S_{\varphi\varphi,d}(e^{\jmath\,\Omega})\,.\,|H(e^{\jmath\,\Omega})|^2 \\ &\quad\text{in which} \;\; |H(e^{\jmath\,\Omega})|^2 = \frac{1}{N^2}\frac{\sin^2(\Omega\,\frac{N}{2})}{\sin^2(\frac{\Omega}{2})}\end{aligned} \tag{27}$$

and N the number of samples in the useful part of the OFDM symbol ($N = T_u/T$).

We may assume that the phase-noise bandwidth $B_\varphi = 2 * f_n$ is smaller than the inter-carrier distance $\Delta f = f_s/N$. Therefore it will be *far* smaller than the sample frequency and we may assume that the phase noise is bandlimited for our purposes. In case the phase-noise is strictly bandlimited ($S_{\varphi\varphi,a} = 0$ for $|\,f\,| \geqq f_s/2$), the relation between continuous-time and discrete-time PSD simplifies to:

$$S_{\varphi\varphi,d}(e^{\jmath\,2\pi f\,T}) = \frac{1}{T}\,S_{\varphi\varphi,a}(f) \tag{28}$$

and thus, restating (26) using (28):

$$\sigma^2_{\tilde{\varphi}_0} = \int_{-f_s/2}^{f_s/2} \frac{\mathrm{sinc}^2(f.T_u)}{\mathrm{sinc}^2(f.T)}\,.\,S_{\varphi\varphi,\,a}(f)\,df \tag{29}$$

This equation may be directly compared to (20).
For determination of the ICI-induced variance, we aim at expressions equivalent to (21) and (22). The analysis is not shown here, but the important observation is that, contrary to the Fourier series case,

[3] The $\sim$ is used to denote the symbol values and phase-noise metrics that result from the discrete-time character of the signal involved.

the weighting functions for all carriers are equal to one-another and are thus independent of the carrier-number:

$$\forall\, r : W_{ICI,\,d}^{(r)}(e^{\jmath\,\Omega}) = W_{ICI,\,d}(e^{\jmath\,\Omega}) = 1 - |H(e^{\jmath\,\Omega})|^2 \quad (30)$$

This weighting function is directly comparable to the ICI weighting function resulting from the Fourier-series analysis (22) and the approximate result in (24).

C. Overview of DFT and FS analysis

An overview of the derived phase-noise variances and weighting functions is given in table II and table III. The relation between the phase-noise variances and the power of the received complex symbol $E[|\hat{B}_r|^2]$ is given in (19). Evaluation of these functions show that, for all practical purposes (for which $f_n << \Delta f$) the results are identical. A difficulty is in the numerical computation of the highly oscillatory functions, so this may be a reason for further approximation. Moreover, as the exact results are known, the merits of further simplification can be assessed.

	Phase-noise Variances
CPE (23)	$\sigma^2_{\varphi_0} = \int_{-\infty}^{\infty} W_{CPE,\,a}(f)\,.\,S_{\varphi\varphi,\,a}(f)\,df$
ICI, Exact (22)	$\sigma^2_{\varphi_\Sigma,r} = \int_{-\infty}^{\infty} W_{ICI,\,a}^{(r)}(f)\,.\,S_{\varphi\varphi,\,a}(f)\,df$ $= \sum_{k=0,\,k\neq r}^{N-1} \text{sinc}^2(f.T_u - (r-k))$
ICI, Appr. (24)	$\sigma^2_{\varphi_\Sigma} = \int_{-\infty}^{\infty} W_{ICI,\,a}(f)\,.\,S_{\varphi\varphi,\,a}(f)\,df$
	Weighting Function
CPE (23)	$W_{CPE,\,a}(f) = \text{sinc}^2(f.T_u)$
ICI, Exact (22)	$W_{ICI,\,a}^{(r)}(f) =$ $= \sum_{k=0,\,k\neq r}^{N-1} \text{sinc}^2(f.T_u - (r-k))$
ICI, Appr. (24)	$W_{ICI,\,a}(f) = 1 - W_{CPE,\,a}(f)$

TABLE II
PHASE-NOISE METRICS FOR OFDM SYSTEMS (FOURIER-SERIES ANALYSIS).

	Phase-noise Variances
CPE (29)	$\sigma^2_{\tilde{\varphi}_0} = \int_{-f_s/2}^{f_s/2} W_{CPE,\,d}(f)\,.\,S_{\varphi\varphi,\,a}(f)\,df$
ICI, Exact	$\sigma^2_{\tilde{\varphi}_\Sigma} = \int_{-f_s/2}^{f_s/2} W_{ICI,\,d}(f)\,.\,S_{\varphi\varphi,\,a}(f)\,df$
	Weighting Function
CPE (29)	$W_{CPE,\,d}(f) = \frac{\text{sinc}^2(f.T_u)}{\text{sinc}^2(f.T)}$
ICI, Exact	$W_{ICI,\,d}(f) = 1 - W_{CPE,\,d}(f)$

TABLE III
PHASE-NOISE METRICS FOR OFDM SYSTEMS (DFT ANALYSIS).

V. THE HIPERLAN/2 CASE

For our HiperLAN/2 system we assume that the Common Phase Error can be sufficiently corrected and that the main performance degrading phase-noise contribution is in the added noise due to the ICI. As the performance metric we therefore use the total ICI phase-noise power in relation to received carrier-symbol power SNR_{ICI} from (12), however with the total ICI power $\sigma^2_{\tilde{\varphi}_\Sigma}$ that resulted from the DFT analysis (see table III). In figure 1 the results are presented for different values of the total phase-noise power.

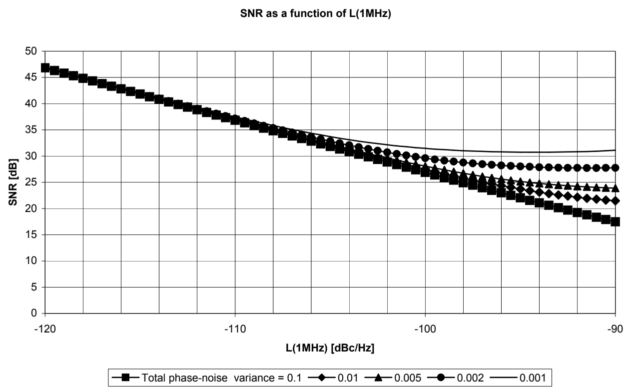

Fig. 1. Phase-noise degradation due to ICI as a function of the SSB phase-noise to carrier ratio $\mathcal{L}_{dB,1}$ (10) for a HiperLAN/2 system ($N = 64$ carriers).

As can be seen, for an ICI SNR requirement of, say, 25 dB, an SSB phase-noise to carrier ratio $\mathcal{L}_{dB,1}$ in the order of -95dBc/Hz is necessary.

One can either resort to an $\mathcal{L}_{dB,1}$ (from (10)) for which the total phase noise power is small (e.g. $\sigma^2_{\varphi,a} = 0.005$) with a large cutoff frequency, or one can use a larger total phase noise power (e.g. $\sigma^2_{\varphi,a} = 0.01$) with smaller cutoff frequency. What is "cheaper" from a oscillator design perspective is an issue for further study.

VI. CONCLUSION

In this paper we surveyed literature on the effects of phase-noise in OFDM.

Wiener-phase noise and small-angle phase-noise descriptions were used. In order to be in-line with the solid-state oscillator literature on can use the Wiener phase-noise *shape* but not Wiener phase-noise itself.

The two types of analysis used, an FS-based analysis and a DFT-based analysis lead to identical results in case the cutoff frequency f_n is far smaller than the inter-carrier distance Δf. As this is the interesting area, one may conclude that results of both analysis are identical. However, while in the

FS analysis the (exact) weighting function for the ICI depends on the carrier number, this is not the case in the DFT analysis.

The presented phase-noise requirements for the HiperLAN/2 case lead, under the assumption of good CPE correction, to an SSB phase-noise to carrier ratio $\mathcal{L}_{dB,1}$ in the order of -95dBc/Hz for an ICI SNR of approximately 25dB.

It was observed that different $\{\sigma^2_{\varphi,a}, f_n\}$ combinations are possible for a particular ICI SNR. What is most convenient for analog designers is an issue for further study.

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