

## In situ sound absorption measurement: investigations on oblique incidence

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### Introduction

In [1, 2], a new method for the measurement of the in situ sound absorption coefficient is proposed. This method, here referred to as the LPW-method, estimates the incident sound intensity by applying a local plane wave assumption. We have investigated the accuracy of this method for oblique incidence by means of a numerical model. This paper discusses the results of these investigations.

### Theory of the LPW-method

The spatially averaged absorption coefficient of a surface area  $S$  is defined as

$$\alpha(\omega) \equiv \frac{W_{ac}(\omega)}{W_{in}(\omega)} = \frac{\int_S \mathbf{I}_{ac}(\omega) \cdot \mathbf{n} dS}{\int_S \mathbf{I}_{in}(\omega) \cdot \mathbf{n} dS}, \quad (1)$$

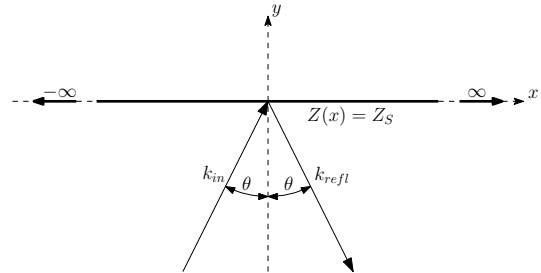
where  $W_{ac}(\omega)$  is the time-averaged active (or: net) sound power, and  $W_{in}(\omega)$  is the time-averaged incident sound power flowing through surface  $S$ .  $\mathbf{n}$  is the direction vector, oriented towards the surface, and pointing towards the surface. To determine the incident sound intensity, we assume that, near a sound absorbing surface, the sound field can be approximated by two oppositely directed plane waves (local plane wave assumption), propagating normal to the surface. Then, the incident sound intensity in direction  $\mathbf{n}$  becomes, [1, 2]

$$I_{in,n}(\omega) = \frac{1}{2} [I_{tot}(\omega) + I_{ac}(\omega)]. \quad (2)$$

Where  $I_{tot}(\omega)$  is the *total intensity* in direction  $\mathbf{n}$  given by

$$I_{tot,n}(\omega) = \frac{1}{4} \left[ \rho_0 c_0 |(\mathbf{U}(\omega) \cdot \mathbf{n})|^2 + \frac{|P(\omega)|^2}{\rho_0 c_0} \right], \quad (3)$$

where  $\mathbf{U}(\omega) \cdot \mathbf{n}$  is the component of the complex particle velocity in direction  $\mathbf{n}$ , and  $P(\omega)$  the complex sound pressure.



**Figure 1:** 2D space with an infinitely extending impedance plane (x-z-plane). The wavenumber vectors  $k_{in}$  and  $k_{refl}$  represent the propagation directions of the incident and reflected wave.

### Oblique incidence

The method is implemented in a numerical model, representing the 2D sound field acc. figure 1. In this sound field, plane waves are specularly reflected by a locally-reacting surface, see [3] for a description of the local reaction model. The complex sound pressure is equal to

$$P(x, y, \omega) = C(\omega) e^{-ik(x \sin \theta + y \cos \theta)} + D(\omega) e^{-ik(x \sin \theta - y \cos \theta)}, \quad (4)$$

where  $C(\omega)$  is the complex amplitude of the incident wave and  $D(\omega)$  the complex amplitude of the reflected wave.  $\theta$  is the angle of incidence and  $Z_0 = \rho_0 c_0$  is the specific acoustic impedance of air.  $D(\omega)$  is related to  $C(\omega)$  by the complex sound pressure reflection coefficient  $R(\theta, \omega)$  given by

$$R(\theta, \omega) = \frac{Z_S(\omega) \cos \theta - Z_0}{Z_S(\omega) \cos \theta + Z_0}, \quad (5)$$

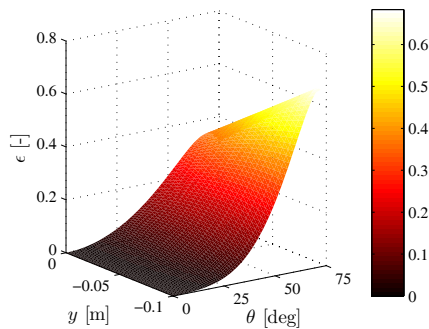
where  $Z_S(\omega)$  is the normal specific acoustic surface impedance. Substituting eq. (4) the associated particle velocity in eqs. (2) and (3), the estimated incident sound intensity in the y-direction becomes

$$I_{in,y,LPW} = \frac{1}{8Z_0} [1 + \cos \theta]^2 |C(\omega)|^2 + \frac{1}{8Z_0} [1 - \cos \theta]^2 |D(\omega)|^2 + \frac{1}{4Z_0} \left[ \sin^2 \theta \operatorname{Re} \left( \overline{C(\omega)} D(\omega) e^{2iky \cos \theta} \right) \right] \quad (6)$$

Expression (6) depends on many parameters and differs much from the exact expression (7) below. For normal incidence of plane waves,  $\theta = 0^\circ$ , eq. (6) is equal to the exact incident intensity given by eq. (7).

$$I_{in,y}(x, y, \omega) = \frac{|C(\omega)|^2}{2Z_0} \cos(\theta). \quad (7)$$

The resulting relative error  $\epsilon = \frac{I_{in,y,LPW} - I_{in,y}}{I_{in,y}}$  for a, purely theoretical, frequency-independent, surface impedance  $Z_S = 2Z_0(1 + i)$  is shown in figure 2 as a function of the angle of incidence (up to  $75^\circ$ ) and the distance to the surface, represented by the  $y$ -coordinate, for a frequency of 1000 Hz. Please note that the maximum distance (10 cm) is chosen to accommodate sound intensity measurement with a pp-probe, in case a large spacer is used.

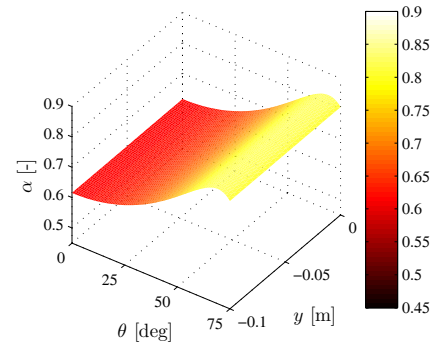


**Figure 2:** Rel. error of the estimated incident intensity  $I_{in,y,LPW}$ .

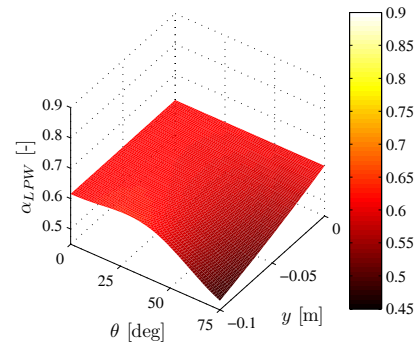
The relative error  $|\epsilon| < 5\%$  for  $|\theta| < 20^\circ$ , even at significant distances from the surface, see figure 2. Although not shown here, this accuracy limit is also valid for other frequencies in the range 10 ... 10000 Hz. To ensure  $|\epsilon| < 1\%$ ,  $|\theta| < 10^\circ$ . For larger angles of incidence the relative error increases.

The resulting inaccuracy in the estimated sound absorption coefficient  $\alpha_{LPW}$  can be evaluated by comparing figure 3, the true sound absorption coefficient, with figure 4, showing the estimated sound absorption coefficient. (Note that the sound absorption coefficient can be calculated by dividing the active sound intensity by the incident sound intensity, as all involved quantities do not depend on the  $x$ -coordinate.)

The estimated sound absorption coefficient varies with the  $y$ -coordinate and the angle of incidence. The true absorption coefficient is independent of the  $y$ -coordinate but displayed here as a function of the  $y$ -coordinate to enable visual comparison. The surface of  $\alpha_{LPW}$  in figure 4 differs much from the true surface in figure 3. This difference is caused by the increasing overestimation of the incident sound intensity at increasing angles of incidence. This overestimation remains, even if the sound pressure  $P$  and the particle velocity in the  $y$ -direction  $U_y$



**Figure 3:** True sound absorption coefficient.



**Figure 4:** Estimated sound absorption coefficient.

are measured near the impedance plane. Nevertheless, there is good agreement for small angles of incidence.

## Conclusions

The accuracy of the LPW-method with respect to oblique incidence of plane waves upon an impedance plane in a free field has been investigated. We found that the estimated incident sound intensity is accurate for small angles of incidence for a surface impedance  $Z_S = 2Z_0(1 + i)$ . It is advised to apply the LPW method in cases with mainly normal sound incidence.

## References

- [1] Y.H. Wijnant, E.R. Kuipers and A. de Boer, *Development and application of a new method for the in-situ measurement of sound absorption*, Proc. of ISMA 31, Leuven, Belgium, (2010).
- [2] E.R. Kuipers, Y.H. Wijnant and A. de Boer, *Theory and application of a new method for the in-situ measurement of sound absorption*, DAGA 2011, Duesseldorf (2010).
- [3] F.J. Fahy, *Foundations of Engineering Acoustics*, Elsevier Academic Press, (2001).