

## Predicting the occurrence of dunes under supply limited conditions

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**ABSTRACT:** If only a part of the bed sediment is being transported by a river erosion resistant layers can develop. The presence of erosion resistant layers in the river bed can limit the volume of sediment available for bedform formation. These conditions develop if the flow is not capable of transporting all present grain size fractions. Different bedform types develop depending on the volume of sediment that is mobile. In this paper we combine data from previously published experiments to explore the possibility to predict the occurrence of dunes under partial transport conditions using a parameter  $s^*$ , the relative sediment transport rate. Herein, we use a model to make reliable estimations of  $s^*$ . We find that a critical  $s^*$  exists so that dunes only occur for  $s^* > s^*_{cr}$ . The results of the model and the critical  $s^*$  for dune occurrence are validated quantitatively and qualitatively using two sets of experiments, experiments conducted by Van der Zwaard (1974) and a recently conducted set of experiments at the Leichtweiß-institute, Germany.

### 1 INTRODUCTION

Many river beds show bedforms. The shear stress caused by the flowing water moves the bed sediment, the resulting topography in turn determines the flow pattern. This two way influence results in the development of certain bedform types that are in dynamic equilibrium. The type and characteristics of the equilibrium bedforms that develop depend on the conditions. The most important parameters are the flow velocity, water depth and the grain size distribution of the sediment. The presence of bedforms, in particular river dunes, is an important factor determining the total hydraulic roughness and therefore they influence the water level. Knowing what water levels to expect is very important for river management since the water levels affect the safety and navigability of the river. Since the overall roughness also determines the flow it also controls the large-scale erosion and accretion of the river bed. Forecasting of the occurrence of bedforms is therefore also important for morphodynamical river models. Therefore we would like to know under which conditions dunes form and what their roughness is.

If uniform sediment is being transported all grains are mobile and therefore all sediment on the river bed can contribute to bedform formation. However, this is not always the case if the sediment is a widely graded mixture. The shear stress that is required to initiate motion (the critical shear stress) varies with the grain size so that only a part of the mixture is mobile. These conditions are called partial transport conditions. The coarsest immobile part of the bed sediment forms a

protective layer that prevents further erosion. The volume of sediment that is moving over the coarse layer can form bedforms. This supply limitation is an additional factor that determines the type and dimensions of the bedforms that develop under partial transport conditions.

In Kleinhans et al. (2002) it was shown that the bedform stability diagrams for uniform sediment (e.g. Southard & Bochuwal 1990, Van den Berg & Van Gelder 1993) have a limited predictive capability to discriminate between bedform types under supply limited conditions. This is mainly caused by the fact that the factor supply limitation was not included in the analysis leading to these stability diagrams. From studies by Van der Zwaard (1974), Dietrich et al. (1989) and Kleinhans et al. (2002) it appears that the bedform type that develops varies with the ratio of the actual sediment transport rate to the transport capacity for the transported sediment.

In this paper we explore the possibility to predict the occurring bedform type using the ratio of the actual sediment transport rate to the transport capacity. In section 2 we use various previously published experiments to derive a method to predict the occurrence of dunes under partial transport conditions. In section 3 two sets of experiments are presented which can be used to validate this method. One set of data is a new series of flume experiments in which the supply limitation was systematically varied in order to study the effect of supply limitation on the bedform type, the dimensions and the resulting roughness. The other data set is from experiments by Van der Zwaard (1974) in which the supply limitation was varied in order to study the effect

on the roughness. In the section 4 the criterion for dune formation found in section 2 is compared with these data sets. Finally, conclusions are drawn in section 5, regarding the possibility to predict the occurrence of dunes under partial transport conditions.

## 2 PREDICTING DUNE OCCURRENCE UNDER PARTIAL TRANSPORT CONDITIONS

The bedform type has been found to vary with the ratio of the actual sediment transport rate to the transport capacity for the transported sediment (Van der Zwaard 1974, Dietrich et al. 1989, Kleinhans et al. 2002). This transport capacity is the sediment transport rate that would exist if all of the bed would consist of the sediment that is being transported. The ratio of the actual sediment transport rate ( $s$ ) to the transport capacity ( $s_0$ ) will be called 'relative sediment transport rate' and is indicated with  $s^*$ .

The volume of mobile sediment is an important factor determining the occurring bedform type. This parameter could not be used directly because of a lack of data. The volume of mobile sediment per square meter is linked to the value of  $s^*$ . Therefore we used  $s^*$  to search for an empirical predictor of the occurrence of dunes. This link between  $s^*$  and the volume of mobile sediment available for bedform formation can be explained as follows:

If the erosion resistant layer is not much coarser than the bedload sediment the mobile volume is locally determined by the shear stress. If the shear stress increases mobilises a larger fraction of the bed sediment, the volume of mobile sediment increases and the local sediment becomes a source for bedform formation. With an increase in shear stress  $s^*$  increases because the actual transport rate increases faster than the transport capacity, due to the increased bed coverage and the coarser composition of the bedload sediment.

If the critical shear stress of the erosion resistant layer is much larger than the present shear stress the local bed can not supply sediment for bedform formation. In this case the supply limitation is controlled by upstream supply. If the supply is small the volume of transportable sediment per unit area is small; the transportable sediment hides in the coarse layer and a large area of the bed consists of the immobile bed. The transport capacity can be much larger, resulting in a low  $s^*$ . If the upstream supply increases the volume of transportable sediment per unit area increases until the sediment transport rate matches the upstream supply due to a increasing bed coverage and a decreased hiding. Therefore the volume of sediment that is available for bedform formation increases with a decreasing supply limitation ( $s^* >$ ).

Table 1. Used data with characteristic parameters.

Source	Grain size distribution		Shear stress $\theta_{b,50}$ [-]
	$D_{avg}$ [mm]	$\sigma_g$ [-]	
Livesey et al. (1998)	1.2	1.6	0.08–0.14
McLelland et al. (1999)	1.3	1.6	0.036
Kleinhans et al. (2002)	1.6	3.3	0.03–0.17
Blom et al. (2003)	1.9	2.7	0.07–0.2
Kuhnle et al. (2006)	1.8	4.0	0.12–0.33
Carling et al. (2005)	4.8	1.4	0.039–0.075
Pender et al. (1999)	3.0	2.7	0.055–0.060

$\sigma_g$  is the geometric standard deviation.

$\theta_{b,50}$  is the Shields parameter based on the median grain size of the bulk grain size composition.

In this section the relation between  $s^*$  and the bedform type is explored using data from literature (see section 2.1). For most of this data no measured value for  $s^*$  was known because either the actual transport rate or the transport capacity or both rates were unknown. Therefore a method to calculate  $s^*$  from the grain size distribution and shear stress is used. This method is explained in section 2.2. In section 2.3 the results from the model are presented and it is shown how the model can be used to predict the occurrence of dunes under partial transport conditions.

### 2.1 Used data

Data has been gathered from various publications (see Table 1) where bedforms were observed under partial transport conditions. The data that is available for all sources consists of the composition of the bed sediment before the experiment, i.e. the bulk sediment composition, the shear stress during the experiment and the bedforms observed during the experiments.

### 2.2 Method to calculate the relative sediment transport rate

In order to calculate  $s^*$  both  $s$  and  $s_0$  need to be determined. They are calculated using a sediment transport model. Wilcock & Crowe (2003) has been selected because it has been calibrated using a widely graded sediment mixtures under conditions near incipient motion. These are conditions that are present in many of the analysed experiments; this makes it a suitable choice as sediment transport predictor in this analysis. The best estimate of the actual transport rate was obtained using the bulk sediment composition and the shear stress as input for the sediment transport model. Hereby it is neglected that the surface sediment composition will deviate somewhat in composition as soon as the flow caused some sorting of the bed. The result

of this first step is a sediment transport rate and the composition of the bedload transport. The transport capacity is calculated by using the transport model for a second time, this time with the bedload composition as input. This way we pretend that all of the present sediment is the bedload sediment. The resulting transport capacity for the bedload sediment is higher than the actual transport rate because the bedload sediment composition is finer. The relative sediment transport rate  $s^*$  is now calculated by dividing both transport rates.

In the following section the calculated value of  $s^*$  is used to find a method to predict whether dunes will develop under a certain supply limitation.

### 2.3 Critical value of $s^*$ for dune occurrence

The bedform types as described in literature have been divided in three groups. The goal is to find out whether there is a critical value of  $s^*$  below which dunes do not develop anymore. Large flow transverse bedforms with a gentle stoss-side and a steep lee-side have been grouped into the category dunes. Therefore this category comprises dunes and supply limited precursors of dunes, like barchans. Flow parallel stripes and the ripple like bedforms as observed in Kuhnle et al. (2006) have been grouped in the category 'low relief bedforms'. These are all bedforms that have a low relief, compared to dunes, and do not scale with the water depth. The third group contains those observations in which the transporting sediment did not form any bedforms.

In Figure 1 the bedform type is plotted against the relative transport rate ( $s^*$ ) and the dimensionless shear stress based on the bulk sediment composition ( $\theta_{b,50}$ ). For a low value of  $s^*$ , i.e. where  $0 \leq s^* \leq 0.4$ , we exclusively observe the categories 'No bedforms' and 'Low relief bedforms'. Apparently, the supply limitation is too strong here for dunes to develop. If  $s^*$  is larger than 0.50 only dunes are observed. For  $s^*$  between 0.4 and 0.5 both dunes and low relief bedforms occur. In this figure the conditions where dunes occurred are well separated from conditions where dunes did not occur. Therefore the relative transport rate  $s^*$  can be used to predict the occurrence of dunes under partial transport conditions. The critical value for presence of dunes is  $s^* = 0.45 \pm 0.05$ . In section 4 this result will be tested against two sets of experimental data, which are presented in section 3.

## 3 EXPERIMENTS WITH VARYING SUPPLY LIMITATION

This section presents two sets of experiments in which the supply limitation was systematically varied. One set of data consists of the experiments by Van der

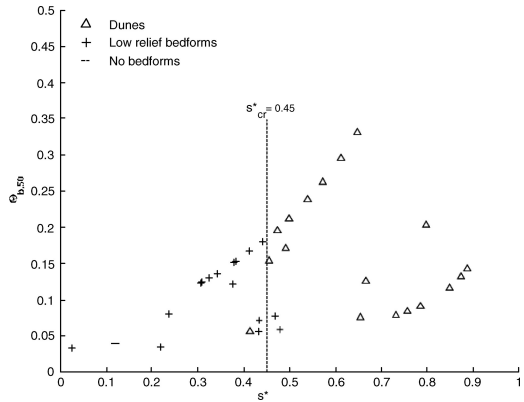


Figure 1. Observed bedforms plotted against the calculated  $s^*$  and  $\theta_{b,50}$ . The dimensionless shear stress based on the  $D_{50}$  of the bulk sediment composition.

Zwaard (1974). These experiments were conducted to study the roughness effects of bedload transport over an artificial armour layer. This is an erosion protection for the river bed consisting of a layer of coarse sediment designed to remain stable at all occurring discharges. This data set can also be used to study the relation between the occurrence of bedform types and the supply limitation due to the extensive photographic documentation that is available.

Recently we performed a series of experiments in which the relation between the bedform type and dimensions and the supply limitation is studied. Between January and March 2007 a series of experiments was executed at the facilities of the Leichtweißinstitut at the University of Braunschweig, Germany. In this section the bedform types observed during these experiments can be directly related to the supply limitation. Furthermore the effects of the presence of dunes for the bed roughness are illustrated.

### 3.1 Van der Zwaard experiments

In this experiment a graded sediment ( $D_{50} = 0.9$  mm  $D_{90} = 2$  mm) was fed to a flume with an a priori installed bottom of coarse, immobile sediment. The feed rate was varied while the discharge was kept constant, thus the supply limitation was varied. This experiment was repeated for different discharges, resulting flow velocities between 0.5 and 0.7 m/s. Two grain sizes were used for the coarse layer: 14 mm and 40 mm.

It appeared that the bedform characteristics varied systematically with the ratio of the actual sediment transport rate ( $s$ ) divided by the sediment transport capacity ( $s_0$ ). In Van der Zwaard (1974) two regimes were identified namely, a regime in which sand was

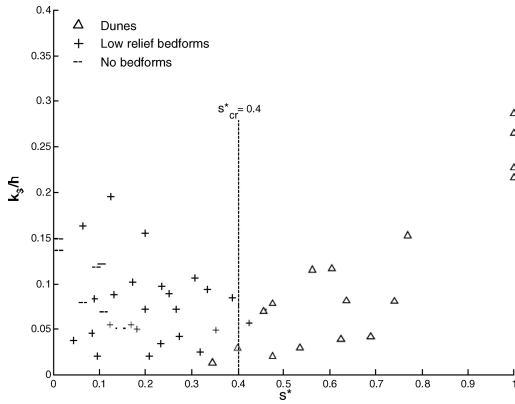


Figure 2. Bedform types observed in experiments by Van der Zwaard plotted againsts  $s^*$  and  $k_s/h$ .

transported without dune formation and a regime in which the sand was transported with dune formation.  $s^* = 0.4$  was proposed as critical value between these regimes.

Van der Zwaard (1974) did not describe the bedform types in detail. However, the report (Van der Zwaard 1974a,b) contains photographed bed configurations after most of the experiments. This enables visual identification of bedforms and classification in the groups ‘no bedforms’, ‘low relief bedforms’ or ‘dunes’. This classification is similar to the one used in Section 2. If no bedforms are visible in the transporting sediment ‘no bedforms’ is obviously assigned. If ‘flow parallel sand ribbons’ and ‘ripples’ are present the bed configuration is classified as ‘low relief bedforms’ because these bedform have a low relief and generally result in a small roughness height  $k_s$ . Bedforms with a steep leeside and a low height-length ratio are classified as dunes. The classified bedforms from the photographs have been plotted in Figure 2. The dotted line indicates the critical value of  $s^*$  as found by Van der Zwaard. Figure 2 shows a transition zone between  $s^* = 0.35$  and  $0.45$  in which bedform types co-occur.

### 3.2 New experiments with varying supply limitation

New experiments have been conducted in order to provide data to study the relation between bedform dimensions and the supply limitation due to partial transport conditions (Tuijnder, in prep.). In this paper the results from these experiments are used to verify the bedform type prediction method. Therefore the observed bedform types have been classified the same way as previously presented data. These experiments provide an extension to the range of conditions of observations of bedform development under supply limited conditions.

#### 3.2.1 Set up of experiments

The experiments were executed in a flume with a measuring length of 6.5 meter and a width of 0.3 m. In this flume a coarse layer was installed at a predefined slope. Uniform fine sand ( $d = 0.8$  mm) was recirculated over this coarse bed. During the experiments the discharge, water level, bed level and sediment transport rate was measured. The discharge was constant throughout the experiment and capable of dune formation with the recirculated sediment if no supply limitation would exist. The boundary conditions during the experiments are shown in Table 3.

The amount of sand in recirculation was increased in steps during each experiment. On average 10 kg of dry sand was evenly distributed over the flume per increase. At the beginning of each experiment no sediment was present. At the end the coarse layer was completely covered with sand dunes, with the coarse layer only incidentally being exposed in the deepest troughs. The amount sediment added was larger for the first steps and the last steps because the system is the most sensitive for changes in the amount of sand where  $s^*$  is between 0.3 and 0.7. This way the supply limitation for bedform formation was gradually reduced.

Equilibrium flow was realised by adjusting the downstream weir after every increase of sand volume. This was done to adjust for the bed roughness and the rise in bed level because of the added fine sediment. From the bed level, water level and discharge measurements the slope of the energy line has been calculated. The energy slope over a uniform measurement section has been used to calculate the bed roughness. We corrected for side wall influences using the method of Vanoni & Brooks (1957).

#### 3.2.2 Relation between supply limitation and the occurrence of dunes.

In Figure 3a and 3b the results are shown from individual experiments. Within one experiment the discharge and the grain size of the sediment were constant, only the supply limitation varied by stepwise adding sand. Each point represents one supply limitation level.

With an increasing volume of fine sand in the flume the supply limitation decreased and different bedform types developed. In Figure 3 the bedform types present at every supply limitation level have been plotted as symbols against the relative transport rate  $s^*$  and the bed roughness. The roughness values along the vertical axis are presented as a Nikuradse roughness lengths ( $k_s$ ) divided by the water depth ( $h$ ).

Figure 3 shows that for low  $s^*$  values the category ‘no bedforms’ and ‘low relief bedforms’ occur and for  $s^* > 0.5$  only dunes occur. The division between the presence and absence of dunes can unfortunately not be determined very precise from these figures because of the lack of observations between 0.3 and 0.5.

Table 2. Conditions during experiments.

Exp number	Grains size coarse layer		Water depth		Flow velocity		Slope average[-]
	D50[mm]	$\sigma_g[-]$	min [m]	max [m]	min [m/s]	max [m/s]	
2	6.5	1.1	0.06	0.07	0.50	0.60	0.003
3	13.5	1.1	0.12	0.13	0.39	0.45	0.002

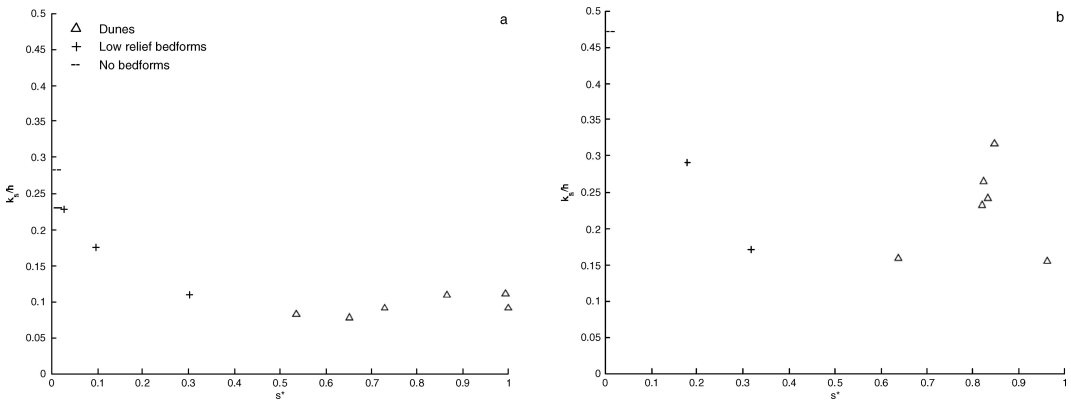


Figure 3. Variation of bedform type (symbol) and roughness ( $k_s/h$ ) with  $s^*$  for experiment 2 (fig. 3a) and for experiment 3 (fig. 3b).

In order to determine the critical value of  $s^*$  for the occurrence of bedforms only the horizontal axis and the bedform type symbols are required. The roughness is introduced on the vertical axis to show the effects of the changes in supply limitation on the bed roughness.

### 3.2.3 Observed relation between roughness, bedforms and supply limitation

The variation in occurring bedforms leads to variations in the hydraulic roughness. If  $s^*$  is low no bedforms are present and the roughness is the roughness of the grains at the surface. The roughness was larger in experiment 3 than experiment 2 due to the use of a coarser sediment for the coarse layer. With  $s^*$  increasing between 0 and 0.4 the pore space between the large grains is filled up with fine sand and the roughness reduces. The minimum roughness is observed for an  $s^*$  of around 0.5 because the coarse layer is largely covered with fine sand but no large dunes can develop due to a supply limitation.

From  $s^*$  is 0.5 to 1 the roughness increases for both experiments. In experiment 2 the dunes that developed remained lower and smaller than in experiment 3. This is reflected in the lower roughness in experiment 2. Most likely, this is caused by the higher Froude number in experiment 2. Therefore only little form roughness is generated in experiment 2, while in experiment 3

the roughness increased considerably due to the form drag of dunes.

## 4 VERIFICATION OF THE PREDICTION METHOD AND DISCUSSION

In this section the result from Section 2 is compared with the data from Section 4. In Figure 4 the observations from the new experiments have been combined with the Van der Zwaard data. The dotted line indicates the critical value for the presence of dunes derived from the literature data in Section 2. We observe that, in general, the criterion derived using the calculated values of  $s^*$  fits the data well. Only two points are classified as low relief bedforms, using the critical  $s^*$  value of 0.45, which were dunes by observation.

A reason for the occurrence of a few dunes below  $s^* = 0.45$  may lie in differences in the classification of the strong supply limited dunes that plot near the critical value of  $s^*$ . The exact bedform type that was present in the experiments by Van der Zwaard is difficult to determine from the photos. Because no lengths or heights are known for the Van der Zwaard data they can not quantitatively be compared to the ripple like bedforms described in Kuhnle (2006). Therefore, some points may have been classified as dunes by visual

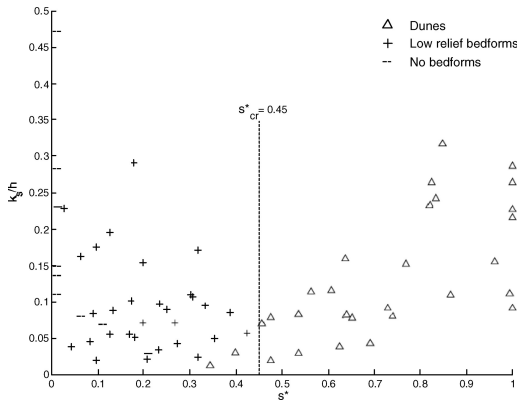


Figure 4. Variation of bedform type (symbol) and roughness ( $k_s/h$ ) with  $s^*$  for data of the new experiments together with the Van der Zwaard data.

observation while they actually are ripple like bedforms and should have been classified as low relief bedforms.

The classification of the ripple like bedforms that are observed near the critical value of  $s^*$  can be discussed. It can be reasoned that these bedform are no dunes because they do not scale with the water depth. They appear to be similar to ripples, apart from the fact that they develop under fully rough flow conditions (Kuhnle et al. 2006). Reversely these bedforms have the same geometry as dunes, a gentle stoss side and a steep lee side and exhibit the same sand over passing mechanism as dunes. The fact that they do not scale with the water depth can be attributed to the supply limitation. These bedforms grow in dimensions when the supply of movable sediment is increased. Only if the supply limitation vanishes the dunes can scale with an increasing water depth. Therefore they can be classified as dunes as well, yet they are strongly supply limited dunes.

Using both the method from section 2 and both data sets the same result is found: If  $s^*$  is lower than 0.4 the supply limitation is too strong for dunes to develop. The critical value for discriminating between presence and absence of dune lies between 0.4 and 0.45. In Figure 4 it can be seen that the critical value from Section 2,  $s^* = 0.45$ , is a bit too high. A value of 0.4 would fit the verification data better.

The experiments used for the model development in section 2 are predominantly experiments where the supply limitation is locally determined by the shear stress. An increasing shear stress mobilises a larger part of the bed and decreases the supply limitation ( $s^* >$ ). In the new experiments and the Van der Zwaard experiments on the other hand, the pre-installed coarse layer exists which is much coarser than the bed load sediment. An increase in shear stress does not lead to

the mobilisation of this coarse layer and a larger volume of transported sediment. The value of  $s^*$  even decreases if the shear stress increases because the transport capacity increases.

Similar bedforms develop for equal relative transport rates despite of the nature of the supply limitation, a shear stress limitation or an upstream supply limitation. This indicates that the method to use a critical value for  $s^*$  for forecasting dune occurrence is applicable for both situations.

## 5 CONCLUSION

Plotting the bedform type against the relative sediment transport rate results in a good separation between conditions where dunes can form and where dunes can not form because of the supply limitation of sediment. This makes the relative sediment transport rate a useful tool to predict the presence of dunes under partial transport conditions.

The relative transport rate needed to be calculated because no measured  $s^*$  values are available for the data set from Section 2. Because the measured  $s^*$  values are not available the model for  $s^*$  calculation can not be verified directly. However, the critical value for the occurrence of dunes calculated from the model and found in both experimental sets is approximately equal. This indicates that the bedform types predicted using the modelled are reliable.

It can be concluded from the modelled  $s^*$  as well as from both experimental data sets that when  $s^*$  is lower than 0.4 dunes do not develop. In this regime the sediment moves without bedform formation between the immobile coarse grains, or it is transported along flow parallel stripes, or it forms small ripple like bedforms but all of these bedforms have a low relief and do not result in a large form related roughness. When  $s^*$  is larger than 0.5 dunes can build. Between these values small dunes may form.

With respect to the observed roughness in the new experiments it can be concluded that a systematic variation with  $s^*$  is visible. In order to explain the roughness variations for  $s^* > 0.5$  insight in bedform dimensions under supply limitation is required. In order to explain the roughness variations for  $s^* < 0.5$  the degree to which the coarse layer is exposed and to which the pore space between the surface grains in the top layer is filled with fine sediment needs to be taken into account.

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