Hydraulic resistance of submerged cylindrical elements. A two-layer scaling approach

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Abstract
The hydraulic resistance of vegetation can play a major role in the hydrodynamics of rivers with extensive natural floodplains. Contrary to commonly used wall roughness methods, vegetation penetrates the flow field and thereby causes drag and, subsequently, additional energy losses. In this study, these processes are treated in an idealized form by replacing vegetation with cylindrical elements with homogeneous geometrical dimensions. Based on scaling considerations of the forces involved, depth-averaged flow velocities within the roughness layer and in the free flowing layer above the roughness elements are estimated. Consequently, conditions in the two separate flow layers yield a new description of the overall average flow field, which is entirely determined by measurable geometrical boundaries. The new description shows to give good agreement with laboratory flume experiments.

Two layer model
Flow in presence of submerged cylindrical elements is described by defining two distinct flow layers:

(i) the cylinder layer, or 'vegetation layer'.
(ii) the surface layer.

Fig. 1 gives a situation sketch together with the relevant geometrical parameters.

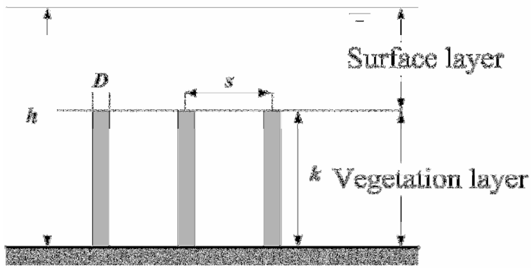

Figure 1. The homogeneous field of rigid cylinders is characterized by a separation s, a stem diameter D and a cylinder height k. The depth of flow is represented by h.

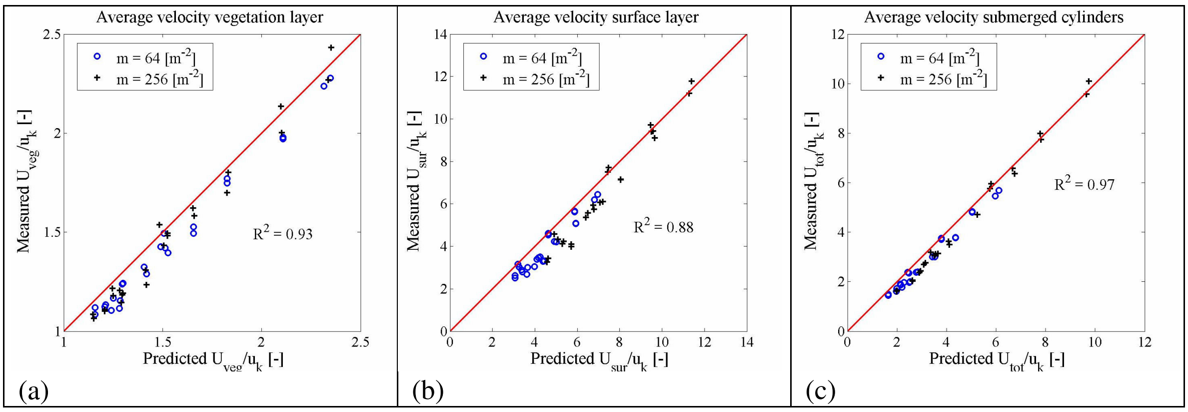

Figure 2. The proposed velocity scaling expressions for (a) the vegetation layer, (b) the surface layer and (c) for the total depth as compared to results of the HKV flume experiments (Meijer 1998). The surface density of the cylinders is given by m (no. of cylinders per m2).

The vegetation layer

In the vegetation layer flow resistance is primarily determined by drag due to the cylindrical elements. In absence of a surface layer (i.e. when h = k) the average velocity uk is determined by balancing the gravitational driving force against the drag force, the result is:

$$u_k = \sqrt{\frac{2gi}{mDC_D}}$$

Where m is the surface density and D the diameter of the cylindrical elements, CD the (dimensionless) drag coefficient and i the channel slope.

However, if a surface layer exists than a shear stress acts at the top of the vegetation layer and the flow in the surface layer will drag along the flow below. The corresponding shear force (Fshear) is estimated by assuming that the shear stress produced above is distributed linearly over the depth of the vegetation layer:

$$F_{shear} = \frac{\rho g(h-k)i}{k}$$

Consequently, a new scaling expression for the average velocity in the vegetation layer is found:

$$U_{veg} = u_k \sqrt{\frac{h}{k}}$$

Data from flume experiments carried out in 1997 by HKV Consultants (Meijer 1998) are used to validate this expression, see Fig. 2.

Surface layer

Resistance in the surface layer originates at the interface with the vegetation layer. Gioia & Bombardelli (2002) used the turbulent kinetic energy to link the average velocity in the surface layer to a characteristic velocity in between the roughness elements. Along the same line of reasoning the velocity in the surface layer may be shown to scale to the depth of the surface layer as:

$$\frac{U_{top}}{u_k} = \left(\frac{h-k+\ell}{\ell}\right)^{2/3}$$

Here the unknown length scale l represents
(i) the degree of penetration of the velocity profile of the surface layer into the vegetation layer and
(ii) the characteristic size of turbulent eddies near the top of the cylinders.
A suitable closure parameter seems the separation between cylinders s. Fig. 2 shows that the proposed scaling expression agrees well with experimental data.

Overall average velocity and resistance parameter

Combining the velocity scaling expressions for the surface and the vegetation layer yields an expression for the total-depth averaged flow velocity.

$$\frac{U_{tot}}{u_k} = \left(\frac{k}{h}\right)^{1/2} + \frac{h-k}{h}\left(\frac{h-k+s}{s}\right)^{2/3}$$

Which in case that the water depth is much larger than the cylinder height reduces to Manning's equation for rough channel flow. Also, Chézy's resistance coefficient C can be found as:

$$\frac{C}{\sqrt{g}} = \frac{2s^2}{C_D D h}\frac{U_{tot}}{u_k}$$

Conclusions

Based on a two-layer flow model, a new scaling expression is proposed that describes the average flow field over a homogeneous field of rigid cylinders.
The separation between cylinders (s) represents the unknown scaling length in the surface layer reasonably well.
Manning's equation is found when the cylinder height becomes very small.

References

Gioia, G. & Bombardelli, F. A., 2002. Scaling and similarity in rough channel flows, Physical Review Letters, 88(1), 14501-14504.

Meijer D.G., 1998. Modelproeven overstroomde vegetatie, HKV Consultants Technical Report PR121.