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NUMERICAL MODELLING OF RUBBER VIBRATION ISOLATORS:
identification of material parameters

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Abstract

Rubber vibration isolators are used for vibration isolation of engines at high frequencies. To make a good prediction regarding the characteristics of a vibration isolator in the design process, numerical models can be used. However, for a reliable prediction of the dynamic behavior of the isolator, the rubber material parameters have to be known. In practice the material parameters have to be determined with help of experiments. Normally a Dynamic Mechanical Analyzer (DMA) is used to measure the dynamic material properties of test samples of the rubber. However, test samples of the rubber part of the mount are often not available.

For this reason a procedure is developed to determine the parameters that describe the material behavior of an isolator with the help of measurements of the entire isolator. Two measurements are used: a static force-displacement and a dynamic transfer stiffness measurement. A finite element model (ABAQUS) of the mount is made and the material parameters are determined in an optimization procedure to fit the numerical response on the measured response. The static force-displacement measurement is used to determine the coefficients of the polynomial fit of the strain-energy function for the correct static behavior. After that a similar optimization procedure is performed to obtain the dynamic shear modulus of the isolator as function of the frequency with help of the measured dynamic transfer stiffness data. The procedure described above is illustrated for a cylindrical mount using a static and dynamic measurement in the transverse direction.

INTRODUCTION

For good characterization of the isolation behavior of rubber mounts with numerical simulations, it is necessary to have a good description of the material behavior of the rubber. Since the material behavior depends on pre-deformation, temperature and excitation frequency, it is difficult to represent the material behavior accurately. To describe the isolation behavior of the isolator for structure-borne sound purposes, it is necessary to have a correct material description for high frequencies. This data is often not available or confidential.

When the rubber is mixed it is possible to make standardized samples which can be analyzed with a dynamic mechanical analyzer, a so-called DMA test. The static material parameters can be determined in this way (to describe the static pre-deformation due to the weight of the engine) as well as the dynamic material parameters (shear and loss modulus) as function of the frequency. Most dynamic analyzers can only be used to identify the material parameters for relative low frequencies. The material parameters for frequencies beyond the measured frequency range can be obtained with use of the time-temperature superposition principle [2]. However, this principle is not valid for rubber with a high filler content like carbon black or silica. For characterization of the rubber material at high frequencies, low temperature measurement data is necessary. However, the glass transition temperature of filled natural rubber may already be situated at a temperature of minus ten degrees centigrade. For temperatures lower than the glass-transition temperature, the time-temperature superposition principle is not valid. Besides that, in most cases only a finished product is available and it is not possible to make test samples of the rubber material.

To overcome these difficulties, a procedure is described in this paper to determine the material behavior of the rubber with use of measurements of the total isolator. The material parameters are defined in such a way that it can be used for finite element packages.

PROCEDURE

The isolation behavior of isolators is in general characterized by a dynamic impedance or stiffness matrix, measured or simulated as function of the frequency and pre-deformation. For analysis of the power flow through the isolators into the supporting structure (e.g. the vehicle or ship), the so-called blocked dynamic transfer stiffness plays the greatest role [5]. The blocked transfer matrix is the transfer between the displacements (six degrees of freedom) at the top of the isolator to the forces (six degrees of freedom) at the clamped bottom of the isolator. The isolation behavior of a rubber isolator depends on the pre-deformation and the excitation frequency. For this reason the numerical analysis is split in two parts: first a nonlinear static analysis is performed to determine the deformed shape and change in material properties due to the pre-deformation caused by the weight on the mount. Subsequently a linear harmonic analysis is superposed on the pre-deformed isolator to determine the blocked dynamic transfer matrix. It is assumed that the amplitude of the vibrations is small in comparison with the pre-deformation.

Rubber material is characterized generally in a finite element package in two ways:

- The static material properties are described by a fit of the strain-energy function. For filled rubbers, usually the Yeoh model is used.
- The dynamic material properties are described by a complex shear modulus as function of the frequency.

Now the procedure will be described how the static and dynamic material properties are determined with help of measurements of the total vibration isolator.

First the static material parameters (described by the parameters of the Yeoh model) are determined with a static force-displacement measurement. The isolator is loaded on top with a set of prescribed displacements, after which for each displacement increment the static reaction force is measured. This yields a reference force vector $\mathbf{f}_{ref}$, which is a vector with reaction forces for the different prescribed displacement increments. After that a numerical model is made with a finite element package (in this paper ABAQUS is used). The displacement is described for the same increments and the resulting reaction force vector is calculated, yielding the solution vector $\mathbf{f}_{upd}$. This process is repeated by updating of the static material parameters of the numerical model in such a way that the quadratic error J_{stat} between the measured reference solution and the simulation is minimized:

$$J_{stat} = (\mathbf{f}_{ref} - \mathbf{f}_{upd})^T(\mathbf{f}_{ref} - \mathbf{f}_{upd}). \tag{1}$$

For the determination of the dynamic material properties, a measurement of the dynamic transfer stiffness is needed. This yields a dynamic stiffness value $k_{ref}(\omega)$ for each frequency step. The isolation behavior of a rubber isolator is strongly dependent on the frequency ω due to the dynamic behavior and due to the change of the material behavior of rubber. Again the numerical model of the isolator is used, but now the complex shear modulus is updated. Also the pre-deformation influences the dynamic material properties, and is taken into account by determining the pre-deformation with help of the Yeoh parameters. The numerical model calculates the dynamic stiffness $k_{upd}(\omega)$ in such a way that the error function J_{dyn} is minimized, see [7]:

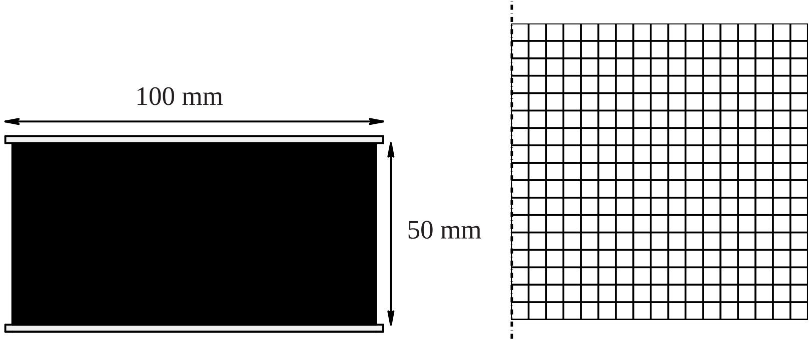
$$J_{dyn} = (\mathrm{Re}(k_{ref}(\omega)) - \mathrm{Re}(k_{upd}(\omega)))^2 + (\mathrm{Im}(k_{ref}(\omega)) - \mathrm{Im}(k_{upd}(\omega)))^2 \,. \tag{2}$$

This shear modulus is updated for each frequency step to determine the minimum value of J_{dyn}. After the first frequency step the initial guess values, needed for the minimization procedure, can be extrapolated from the previous converged values of the shear moduli. This procedure is repeated for each frequency step, finally resulting in an identification of the shear modulus as function of the frequency.

The minimization procedure is implemented in MATLAB with a standard unconstrained nonlinear optimization procedure.

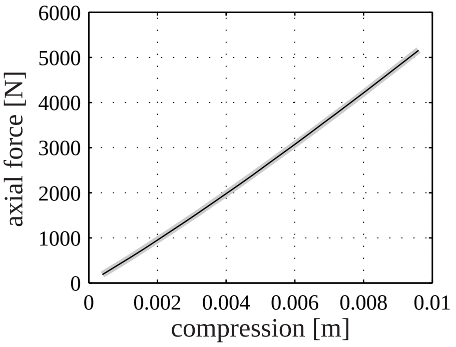
VALIDATION

First the procedure as described in the previous section is validated. A rubber mount analyzed in the literature will be simulated in ABAQUS with the known material parameters and used as the reference solution [4], [3], see Figure 1. First a static force-displacement curve will be determined and the axial dynamic transfer stiffness will be simulated. More details on the numerical model and results can be found in reference [1]. The numerical results are used as the reference solutions $\mathbf{f}_{ref}$ and $k_{ref}(\omega)$ respectively. After that the same numerical model is used, but now to determine the material properties with the help of the described optimization procedure. The static and dynamic material properties are updated in such a way that the responses fit with each other. The material behavior found for the static and dynamic simulations after the optimization procedure must be equal to the reference solution. In Figure 2(a) the static force-displacement curve is depicted. It is seen that the stiffness of the mount increases with increasing pre-deformation due to the change in shape and material properties of



(a) (b)

Figure 1: Considered test case (a) and the finite element mesh (b).

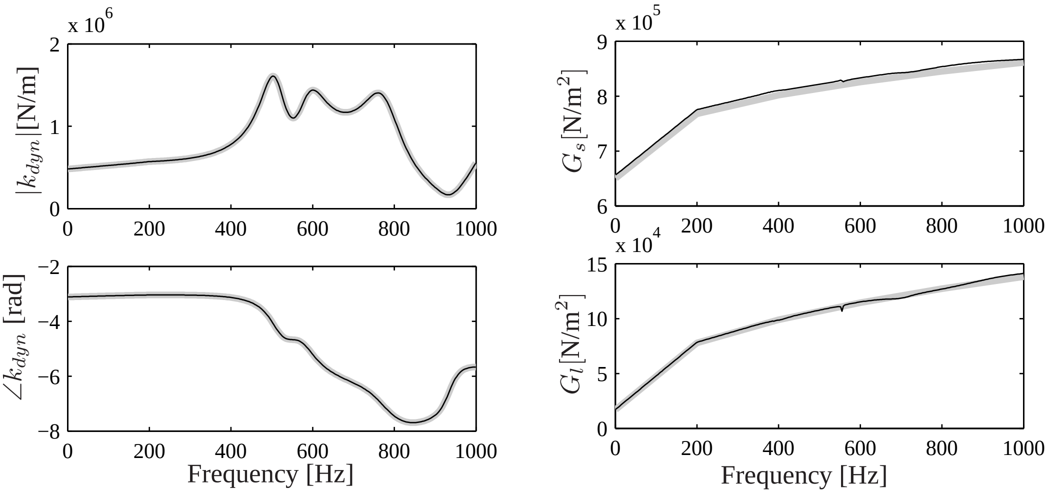


(a)

Yeoh param.	Reference value	Updated value
C_{10}	$2.98 \cdot 10^5$	$2.96 \cdot 10^5$
C_{20}	$-4.5 \cdot 10^4$	$-3.8 \cdot 10^4$
C_{30}	$1.5 \cdot 10^4$	$6.33 \cdot 10^3$

(b)

Figure 2: Static force-displacement response and identified Yeoh material parameters of the considered test case, (—) reference solution, (—) solution with updated material parameters.



(a) (b)

Figure 3: The dynamic response (a) and dynamic material properties (b) as function of the frequency (—) reference solution, (—) solution after updating.

the mount. The reconstructed curve resembles the reference curve very well. However, looking at the coefficients of the Yeoh model in Table 2(b), some deviations are distinguished. The coefficient C_{10} is estimated well, but considerable difference exists for the coefficient C_{20} and especially the coefficient C_{30}. The influence of the second and third coefficient on the total static force-displacement curve is very small, so the influence on the total behavior (thus also the dynamic behavior) of the rubber mount can still be estimated well.

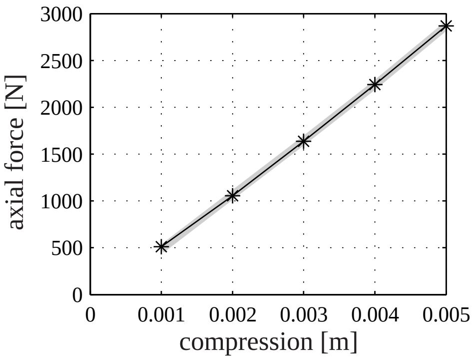
After identification of the Yeoh parameters to describe the strain-energy function, the dynamic material parameters are identified. In Figure 3(a) the dynamic axial stiffness is depicted for the case without pre-deformation. The dynamic stiffness after updating and identification of the material parameters matches the reference solution very well. As seen in the figure, the dynamic stiffness depends strongly on the frequency. Waves occur in the rubber material and cause a considerable influence on the dynamic stiffness due to the mass effects of the mount. The dynamic behavior results in a strong variation of the isolation characteristics. An increase of the dynamic stiffness indicates a stiffer behavior of the mount with less good isolation characteristics. The identified dynamic material parameters itself (the shear storage modulus G_s and shear loss modulus G_l) are depicted in Figure 3(b). The reconstructed material parameters resemble the reference solution well, some deviations are distinguished caused by the little difference of the Yeoh parameters.

EXAMPLE

The optimization procedure is tested with the vibration isolator as shown in Figure 4. The isolator is manufactured at the Rubber Technology department at the University of Twente and is made of silica reinforced rubber (50 phr) representing a realistic rubber

Figure 4: Photo of the analyzed rubber mount.

compound for vibration isolators. Due to the relatively large amount of filler, the mount is relatively stiff and has a hardness of approximately 65 Shore A. The diameter and height of the mount are 62 mm and 30 mm respectively and the density is 1200 kg/m^3. First the Yeoh parameters are determined with help of a static force-displacement measurement. The measurement and the fitted curve are displayed in Figure 5(a). The corresponding Yeoh parameters are shown in Table 5(b). It is noticed that this mount behaves much stiffer as the mount described in the previous section due to the much larger filler content.



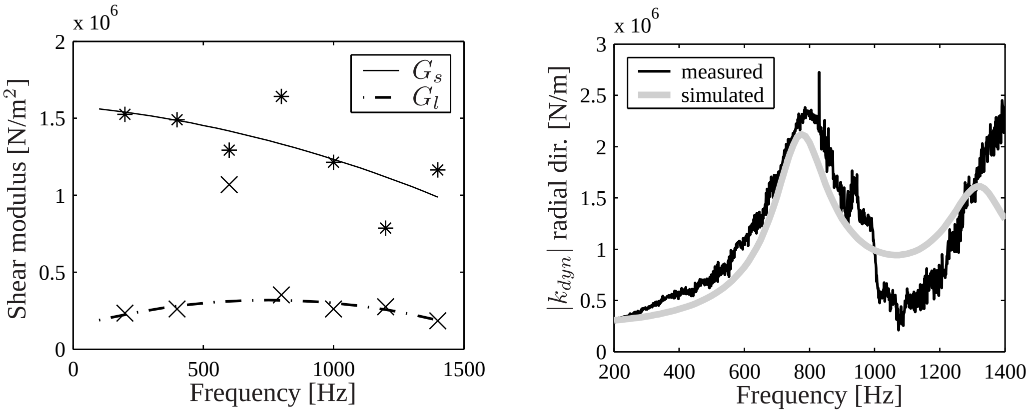
(a)

Yeoh param.	Updated value
C_{10}	$5.11 \cdot 10^5$
C_{20}	$-6.38 \cdot 10^4$
C_{30}	$1.16 \cdot 10^4$

(b)

Figure 5: Static force-displacement response and identified Yeoh material parameters of the considered test case, (—) reference solution, (—) solution with updated material parameters.

After determination of the static material parameters, the dynamic shear moduli are determined. The dynamic stiffness of the isolator is measured in the transverse direction with help of the so-called indirect method, see [6]. The magnitude of the measured dynamic stiffness of the mount is shown in Figure 6(b). It is seen that the dynamic stiffness varies considerably due to the dynamic behavior of the mount. The increase at around 800 Hz corresponds with an anti-resonance frequency. The trough after the anti-resonance frequency is caused by a resonance in the mount. Finally, the dynamic stiffness rises again, corresponding with the second anti-resonance frequency. This measurement is used for the estimation of the dynamic shear storage modulus G_s and the dynamic shear loss modulus G_l. The optimization procedure as described in



(a) (b)

Figure 6: Estimated dynamic material parameters (a) and measured and simulated dynamic transverse stiffness (b) of the silica reinforced vibration isolator.

the previous section is executed at seven frequencies (200, 400, 600, 800, 1000, 1200 and 1400 Hz). The determined values are depicted in Figure 6(a), marked by a $*$ for the storage modulus and by a $\times$ for the loss modulus. A description of the shear moduli over the whole considered frequency range is obtained by a (parabolic) curve fit through the determined discrete points. The value of the loss modulus at 600 Hz is omitted for the curve fit. With the fitted material parameters the transverse dynamic stiffness is simulated in ABAQUS and compared with the measurements in Figure 6(b). As can be seen the dynamic stiffness is described rather well till a frequency of about 1000 Hz. At higher frequencies some deviations occur. It is stressed that the material behavior of the considered rubber compound is quite complicated and certainly not all effects are included with the considered material description. Rubbers with a large filler content behave nonlinear (e.g. friction effects occur between the filler particles and natural rubber molecules) and the question remains if it is possible to describe such materials accurately with dynamic shear moduli.

CONCLUSION

A method is presented to determine the static and dynamic material parameters of the rubber material of an actual vibration isolator. The method is based on an optimization procedure in which the material parameters of a numerical model are updated in such a way that a fit is accomplished with experimentally determined measurement data of the complete vibration isolator. The measurements that are necessary are a static force-displacement for updating of the static material properties (the Yeoh parameters) and a dynamic transfer stiffness measurement for updating of the dynamic material parameters (the dynamic shear loss and storage moduli). First a simple test case is considered to test the procedure, and it can be concluded that it works quite well. As test case a silica reinforced rubber vibration isolator is analyzed and its static and dynamic material parameters are determined. The determined material parameters describe the dynamic behavior quite well in the low-frequency region for the dynamic transverse stiffness. In the higher frequency region some deviations are distinguished between the measurements and the simulated results. Also more fluctuations occur in the estimated material parameters. This is probably due to the effect that effects occur that are not described probably with the adopted material model.

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