Propagation of Discharge Uncertainty in A Flood Damage Model for the Meuse River

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Abstract. Uncertainty analysis plays an important role in the decision- making process. It can give decision makers a complete idea of how different measures will affect the whole river system. Thus it helps decision makers to make a better choice among measures in a more systematic manner. In case of flood damage reduction projects, uncertainty analysis helps to evaluate the main decision criterion – expected annual damage. The aim of this paper is to investigate the propagation of discharge uncertainty, which is one of the main uncertainty sources in a damage model, into expected annual damage. The discharge uncertainty considered here includes model uncertainty (choice of different probability distributions) and sampling errors due to finite gauge record lengths. The calculated uncertainty in the discharge varies between 17 percent for a return period of 5 year and 30 percent for a return period of 1250 year. A first order method is used here to explore the role of discharge uncertainty in the expected annual damage model. The results from the damage model indicate that both model uncertainty and sampling errors are important, with the latter being somewhat more important. The Log-Pearson Type 3 gives a much smaller uncertainty range of the expected annual damage than the other three distribution models used. The uncertainty is aggravated when propagated into the damage results. The uncertainty in the damage reduces a great amount when the sample size increases to n=80. The results derived from the first order method in fact give two bounds of uncertainty, which is an overestimate in this case.

Key words. Flood frequency analysis; expected annual damage; first order analysis; uncertainty; Meuse River

1. Introduction

Flood damage estimation is an important part in the development of a decision support system for river basin management. Recently, in the Dutch part of the Meuse River in Europe, the serious 1993 and 1995 floods stimulated the start of a new project “De Maaswerken”, which mainly aims to alleviate the effects of floods. One of the main decision criteria in the evaluation of this project is the expected annual damage. In order to have a complete idea of the effects of the different measures, it is necessary to investigate the uncertainty sources in the damage model and how they propagate into the final damage results.

In general, there are several sources of uncertainty in a model. Important sources include data uncertainty, model uncertainty and parameter uncertainty. For more detailed information, see e.g. the taxonomy of uncertainty of Suter et al. (1987). The uncertainties in a damage model originate from e.g. the river discharge, river cross-section data, schematization of rivers, spatial resolution and damage functions. For illustration purposes, this paper only considers the effect of river discharge uncertainties on the damage model applied to the Dutch Meuse River. The effects of other uncertainties are described in other papers, for example the effect of spatial resolutions on flood damage is explored by Xu et al. (2002).

The uncertainties in the river discharge consist of uncertainties due to the choice of different probability distributions, which are used to describe flood frequency, and parameter uncertainties caused by sampling errors due to finite gauge record lengths. The availability of data is an important aspect in flood frequency analysis. The estimation of the exceedance probability of floods is an extrapolation based on limited available data. From a statistical point of view, the larger the available data set, the more accurate the estimates of exceedance probabilities of floods will probably be.

The uncertainties in the discharge have already been explored by Wood and Rodriguez-Iturbe (1975), Stedinger (1983), Al-Futaisi and Stedinger (1999). They proposed different methods to take into account the uncertainty in the river discharge. However, the propagation of discharge uncertainty into damage models has not been often investigated. Beard (1997) and Stedinger (1997) used Monte Carlo Methods to investigate the role of uncertainty in the expected annual damage from which the results are difficult to interpret. However, they mainly emphasized the advantages and disadvantages of different parameter estimators while they ignored the effects of different probability models.

The objective of this paper is therefore to investigate the effects of both probability distributions and sample size on discharge and damage results. This is done by a first order method. Section 2 describes the methods used including the first order method. Section 3 gives the application of the methods to the Dutch Meuse River. Section 4 shows the uncertainty effects on the model results and the conclusions are given in

Section 5. This paper gives a complete idea of how the uncertainty will affect the decision criteria in evaluating the different measures and thus support decision makers to make sound decisions.

2. Flood damage and uncertainty

2.1. Flood frequency analysis

Flood frequency analysis is an essential part when the expected annual damage is calculated. The primary objective of flood frequency analysis is to relate the magnitude of extreme events to their frequency of occurrence through the use of probability distributions (Chow et al.1988). Flood frequency analysis is often used to calculate the expected annual damage, design flows for dams, bridges, culverts, flood control structures, flood plain planning etc. The basic assumptions of flood frequency analysis are:

1) Historical events represent future events
2) Events are independent
3) Distributions fit entire data sets and future data
4) Space and time independency

In fact this is rather a simplification, for example when climate change and land use change play a role in flood frequency analysis. In this paper, our key point is to explore the role of uncertainty in the estimation of flood flows for different frequencies and in the estimation of the expected annual damage. Therefore, the main features in flood frequency analysis are described, namely extreme value distributions, parameter estimations, estimations of T-year event discharges and confidence intervals for T-year event discharges in subsection 2.1.1 through subsection 2.2.4.

2.1.1. Extreme value distributions

In flood frequency analysis, an important aspect is to choose a certain distribution that will be used to describe flood flows. The most often used are the Lognormal distribution (LN), Gumbel Extreme Value distribution (GEV), Pearson Type 3 distribution (P3) and Log Pearson Type 3 distribution (LP3) (Chow et al. 1988). The choice of the distribution is one of the main sources of uncertainty because it is unknown which of the above distributions is the true distribution for flood flows. This is important because the sample events available are usually for relatively low return periods (i.e. around the center of the probability distribution) while the events for which estimations are required are associated with large return periods (i.e. in the tail of the distribution) (Kite 1977). Many probability distributions have very similar shapes in their centers, but differ widely in their tails. It is thus possible to fit several distributions to the sample data and end up with several different estimates of the T-year event discharge. Some goodness-of-fit tests like the Chi-squared test or the Kolmogorov-Smirnov test can be used in this case. However, this does not solve the basic problem of different tails (Kite 1977). In this paper, as mentioned four different probability distributions are investigated and compared.

2.1.2. Parameter estimations using method of moments

When fitting a probability distribution to data sets, an estimation of the parameters of that distribution is needed. Several approaches are available for estimating the parameters of a distribution, such as the method of moments, maximum likelihood estimators, expected probability estimators and the Bayesian estimation procedure. The advantages and disadvantages of these methods have been discussed by Al-Futaisi and Stedinger (1999) and Stedinger (1997). The choice of the parameter estimation method is not our point in this paper. Here the traditional method of moments is used.

The method of moments states that the *k*-th sample moment about the origin is an unbiased estimator for the *k*-th population moment. Thus in order to estimate the parameters of a proposed probability distribution, we assume that the first and second population moments about the origin equal to the first and second sample moments. Based on this, we have:

$$\mu_1(0) = M_1(0) \qquad\qquad \mu_2(0) = M_2(0) \tag{1}$$

Here $\mu_1(0)$ and $\mu_2(0)$ are the first and second population moments about the origin respectively and they are functions of the population mean μ and standard deviation $\sigma, M_1(0)$ and $M_2(0)$ are the first and second sample moments respectively (Shahin et al. 1993).

2.1.3. Estimation of T-year event discharges

After the estimation of the distribution parameters, the T-year event discharges need to be calculated. A T-year event discharge is a discharge for a specific exceedance probability. Chow et al. (1988) expressed the T-year event discharge as:

$$X_T = \mu + K_T \sigma \quad (2)$$

Where K_T is a frequency factor that is a function of the return period and the distribution parameters. Shahin et al. (1993) has given the detailed information about the calculations of K_T. The event magnitude X_T can be estimated as soon as the mean μ and standard deviation σ of the underlying probability distribution are estimated from Equation (1).

2.1.4. Confidence intervals of T-year event discharges

The common way to express the uncertainty in the T-year event discharges is to estimate the confidence intervals. According to Shahin et al. (1993), the confidence intervals for the T-year event discharge X_T are:

$$X_T \pm z S_T \quad (3)$$

If the probability distribution is lognormal, z is taken from the table of the standard normal distribution assuming a certain confidence level (e.g. 95%). For GEV, P3 and LP3 distributions, z can be taken from the Student's t-table.

In Equation (3), the standard error S_T is a measure of the variability of the resulting T-year event discharges. Equations of S_T for different distributions are given by Shahin et al. (1993).

2.2. Expected Annual Damage (EAD)

The U.S. Army Corps of Engineers' framework of EAD estimation will be used and is explained below (National Research Council 2000). This approach also applies to the development of a decision support system that can be used as a tool to compare different measures, including flood damage reduction projects. According to the

National Research Council (2000), the expected annual damage is the average damage determined from floods of different annual exceedance probabilities over a long period. The mathematical equation of *EAD*, according to the National Research Council (2000), is:

$$EAD = \int_0^1 D(F)dF \tag{4}$$

Here $F = F(X \geqslant X_T)$ is the probability that the discharge X_T is equaled or exceeded in any given year and is the reciprocal of the return period T. *D(F)* is the damage for a flood with annual exceedance probability *F*.

The procedure to estimate EAD is well illustrated in figure 1. In this figure, the solid line is for the current situation without measures and the dashed line is for the improved state after measure implementation. The EAD by definition is the area below the exceedance probability -damage relationship curve in (4). The shaded area in (4) represents the annual possible savings in damages after the completion of the measures.

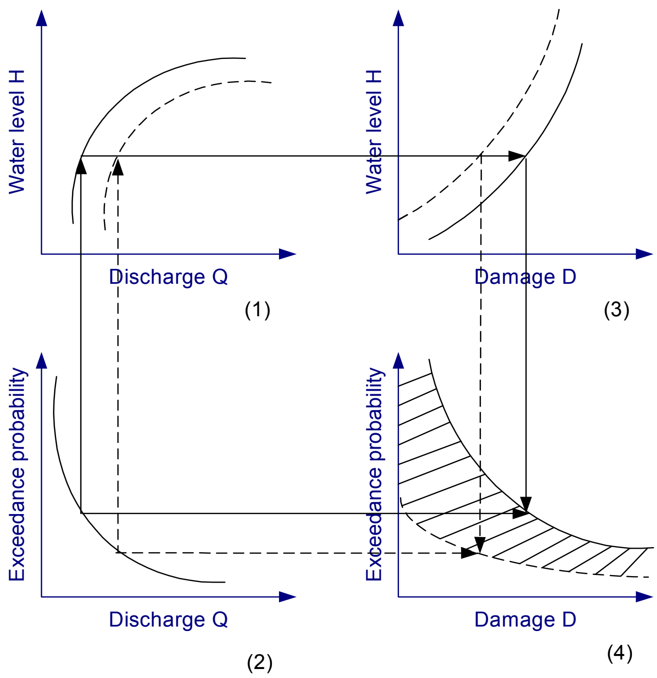

Figure 1 The procedure to estimate EAD (based on Shaw 1994)

The EAD model used in this paper is a modified version of the INUNDA model which has been used to calculate the flood damage caused by the 1993 and 1995 floods in the Dutch Meuse River (De Blois 2000). The main difference between the EAD model and the INUNDA model is that the former uses less detailed damage functions. This is appropriate for the current research purpose.

2.3. Propagation of uncertainty

The Monte Carlo method is the often-used approach to analyze the uncertainty in model inputs and parameters. To simplify the procedure but still have a good idea of the order of magnitude of the uncertainties, a first order method is used here (See e.g. Bevington and Robinson 1992).

Assume that x is a function of variables u, v, … etc. Here u and v are inputs and parameters in a specified model.

$$x = f(u,v,...) \tag{5}$$

Based on Taylor series expansion, the approximation for the standard deviation σ_x of x becomes

$$\sigma_x^2 \approx \sigma_u^2 \left(\frac{\partial x}{\partial u}\right)^2 + \sigma_v^2 \left(\frac{\partial x}{\partial v}\right)^2 + 2\sigma_{uv}^2 \left(\frac{\partial x}{\partial u}\right)\left(\frac{\partial x}{\partial v}\right) + \cdots \tag{6}$$

Here σ_u^2 is the variance of u, σ_v^2 is the variance of v, and σ_{uv}^2 is the covariance between the variables u and v.

If the fluctuations in u and v, … are not correlated, the higher order terms can be neglected. Then the equation (6) reduces to

$$\sigma_x^2 \approx \sigma_u^2 \left(\frac{\partial x}{\partial u}\right)^2 + \sigma_v^2 \left(\frac{\partial x}{\partial v}\right)^2 + \ldots \tag{7}$$

In this paper, Equation (7) is used to investigate the effects of uncertainties in T-year event discharges on the damage results. Equation (7) is based on two assumptions, namely independence among variables and model linearity. This equation is used to estimate the uncertainty due to parameter uncertainties caused by sampling errors. Therefore, u and v in Equation (7) represent the distribution parameters and x in equation (7) is the EAD. Additionally, the model uncertainty is investigated by comparing different types of distribution models in flood frequency analysis when affecting the damage calculation.

3. Case study

3.1. The Meuse River

The Meuse River has a total length of about 900 km. The basin covers an area of about 33,000 km^2. After passing through France, Luxembourg and Belgium, the river enters the Netherlands at Eijsden, south of Maastricht (see figure 2). The Meuse River can be subdivided into three major zones: the Lotharingian Meuse, the Ardennes Meuse and the lower reaches of the Meuse.

The Meuse River is fed by rainfall all year round. Large seasonal variations in river discharge can be observed (figure 3). The average discharge of the river at Borgharen, which is located upstream of the Dutch Meuse River, is about 200 m^3/s (Booij 2002). Most high discharges occur in winter, when evaporation levels are lowest. For example, the discharges during the 1993 and 1995 floods were about 3050 m^3/s and 2750 m^3/s respectively at Borgharen.

The Dutch part of the Meuse River is selected here as the case study. The sub-sequent Dutch sections of the Meuse include the Grensmaas, the Zandmaas and the Getijdemaas. The expected annual damage is estimated for the province of Limburg, which includes two sections of the Grensmaas and the Zandmaas. The approximate coverage of the study area is indicated in figure 2. The total length of this area is about 175 km.

Figure 4 shows a series of annual maximum discharges from year 1911 to 1997 at Borgharen. The average annual maximum discharge is about 1450 m^3/s.

3.2. Data used

Discharge data, elevation, land use, maximum economic values and damage functions for different land use types and QH curves are used in this study. Daily discharge data at Borgharen are provided by Rijkswaterstaat for 1911-1997. The elevation and land use data are the same as those used in the INUNDA model (De Blois 2000). The spatial resolution of the elevation and land use data is 150 meter. The QH curves are available in 22 gauging stations along the Dutch Meuse River. The maximum economic values and damage functions for different land use types have been obtained from Rijkswaterstaat (2002). There are 8 types of land use, namely households, industry, construction, trade, agriculture, greenhouse, institution and

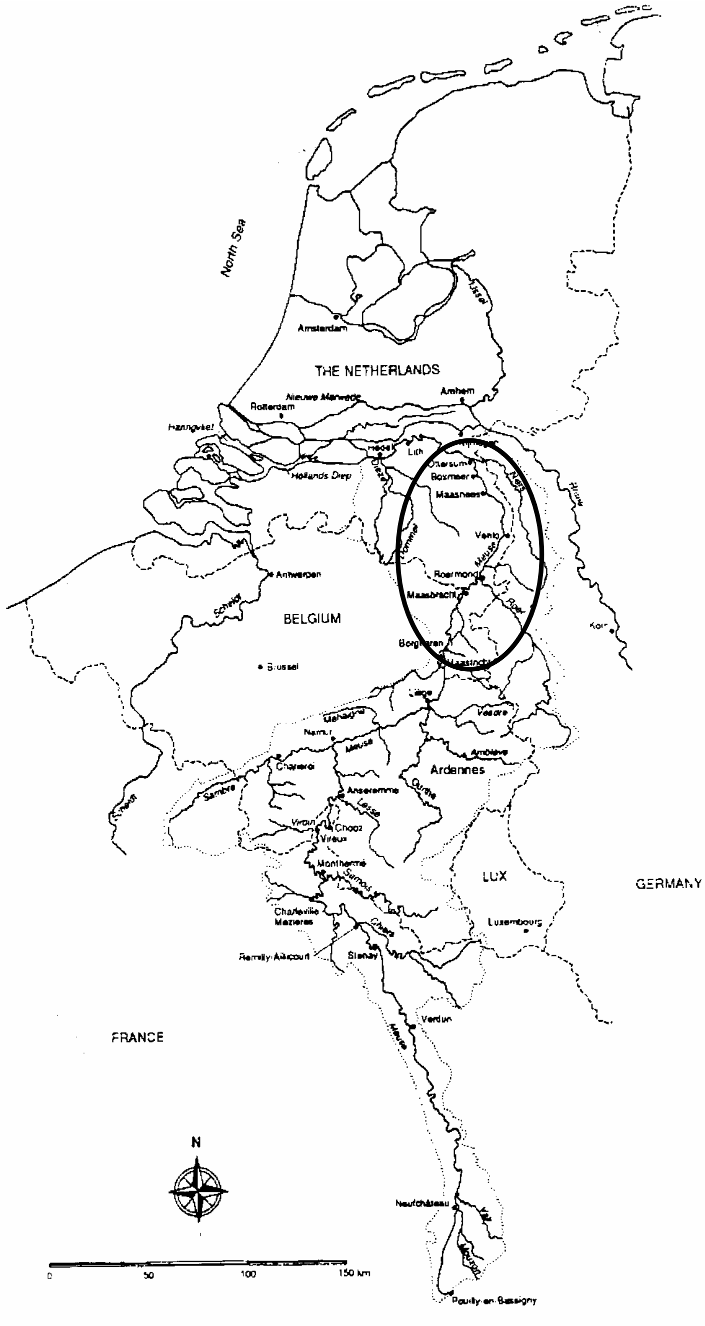

Figure 2 Catchment area of the River Meuse (Berger 1992)

others. The land use types from Rijkswaterstaat have been somewhat adjusted according to the land use types used in the INUNDA model for consistency. The damage functions are simple relations between inundation and damage for different land use types. They are either step functions or linear functions. As an example, a damage function for households is given in figure 5. The damage factor in this figure is the damage expressed as a fraction of the maximum economic value for a certain inundation level.

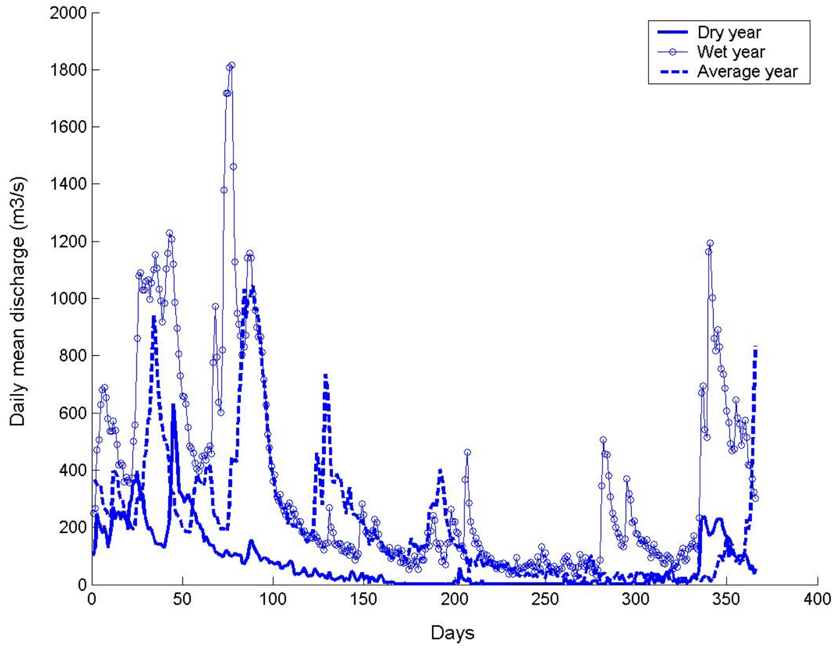

Figure 3 Daily average discharges at Borgharen

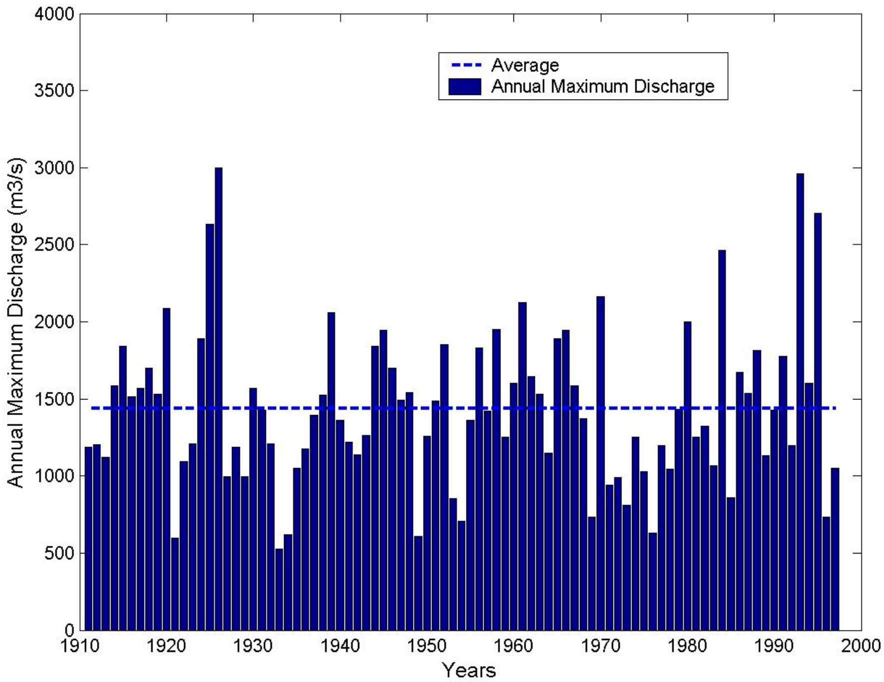

Figure 4 Annual maximum discharges at Borgharen

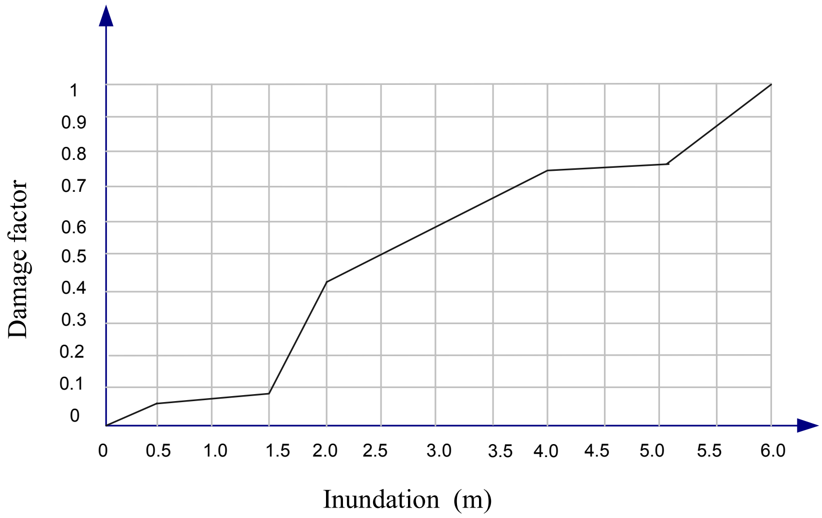

Figure 5 Example of a damage function for households

4. Model results and discussion

4.1. Uncertainty analysis of T-year event discharges

As mentioned in Section 2.1.1, the main difference among distributions can be found in their tails. Figure 6 shows the exceedance probability — discharge relationships for the four distributions considered here. It is clearly shown in this figure that for a small probability, the P3 model and the LP3 model resulted in much smaller discharges than the other two models.

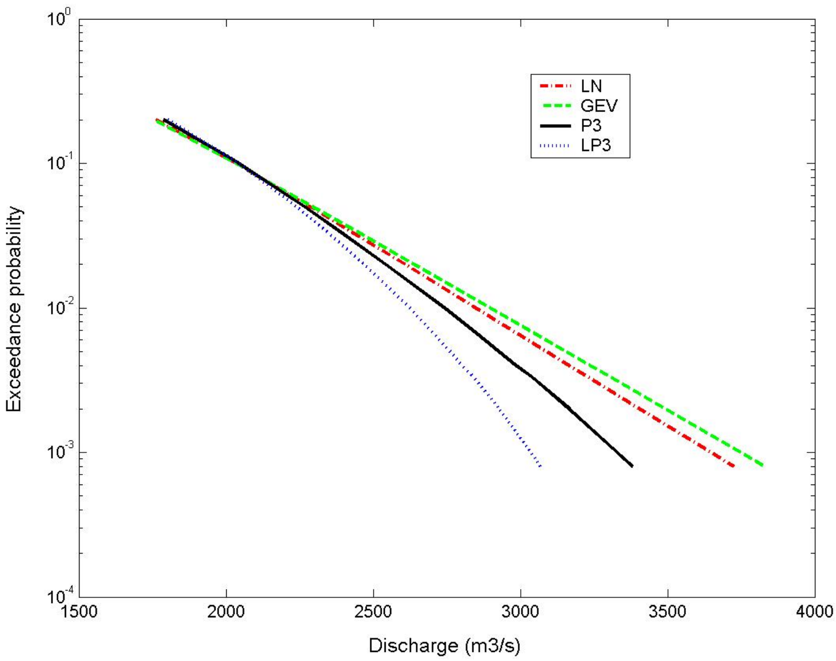

Figure 6 Discharges vs. exceedance probability for the four distribution models

4.1.1. Effects of different distribution models

First the results are given under the assumption that the flood frequency obeys the LN model. Annual maximum discharge data for 1911-1940 (sample size *n* = 30) are used. The relationship between the discharges and return period and the related uncertainty according to the LN model is shown in figure 7 given a confidence level of 95%.

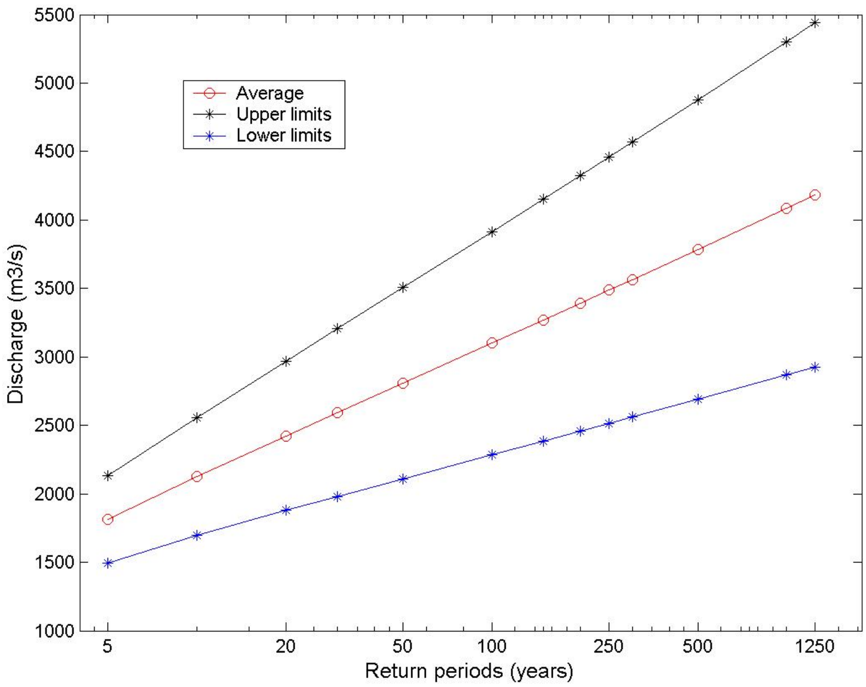

Figure 7 Return periods vs. T-year event discharges

Figure 7 shows that the uncertainty due to the natural variability can be quite large. The larger the return period, the larger the uncertainty for a T-year event discharge. Let's take as an example the 5-year return period event. The average discharge here is about 1820 m^3/s with uncertainty bounds from 1500 m^3/s and 2130 m^3/s, which is approximately 17% uncertainty involved. For the 1250-year return period, the average discharge is 4180 m^3/s with an uncertainty range around 30%. Note that the discharge for the 1250-year event calculated here (4180 m^3/s) is much higher than the value given by Rijkswaterstaat (2000) of about 3800 m^3/s because of the effect of sample size (see Section 4.1.2). For the other three distribution models (GEV, P3 and LP3), similar results can be derived.

Figure 8 shows the uncertainty for the different distributions for a sample size *n*= 30 and return periods of 50 and 1250 years. This figure shows that the average discharges for each distribution model are more or less comparable except for the LP3 model that results in smaller values. The uncertainty is different for the four distribution models. The averages and uncertainty involved in the LN model and the GEV model almost overlap while the P3 model results in the largest amount of uncertainty. Note that the results given here are without goodness-of-fit tests due to the reason that these tests still cannot solve the problem of different tails as introduced in section 2.1.1. Figure 8 also indicates that for larger return periods, the uncertainty of T-year event discharges increases due to extrapolation.

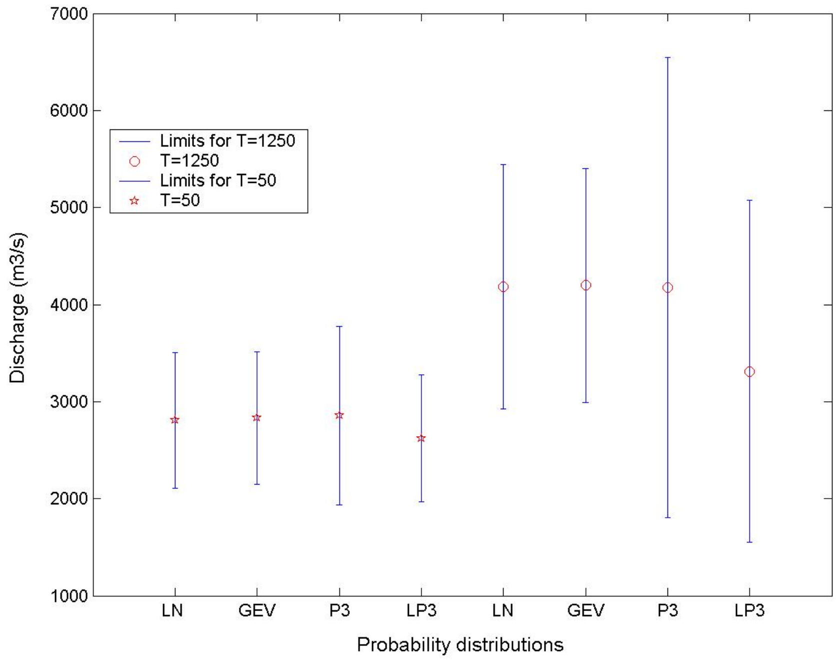

Figure 8 95 % confidence intervals for 50- and 1250- year event discharges for four distribution models

4.1.2. Effects of sample size

The relationship between discharges and return periods for different sample sizes is shown in figure 9. This figure illustrates the average discharges for different sample sizes for the LN model. The samples are taken from the same series and start in 1911. Figure 9 shows that, for small return periods, the differences between T-year event discharges for different sample sizes are small, while for big return periods the differences are much larger. For example, for n=30 and n=80, the average discharges for the 5 year return period are 1810 m^3/s and 1760 m^3/s, while for the 1250 year return period the averages are 4180 m^3/s and 3720 m^3/s respectively. Here smaller values of T-year event discharges for larger sample sizes are due to the reduced uncertainty in discharges associated with larger sample sizes, which changes the parameter values of distribution models. This characteristic depends more on data sets.

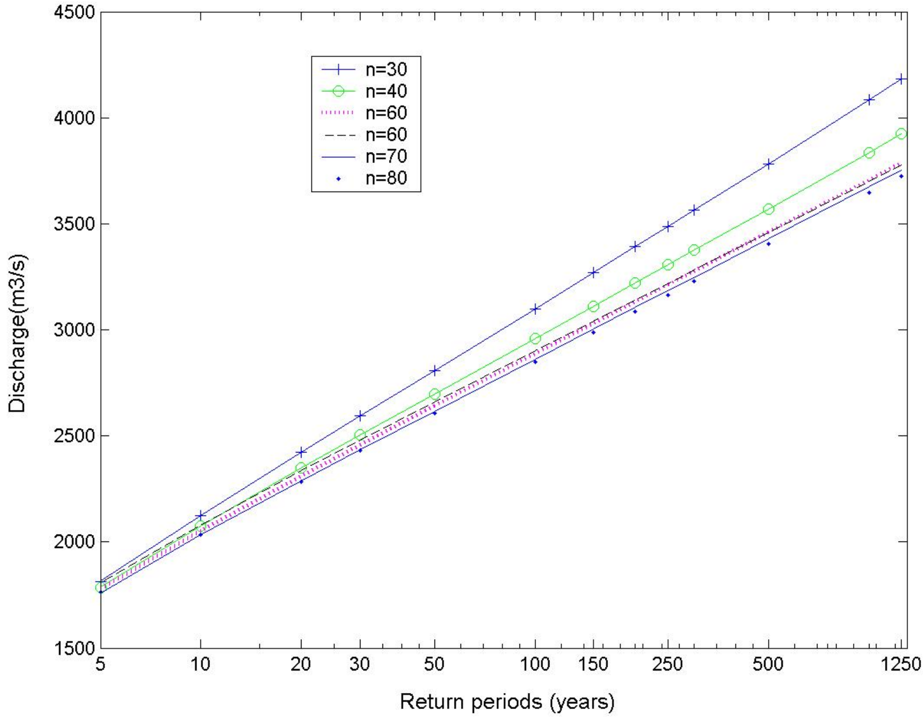

Figure 9 Discharges vs. return periods for different sample sizes

4.1.3 Combined effects of sample size and model choice

Figure 10 gives the 1250-year event discharge for three sample sizes (*n*=30, 50 and 80) for the four distribution models. This figure shows when the sample size becomes bigger, the uncertainty bounds become smaller as well as the averages. For example for a sample size *n*=80,for the LN model and the GEV model, the average values are 3720 m^3/s and 3830 m^3/s respectively, while for P3 and LP3 the values are much smaller.

4.2. Uncertainty in damage

The uncertainty described in Section 4.1 is then propagated into the damage results. Here the first order method introduced in Section 2.3 is used to investigate the effect of the uncertainty on flood damage. Because the uncertainty considered here is mainly caused by the same natural variability in the discharge, similar conclusions can be derived as those in Section 4.1. Figure 11 shows the relationship between the damage

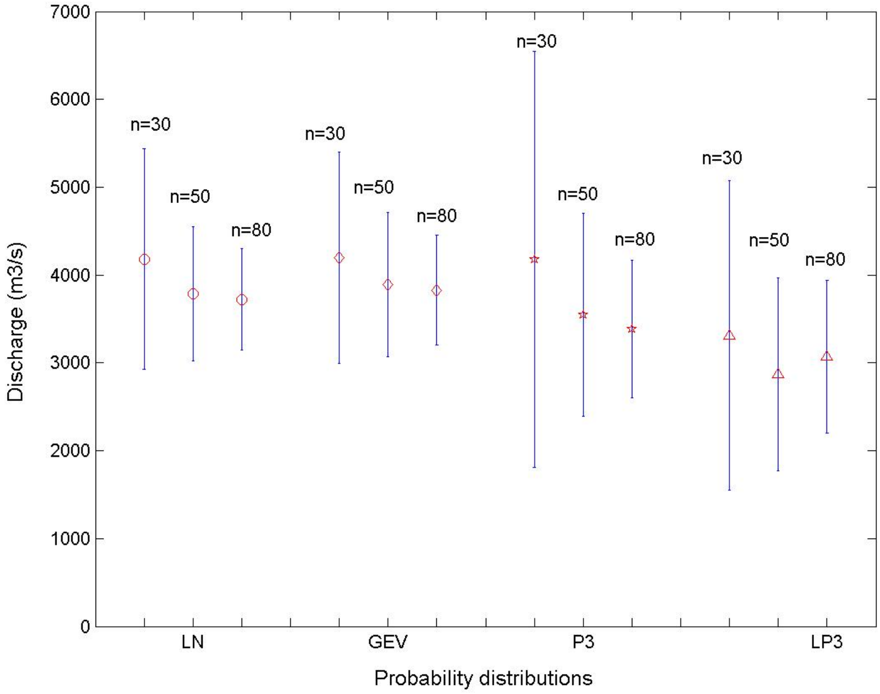

Figure 10 95 % confidence intervals for 1250- year event discharge for different sample sizes and probability distributions

and return periods for the LN model for sample sizes n=30 (left) and n=80 (right). This figure tells that the uncertainty in damage could be two times as much as the uncertainty in the T-year event discharges. That means the uncertainty has been aggravated when propagated into damage results. This is because of the non-linearity in the damage model. The results here indicate that if reducing the large uncertainty in flood damage due to extrapolation for large return periods are required, for example in case of helping decision makers distinguish the different measures, more data are necessary (increasing the sample size).

Figure 12 gives four graphs describing the relationship between the EAD and the sample size for different distribution models. With the increase of sample size, there is a trend that the values of EAD decrease and so does the uncertainty. From figure 12 it is shown that the results from the LN model and the GEV model are comparable. There is more uncertainty involved in the results based on the P3 model and thus for this model the largest reduction of the uncertainty can be observed when the sample size increases. The EAD based on the LP3 model is smaller than the results based on the other three models.

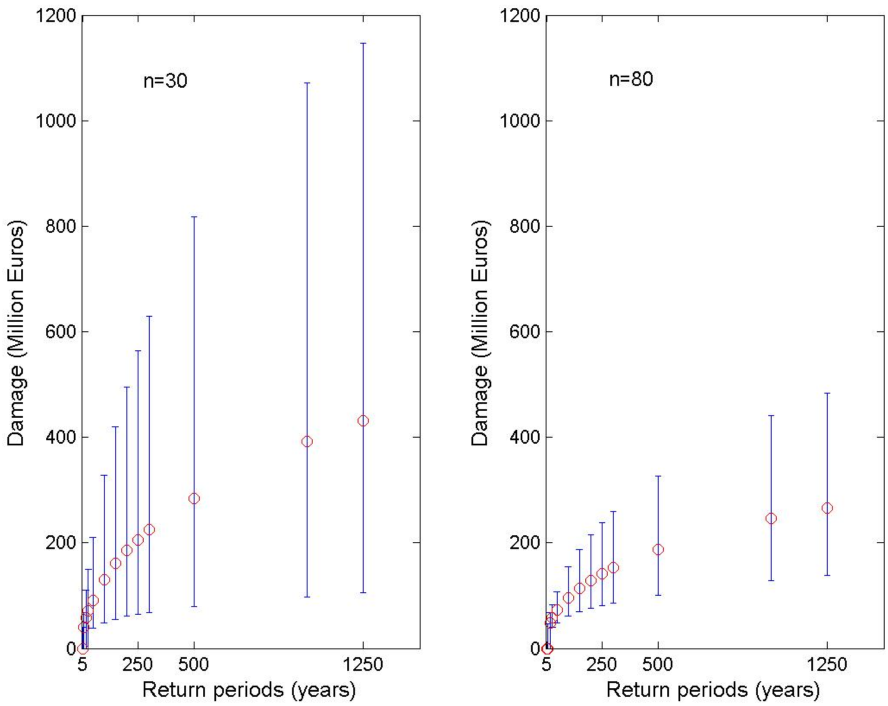

Figure 11 Damage vs. return periods for 95 % confidence intervals for *n*=30 (left) and *n*=80 (right)

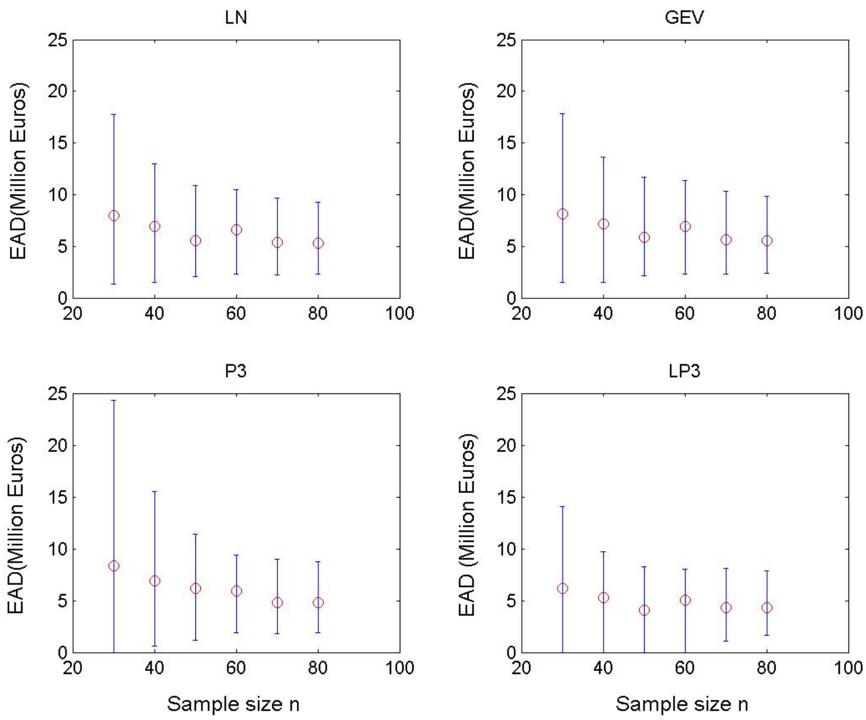

Figure 12 The effect of different distribution models and sample sizes on the EAD

If we change the sample size from n=30 to n=80, the average values reduce 33% for the LN model, 32 % for the GEV model, 42 % for the P3 model and 30 % for the LP3 model. From figure 12, it can be seen that the effect of sample size on the average amounts of EAD is smaller than the effect on the uncertainty. Thus increasing the size of data sets is an effective way to reduce the uncertainty in damage results.

4.3. Discussion

The first order method used in this paper approximates the damage model by a linear function that is locally a good approximation. The damage model here is not really linear due to the non-linearity in the QH functions and the damage model itself. Thus the first order method in fact overestimated the uncertainty in the damage results. However the results still give useful preliminary answers. Moreover, the discharge uncertainty considered here is not the only uncertainty in the damage model. The first order method can also be applied to situations where other uncertainties need to be investigated. According to the authors' experience, the uncertainty caused by some hydraulic parameters and damage functions also have significant contributions to damage.

As shown in Section 4.1, the effect of the natural variability on the T-year event discharges could be as large as 17% to 30%. In order to reduce the uncertainty, Eberle et al. (2000) suggested to use a stochastic rainfall generator coupled to precipitation-runoff models for the generation of long time series. This is perhaps a good alternative when more data could not be obtained.

It is clear that the natural variability in this study is in fact based on historic data. The future natural variability (climate change, land use change) may even have larger effects on the river discharge (see e.g. Booij 2002).

5. Conclusions and recommendations

The results of this study show that the effect of the uncertainty in the river discharge (17%-30%) is aggravated when it is propagated into the damage results. The uncertainty in the damage results could be more than 100% for small sample sizes. It is also shown that both probability distribution models and sample size have important effects on the calculation of expected annual damage. It is believed that the sample size has a larger effect on the damage results than the probability distribution models except for the LP3 model. Possibly, this could be improved by further goodness-of-fit tests.

Solutions to the considerable uncertainty in expected annual damage are by reducing the uncertainty in river discharge through obtaining more observation data or using a stochastic rainfall model coupled to a precipitation-runoff model for the generation of long time series.

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