Multi-Scale permeability of particulate and porous media

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ABSTRACT

In this paper, a finite element (FE) based model for the viscous and incompressible fluid flow through a regular porous media composed of rigid (immobile) particles/fibers is considered, and an analytical-numerical approach is employed to calculate the associated transverse permeability. The effects of an isotropy, i.e., the particle shape or orientation, as well as porosity, i.e., the volume fraction, on the overall permeability are discussed in detail. The results from this study can be used for verification and validation of advanced models for particle-fluid interaction and for the coupling of the discrete element method (DEM) with FEM or CFD.

1. INTRODUCTION

The problem of creeping flow between solid bodies arranged in a regular array is fundamental in the prediction of seepage through porous media and has many applications, including: composite materials, rheology, geophysics, colloid science, etc. A compelling motivation for such studies concerns the understanding, and eventually the prediction, of the single and multiphase transport properties of the porous structure. Prediction of the permeability of fibrous media dates back to experimental work of Sullivan [1] and theoretical works of Hasimoto [2] and Happel [3]. More recently, Sobera and Kleijn [4] studied the permeability of random 1D and 2D fibrous media both analytically and numerically. Their analytical model was based on scale analysis and the proposed relationship was a function of fiber distance and a non-dimensional randomness number. Tomadakis and Sotirchos [5] proposed a model that enables the prediction of anisotropic permeability through 1-D, 2-D, and 3-D random fibrous beds. The Darcy law for permeability of a dense network of fibers with general orientation was examined experimentally and analytically by Håkanson et al. [6]. Bechtold et al. [7] examined the influence of the fibre distribution characteristics on the transverse permeability using 2-D finite element simulations. Ogata [8] approximated the solutions of steady three-dimensional Stokes flow past obstacles in a planar periodic array by linear combinations of the periodic fundamental solutions presented by Ishii [9]. Tamayol et al. [10] determined analytically the permeability of touching and non-touching ordered fibrous media towards normal and parallel flow.

In this study, periodic arrays of parallel cylinders perpendicular to the flow direction are considered and the effects of shape and orientation of cylinders as well as their porosity on the macroscopic permeability of the porous media are discussed in detail. For verification of our model, the results are compared with previous theoretical and numerical data for square and hexagonal packing configurations.

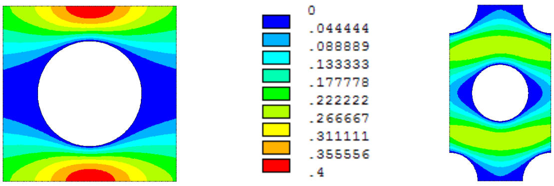
2. MODEL DEVELOPMENT AND NUMERICAL RESULTS

Both hexagonal and square arrays of parallel cylinders perpendicular to the flow direction are considered in this section. The basis of such model systems lies on the assumption that the porous media can be divided into representative elements or unit cells. The permeability is then determined by modeling the flow through these, more or less, idealized cells. At the left and right pressure- and at the top and bottom periodic-boundary conditions are applied. No-slip boundary conditions, i.e., zero velocity are applied on the surface of the particles/fibers.

Under laminar, steady state condition, flow through porous media is governed by Darcy's law, which for one dimensional flow, is expressed as:

$$\langle u \rangle = -\frac{K}{\mu}\frac{dp}{dx} \qquad (1)$$

where $\langle u \rangle$, K, μ and p are volume averaged (superficial) flow velocity, permeability in the x direction, viscosity and pressure, respectively. The horizontal velocity field for both hexagonal and square packing configurations is shown in Figure 1.



(a) (b)

Fig. 1: The horizontal velocity field for (a) square and (b) hexagonal packing at ε = 0.71 (pressure boundary at right and left and periodic boundary at top and bottom)

By calculating the average velocity from our FE simulation, knowing the pressure drop Δp over the length of the unit cell ΔLx, and using Eq. (1), the normalized permeability K/r^2 is obtained. The variations of normalized permeability (with respect to the radius cylinder) versus porosity, for square and hexagonal packing, are shown in Figure 2. The lubrication theory presented by Gebart [11] and Bruschke [12] agrees well with our numerical results at low porosities whereas, for high porosities, the results of Drummond [13] fits best our data.

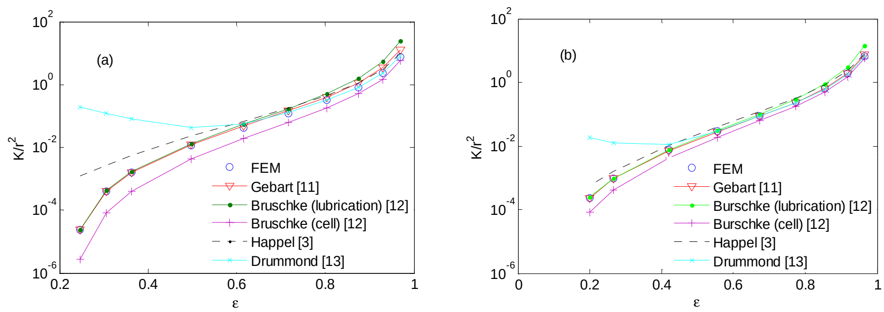


Fig. 2: Variation of normalized permeability versus porosity for (a) square and (b) hexagonal packing for circular shape particles/cylinders with radius r

In order to be able to compare different shapes and orientations, the permeability can also be normalized with respect to the pore length, which is defined as:

L_p = 4 area / circumference

$L_p = 2r = d$ (for circle), $L_p = c$ (for square), $L_p = 4\pi ab / A_L$ (for ellipse) (2)

where r, c, a and b are the radius of the circle, length of the square, major and minor radius of ellipse respectively. By applying the same procedure, the normalized permeability is calculated for different shapes on a square configuration. The comparison of normalized permeability for different shapes is shown in Figure 3. At high porosities the shape of particles does not affect the permeability too much, but at low porosities the effect is much more pronounced. Circles have the lowest and ellipses the highest normalized permeability.

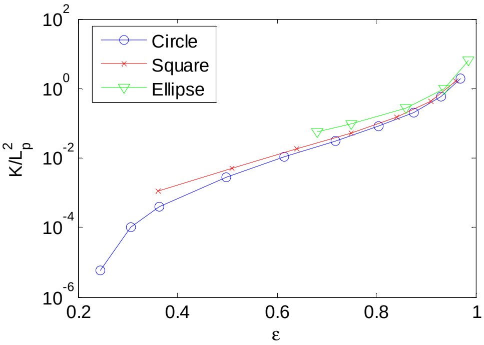


Fig. 3: Effect of shape on the normalized permeability from a square packing configuration of circles, squares and ellipses (*a/b*=2, major axis in flow direction)

By changing the orientation (θ), the angle between the major axis of the obstacle and the horizontal axis, the permeability tensor becomes anisotropic. The effect of orientation on the normalized permeability for squares and ellipses (*a/b*=2) is shown in Figure 4. For ellipses, at high porosity, the orientation does not affect the permeability; however, at low porosities the effect is strong: by increasing the rotation angle the permeability is reduced. For squares, again, at high porosities the orientation does not much affect the permeability, but at low porosities it affects the permeability a lot. At θ = 45^{O} we observe a drop in permeability, because we are close to the blocking situation, i.e. zero permeability, at $\varepsilon \approx 0.5$.

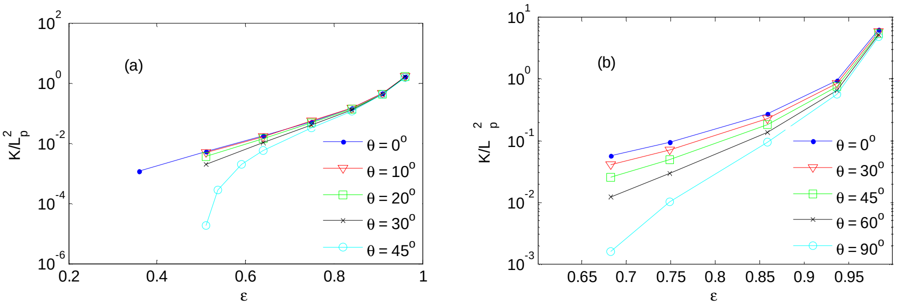


Fig.4: Effect of orientation (θ) on the normalized permeability for (a) square and (b) ellipse (*a/b*=2) from a square packing configuration

3. CONCLUSIONS

Permeability is an important property that characterizes porous media; however, its determination is challenging due to its complex dependence on the microstructure of the media. In this study, transverse flow in aligned fibrous porous media has been investigated by detailed FE simulations. The effects of porosity, shape and orientation of the particles have been studied. Our numerical results show that the circles and ellipses have the lowest and highest normalized permeability, respectively, when the major axis of ellipses are aligned with flow direction, i.e. horizontal ellipses. For a vertical ellipse, it has lower normalized permeability than the circle. By changing the orientation of the particles, the permeability tensor becomes anisotropic and it decreases when increasing the density or the orientation angle until the channel is blocked. Our results are in good agreement with previous numerical and theoretical data.

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