The Work of Otto Fischer and the Historical Development of His Method of Principal Vectors for Mechanism and Machine Science

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Abstract. This article gives an overview of the distinctive work of Otto Fischer (1861-1916) on the motion of the human musculoskeletal system. In order to be able to derive the individual muscle forces for human in motion, he invented *the method of principal vectors* to describe the motion of the centers-of-mass and the inertias of body segments. This method has proven to be successful, not only for the studies on biomechanics but in particular also for mechanism and machine science. A historical development of the application of the method is presented for today's potential.

Keywords: Otto Fischer, center-of-mass, biomechanics, static balancing, dynamic balancing.

1 Introduction

With time the amount of knowledge increases at a rapid pace. For research it is a challenge to both remain up-to-date and to not forget the past. Particularly this is a challenge when old research results and ideas lose interest for a certain time, for decades or for ages. To retrieve the results, and more importantly to retrieve the philosophy with which they were obtained, often investigation of the original sources is needed. Citations of old literature in current articles often tend to become rather 'automatic' or indirect, e.g. because of language differences or unavailability, and therefore do not reveal the essence and the potential of the original sources. This however is necessary to discover the value old methods may have for contemporary research goals.

With this perspective, the work of the physiologist, physicist, and medical doctor (Physiologe, Physiker, Arzt) Otto Fischer[1] is summarized and the philosophy and application of his *method of principal vectors* is investigated. In addition to his still relevant results on the mechanical properties of the human musculoskeletal system and of his investigation of the human gait, this method has been essential for studying the individual muscle forces for human motion in the pre-computer era. Although invented for biomechanics, Fischer emphasized the potential of his method for mechanism and machine science, which was taken over by various researchers. This article includes a historical development of his method in mechanism and machine science to illustrate its current potential.

[1] *26-04-1861 Altenburg (Thüringen), Germany †16-12-1916 Leipzig, Germany.

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2 Otto Fischer and his Works

[1][2][3] Otto Fischer studied mathematics and physics at the German universities of Jena, München, and Leipzig and obtained his PhD-degree (Dr. phil.) in mathematics from the University of Leipzig in 1885, supervised by Prof. Felix Klein. In 1887 he became teacher of mathematics and physics at the business school (Handelslehranstalt) of Leipzig. He continued as a teacher at the Petri-Realgymnasium in Leipzig in 1895, where he would teach until the end of his life, from 1912 onward at the position of rector.

At the same time Fischer worked on medical and biological research together with the anatomist Prof. Christian Wilhelm Braune at the Anatomic institute of the University of Leipzig. In 1893 he obtained his habilitation on physiological physics (Physiologische Physik) at the University of Leipzig and became teacher at this topic at the Faculty of Philosophy. In 1896 he became Professor extraordinarius on medical physics (Medizinische Physik) at the Medical Faculty of the University of Leipzig where he remained until his early death in 1916.

Together with Braune, Fischer was pioneering the research on biomechanics of the human musculoskeletal system in motion. While anatomists at that time investigated the mechanics of the human body empirically, he understood the necessity of theoretical investigation. He stated that only with an accurate kinematic model of the human skeletal system the functioning of human motion and of the muscles could be investigated to achieve full understanding. Being a mathematician, he stressed in probably his first publication in 1885 that a mathematical method was needed to gain exact and reliable results in human kinematics [4].

The investigation in [4] consists of three-dimensional measurements of the motion of the forearm. At three non-collinear points at the bone wooden needles were attached. The motions of the tips of the needles were recorded by drawing their projections on millimeter-paper and by recording the distance from the needle to the plane of the paper. With these measurements he became the first to make three-dimensional motion analysis of human. A mathematical description of the measured motions was derived by screw motion, for which he determined the screw-axes of each body segment. Based on the motion of the screw-axes the real motion could be interpreted accurately.

His results led to important corrections on the results from purely two-dimensional empirical analysis by others. For example in 1887 he reexamined the motion of the elbow and the hand with a 'rigorous mathematical analysis' gaining new insight at a topic that had been considered known [5].

In [6] Fischer considered the importance of measurements on the living hand as compared to a cadaver hand, because of the significant influence muscles have on the real motion capabilities in addition to the joint geometry. To measure the motions, he placed metal tubes around the finger to which the wooden needles and steel wires for measuring were attached. In 1888 he started analyzing the combined motion of multiple body segments by considering the shoulder with the humerus [7] and in 1889 the functioning of the flexor muscles of the elbow was investigated [8].

For the first time in 1889, Fischer considered the motion of the center of mass (CoM) of the complete human body [9]. As compared to statics and mechanics of the human body at a certain pose, which can be analyzed from individual body segments, he stated that for deriving the muscle forces for certain human motion knowledge is needed about the motion of the body CoM. One argument was that while each body segment has various possible motions, the body CoM always moves in a very specific manner. Fischer and Braune determined the positions of the CoM of human bodies and of parts of human bodies by using rigidly frozen cadavers. They verified their results with the results of anatomists for individual body segments and presented them as ratios or coefficients in order to account for differences of individuals. Their results remained important well into the computer era [10,11].

In 1891 advanced kinematic measurements and analysis of the knee joint were published [12] followed in 1892 by measurements of the inertia values of body segments [13].

One year after the death of Braune, Fischer published in 1893 his habilitation work on determining the muscle forces for human motion [14]. He noted that the human body had become a mechanically known object. At anatomic level the properties of each body segment were well known and at kinematic level the relative motion of the body segments had become well known too. With the rich knowledge on the functioning of individual muscles at an anatomic level, the next new step had to be the investigation of the combined muscle forces that cause human motion. He distinguished human motion by the absolute motion of the body CoM and by the relative motion of the body segments with respect to the body CoM. For the analysis of the relative motions and to derive muscle forces he invented 'The method to derive kinetic energy' ('Die Methode der Ableitung der lebendigen Kraft'), of which a part would be named later 'The method of principal vectors' [15]. With this method, Fischer investigated the muscle forces of the lower arm in 1895 [16] and he made other contributions to the statics and dynamics of muscles in 1896 [17] and 1897 [18].

From 1895 to 1904 Fischer published a series of six works on human gait (Der Gang des Menschen). The first part consists of the measurements of the motion of unloaded and loaded humans within a spatial co-ordinate system with high accuracy [19]. The results of these measurements were used for analysis in the following five parts for which Otto Fischer is said to be the first to conduct three-dimensional gait analysis [20]. In part two in 1899 the precise motion of the body CoM based on the CoM of individual body segments is investigated in relation with the external forces that apply [21]. In part three in 1900 the lower extremities are investigated [22], while part four in 1901 treats the forces and moments in the foot [23]. In both part five in 1903 [24] and part six in 1904 [25] the motion of the upper-leg is treated with which he became the first to give the exact proof that for walking the upper-leg does not solely swing forward as a passive pendulum.

In 1902 Fischer studied the influence of muscles on one another to stress, again, the importance of considering the motion of the complete human musculoskeletal system in order to investigate the functioning of the individual muscles [26]. In 1905 he extended his method of principal vectors to derive the spatial equations of motion of spatial chains [27].

In 1906 Fischer published his book 'Theoretical fundamentals for mechanics of moving bodies' ('Theoretische Grundlagen für eine Mechanik der lebenden Körper') [28]. In the first part of what would become a classical work [1] all of his achievements on the method of principal vectors are presented and developed from the very beginning. The second part summarizes his investigations on human motion to which the method had been applied. The book ends by illustrating the potentials of his method for mechanism and machine science.

Fischer published also a book on descriptions of human joints from a rather kinematical point of view instead of from the common anatomical point of view in 1907 [29]. In 1909 he published a work on spherical kinematics of Listing's law (das Listinsche Gesetz) [30] and of the humeroradial joint [31].

Most of the cited publications of Otto Fischer were published in the proceedings of the Mathematisch-Physikalischen Classe der Königlich-Sächsischen Gesellschaft der Wissenschaften in Leipzig of which he was extraordinary member since 1893 and ordinary member as of 1905. All cited literature is freely accessible on the internet in the DMG-library[2] or in the SLUB-library of Dresden [3].

3 The Method of Principal Vectors

To derive individual muscle forces for a human in motion, the motion of all body segments needs to be considered. For instance the motion of the elbow affects the forces at the shoulder or motion of the arm may affect to forces in the leg. However, calculations with the inverse dynamic model of the human at each instant are cumbersome to do by hand. Therefore Fischer developed a method that is capable of reducing the mechanics of the human mechanism to solely the element of interest, e.g. the elbow, knee, or foot. From the resulting reduced mass and inertia model the equations for the kinetic energy and the equations of motion could be derived.

The essential choice Fischer made was to investigate the motion of the body CoM independently from the relative motions of the body segments with respect to the body CoM. Since the motion of the body CoM is determined by external forces while the relative motion of body segments is determined by internal (muscle) forces, then also these can be investigated independently. The relative motion of the body segments about the body CoM is determined from the kinematics for which the dynamic model can be reduced. It is exactly this reduction step which is named *the method of principal vectors*, which is part of Fischer's complete 'method to derive kinetic energy' [14].

Fischer explained his method for the linkage of three elements shown in Fig. 1. The elements are linked with revolute pairs at $G_{1,2}$ and $G_{2,3}$ and each element has a mass m_i centered at position S_i. It is possible to define *principal points* H_i at each element. At element 1 principal point H_1 is defined as being the CoM of mass m_1 at S_1 and mass $m_2 + m_3$ at joint $G_{1,2}$. This is a projection of the mass of elements 2 and 3 at element 1. Equivalently, at element 3 principal point H_3 is defined as being the CoM of mass m_3 at S_3 and mass $m_1 + m_2$ at joint $G_{2,3}$. At element 2 principal point H_2 if defined as being the CoM of m_2 at S_2, m_1 at $G_{1,2}$, and m_3 at $G_{2,3}$.

With the principal points, lengths $H_1G_{1,2} = d_1$, $H_2G_{2,3} = d_2$, $G_{1,2}H_2 = c_2$, and $H_3G_{2,3} = c_3$, are determined which are the principal dimensions of the linkage. With

[2] http://dmglib.org/dmglib/handler?biogr=34004 (Dec. 2011).

[3] www.slub-dresden.de/sammlungen/digitale-sammlungen (Dec. 2011).

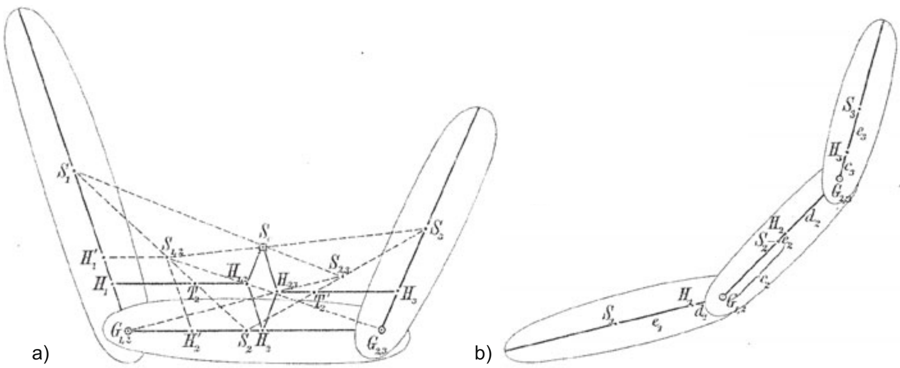

Fig. 1. Derivation of the principal points H_i that define the principal dimensions of a linkage of three elements with their CoM at S_i. The linkage CoM S_0 then is found geometrically [28].

these lengths the CoM of all three links S_0 can be geometrically found by parallelograms as indicated in Fig. 1a. Since the principal dimensions are independent of the motion of the linkage, these parallelograms trace the linkage CoM for all motion of the links. With respect to a fixed reference frame, the vectors describing the principal dimensions are named the principal vectors.

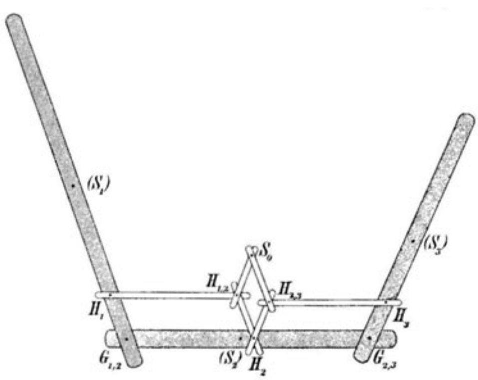

Fig. 2. Mechanism by Fischer to trace the CoM of three links at S_0 by additional links [28].

The geometric result of Fig. 1 can be seen as a linkage as shown in Fig. 2. Assuming the linkage CoM S_0 to be stationary, the position of the linkage is completely defined with the rotation of each element φ_i and its motion by their derivatives. Then for $\dot{\varphi}_2 = \dot{\varphi}_3 = 0$ element 1 rotates about H_1 while elements 2 and 3 solely translate. The inertia of this motion is the reduced inertia of the mechanism with respect to element 1. For each of the three elements this can be done with which the motion of the linkage consists of three reduced subsystems. For $\dot{\varphi}_1 = \dot{\varphi}_2 = 0$ element 3 rotates about H_3 while elements 1 and 2 solely translate and for $\dot{\varphi}_1 = \dot{\varphi}_3 = 0$ element 2 rotates about H_2 while elements 1 and 3 solely translate.

Based on these subsystems Fischer was able to formulate the kinetic energy equation of the motion of the complete linkage to depend solely on the total mass $m_0 = m_1 + m_2 + m_3$, the three reduced inertias k_i, and the principal lengths. The kinetic energy of the linkage relative to the linkage CoM then is written as

$$T_r = \frac{m_0}{2}(k_1^2\dot{\varphi}_1^2 + k_2^2\dot{\varphi}_2^2 + k_3^2\dot{\varphi}_3^2) + m_0 d_1 c_2 \cos(\varphi_1 - \varphi_2)\dot{\varphi}_1\dot{\varphi}_2 + m_0 d_1 c_3 \cos(\varphi_1 - \varphi_3)\dot{\varphi}_1\dot{\varphi}_3 + m_0 d_2 c_3 \cos(\varphi_2 - \varphi_3)\dot{\varphi}_2\dot{\varphi}_3 \quad (1)$$

Including also the kinetic energy of the motion of the linkage CoM, which depends on m_0 and on the velocity of the CoM $\dot{x}_0$ and $\dot{y}_0$, results in the equation of the kinetic energy for both relative and absolute motion of the linkage

$$T = \frac{m_0}{2}(\dot{x}_0^2 + \dot{y}_0^2 + k_1^2\dot{\varphi}_1^2 + k_2^2\dot{\varphi}_2^2 + k_3^2\dot{\varphi}_3^2) + m_0 d_1 c_2 \cos(\varphi_1 - \varphi_2)\dot{\varphi}_1\dot{\varphi}_2 + m_0 d_1 c_3 \cos(\varphi_1 - \varphi_3)\dot{\varphi}_1\dot{\varphi}_3 + m_0 d_2 c_3 \cos(\varphi_2 - \varphi_3)\dot{\varphi}_2\dot{\varphi}_3 \quad (2)$$

Deriving the equations of motion with Lagrange then results in five differential equations, two for the motion of the linkage CoM and three for the motion of the links with respect to the linkage CoM. The force components of the first two equations correspond with the external forces applied to the linkage CoM. The force components of the latter three equations correspond to the resultant moment on each link about the principal point H_i. These components are the resultants of all internal and external forces applied at the linkage that apply to an individual element from which the muscle forces can be derived.

4 Applications by Otto Fischer

Fischer applied his method to investigate the muscle activity of various parts of the human body. Figure 3 shows the method applied to the right arm. In this case the forces in the shoulder and elbow were considered with the trunk being stationary and the hand being rigidly attached to the lower arm. In Fig. 3a the position of the CoM of the arm with hand is $S_{8,10}$. Figure 3b shows the same arm but with the hand holding a mass for which the principal points are located closer to the hand.

Equivalently, Fig. 4a shows the method applied to a leg, assuming the trunk to be stationary and the foot to be rigidly attached to the lower leg. Figure 4b shows the upper leg for the investigation of the forward swing of a leg during walking. Here the motion of the complete human was considered as being reduced to the motion of the leg. The reduced system was regarded as a linkage of three elements with the lower leg with rigidly attached foot being element 1, the upper leg being element 2, and the upper body being element 3 as compared to Fig. 1.

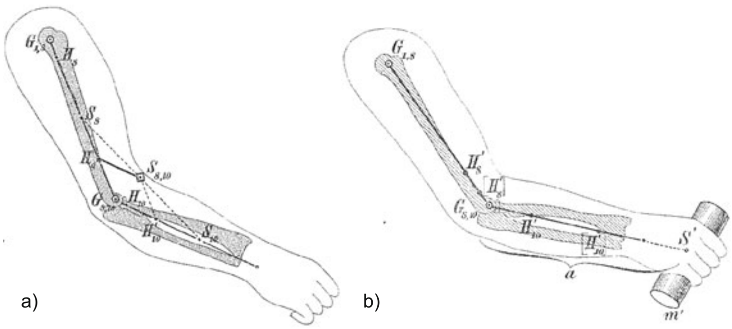

Fig. 3. Fischer's method applied to (a) the right arm, (b) the right arm holding a mass [28].

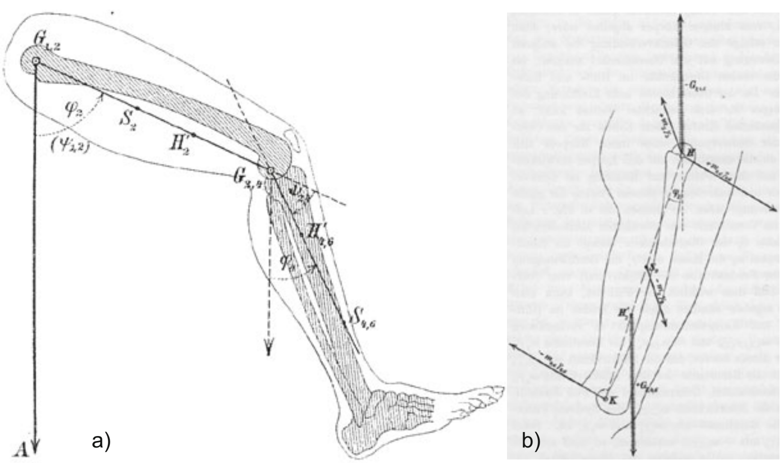

Fig. 4. Investigation of (a) forces in the leg [28] and (b) the swing phase of the leg [24,25].

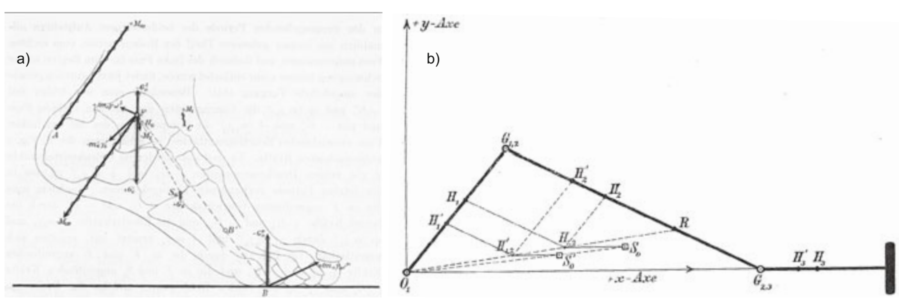

Fig. 5. Investigation of (a) the foot [23] and (b) the shaking force balancing of a crank-slider mechanism [28].

Fischer also studied the forces and moments in the foot during walking. Figure 5a shows the foot being in contact with the ground just before making a step [23]. In [28] Fischer illustrates the potential use of his method for machines and mechanisms by applying it to a crank-slider mechanism as shown in Fig 5b. He investigated the shaking forces, i.e. the resulting dynamic forces at the base, and derived the conditions for which they become zero and the mechanism is shaking force balanced. In [28] he also shows the benefit of his method for deriving the equations of motion with a pendulum used as example. From the reduced system the equations of motions are readily obtained, 'without the need of taking the indirect route by the kinetic energy and the general Lagrange differential equations of motion', with which he concludes his book.

Fischer developed his method in [27] for more complex linkages such as a linkage with six elements in series and a linkage with twenty elements. Also he showed that his method applies to spatial mechanisms for which he investigated a linkage of two elements with spatial motion.

It is likely that Fischer was going to apply his method for the investigation of spatial joints. Therefore he investigated the spherical kinematics of Listing's law [30] and of the humeroradial joint [31] of which Fig. 6 shows a mechanism model and an illustration.

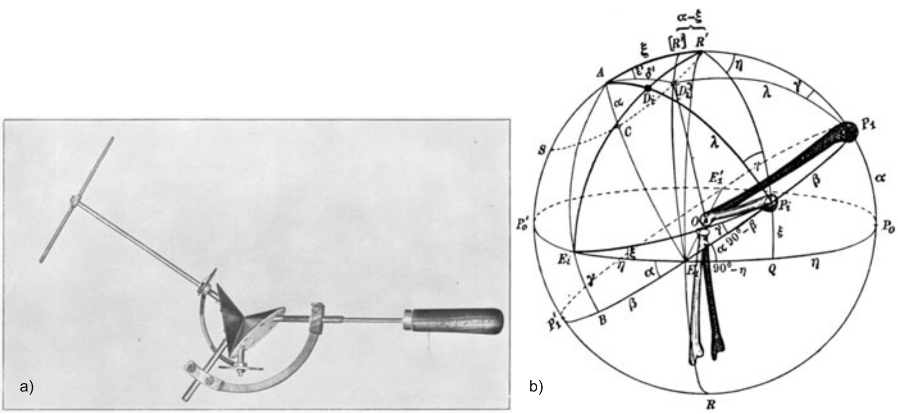

Fig. 6. (a) Spherical mechanism model of the kinematics of Listing's Law [30], (b) Study of the relative motion of the radius at the humeroradial joint [31].

5 Development and Application by Other Researchers

The method of principal vectors is described and applied by various authors. Nerreter in 1912 used the method for an extensive investigation of the shaking forces of a four-cylinder motor [34]. Wittenbauer in 1923 developed the method for investigating the motion of the linkage CoM and applied it to more complex mechanisms among which a parallel linkage [32]. He also applied the method to the linkage with three elements of Fig. 2, but with each element having an arbitrary CoM location as shown in Fig. 7a. Summaries of the method were given by, among others, Beyer in 1931 and 1960 [35,36] and Federhofer in 1932 [37].

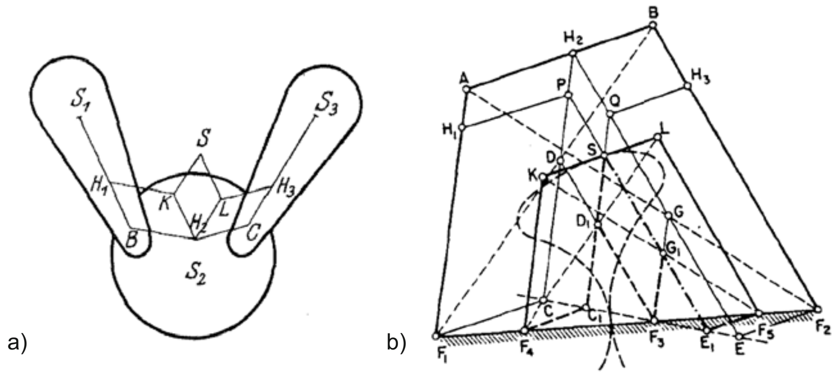

Fig. 7. (a) Application of the method by Wittenbauer for an arbitrary mass distribution of each of the three links [32]. (b) Illustration by Kreutzinger to show that the trajectory of the CoM of a 4R four-bar linkage is similar to a coupler curve of this linkage [33].

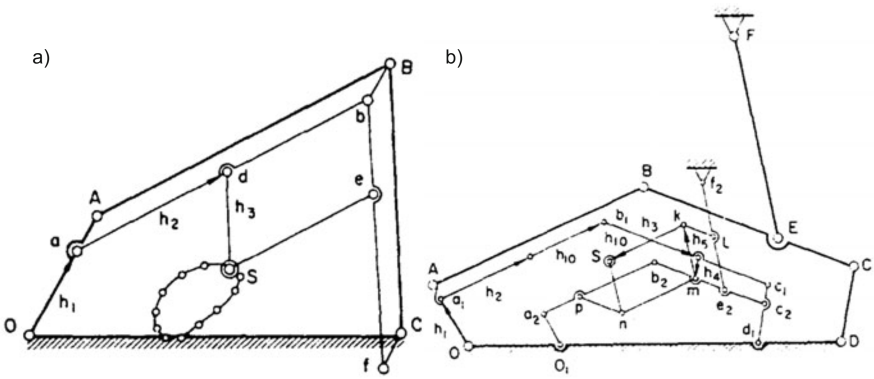

Fig. 8. Alternative geometric solutions by principal vectors to trace the CoM (a) by Artobolevskii and (b) by the *double contour transformation* on a six-bar linkage by Shchepetil'nikov [38].

Kreutzinger in 1942 applied the method of principal vectors to show that the trajectory of the CoM of a 4R four-bar linkage is a curve similar to a coupler curve of this linkage [33]. His solution is illustrated in Fig. 7b.

For the purpose of dynamic balancing, Shchepetil'nikov in 1957 extended the method of principal vectors to the method of *double contour transformation* [38]. He based his research on the findings of Artobolevskii in 1951 [39] who had proposed an alternative geometric solution to trace the CoM with principal dimensions, shown in Fig. 8a. Since for mechanisms with multiple closed chains the geometric solution by parallelograms quickly leads to excessive bulkiness, Shchepetil'nikov showed how the CoM can be traced by similar auxiliary linkages being jointed to the original linkage. One example of the application to a six-bar linkage is shown in Fig. 8b. His approach towards balancing was to have the linkage CoM move along a circular trajectory with a

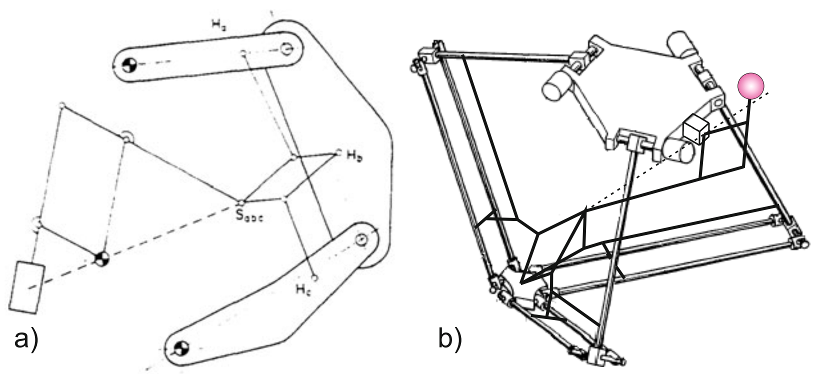

Fig. 9. (a) Hilpert's solution to balance the CoM of a 4R four-bar linkage by a pantograph with countermass [41]. (b) Delta Robot of which the CoM is made stationary by a pantograph with countermass by Van der Wijk and Herder [42].

rotation synchronous to the the driving crank. Then with a single counter-mass moving along with the driving crank the mechanism could be force balanced. By placing the counter-mass at an additional link elsewhere at the base, also the first harmonic of the shaking moment could be balanced. Having applied his method solely to linkages with mass symmetric links, he extended his method in 1975 to linkages with general mass distributions [40].

Hilpert in 1965 considers the balancing of a 4R four-bar linkage for which he uses Fischer's geometrical solution together with a pantograph with counter-mass to bring the linkage CoM at a stationary position with respect to the base as shown in Fig. 9a [41].

When the geometric solution to trace the CoM becomes a real linkage as in Fig. 9a, the masses of the auxiliary links have to be considered too. Agrawal et al. in 2001 showed by experiments that the mass of the auxiliary links of Fischer's original linkage of Fig. 2 can be included for which this linkage traces the CoM of all links together [43]. Then by having the CoM be stationary with the base results in a balanced manipulator as shown in Fig. 10a. In 2004 Agrawal and Fattah showed that also for a spatial manipulator the mass of all links can be included, of which a prototype is shown in Fig. 10b [44]. In this case the linkage CoM was statically balanced by a spring.

Van der Wijk and Herder in 2009 showed how the approach by Hilpert can be applied to balance a Delta Robot [42]. Figure 9b shows how with a spatial linkage the CoM of the moving arms and platform of the robot is derived and made stationary with the base by a pantograph with countermass.

Van der Wijk and Herder in 2010 considered Fischer's linkage of Fig. 2 with an arbitrary mass distribution of all links [45]. Since Fischer's approach on deriving the principal points and the principal dimensions then becomes cumbersome, they proposed a more fundamental approach based on linear momentum equations. Linkages with three and four elements in series were investigated of which the latter is illustrated in Fig. 11. Fig. 12a shows the mechanism model of a balanced 5-DoF chain with five arbitrary elements and mass symmetric auxiliary links which was presented.

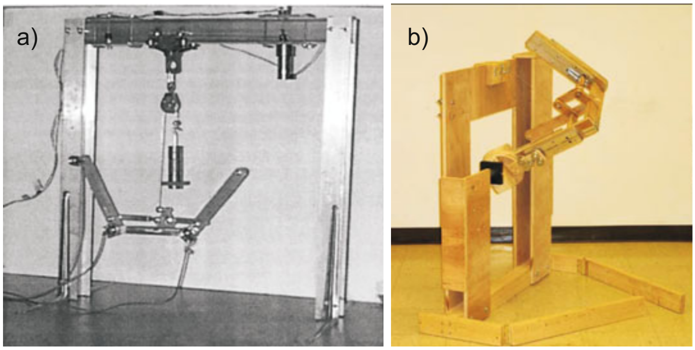

Fig. 10. Applications by (a) Agrawal et al. to include the mass of all links and [43] (b) by Agrawal and Fattah to derive a statically balanced spatial manipulator [44].

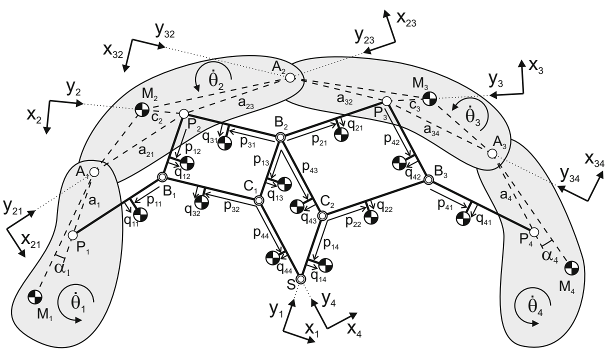

Fig. 11. A 4-DoF chain of 16 links with arbitrary CoMs of which invariant link point *S* is the CoM of the complete mechanism [45,46].

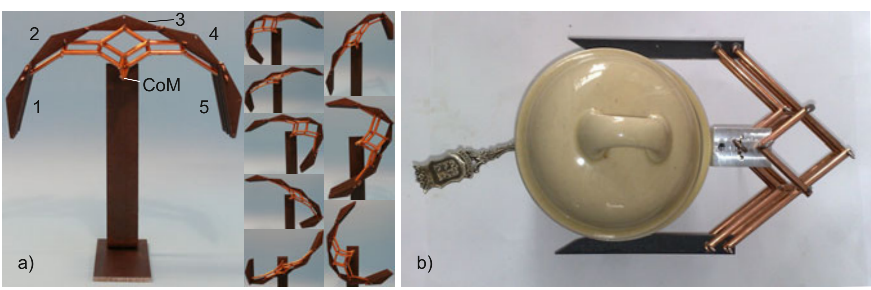

Fig. 12. (a) Mechanism model of a balanced 5-DoF chain [45] and (b) Prototype of a 2-DoF dynamically balanced grasper mechanism [46] by Van der Wijk and Herder.

In 2011 Van der Wijk and Herder extended their approach to parallel chains of links with all having an arbitrary mass distribution [46]. They also showed how Fischer's geometric solution of Fig. 2 can be found from a union of three pantographs which resulted in an explanation of the principal points from another perspective. With the focus on the design of *inherently balanced mechanisms*, mechanisms of which the CoM of all elements is at an invariant point at one of the links, low mass and low inertia balanced mechanisms were aimed at. As a result they presented the grasper mechanism of Fig. 12b which has 2-DoF grasping motion with complete dynamic balance.

6 Conclusion

An overview of the work of Otto Fischer was presented showing his contributions to three-dimensional human gait analysis and in particular the investigation of muscle forces for human motion. His method of principal vectors was summarized and some of Fischer's applications of the method were shown. The historical development of the method for mechanism and machine science was investigated, showing a clear focus on the motion of the center of mass of mechanisms for the purpose of static and dynamic balancing. Mechanisms based on Fischer's solution have led to inherently balanced manipulators. Surprisingly enough, research on shaking moment balancing by Fischer's method has not been found, although he invented his method for calculating the reduced inertia moments within the system.

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