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Dynamic optimization of dead-end membrane filtration

Bastiaan Blankert, Ben H.L. Betlem and Brian Roffel

Faculty of science and technology, University of Twente, P.O. Box 217, 7500 AE Enschede, The Netherlands

1. Abstract

An operating strategy aimed at minimizing the energy consumption during the filtration phase of dead-end membrane filtration has been formulated. A method allowing fast calculation of trajectories is used to allow incorporation in a hierarchical optimization scheme. The optimal trajectory can be approximated closely by constant power filtration, which allows robust implementation of the optimal strategy. Under typical conditions, the relative saving in energy, is small compared to constant flux (0.1%) or constant pressure filtration (4.1%).

Keywords: Dynamic optimization, dead-end filtration

2. Introduction

Dead-end ultra filtration is applied in the purification of surface water to produce either process water or drinking water. Due to its high selectivity, economic scalability and low chemicals consumption, it is a promising technology in this field.

However, the performance of membrane systems is often limited by fouling phenomena. Accumulation of retained particulates at the membrane surface increases the hydraulic resistance of the system. This increases the operating costs due to extra energy consumption and the necessity of periodic cleaning. Hence, dealing with membrane fouling is one of the main challenges in the application of this technology. Since the process settings are currently based on rules of thumb and pilot plant studies, it is believed that optimization will result in a reduction of the operational costs.

Dead-end filtration is a cyclic process which consists of three phases. During the filtration phase clean water is produced and the membrane is subject to fouling. This is followed by the backflush phase, in which the flow is reversed in order to remove the fouling. After a number of alternating filtrations and backflushes, chemical cleaning is performed to remove "irreversible" fouling. The evolution of the fouling state during the sequence of filtrations and backwashes is illustrated in the left of fig. 1. This study is concerned with the sequence of alternating filtrations and backflushes.

Since filtration and backflushing are fundamentally different, they need to be described by separate models. Both have the flux as control variable and the amount of fouling as state. Since the filtration and backflush phases alternate, the initial state and the cost of the final state are difficult to determine.

Therefore, a hierarchical structure with two layers is used. The top level coordinates the initial and final states of subsequent phases. It searches for a trajectory of initial and final conditions for which the total costs of the series of subsequent phases are minimal. The bottom level is concerned with reaching the final state at minimal costs. This is illustrated in the right of figure 1.

A bottom up approach is followed to construct the hierarchical structure, starting with optimization of the filtration phase. This is a dynamic optimization problem which aims to minimize the energy consumption. As it is part of the hierarchical structure, a requirement on the final state and time must be satisfied. Furthermore, each iteration towards the total optimum at the top level involves a dynamic optimization at the bottom level. Hence, the optimal trajectory should be calculated fast.

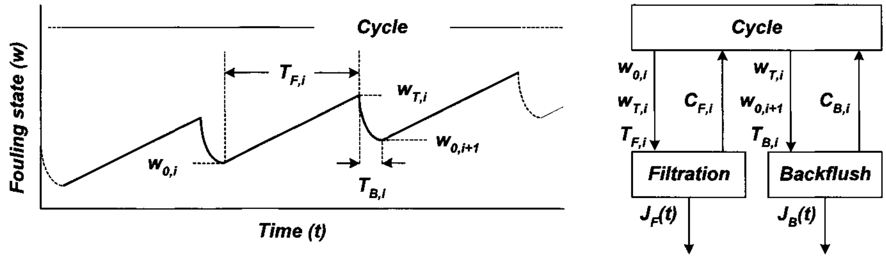

Figure 1. Left: Semi continuous operation of dead-end filtration consists of consecutive filtration and backflush phases. Right: Hierarchical control structure which corresponds to the cyclic operation.

3. Theory

3.1. Model

In dead-end filtration the fouling state (w) is proportional to the filtrated volume. The flux (J) is the control variable, which is also the production rate. The model parameters are given in table 1.

$$\frac{\mathrm{d}w}{\mathrm{d}t} = J \tag{1}$$

The driving force of the filtration process is the trans-membrane pressure, which is related to the flux and the hydraulic resistance of the system by Darcy's law:

$$\Delta P = \eta J R_M \gamma_F \tag{2}$$

in which γ_F is the relative increase in the resistance due to fouling, which can be given by (Blankert):

$$\gamma_F = 1 + \frac{1}{R_M} \alpha w \Phi (1 + \alpha w \Phi \eta \beta J) \tag{3}$$

with Φ a correction factor for the geometry of the fiber, given by (Belfort):

$$\Phi = -\frac{r}{2wx}\ln\left(1-\frac{2wx}{r}\right) \tag{4}$$

The relative increase in energy consumption due to the pump efficiency can be approximated by (Karasik):

$$\gamma_P = \left(\frac{\eta R_M J_x^2}{P_{\max}}\frac{\gamma_F}{J}+\frac{1}{4}\right)\left(\sqrt{\frac{\eta R_M J_x^2}{P_{\max}}\frac{\gamma_F}{J}+\frac{1}{4}}-1\right)^{-1} \tag{5}$$

Table 1. Model parameters and their values

Specific cake resistance	α	m^{-2}	1.00×10^{13}
Compressibility	β	Pa^{-1}	5.00×10^{-5}
Cake volume fraction	x	-	1.00×10^{-3}
Viscosity	η	Pa s	1.01×10^{-3}
Membrane resistance	R_M	m^{-1}	7.00×10^{11}
Fiber radius	r	m	4.00×10^{-4}
Maximum pump pressure	$P_{\max}$	Pa	1.33×10^{5}
Maximum pump efficiency	$\eta_{P,\max}$	-	0.50
Flux at maximum efficiency	J_x	m/s	4.16×10^{-5}

3.2. Optimization

The energy consumption per unit area is equal to the integral over the power per unit area ($J\Delta P\gamma_P$), which can be given by:

$$C_F = \int_0^T \left(\eta\eta_{P,\max}R_M\gamma_F\gamma_P J^2\right)\mathrm{d}t \tag{6}$$

For this process the Hamiltonian can be given by:

$$H(w,J,\lambda) = \lambda J + \eta\eta_{P,\max}R_M\gamma_F\gamma_P J^2 \tag{7}$$

In the Hamiltonian the adjoined state (λ) is introduced. The first necessary condition for optimality states that the optimal flux minimizes the Hamiltonian, thus:

$$\frac{\partial H}{\partial J} = \lambda + \eta\eta_{P,\max}R_M\gamma_F\gamma_P J\left(2+\frac{J}{\gamma_P}\frac{\partial\gamma_P}{\partial J}+\frac{J}{\gamma_F}\frac{\partial\gamma_F}{\partial J}\right)=0 \tag{8}$$

This equation allows us to calculate the optimal flux as function of the state and the adjoined state. However, here it is used to eliminate λ from eqn. 7. The result is the minimum value of the Hamiltonian as function of the flux and the state.

$$H_{\min}(w,J) = -\left(\eta\eta_{P,\max}R_M\gamma_F\gamma_P J^2\right)\left(1+\frac{J}{\gamma_P}\frac{\partial\gamma_P}{\partial J}+\frac{J}{\gamma_F}\frac{\partial\gamma_F}{\partial J}\right) \tag{9}$$

This equation leads to two approaches which are discussed in the following paragraphs.

3.2.1. Simplified system

One approach is simplification of the system. The effect of compressibility and pump efficiency are neglected ($\beta = 0$, $\gamma_P = 1$). In that case the right-hand term of eqn. 9 vanishes and the Hamiltonian is proportional to the power. Since the Hamiltonian is constant along an optimal trajectory, constant power filtration is optimal.

From this consideration, constant gross power filtration is introduced as alternative for the dynamically optimal trajectory. The main advantage of this approach is that it can be implemented in a robust way. The power, which can be easily measured, can be kept constant by a feedback controller, which is part of a cascade control structure. The master controller uses the setpoint of the power to ensure the final condition (produced volume) is met.

3.2.2. Predefined trajectories

The second approach also makes use of the consideration that for optimal trajectories the Hamiltonian is constant. With eqn. 9 the Hamiltonian is calculated for a large number of states and fluxes on a grid. The calculated points (J, w, H) are sorted in a table such that each row corresponds to a value of the state and each column corresponds with a value of the Hamiltonian.

Each column contains a trajectory, which is optimal for some final condition. Since the state and the flux are known, the time and costs of each point in a column can be calculated and added to the table. This is done at the moment the model parameters are estimated. Then at each filtration phase, for a given final time and final state, the optimization problem is reduced to finding the correct row indices and column index. The row indices follow directly from the initial and final condition for the state. The column index follows from the final time (duration) condition. It is equal to the difference between the initial and final column. The costs can be found in a similar way.

4. Results

The optimal trajectories were calculated for model parameters shown in table 1. Fig. 2 shows these trajectories for w_0 = 0m, w_T = 0.0375m and T = 1800s. The common operating strategies, constant flux (flow) and constant trans-membrane pressure (driving force), are shown in the figure as well. It can be seen that the constant gross power trajectory is close to the optimum. The constant flux trajectory is also close. The relative difference in costs between the optimal and suboptimal strategies are shown in table 2.

Table 2. Potential savings of reference strategies

	Complete model		Simplified model	
Final time (s)	1800	3600	1800	3600
Final state (m)	3.75×10^{-2}	7.50×10^{-2}	3.75×10^{-2}	7.50×10^{-2}
Constant flux	0.1 %	1.1 %	0.4 %	1.0 %
Constant pressure	4.1 %	16.0 %	0.4 %	1.0 %
Constant gross power	< 0.1 %	<0.1%	0	0

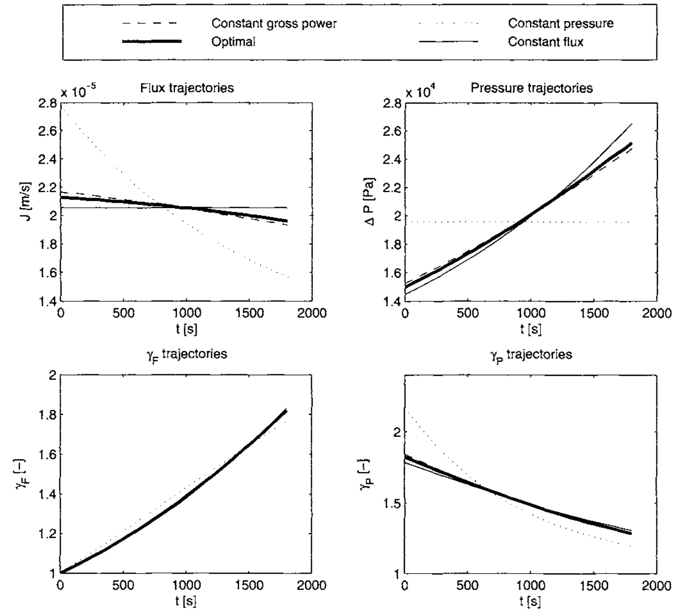

Figure 2. Top left: flux trajectories, top right: trans membrane pressure trajectories, bottom left: relative increase in resistance due to fouling, bottom right: relative increase of energy consumption due to pump efficiency.

5. Conclusion

Constant flux and constant pressure filtration are equally expensive according to the simplified model. However, when the pump efficiency, compressibility and cake volume are taken into account, constant pressure filtration consumes more energy than constant flux filtration. There is no significant difference (<0.1%) between constant gross power filtration and the optimal trajectory. Hence, constant power filtration can be a robust way to implement the optimization. Under typical conditions, the relative saving in energy, is small compared to constant flux (0.1%) or constant pressure filtration (4.1%).

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