Numerical Observations of Negative Group Velocity in a Two-port Ring-resonator Circuit and Its Potential for Sensing Applications

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A two-port ring-resonator circuit is studied theoretically. It is shown that superluminal and negative group velocity (v_g) phenomena can occur in both a passive two-port ring-resonator circuit with loss and an active one with gain. We present numerical observations of temporal behavior of a Gaussian pulse passing through circuits having such properties. We also show that when the negative v_g is 'slow' ($|v_g|<<c$), there is enhanced sensitivity of the phase shift to the ring effective index changes, which suggests its potential for highly sensitive optical sensing applications.

1. Introduction

Besides the possibility to travel with low group velocity (v_g), light can also travel with large v_g (larger than the light velocity in vacuum c) or even with negative v_g [1]. In fact, the phenomenon where light travels with negative v_g has been theoretically studied by Brillouin and Sommerfeld [2] and recently experimentally demonstrated in active optical fibers [3]. Its consistency with causality has also been experimentally verified [4].

Recently, Heebner and Boyd [5] briefly reported that a lossy two-port ring-resonator (TPRR) circuit (as depicted in Fig. 1) can also exhibit such negative v_g phenomenon when operated in an undercoupled condition. However, since the group velocity is not the signal velocity [2] in such a phenomenon, there is no true signal advancement and hence regarded as physically meaningless [2] and can not be used in applications like telecom delay lines [6]. While at one side, the peculiar pulse temporal behavior in such a structure is interesting for scientific curiosity, at the other side, we believe that it is still can be beneficial for applications where an analytic form of input signal is used. In this paper, we studied TPRR circuit theoretically and observe the temporal behavior of Gaussian pulses passing through such structure in various regimes including the negative v_g and superluminal v_g regimes. We also show, that when the negative v_g is 'slow' (i.e. when $|v_g|<<c$), there is enhancement of sensitivity of the optical properties (like phase shift) to the effective index changes in the ring, which suggests the potential of such devices for sensing applications.

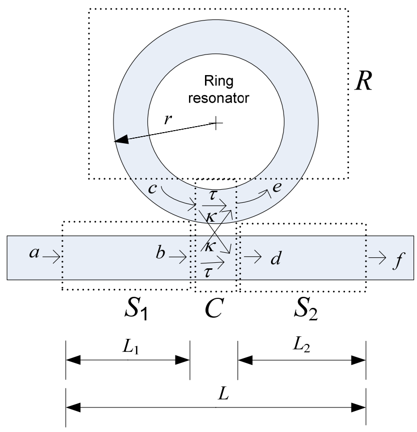


Figure 1. The two-port ring-resonator (TPRR) circuit and the notation used in the modeling.

2. Modeling method

To study the TPRR, we divided the structure into 4 sections as shown in Fig. 1. In this study, we have assumed that the coupler and straight waveguide sections are lossless, while the resonator can have loss or gain. For simplicity, we assume that both the ring resonator and straight waveguides are single mode waveguides.

By assuming a time dependence of exp(iωt), the transfer function of the straight waveguide section for a wave traveling from left to right in Fig. 1 can be expressed as $S_1 = b/a = \exp(-i\beta_{\text{straight}}L_1)$ and $S_2 = f/d = \exp(-i\beta_{\text{straight}}L_2)$, where a to f are variables representing the fields at corresponding positions as illustrated in the figure; β_{straight}, L_1, and L_2 are the propagation constant of the mode of the straight waveguide, the length of the first and the second sections of the straight waveguide, respectively.

The properties of the coupler section can be described using a scattering matrix

$$\begin{bmatrix} d \\ e \end{bmatrix} = \mathbf{S}\begin{bmatrix} b \\ c \end{bmatrix} = \begin{bmatrix} \tau & \kappa \\ \kappa & \tau \end{bmatrix}\begin{bmatrix} b \\ c \end{bmatrix} \tag{1}$$

with τ and κ representing the through and cross port amplitude coupling constant of the directional coupler, respectively. The transfer function of the ring is

$$R = c/e = \exp(-i\theta) \tag{2}$$

with $\theta = (\beta_{\text{res}} - i\alpha_{\text{res}})L_{\text{round-trip}} = (\beta_{\text{res}} - i\alpha_{\text{res}})2\pi r$, where β_{res}, α_{res}, $L_{\text{round-trip}}$, and r represent the (linear) propagation constant, the attenuation constant, the effective round trip propagation length, and the effective radius of the ring resonator, respectively. Using eq. (1) and (2), we obtain $C = d/b = \left[\tau + (\kappa^2 - \tau^2)\exp(-i\theta)\right]/\left[1 - \tau\exp(-i\theta)\right]$. Hence, the transfer function of the TPRR circuit can be written as

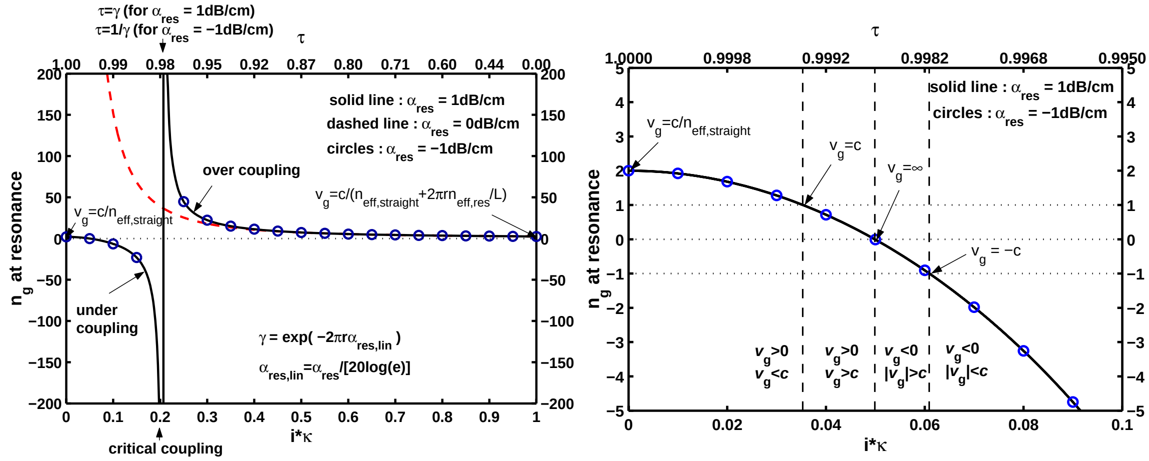
$$T = f/a = (f/d)(d/b)(b/a) = S_2CS_1 = \exp\left[-i\beta_{\text{straight}}(L_1 + L_2)\right]\left[\tau + (\kappa^2 - \tau^2)\exp(-i\theta)\right]/\left[1 - \tau\exp(-i\theta)\right].$$

By making use of the unitary property ($\mathbf{SS}^{\text{H}}=\mathbf{I}$, where superscript H denotes conjugate transpose) of the scattering matrix as a consequence of power conservation of the lossless coupler, with purely imaginary κ and real τ, the transfer function of the structure can then be written as $T = \exp(-i\beta_{\text{straight}}L)C$, where $L = L_1 + L_2$ and $C = \left[\tau - \exp(-i\theta)\right]/\left[1 - \tau\exp(-i\theta)\right]$.

Using the complex transmission coefficient approach [7], we can treat T as a complex quantity and rewrite it as $T = |T|\exp[-i\phi]$, where the effective phase shift $\phi = -\arctan\left[\text{Im}(T)/\text{Re}(T)\right] \pm 2\pi p$ can be used to calculate the group velocity defined as $v_g \equiv \{\partial\beta_{\text{eff}}/\partial\omega\}^{-1} = \{\partial(\phi/L)/\partial\omega\}^{-1}$ and other phase related parameters of the structure. In the ϕ expression, p is an integer. The complex transmission spectrum $T(\lambda)$ of the TPRR can then be calculated. For a known input pulse $a(t)$, by the help of Fourier transform, we can then get the shape of the output pulse $f(t)$ to study the pulse temporal behavior in the TPRR.

3. Numerical observations and discussions

For numerical observations, we take a TPRR circuit with $n_{\text{eff,straight}}=n_{\text{eff,res}}=2$ for the effective indices of the straight and ring waveguides, respectively; α_{res} 1dB/cm, 0dB/cm, and -1dB/cm for lossy TPRR, lossless TPRR, and TPRR with gain, respectively; r= 300µm, and L=1 cm.



(a) (b)

Fig. 2. (a). n_g at resonant wavelength as function of the coupling constant for lossy TPRR, lossless TPRR, and TPRR with gain as specified in the text. (b). A zoomed plot of part of figure (a) showing the four possible regimes of v_g. The plot for α_{res}=1dB/cm coincides with α_{res}=-1dB/cm.

Fig. 2a shows the group index $n_g = c/v_g$ at the resonant wavelength as one varies the coupling constant. The figure clearly shows that negative v_g can occur in both TPRR with loss and gain operating in the under coupling condition, but can not occur in lossless TPRRs. It also shows that for TPRR with loss or gain, in the over coupling condition, v_g is always positive and slow i.e. $0<v_g<c/(n_{eff,straight}+2\pi r n_{eff,res}/L)$ (see [8]). However, Fig. 2b shows that in the under coupling condition, there are 4 possible operating regimes, i.e. $c/n_{eff,straight}<v_g<c$, $v_g>c$, $v_g<(-c)$, and $0>v_g>(-c)$ which we referred to as 'slow' light with positive v_g, 'fast' light with positive v_g, 'fast' light with negative v_g, and 'slow' light with negative v_g, respectively. Note that our definitions are slightly different than the widely accepted definitions [1]. Detail analysis on classification of such regimes is reported elsewhere [8].

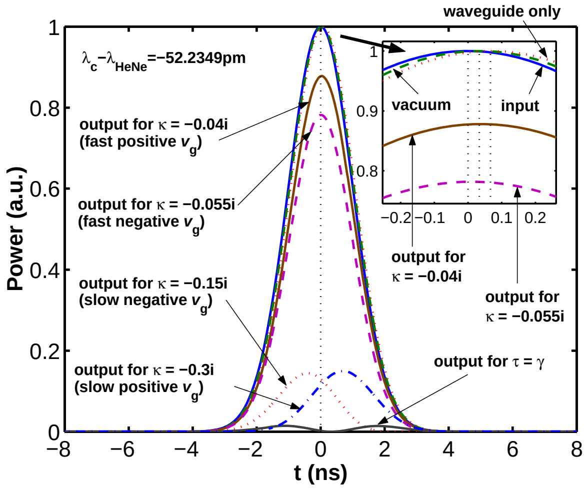


Fig. 3. The power of output pulses of the lossy TPRR circuit operating in various regimes, exited by a Gaussian pulse. For reference, the input pulse, the output pulse if it would travel through vacuum and straight waveguide only of length L, are also plotted together with vertical dotted lines to indicate their peak positions.

Fig. 3 shows the temporal behavior when the lossy TPRR is excited with a Gaussian input pulse $a(t) = \exp\left[-(t/t_d)^2\right]\exp(i\omega_c t)$ with t_d=2ns, $\omega_c = 2\pi c/\lambda_c$ where λ_c is the resonant wavelength of the ring nearest to 6328nm (λ_{HeNe}) at several sampled points representing the four operation regimes. Here, we have assumed that the straight waveguide is not dispersive. For 'slow'-light regimes, the pulses experience considerable delay and insertion loss (as an indication of intensive light-matter interaction). For 'slow' negative v_g, the pulse experiences negative delay, where the peak of the output pulse appears earlier than the peak of the input pulse. Since the leading edge of the strong input pulse already exists before the weak output pulse with negative v_g, the energy of such output pulse indeed comes from the energy of the input pulse as has been discussed in the literature [1]. Besides, the leading edge of the output pulse if it would travel in vacuum also already exists before the output pulse with negative v_g. Hence, the energy velocity is positive and not faster than c. Since the Gaussian pulse is analytic, it is infinitely differentiable. Through Taylor's expansion, it is possible to exactly predict the pulse in the 'future' using the information available in the neighborhood of a point in the 'past'. So, the information velocity is also positive, since the information is in fact already available in the 'past'. Hence, there is no violation to the causality in such negative v_g phenomenon. Fig. 4 shows that as the light is 'slow' (either with positive or negative v_g), there is enhanced sensitivity of the phase shift to the ring effective index change $S \equiv \partial\phi / \partial n_{eff,res}$, which suggests its potential for sensing applications. The S of the 'slow' light in the figure corresponds to sensor effective interaction length of around 10 cm. Since we can opt to work with continuous wave (which is analytic), we believe that the exploitation of 'slow' light with negative v_g for sensing application is possible. The experimental verification of this application will be part of our future topics.

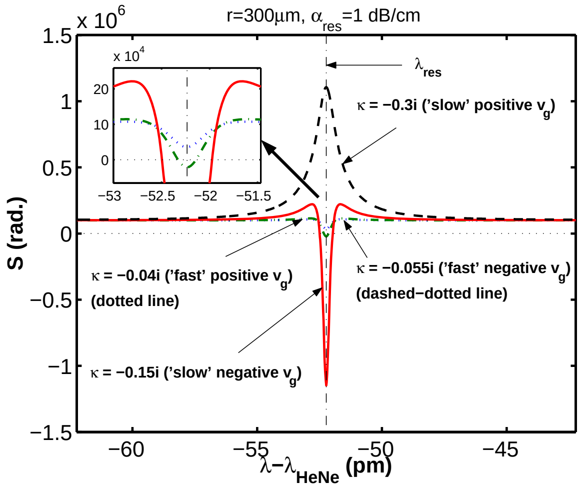


Fig. 4. The sensitivity of the phase shift to the changes in the effective index of the resonator.

Acknowledgements

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