Accelerating solving the dynamic multi-objective network design problem using response surface methods

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ABSTRACT

Multi objective optimization of externalities of traffic solving a network design problem in which Dynamic Traffic Management measures are used, is time consuming while heuristics are needed and solving the lower level requires solving the dynamic user equilibrium problem. Use of response surface methods in combination with evolutionary algorithms could accelerate the determination of the Pareto optimal set. Three of these methods are compared with employing the SPEA2+ evolutionary algorithm without use of these methods. The results show that the RSM methods accelerate the search considerably at the start, but tend to converge faster and therefore loose their head start.

Keywords: *NDP, Externalities, DTM measures, EMOA*

INTRODUCTION

Although a significant portion of research on optimization in traffic and transport considers a single objective related to accessibility [*1*,*2*], it may no longer suffice to neglect externalities of traffic. However, the multi-objective network design problem (MO NDP) is challenging to solve. One specific example of a MO NDP is to optimize a network through the implementation of dynamic traffic management (DTM) measures that can influence the supply of infrastructure dynamically (e.g. traffic signals and rush hour lanes) and minimization of externalities are the objectives. The presence of multiple conflicting objectives makes the optimization problem interesting to solve. Since no single solution can be termed as an optimum solution, the resulting multi-objective (MO) optimization problem resorts to a number of trade-off optimal solutions, known as Pareto optimal solutions.

Mathematical modeling of such a highly complex socio-technical system provides insight in the extent to which objectives are conflicting or not and the consequences related to weights used concerning the trade-offs, which is very useful in the decision making process. The NDP is usually formulated as a bi-level problem in which the lower level describes the behavior of road users that optimize their own objectives (travel time and travel costs), modeled by solving the user equilibrium problem. While DTM measures are the decision variables and traffic dynamics are important explanatory variables assessing the effects on externalities, a dynamic traffic assignment (DTA) to solve the lower level is preferred. The upper level consists of the objectives that have to be optimized for solving the NDP. Because of the non convexity of the problem [*2*,*3*], often heuristics are used to optimize the total system. In bi-level optimization studies, the solution approach using evolutionary algorithms has been proven successful and a comparison of different evolutionary algorithms has shown that the strength Pareto evolutionary algorithm 2+ (SPEA2+) algorithm performs well for the dynamic MO NDP [*4*].

While the evaluation of a possible solution requires solving the lower level using a DTA and heuristics are used, computation time can become extremely large. A possible solution for accelerating the search is combining response surface methods (RSM) with the SPEA2+ algorithm. RSM are methods in which a surrogate model is estimated based on available exact evaluations of solutions (e.g by fitting a model using regression). This estimated surrogate model can be used in different ways within the optimization process. A similar approach is for example the trust region optimization method which can be used for single objective optimizations and applied in research by Osorio [*5*] or the multi-objective radial basis function algorithm developed and applied by Chow [*6*] which are two of the rare studies in which RSM methods are used within research on optimization in traffic and transport. Earlier research [*7*] has shown that simple regression methods in which a full quadratic function is estimated shows promising results. While the SPEA2+ shows more diversity in solution and objective space then other tested algorithms and diversity is relevant for the estimation of the surrogate model this algorithm is used within this research as a starting point. In this research we compared three possible algorithms in which RSM are used with employing the SPEA2+ algorithm without using these methods.

NETWORK DESIGN PROBLEM

The NDPs are typically grouped into discrete problems (DNDP), in which the decision variable is a discrete variable [*2*,*11*,*26*], continuous problems (CNDP), in which is assumed that the decision variable is a continuous variable *[3,12,13,14,15*], and mixed problems, which is a combination of both [*16*]. Based on demand, NDPs can be grouped into fixed demand [*14*], stochastic demand [*18,19*] and (stochastic) elastic demand [*20*]. Based on the way time is considered, NDPs can be classified into static, in which stationary travel demand and infrastructure supply is assumed (used in all but one above mentioned studies), or dynamic, which is rarely used [*18,21]*. Traditionally, the

NDP is associated with the minimization of the total travel time using infrastructural investment decisions under a budget constraint. Most of the previous works consider fixed demand, and use a static user equilibrium to model the lower level. There are also other design variables of networks that can be considered as an NDP. Brands et al. [*21*] studied for example optimal tolling and Cantarella et al. [*16,17*] the optimal signal setting in combination with lane layout.

In most cases, single objective network design problems are studied in which accessibility is optimized, where accessibility is expressed as the total travel time in the traffic network [*1,2*]. Different studies incorporated the investment costs within the objective function. Chiou, Meng et al. and Xu et al. [*3,14,15*] optimized total travel time in which the investment was translated in time using a conversion factor. Or in which travel time is translated into cost [*11,22*]. Occasionally other costs, like environmental costs (expressed in money), are added to the travel cost [*16,23*].

There are less papers that use multiple objective functions in the upper level. Chen et al. [*19*] use travel time and construction costs as two separate objective functions and used an evolutionary algorithm. Friesz et al. [*13*] focuses on minimizing the transport costs, construction costs, vehicle miles traveled and dwelling units taken for rights-of-way and used a weighted sum approach in combination with simulated annealing. Sharma et al. [*24*] used a genetic algorithm to minimize total travel time and the higher moment for total travel time i.e. variance. Cantarella and Vitetta considered traveltime, walking time and CO emissions in their optimization using a genetic algorithm [*17*]. Most MO NDP studies consider the minimization of investment cost as second objective as reported in [*24*].

In this research, instead of using static traffic models, focusing on a single objective, we propose an MO NDP in which the externalities of traffic are minimized using DTM measures and in which a DTA model is used to operationalize the lower level. This MO NDP is used to compare the three algorithms.

OPTIMIZATION PROBLEM AND FRAMEWORK

The MO optimization problem is formulated as the following MO MPEC (mathematical problem with equilibrium constraints):

$$\min_{S\in F}\begin{pmatrix} z_1(S)\\ z_2(S)\\ \vdots \\ z_I(S)\end{pmatrix}, \text{s.t.}\left(q(S),v(S),k(S)\right)\in\Gamma^{DTA}\Big(G\big(N,A\big(C(S)\big)\big),D\Big)$$

in which S is a set of applications of strategic DTM measures to be selected from a set of feasible applications F, and called a solution. Time and settings of the DTM measures are discretized, so the upper level then becomes a discrete optimization problem where for each time period a certain DTM measure with a certain setting is implemented. If we assume that there are B different DTM measures available in the network, the application of the DTM measures in time step t is defined by $S^t=\left(s_1^t,\ldots,s_B^t\right)$, where each s_b^t, $b=1,\ldots,B$, can have M_b different settings, which we simply number from 1 to M_b. The set of feasible solutions can therefore be written as $F=\left\{S \mid s_b^t\in\{1,\ldots,M_b\},\forall t=1,\ldots,T\right\}$. Each possible solution S leads to certain dynamic traffic conditions which is the result of the optimization of road users in the lower level of their own objective (travel time). The DTA model Streamline [*25*], which is a fast multiclass model with physical queuing and spillback and easy to connect to Matlab® which is used to program the solution algorithms, is used to solve for this dynamic user equilibrium indicated by Γ^{DTA}, for which the supply of infrastructure is given by G with nodes N and links A (with corresponding characteristics C), and the travel demand D. Output of this model are dynamic flows $q(S)$, speeds $v(S)$, and densities, $k(S)$, for all modes on all links of the networks and are input to calculate the objective functions $z_i(S)$. These objectives in our case concern accessibility, climate, and noise, but could be extended with air quality and safety.

Based on an extensive literature review [*10*], for each objective an objective function $z_i(S)$, is defined, where the input stems from the DTA model. In this research the objectives concerning efficiency, climate and noise are used. Efficiency is defined in terms of the total travel time in the network. Climate is defined as the total emission of CO_2. The emissions are determined based on the ARTEMIS traffic situation based emission model, which means dependent on the level of service of the traffic flows. Finally, noise is calculated as the average weighted sound power level, in which the weights of noise emissions depend on the level of urbanization, and emissions are based on a load and speed dependent emission function of the Dutch RMV noise model [*26*].

SOLUTION APPROACHES

Evolutionary multi-objective algorithm SPEA2+

Kim et al. [*9*] adapted the SPEA2 approach which is originally developed by Zitzler. Within the algorithm, the fitness assignment depends on the level of dominance and fitness sharing based on density to maintain population diversity. SPEA2+ contains elitism by the preservation of good solutions in the environmental selection step. This is a deterministic step in which an archive is maintained containing the best solutions, based on their fitness, considered so far. Within the SPEA2+ approach, two archives are maintained. In one archive the distances between solutions within the solution space, while in the other archive the distances between solution within the objective space are used to truncate the Pareto optimal set if it's size exceeds the pre-defined maximum size. These archives contain solutions used for the mating selection which is done using neighborhood crossover, which crosses over solutions close to each other in the objective space.

Response surface methods

The RSM is introduced by Box and Wilson [8] and was originally intended as a guideline to design experiments. In this case we fit a regression model using a pure quadratic polynomial (single and quadratic terms), which is also recommended in other studies [*5*,*7*,*8*]:

$$z_i(S) = \alpha_0 + \sum_{j=1}^{T}\sum_{k=1}^{B}\alpha_{(j-1)*T+k}s_k^j + \sum_{j=1}^{T}\sum_{k=1}^{B}\alpha_{TB+(j-1)*T+k}s_k^{j2}$$

By fitting a regression model a least square problem is solved using the exact evaluated solutions as input and results in the estimates for the parameters α . To be able to solve the least square problem (finding a unique solution) the number of exact evaluated solutions which form the input should be at least equal to the number of parameters α to estimate. However, to avoid over fitting the number of exact evaluated solutions should be larger. In addition, while the MO NDP is not specifically interested in one part of solution space, the model is used for global approximation and to avoid fast convergence to local optima, diversity of exact evaluated solutions which are used for fitting the regression model is relevant. Using this type of model is easy to understand and can be estimated fast even with a large number of exact evaluated solutions.

Algorithms using RSM

All algorithms use a Latin Hypercube Sample (LHS) optimized for correlation as a starting population. This LHS is used as input (approximation set) for estimating the surrogate model. In all algorithms this approximation set is extended based on new solutions exactly evaluated, although only if these solutions provide information for low dense areas in the solution space. This approximation set is combined with the Pareto optimal set known thus far to estimate the surrogate model which is done every iteration.

Within the first approach (SPEA2+ pre evaluation FA) the surrogate model is used as a pre-evaluation within the SPEA2+ algorithm to determine which children are interesting to evaluate exactly. In addition, the children which are situated in less dense areas are also included to evaluate exactly while these solutions can improve the surrogate model and while the error of the approximation of these solutions is relatively high. If the algorithm tends to converge the pre-evaluation is neglected, which means that the algorithm becomes a regular SPEA2+ algorithm. Within the second approach (FA optimized SPEA2+), the surrogate model itself is optimized using a SPEA2+ algorithm and the resulting solutions are exactly evaluated to determine the Pareto optimal set and used to update the approximation set. Within the third approach (FA seeded SPEA2+) the algorithm of the second approach is only used in the first *h* steps whereafter the algorithm continues as a regular SPEA2+ algorithm. In this algorithm the surrogate model is used to obtain a seeded starting population.

Performance measures

The S-metric, size of dominated space, the C-metric, coverage of two sets, epsilon indicator and the spacing metric [*4,27*] are used as performance measures.

NUMERICAL EXPERIMENT

Case

A case study is conducted on a small hypothesized road network consisting of one OD pair, three routes and four DTM measures (three traffic signals and dynamic speed limit) and a large solution space (11 possible settings, 6 time periods resulting in 24 decision variables and 4.05×10^{21} possible solutions), to compare the solution approaches and the results of a Pareto optimal multi objective optimization of externalities. Although the network was small, it did incorporate the major elements like urban and non-urban routes using DTM measures to optimize the externalities. Moreover, these objectives were modeled in a realistic manner incorporating traffic dynamics.

Parameter settings

In order to restrict computation time, we limit the budget of solutions that can be considered. In the comparison of the approaches, the total number of solutions exactly evaluated is a fixed number of 5,100 solutions (initialization inclusive). For all algorithms we used the same genetic operators, namely uniform crossover and mutation in which the initial mutation rate is 0.2 and decreases with 95% within the first 10 generations. Only small mutations occur, as we assume that mutation results in shifting the DTM application one up or down, i.e., if $s_b(t)$ is selected for mutation, its value after mutation becomes either $s_b(t)-1$ or $s_b(t)+1$. All approaches are repeated 8 times and the archive size was set to be equal to the population size of 100 solutions. In all algorithms the deterministic environmental selection procedure of the SPEA2+ algorithm was used in every iteration to select the 100 Pareto optimal solutions.

RESULTS AND CONCLUSIONS

The results show that the algorithms find similar Pareto optimal fronts and objectives efficiency and climate in this case are strongly aligned. However, both objectives are opposed to the objective noise. Optimizing efficiency aims at avoiding congestion using full capacity of the available routes, which is also good for minimizing CO_2 emissions. Optimizing noise aims at lowering the driving speeds as much as possible and also avoiding traffic using the urban routes.

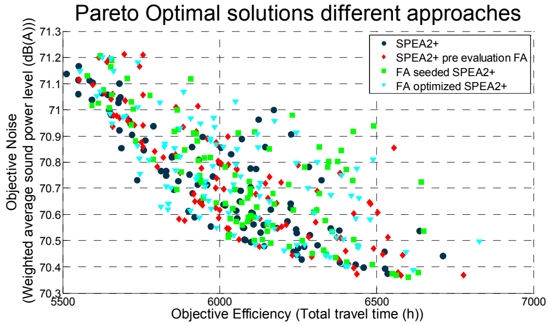

Fig. 1: Pareto optimal solutions

The comparison of the algorithms shows that using RSM methods accelerates the search at the start considerably. With less exact evaluated solutions already good solutions are found. The algorithms using these RSM methods tend to converge faster, possibly to a local optimum and therefore loose their head start, while these algorithms depend largely on the quality of the surrogate model. Therefore, these methods are mainly of interest if a limited number exact evaluations can be done or can be used as a pre phase in a hybrid approach. To avoid premature convergence two algorithms proceeded with regular SPEA2+ in which the FA seeded SPEA2+ has difficulties to find further improvements, whereas the SPEA2+ pre evaluation FA performs at least similar in these generations as the regular SPEA2+ algorithm. Another option, not investigated here, is using neighborhood search [*6*], which is next to other approximation methods like fitness granulation [*28*], incorporation of knowledge of road transport systems and further research using other and more complex networks an interesting research direction.

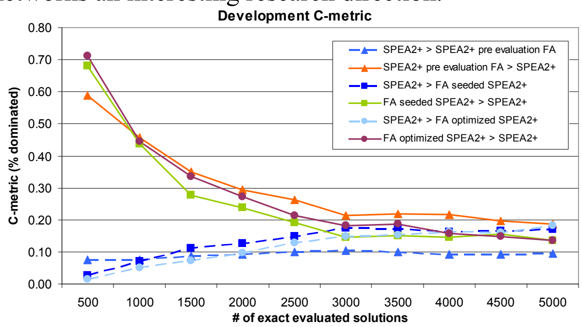

Fig. 2: Development C-metric approaches

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