Modelling absorption in porous asphalt concrete for oblique incident sound waves

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Abstract

A numerical model to predict the sound absorption of porous asphalt has been developed. The approach is a combination between a microstructural approach and a finite element approach. The model used to describe the viscothermal properties of the air inside the pores of the asphalt is the low reduced frequency model (LRF). The geometry of the porous asphalt is implemented directly in the FEM model, where a fast simulation will give both the viscothermal losses of the sound energy within the pores as well as the scattering due to the stones.

1 Introduction

The sound absorption of porous asphalt is an important parameter to reduce tyre road noise. The absorption depends on the material properties of the asphalt grading; like the porosity, the stone size, the type of bitumen, fillers, etc. Asphalt gradings can be optimised for a high absorption coefficient in the frequency range where the contribution of tyre road noise is largest. This optimisation is classically done for sound waves at normal incidence. However, a rolling tyre will radiate noise in all directions. Hence, to model the sound radiation of a rolling tyre correctly, a numerical or analytical model is needed which includes the frequency dependency as well as the angle dependency of the sound absorption of asphalt roads.

One of the methods to predict the sound radiation of tyres is the boundary element method (BEM). Here, the tyre is the (vibrating) source of which the behaviour can be found by finite element models or by some more analytical approximation.

Sound propagation above ground surfaces is often described in the literature, which can be implemented in a BEM model by modelling an impedance plane with the half-space Green's function (e.g. [1], [2]). However, this approach assumes a local reacting surface. According to Attenborough ([3], pp. 39–50) a porous asphalt concrete is best described by an extended reacting surface with hard backing. An extended reacting surface can be implemented using a multi-domain BEM approach, where the air and the porous pavement are modelled as two coupled domains (e.g. [4], [5], [6]).

For both the impedance plane method and the two-domain BEM approach the acoustic properties of the pavement have to be described by an impedance model. Numerous models and approaches are developed in the past to obtain such a model. A distinction is usually made between empirical and phenomenological models and microstructural models. In empirical and phenomenological models the porous medium is considered as a compressive fluid where viscous and thermal dissipation can occur. These models have often adjustable parameters determined via measurements. In microstructural models the porous material is often considered to consist of pores filled with air in an elastic or rigid frame. The geometry of the pores is used to analytically describe the thermal and viscous dissipation.

Most impedance models are used to describe the surface impedance and sound absorption for normal in-

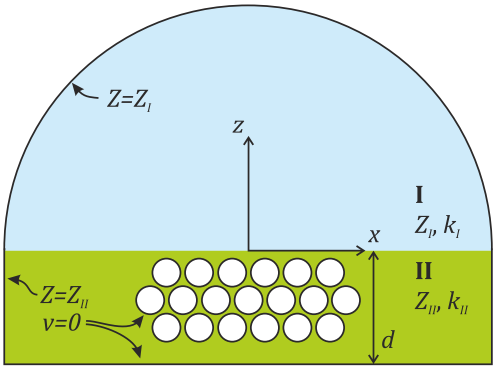

Figure 1: Geometry and boundary conditions of FEM model

cidence. When used to describe the surface impedance and sound absorption for oblique incidence, the behaviour of the porous layer is assumed to be homogeneous. Nonlocally reacting surface effects, such as scattering effects on the individual stones as well as the effect of various layers of different gradings, are omitted. In this paper an approach is described where the geometry of the porous asphalt layer is modelled in a finite element package while using the acoustical properties of a single pore determined with a microstructural model for the fluid medium inside the pores. With this approach the scattering effects are included in the geometry of the FEM model and the viscothermal dissipation is included in the properties of the medium. In this way, one avoids time consuming viscothermal finite element simulations.

2 Modelling approach for nonlocally reacting surface

Using a microstructural model, the scattering effects are not included. The proposed method combines a microstructural model with a finite element model to also capture the scattering effects. The viscothermal behaviour of the air within individual pores is described by the characteristic impedance and the complex wave number. Instead of averaging the characteristic properties of the individual pores using properties as the porosity and tortuosity, the geometry is modelled directly. With this approach, the nonlocally reacting behaviour is included.

There are various microstructural models available. As basis for the here developed method, the low reduced frequency (LRF) model [7], [8] is chosen. The LRF model is described in Appendix A.

The numerical models are made in the multiphysics package Comsol. A schematic example of the FEM model is shown in Figure 1.

2.1 Boundary conditions

The developed approach is illustrated in Figure 2. The solution of the total pressure field is the combination of the incoming sound waves, the reflected sound waves and the scattered sound waves. At the boundaries of the stones the particle velocity is zero. To apply this boundary condition properly, the analytical solution for the acoustic pressure and particle velocity for a geometry without the stones is derived. This analytically derived particle velocity is applied in the opposite direction at the boundaries of the stones We thus use a scattering analysis which includes the viscous effects.

The boundary conditions are shown in Figure 1. An impedance boundary condition is set at the outer boundaries of the modelled domain. The boundary at the bottom of domain II is acoustically hard.

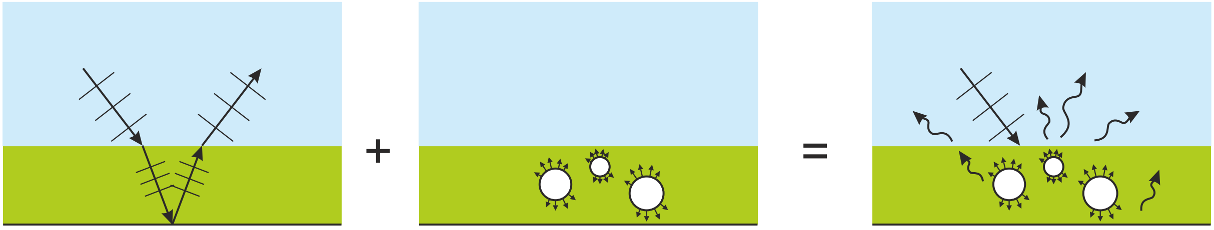

Figure 2: Schematic view of proposed method

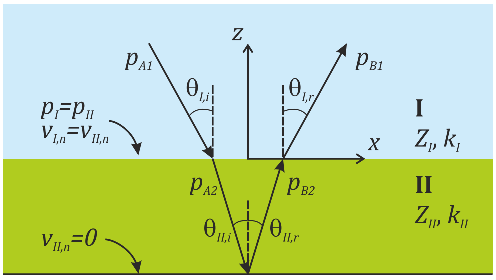

Figure 3: Geometry and boundary conditions used for analytical solution

2.2 Analytical solution for an impedance layer

The analytical solution for the situation shown in Figure 3 is derived in order to find the particle velocity at the boundaries that are used in the scattering analysis. Snell's law yields $\theta_{\mathrm{I},i} = \theta_{\mathrm{I},r}$ and $\theta_{\mathrm{II},i} = \theta_{\mathrm{II},r}$.

The pressure in domain I is given by p_{I}, the pressure in domain II by p_{II}:

$$p_{\mathrm{I}} = A_1 e^{-ik_{\mathrm{I}}(x \sin\theta_{\mathrm{I}} - z\cos\theta_{\mathrm{I}})} + B_1 e^{-ik_{\mathrm{I}}(x\sin\theta_{\mathrm{I}} + z\cos\theta_{\mathrm{I}})} \tag{1}$$

$$p_{\mathrm{II}} = A_2 e^{-ik_{\mathrm{II}}(x \sin\theta_{\mathrm{II}} - z\cos\theta_{\mathrm{II}})} + B_2 e^{-ik_{\mathrm{II}}(x\sin\theta_{\mathrm{II}} + z\cos\theta_{\mathrm{II}})} \tag{2}$$

The particle velocity is:

$$v_{\mathrm{I}} = \frac{A_1}{Z_{\mathrm{I}}} e^{-ik_{\mathrm{I}}(x \sin\theta_{\mathrm{I}} - z\cos\theta_{\mathrm{I}})} - \frac{B_1}{Z_{\mathrm{I}}} e^{-ik_{\mathrm{I}}(x\sin\theta_{\mathrm{I}} + z\cos\theta_{\mathrm{I}})} \tag{3}$$

$$v_{\mathrm{II}} = \frac{A_2}{Z_{\mathrm{II}}} e^{-ik_{\mathrm{II}}(x \sin\theta_{\mathrm{II}} - z\cos\theta_{\mathrm{II}})} - \frac{B_2}{Z_{\mathrm{II}}} e^{-ik_{\mathrm{II}}(x\sin\theta_{\mathrm{II}} + z\cos\theta_{\mathrm{II}})} \tag{4}$$

The relation between angle θ_{I} and θ_{II} follows from the law of refraction: $k_{\mathrm{I}} \sin\theta_{\mathrm{I}} = k_{\mathrm{II}} \sin\theta_{\mathrm{II}}$. Setting the amplitude of the incident wave to $A_1 = 1$, the unknowns are the amplitudes of the reflected wave B_1 in medium I, the transmitted wave A_2 in medium II and the reflected wave B_2 in medium II. The unknowns are solved by the boundary conditions; continuity of the pressure and the particle velocity in normal direction at $z = 0$ and the sound hard boundary at $z = -d$, with d is the thickness of medium II.

$$p_{\mathrm{I}}|_{z=0} = p_{\mathrm{II}}|_{z=0} \tag{5}$$

$$v_{\mathrm{I},n}|_{z=0} = v_{\mathrm{II},n}|_{z=0} \tag{6}$$

$$v_{\mathrm{II},n}|_{z=-d} = 0 \tag{7}$$

Solving the system for an incident wave of unit amplitude, $A_1 = 1$, yields:

$$B_1 = A_1 \frac{Z_{\mathrm{II}} \cos\theta_{\mathrm{I}} \left(1 + e^{-2ik_{\mathrm{II}} d \cos\theta_{\mathrm{II}}}\right) - Z_{\mathrm{I}} \cos\theta_{\mathrm{II}} \left(1 - e^{-2ik_{\mathrm{II}} d \cos\theta_{\mathrm{II}}}\right)}{Z_{\mathrm{II}} \cos\theta_{\mathrm{I}} (1 + e^{-2ik_{\mathrm{II}} d \cos\theta_{\mathrm{II}}}) + Z_I \cos\theta_{\mathrm{II}} \left(1 - e^{-2ik_{\mathrm{II}} d \cos\theta_{\mathrm{II}}}\right)} \tag{8}$$

$$A_2 = 2A_1 \frac{Z_{\mathrm{II}} \cos\theta_{\mathrm{I}}}{Z_{\mathrm{II}} \cos\theta_{\mathrm{I}} (1 + e^{-2ik_{\mathrm{II}} d \cos\theta_{\mathrm{II}}}) + Z_{\mathrm{I}} \cos\theta_{\mathrm{II}} \left(1 - e^{-2ik_{\mathrm{II}} d \cos\theta_{\mathrm{II}}}\right)} \tag{9}$$

$$B_2 = A_2 e^{-2ik_{\mathrm{II}} d \cos\theta_{\mathrm{II}}} \tag{10}$$

This solution is also valid for a situation where the incoming waves are dissipated in medium II, thus for a complex impedance Z_{II} and a complex wave number k_{II}. Note that $\sin\theta_{\mathrm{II}}$ is then a complex number and $\cos\theta_{\mathrm{II}}$ cannot be calculated directly. However, using

$$\begin{aligned} \cos\theta_{\mathrm{II}} &= \pm\sqrt{1 - \sin^2\theta_{\mathrm{II}}} \\ &= \pm i \sqrt{(k_0/k_2)^2 \sin^2\theta_{\mathrm{I}} - 1} \end{aligned} \tag{11}$$

the angle of transmission can be calculated as well as the reduction in amplitude with distance. Note that the minus sign is chosen in order to have an evanescent wave.

3 Sound absorption for porous asphalt concrete

A simple 2D geometry is modelled to demonstrate the developed FEM model. The chosen geometry consist of 4 layers of circular stones of equal diameter. The stones are stacked, but, because the model is 2D, a small opening is left between the stones to represent the pores. The parameters are listed in Table 1. The geometry and packing are a rough estimate of the actual geometry and packing and is solely used here to demonstrate the approach.

Parameters	Value
Radius stone (R_{stone})	8mm
Width between stones (Δw)	1.5mm
Number of stones in 1st layer (N_1)	15
Number of stones in 2nd layer (N_2)	16
Number of stones in 3rd layer (N_3)	15
Number of stones in 4th layer (N_4)	16

Table 1: Parameters used in the FEM model

3.1 Properties of the asphalt layer

The properties of the pores are estimated based on the geometric properties of the structure. The parameters are listed in table 2.

Parameters	Value
Radius single pore (R_t)	1.5mm
Equivalent length single pore (L_t)	44mm
Thickness layer (d)	35mm

Table 2: Parameters used in the LRF model

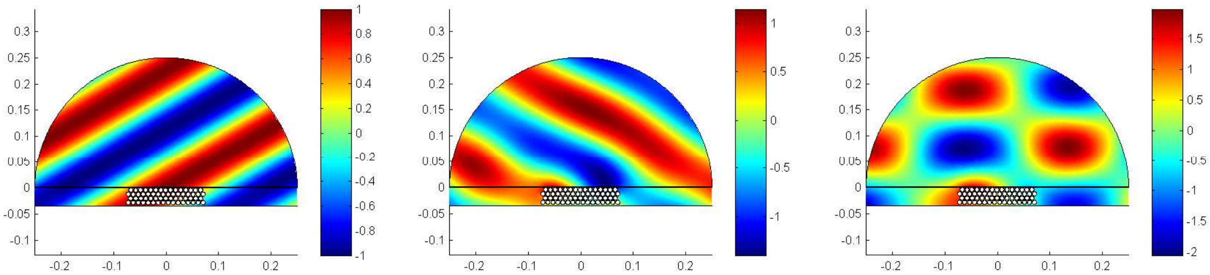

Figure 4: Incoming pressure (left), reflected and scattered pressure (center) and total pressure (right) for simulation with FEM model at $f = 1750$Hz and $\theta_i = 30°$

Using equations 22 and 23, the characteristic impedance and complex wave number of an individual pore are calculated:

$$Z_c = -\frac{\rho_0 c_0}{G} = 418 - 5.5i \tag{12}$$

$$\frac{k}{\omega} = -\frac{i\Gamma k_0}{\omega} = 0.0030 - 0.00010i \tag{13}$$

The impedance and wave number are implemented as properties of medium II in the FEM model together with the chosen geometry. The incoming pressure wave, given by p_{A1} and p_{A2}, the reflected pressure, given by p_{B1} and p_{B2}, the scattered pressure, solved with the FEM simulation, and the total pressure for $f = 1750$Hz and $\theta_i = 30°$ are shown in Figure 4.

3.2 Sound absorption coefficient

The sound absorption coefficient α is determined using the active sound intensity I_{ac} and the incident sound intensity I_{in} integrated over the interface S between domain I and II:

$$\alpha = \frac{\int_S I_{ac} dS}{\int_S I_{in} dS} \tag{14}$$

where:

$$I_{ac} = \frac{1}{2}\text{Re}\left[p\overline{\mathbf{v}} \cdot \mathbf{n}\right] \tag{15}$$

$$I_{in} = \frac{1}{2}\text{Re}\left[p_{in}\overline{\mathbf{v}_{in}} \cdot \mathbf{n}\right] \tag{16}$$

A limitation of this method is that there will be leakage around the edges of interface. This can cause some small anomalies in the absorption coefficient.

3.2.1 Absorption for normal incidence

The absorption coefficient for normal incidence is shown in Figure 5. The results of the FEM model show two peaks in the absorption coefficient, which can be caused by the scattering effects of the stones. The peak around 1800Hz is located at about the same frequency as when calculated with the LRF model using the parameters from Table 2.

The results shown here are the first, preliminary, results and some small anomalies in the absorption coefficient are visible. More research is needed to improve these results.

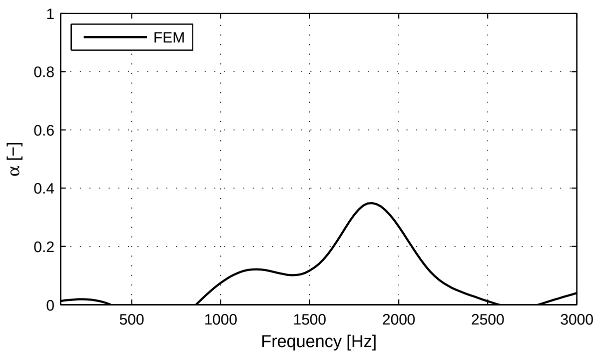

Figure 5: Absorption coefficient for normal incidence

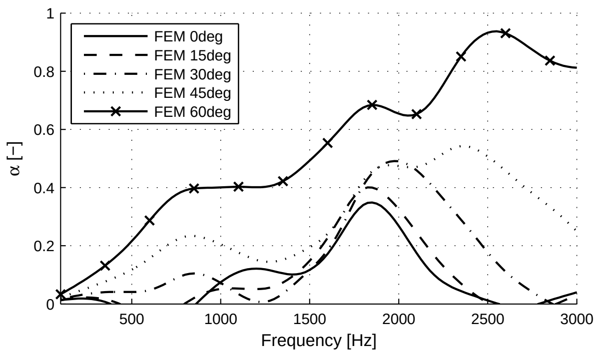

Figure 6: Absorption coefficient for oblique incidence

3.2.2 Absorption for oblique incidence

The method described in this paper is developed to find the absorption coefficient for oblique incidence. Therefore, simulations are performed with several angles of incidence, the results are shown in Figure 6. An increasing angle of incidence leads to an increase in the sound absorption and a shift of the peak value to higher frequencies. As for normal incidence, multiple peaks in the absorption coefficient are visible, possibly caused by the scattering effects.

4 Conclusions and future work

A new method to predict the reflected and scattered sound pressure waves above a porous asphalt concrete is presented. With this method the absorption coefficient for normal and oblique incidence can be found. A simple 2D geometry is used to demonstrate the model. The results show that further research has to be done to improve the computation of the absorption coefficient and to improve the interpretation of the results. Furthermore, the model has to be validated with experimental results.

In future work a more advanced and realistic geometry will be implemented, including different sizes and shapes for the stones. Also, a 3D implementation will be explored.

Acknowledgements

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A Low reduced frequency model

The low reduced frequency solution for the pressure p in a cylindrical tube is:

$$p\left(z\right)=Ae^{\Gamma k_0 z}+Be^{-\Gamma k_0 z} \tag{17}$$

where A and B are the complex amplitudes of the backward and forward travelling waves, z is the displacement directed out of the surface and $k_0=\omega/c_0$ is the wave number. The solution for the velocity is averaged over the cross-section of the tube and is given by:

$$\bar{v}\left(z\right)=\frac{G}{\rho_0 c_0}\left[Ae^{\Gamma k_0 z}-Be^{-\Gamma k_0 z}\right] \tag{18}$$

The LRF model is described in more detail by [7], [8]. The coefficients Γ and G are explained below.

In the low reduced frequency solution, the viscothermal behaviour is described by the viscothermal wave propagation coefficient Γ, which is defined as:

$$\Gamma=\sqrt{\frac{J_0\left(i\sqrt{i}s\right)}{J_2\left(i\sqrt{i}s\right)}\frac{\gamma}{n}} \tag{19}$$

where J_0 and J_2 are Bessels functions of the first kind of order 0 and 2. n is the polytropic coefficient:

$$n=\left[1+\frac{\gamma-1}{\gamma}\frac{J_2\left(i\sqrt{iN_{pr}}s\right)}{J_0\left(i\sqrt{iN_{pr}}s\right)}\right]^{-1} \tag{20}$$

where N_{pr} is the Prandtl number and s is the shear wave number:

$$s=l\sqrt{\frac{\rho_0\omega}{\mu}} \tag{21}$$

with l is the characteristic length scale, which for a cylindrical tube equals the radius of the tube $l=R_t$.

The characteristic impedance Z_c and the complex wave number k for the air within the tube are derived from the low reduced frequency solution:

$$Z_c\left(z\right)=\frac{p\left(z\right)}{u\left(z\right)}=-\frac{\rho_0 c_0}{G} \tag{22}$$

$$k=-i\Gamma k_0 \tag{23}$$

where $G=-i\gamma/\Gamma n$.