Application of the Coupled Mode Theory to the problem of coupling between bent and straight waveguides

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The coupling between high finesse (up to 140) cylindrical microresonators and straight waveguides is analysed using the Coupled Mode Theory and numerically calculated 2D fields of the unperturbed bend and straight waveguides. The calculated coupling constants are in good agreement with both experiments and other numerical methods. Extension of our approach to more advanced geometries, like coupling between bends, is straightforward.

Introduction

Several devices based in optical ring waveguide resonators have been proposed and build in the past [1], [2], [3], [4], [5]. The ring waveguide geometry potentially allows the realisation of the small size very high finesse cavities, necessary for the construction of highly integrated optical devices.

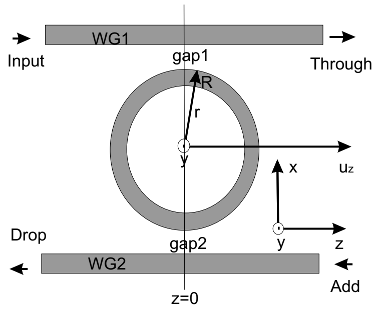

Figure 1: Schematic of a straight waveguide coupled micro ring resonator

Figure 1 shows a typical arrangement; in it, a ring resonator is located at distances $gap1$ and $gap2$ of the straight waveguides $wg1$ and $wg2$. Usually the ring resonator is optimised trying to minimise the losses, while the gaps $(gap1, gap2)$ are chosen according to the coupling coefficients required for the specifications of the device in mind [6].
As a design example figure 2 shows the power in the cylindrical cavity in function of the coupling coefficient with one of the waveguide. For applications in switching

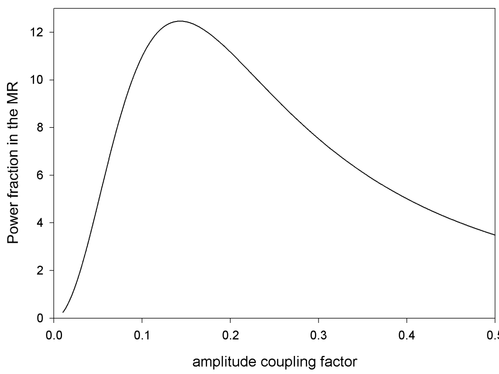

Figure 2: Power inside the Cylindrical microresonator as a function of the coupling coefficients

those coefficients are chosen trying to maximise the intensity of the field in the cavity, i.e., asymmetrical. For applications of filtering they are chosen trying to optimise the power in the drop port (at resonance), given as a result symmetrical arrangements.

Application of numerical modes to the design of ring resonator devices

The problem of the calculation of the modes in bent waveguides has received special interest in the past [7], [8], and several precise approximation models have emerged; as a result, a simulation tool [9] able to numerically calculate the modal profiles of layered bent systems have been recently released. The availability of tools to calculate the modal profiles allows the use of approximations like the Coupled Mode Theory, to precisely estimate the coupling matrix of the ring-straight waveguide system.

In one of the approaches to the last problem, Little *et al* [10] used numerical modes to calculate the coupling coefficients in the point of closest neighbourhood between the waveguide and the ring, and an analytical approximation based on the assumption of exponential decay of the modal profiles in the cladding, to estimate the coupling properties of a ring-waveguide system.

The approach suggested in the present article is based on similar ideas, but instead of using an analytical approximation to calculate the spatial evolution of the coupling coefficients, we use the properties of numerical modes on an uniform grid defined in the plane $z = 0$, and the definition of the angle of propagation of the θ-invariant modes[7]:

$$\theta := \arcsin \frac{z}{R} \tag{1}$$

To find an approximation to the spatial evolution of the numerical modes of the ring resonator:

$$\left\{ \begin{array}{l} \bar{\mathbf{E}}_{ring}\left(r,\theta,y\right) \\ \bar{\mathbf{H}}_{ring}\left(r,\theta,y\right) \end{array} \right\}_{z=zn} \approx \sum_{i=0}^{\infty} \left\{ \begin{array}{l} \mathbf{e_i}\left(x_{j+n},y\right) \\ \mathbf{h_i}\left(x_{j+n},y\right) \end{array} \right\} e^{-i\left(\gamma_i \arcsin \frac{zn}{R}\right)} \tag{2}$$

With:

$$\begin{aligned} z_n &:= \pm\sqrt{R^2 - \left(R - n\Delta x\right)^2} \\ &= \pm\sqrt{2Rn\Delta x - n^2\Delta x^2} \end{aligned} \tag{3}$$

Where $\{\mathbf{e_i}, \mathbf{h_i}\}$ are the components of the electric and magnetic field of the mode with angular propagation constant $\gamma_i = \frac{2\pi}{\lambda} n_i R$; $x_l = l\Delta x$ with Δx the step of the uniform grid. $z_n | n = \{\cdots, -2, -1, 0, 1, 2, \cdots\}$ are planes of evaluation defined in such way that a point of the grid is always placed on the radius of curvature, this is necessary because equation (1) implies the angle of propagation must can be univocally determined.
The modal profiles and evaluation surfaces determined through equations 2 and 3 can be used to calculate the coupling coefficients; using the theory e.g. by Vasallo [12] and a numerical integration of the coupling mode equations.

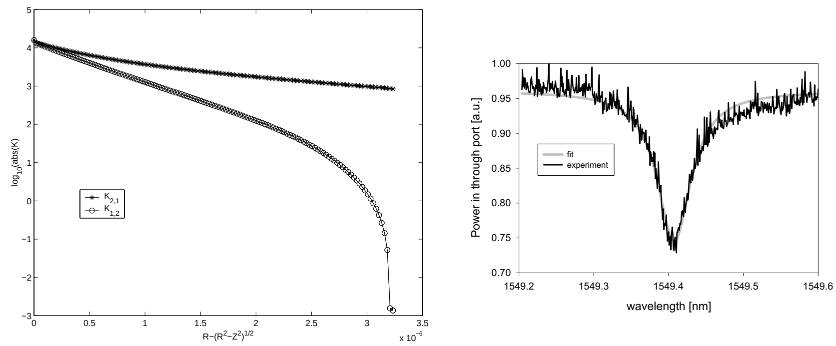

Figure 3: A: Logarithm of the coupling coefficients Vs. the distance to the point of closest approach calculated using the present mode. B: Measured spectra on the cavity of finesse ≈ 120 described in [13].

Figure 3A shows a calculation of the coupling coefficients using the present theory for a symmetrical ridge waveguide-ring system (Air cladding, Si_3N_4 guiding layer, SiO_2 substrate), with geometrical parameters: $R = 25\mu m$, waveguide width=1 μm, ring width=2.5μm.
The linearly decaying region of the curves of the left side corresponds to the range of distances where the modal profiles in the cladding can be approximated by exponentials. The sudden decay of the curve for the perturbation of the mode of the

straight waveguide by the ring is explained by the finite size of the numerical window used to calculate the modal profiles. The coupling coefficients were calculated using numerical integration of the coupled mode equations after the present theory, and were compared against BPM integration, Table 1 shows the values obtained, as well as the measured one.

	Coupling coefficient
Experimental value	5.2 %
BPM	7.9 %
Present approach	10 %

Table 1: Comparison of the coupling coefficients of the ring-waveguide system. The experimental values are obtained form a detailed fitting procedure [13]

We found that the error properties of the present implementation can be significatively improved, if the modal profiles returned for the mode solver [9], are given in an uniform grid, saving an interpolation step.

Conclusions

The use of numerical modes to evaluate the coupling matrix of ring-waveguide systems is not new, but the previous approaches relied in analytical approximations to the evolution of the coupling coefficients. We have succeeded in extending the application of fully numerical modes to study the dynamics of coupling between the mentioned structures.
The numerical experiments performed show the non uniform grid of the bend solvers, and the finite size of their numerical window are key limiting factors of the applicability of this model.
Future extension to the study of coupling between non concentric ring resonators is straight forward.

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