Theory and application of a new method for the in-situ measurement of sound absorption

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Introduction

A new (patent pending, [3]) method for the in situ determination of the sound absorption coefficient was presented in a recent paper of Wijnant [1]. It is based on the synchronous measurement of the active sound intensity and a new quantity called the **total intensity**. In this paper, theory, implementation and experimental results will be discussed.

Theory

The spatially averaged absorption coefficient of an area S is defined as the ratio of the time-averaged active sound power and the time-averaged incident sound power.

$$\alpha \equiv \frac{P_{in} - P_{refl}}{P_{in}} = \frac{P_{ac}}{P_{in}} = \frac{\int\limits_S \mathbf{I_{ac}} \cdot \mathbf{n} dS}{\int\limits_S \mathbf{I_{in}} \cdot \mathbf{n} dS} \tag{1}$$

Where P_{in} is the time-averaged incident sound power and $P_{ac} \equiv P_{in} - P_{refl}$ is the time-averaged nett power or active power flowing through S. Using a sound intensity probe, the active sound power can be determined by measuring the normal active sound intensity $\mathbf{I_{ac}} \cdot \mathbf{n}$ over the surface and integrating spatially. The incident sound power cannot be measured directly but can be determined using the new approach explained in the following paragraphs.

It is assumed that, at a location $\mathbf{r}$ in a certain direction $\mathbf{n}$, see figure (1), the sound field can be decomposed in a plane incident- and reflected wave.

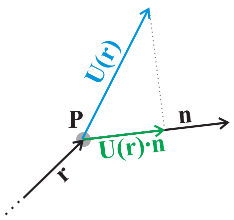

Figure 1: Complex pressure P, complex particle velocity U and its projection in direction $\mathbf{n}$ at an arbitrary point in space

As a result, the active intensity mentioned above is equal to the difference of the incident- and reflected intensity.

$$I_{ac}\left(\mathbf{r},\mathbf{n}\right) = \frac{A(\mathbf{r},\mathbf{n})\,\overline{A(\mathbf{r},\mathbf{n})} - B(\mathbf{r},\mathbf{n})\,\overline{B(\mathbf{r},\mathbf{n})}}{2\rho_0 c_0} \tag{2}$$

Where $A(\mathbf{r},\mathbf{n})$ is the complex amplitude of the incident wave and $B(\mathbf{r},\mathbf{n})$ the complex amplitude of the reflectd wave. For clarity, the explicit dependency on $\mathbf{r}$ and $\mathbf{n}$ will be dropped in the notation from here. Furthermore, the overline symbol as in $\overline{A}$, indicates the complex conjugate of A. Bold printed quantities are vector quantities. The total intensity is defined as the **sum** of the incident- and reflected intensity.

$$I_{tot} = \frac{A\overline{A} + B\overline{B}}{2\rho_0 c_0} \tag{3}$$

if follows that the incident and reflected intensities can be obtained by

$$I_{in} = \frac{1}{2}\left[I_{tot} + I_{ac}\right] \tag{4}$$

$$I_{refl} = \frac{1}{2}\left[I_{tot} - I_{ac}\right] \tag{5}$$

as $I_{in} = \frac{A\overline{A}}{2\rho_0 c_0}$ and $I_{refl} = \frac{B\overline{B}}{2\rho_0 c_0}$ by definition. As shown in Wijnant's paper [1], the total intensity I_{tot} is equal to

$$I_{tot} = \frac{1}{4}\left[\rho_0 c_0 \left(\mathbf{U}\cdot\mathbf{n}\right)\left(\overline{\mathbf{U}}\cdot\mathbf{n}\right) + \frac{P\overline{P}}{\rho_0 c_0}\right] \tag{6}$$

Hence, when one measures P and $\mathbf{U}\cdot\mathbf{n}$, one is capable of determining both the **active** and **incident** sound intensity, thus enabling a rapid analysis of the local sound absorption coefficient at that point, in the direction of probe orientation.

Implementation

Practically, the integrals can be replaced by discrete sums if point by point measurements are performed. Continuous scanning of the surface is also possible, provided that the probe velocity is kept constant or, if this is not the case, a surface weighting procedure in combination with synchronous probe position measurement is applied. The expression for the local sound absorption coefficient can be written in terms of single-sided auto- and cross-power spectra.

$$\alpha = \frac{4\rho_0 c_0 \Re\left(G_{pu}\right)}{G_{pp} + 2\rho_0 c_0 \Re\left(G_{pu}\right) + \rho_0^2 c_0^2 G_{uu}} \qquad (7)$$

Where G_{pu}, G_{pp} and G_{uu} can be calculated from G_{11}, G_{22} and G_{12} when a 1D pp-probe is applied, see [2].

Experimental results

The method was successfully validated by measuring the sound absorption coefficient of a sample in an impedance tube with the transfer function method and the new method, see [1]. In a next step, the new method was investigated with respect to the spatially averaged sound absorption coefficient of a large tube that radiates sound to the exterior. We can calculate an absorption coefficient for any cross section in the tube as if this cross section would be an absorbing sample. At the closed end of tube, a small source was introduced off-axis, see figure (2).

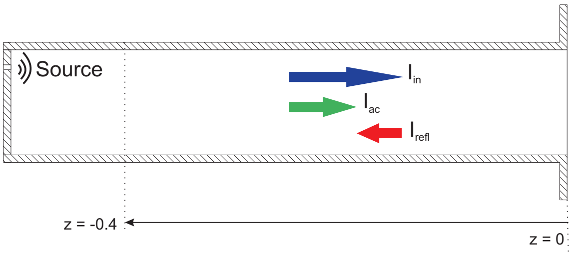

Figure 2: Schematic of the tube set-up with two measurement planes at z = -0.4 m and z = 0 m and intensity direction convention.

The right end of the tube is open and has a small flange. The source emitted band-limited white noise in the range 250-4000 Hz. The results for the measurement planes z = -0.4 m and z = 0 m will be discussed here. The sound field in the duct was simulated using the finite element software COMSOL. An example of the sound field at 1000 Hz is shown in figure (3).

The measurements were performed with a pp-probe with a 12 mm spacer. Measures were taken to ensure that the probe could only move in the plane of measurement. Assuming a constant scanning velocity and uniform scanning over the cross-section of the tube, usual procedures for averaging of auto- and cross-power spectra could be employed. The uniform scanning assumption proved to be correct, as the results of three independent measurements were almost identical.

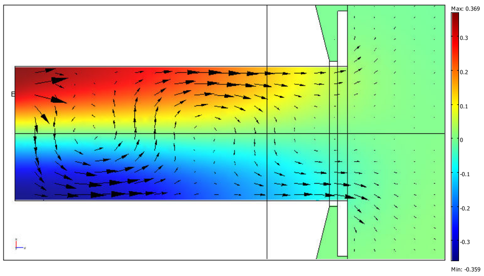

Figure 3: Sound field in the tube at the 1^{st} cut-on frequency at 1000 Hz; Arrows: active intensity; Color: sound pressure

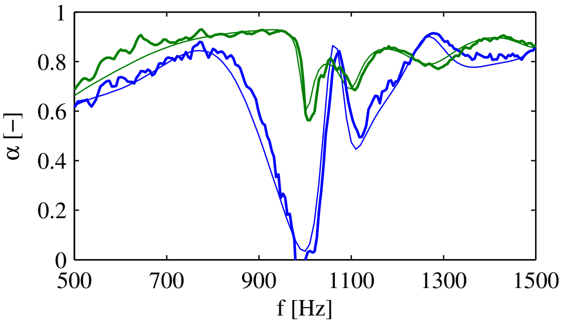

Figure 4: Sound absorption coefficient for z =-0.4 m (blue) and z = 0.0 m (green); Thin curves: simulation, thick curves: measurement

The sound absorption coefficient curves obtained by simulation and by measurement are shown in figure (4). Below the cut-on frequency (approx. 1000 Hz) of the tube the curves of both measurement planes do not coincide as the tube is too short for 1D wave propagation to occur. So, 3D wave propagation occurs and the spatially averaged absorption coefficient varies due to variations of the incident power P_{in} with the z-coordinate. At cut-on, the sudden increase of P_{in} causes a clear dip in the curves. Furthermore, the agreement between measurement and simulation is very good.

Conclusions

A new method for the in situ measurement of the sound absorption coefficient is developed. The concept of total intensity enables the direct calculation of the incident- and reflected sound intensity. The measurement method is validated and brought into practice with promising results. Further research will be directed towards measurements outdoors.

References

[1] Y.H. Wijnant, E.R. Kuipers and A. de Boer, *Development and application of a new method for the in-situ measurement of sound absorption*, Proc. of ISMA 31, Leuven, Belgium, (2010).

[2] F. Jacobsen, *Active and reactive, coherent and incoherent sound fields*, Journal of Sound and Vibration 130(3), pp. 493-507 (1988).

[3] Y.H. Wijnant, Patent (pending) - NL 2004628, University of Twente (2010).