Numerical studies on dc-SQUID sensors with tightly coupled input coil

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Abstract. We investigated the behavior of two low-T_c direct current superconducting quantum interference device (dc-SQUID) sensors with integrated input coil. A model including the capacitance of the Josephson junctions, thermal noise of the integrated shunt- and damping-resistors as well as a frequency dependent inductance of the SQUID loop was determined and numerically simulated. The SQUID inductance is found to be mainly influenced by parasitic elements introduced by the integrated coils. The simulated characteristics of the examined SQUIDs show many features also seen in experiments, including a hysteresis due to the frequency dependent washer impedance. The measured sensitivity of one of the designs fits well to the simulated value.

1. Introduction

We designed and characterized low-T_c dc-SQUID current sensors for the readout of a resonant input load with a high quality factor [1]. During the design process attention had to be paid to reducing back-action effects of intrinsic or external origin to prevent loss in accuracy or unstable operation.

Besides a dc-SQUID with integrated flux transformer, we made one highly symmetric design with minimal direct mutual inductance between the feedback- and signal coils, the dc-SQUID in a parallel washer configuration [2]. This is believed to prevent additional back-action due to the operation in flux-locked loop. Both designs have a gradiometric layout to reduce interferences. Furthermore, a good sensitivity at the target frequency band in the kHz-range is desirable. We therefore integrated the input coils on-chip to reach a better equivalent noise energy referred to the input current. The planned operation is in the mK temperature range.

The measured behavior of the developed SQUIDs strongly deviates from the predictions by standard theory [3]. Especially below 4.2 K we partly observed distorted and extremely steep flux-to-voltage characteristics [1]. Parasitic elements in the SQUID design were found to most likely cause this behavior [4] and therefore we performed numerical simulations to study their effect.

2. Modeling the washer structures at high frequencies

The used configuration to convert the signal current into a flux in the SQUID loop is the widely used square washer structure [5] with integrated input coil. Lots of work has been done to explain the

behavior of this configuration at the operation frequency of the SQUID. Especially the work of Enpuku and co-workers has to be mentioned here, their analytic model from [6] was used to estimate the influence of all turns of the integrated coils on the washer impedance.

To estimate the hole inductance L_H of the SQUIDs we used the well known formulas from [5]. The properties of the microstrip line (MSL) formed by the turns of the 3 µm wide coil on top of the washers of all designs were estimated using the analytic model introduced by Chang [7]. The geometry given in the design rules [8] and a relative permittivity of 6.5 for the SiO insulator led to characteristic values of $2.45 \cdot 10^{-7}$ H/m and $3.60 \cdot 10^{-10}$ F/m respectively. The inductance of each slit configuration L_{SL} was estimated using the numerical inductance calculation software Fasthenry [9]. With the width of all turns and the spacing between them being 3 µm, the slit length is 6 µm times the number of turns N. The coils are not shorted to the washer. In table 1 and in figure 1 one can see the calculated data and impedance characteristics for the three washers included in our designs.

Table 1. Parameters of the washers and their fitted simplified models (see figure 2).

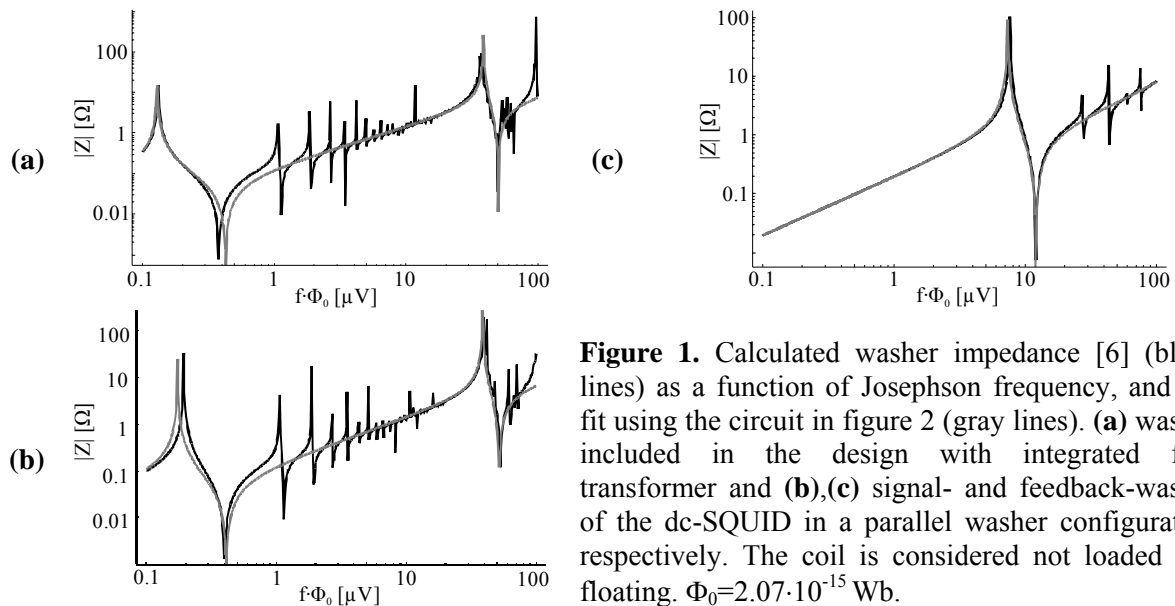|	N	L_H	L_{SL}	$L_{F,C}$	$C_{F,C}$	$L_{F,W}$	$C_{F,W}$	L_R
Flux transformer	60	369 pH	141 pH	464 pH	19 nF	19 pH	3.8 pF	28 pH
Parallel signal	70	94 pH	161 pH	210 pH	17 nF	20 pH	3.6 pF	25 pH
Parallel feedback	15	16 pH	49 pH	40 pH	50 pF	12 pH	160 fF	13 pH

(a)

(b)

(c)

Figure 1. Calculated washer impedance [6] (black lines) as a function of Josephson frequency, and the fit using the circuit in figure 2 (gray lines). **(a)** washer included in the design with integrated flux transformer and **(b)**,**(c)** signal- and feedback-washer of the dc-SQUID in a parallel washer configuration respectively. The coil is considered not loaded and floating. $\Phi_0 = 2.07 \cdot 10^{-15}$ Wb.

As pointed out in [6], basically there are two characteristic resonances and their harmonics. At lower frequencies, with a wavelength in the order of the length of the coil, L_H and a part of L_{SL} is shorted and thus the inductance of the washer is dropping. Above frequencies corresponding to wavelengths in the order of the dimensions of the washer the so-called washer-resonances start.
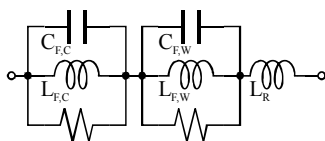

Figure 2. Lumped circuit element model for the washer impedance. See table 1 for values. The resistors determine the Q, see text for details.

A simplified model using two LC-resonance circuits for the coil- and the washer-resonance is shown in figure 2. Another inductance in series L_R qualitatively models the inductance above the washer resonance frequency. The elements of the simplified model were determined by fitting. Their

values can be found in table 1 and the resulting impedance of the simplified washer model is shown in figure 1. The parallel resistors set the quality factor $Q=R/\sqrt{(L/C)}$ of the resonance. We usually used a value of 1000, any differences are pointed out.

The advantage of such a simple model is besides a low computation time that one can easily study the effects by changing parameters and testing their effect on the behavior of the complete SQUID.

3. Description of the simulation procedure

The software JSIM including noise sources [10] was used to conduct all presented simulations. The model of the Josephson junctions included in JSIM is based on the resistively capacitively shunted junction model. The value of the critical current I_C of each junction was chosen equally to its design value, which was close to the measured value. The capacitance of each Josephson junction was calculated using the design rules [8]. The values of the measured resistances were about 90% of their design values for the characterized SQUIDs, so we also adapted them in our simulations. Noise sources modeling the Johnson noise at the temperature T were connected to the shunt resistors of the junctions and to the used shunt resistors in the SQUID loop.

To estimate the characteristics the SQUID was set up at different bias currents and flux values. To investigate the later shown hysteresis, the SQUID was first switched into the voltage-state before adjusting the desired working point. The voltage of the SQUID was filtered using a RC low pass element with a resistance of 1 MΩ and then recorded for some time. For the simulation of the sensitivity in terms of the flux noise amplitude spectral density (ASD), we used the voltage noise ASD and the flux-to-voltage transfer. The voltage noise ASD was determined using the standard deviation of the output voltage and the effective bandwidth of the connected low pass. The flux-to-voltage transfer function was determined by a negligible small excitation on the flux input and its correlation with the output voltage.

The whole simulation procedure was tested on other simulation results from literature [3] and showed good agreement.

4. Numerical studies on the dc-SQUID with integrated flux transformer

The estimated model of the washer was now used in a complete model of the SQUID with integrated flux transformer, which can be seen in figure 3. We introduced a few more elements we found to be of importance. The values of all elements were estimated using the analytic formulas for the square washer inductances [5], MSL calculations [7] and numerical inductance calculations of simple test geometries [9].
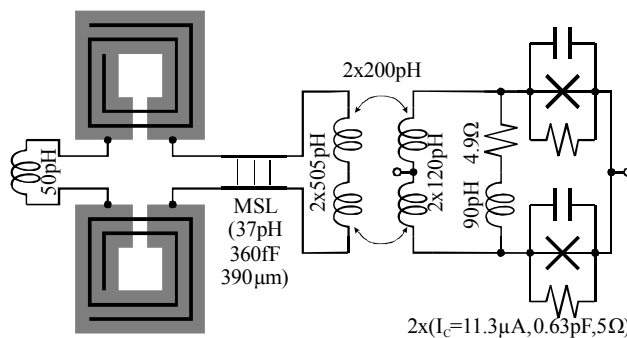

Figure 3. Model of the dc-SQUID with integrated flux transformer. The washers sketched here were replaced by the model shown in figure 2 with values from table 1. Thermal noise sources not shown here are connected to the shunt resistors and the resistor shunting the SQUID inductance.

The inductance in series with the resonance-damping resistor in figure 3 results from a MSL on top of the SQUID washer as well as the crossing of the slit. The transformer loop is coupled to the SQUID square washer via two turns.

In the transformer loop two elements had to be introduced. The line connecting the SQUID with the transformer coil was designed in form of a MSL and was thus included in the model. Furthermore the slit inductance of the connection of the two gradiometric washers was added. More detailed information on this design can be found in [1].

Because of the low resonance frequency of the coil resonance we reduced its Q to 0.5 to lower the characteristic time constant and save computation time. The results of the simulation are shown in figure 4. One first remarkable feature is a hysteresis we observed close to the superconducting state of the SQUID. Using different directly coupled SQUID electronics, we conducted new measurements at a temperature of 0.1 K. A similar behavior as already published in [1] was observed, but now we were able to see the same hysteresis as in the simulations and other unstable points.
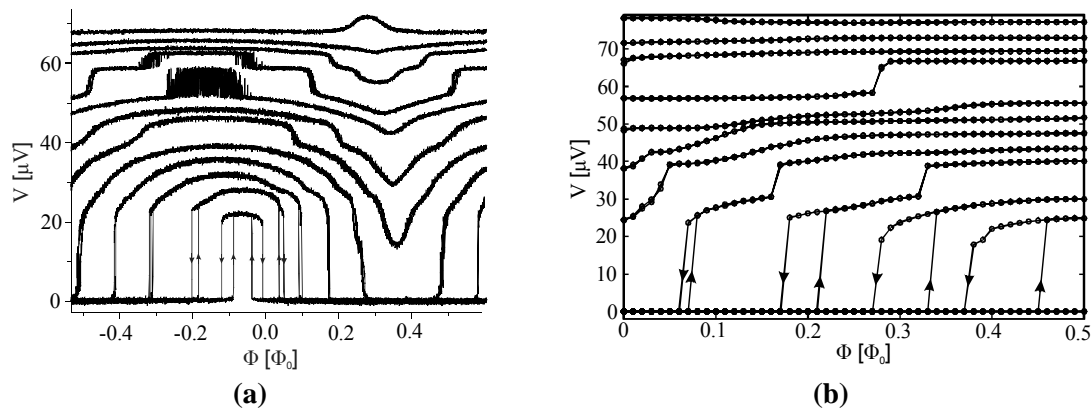

Figure 4. (a) Measured and **(b)** simulated characteristics of the dc-SQUID with integrated flux transformer at a temperature of 0.1 K. Hysteretic paths are indicated by arrows. **(a)** A single flux sweep in both directions in the range of 1 kHz for each bias current is shown. Applied bias currents, from bottom to top: 16.6, 18.4, 20, 21.7, 23.3, 25, 26.6, 28.9, 30, 31.7, 33.3 and 35 μA. One 50Ω-1nF shunt was connected in parallel to the input coils. **(b)** Each working point was averaged for 0.1μs with a bandwidth 100MHz. The applied bias currents are from bottom to top 12 μA (superconducting) till 36 μA in steps of 2 μA.

We performed some test simulations to study the observed hysteretic behavior. The critical current of the SQUID for all flux values as well as the abruptly reached voltage were explained using simulations without the coil resonance of the signal washer, once including and once excluding the inductance $L_{F,C}$ normally being shortcut. The SQUID is switching to the voltage state at a point determined with its screened dc-inductance. The resulting voltage after the switching event equals for wide flux ranges the voltage of a SQUID in the same working point but having a reduced inductance, the effective inductance at frequencies in this operation range (see figure 1a). The current in the SQUID loop is oscillating in the voltage state and so the inductance at the operation frequency becomes important. This effect has already been shown in simulations [11, 12]. The exact mechanism of switching back from the voltage state to the superconducting state, basically the width of the hysteresis shown in figure 4, we currently do not understand completely.

At voltages between 30 and 40 μV, the measurement and the two simulations show irregularities that originate from the washer resonance. This was identified by test simulations excluding this resonance from the simplified washer model. The effect seems to be more extreme in our simulations, suggesting that the used Q of 1000 of this resonance is too high.

The irregularities at voltages between 50 and 60 μV originate from a resonance between the capacitance of the connecting MSL included in the transformer loop and the effective inductance of the two washers plus the parasitic inductance of the connection.

The resonance between the effective SQUID inductance and the capacitances of the Josephson junctions, recognized by a "flipping" of the SQUID V-Φ-characteristics [13], can be seen in both the simulated model and the measurement at voltages around 70 μV.

We do not have data of a two-stage SQUID measurement on the sensitivity of the dc-SQUID with integrated flux transformer [1] and so we did not conduct simulations in this direction.

5. Numerical studies on the dc-SQUID in a parallel washer configuration

The design consists of two identical branches in parallel to the junctions. Those branches consist of a series connection of one signal- and one feedback-washer [2] whose properties are shown in figure 1bc and table 1. More details on the design can again be found in [1].

The simulation model is shown in figure 5. The effective impedance of the SQUID loop was calculated for the washer configuration using the models shown in table 1 and figure 1. This impedance was then distributed symmetrically over the two branches of the SQUID. The values of each resonating inductance shown in figure 5 equals one quarter of the values listed in table 1, a factor two for the parallel connection and another factor two for distributing the inductance. The shown values for the resonating capacitances are thus a factor of four higher than the values from table 1. Because of the low resonance frequency of the coil resonance we again reduced its Q to 0.5.
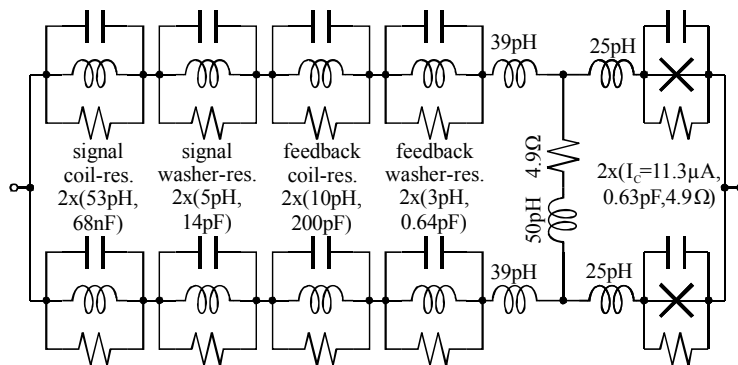

Figure 5. Schematic of the simulated model of the dc-SQUID in a parallel washer configuration. Thermal noise sources not shown here are connected to the shunt resistors of the junctions and across the SQUID loop, but not for the resistors setting the Q of the resonances.

Parasitic inductances originating from MSLs were added in series with the junctions and in series to the damping resistance. The effective slit inductances of the connection of all the four washers was added in series to the models of the washers, 30 pH per branch – in figure 5 added to the effective $L_R/4$ of both washer models (9 pH).

The calculated characteristics and the measurement data from [1] are shown in figure 6. The simulation shows a hysteresis which is in qualitative agreement to the one observed before. Currently we did not conduct another experiment to examine the hysteretic behavior of this SQUID.
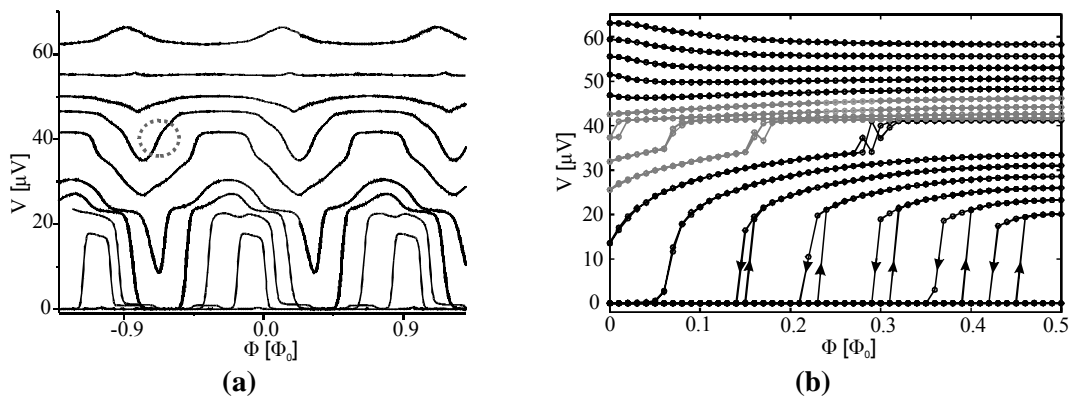

(a) (b)

Figure 6. **(a)** Measured (from [1]) and **(b)** simulated characteristics at a temperature of 0.6 K. The applied bias currents are from bottom to top: **(a)** 18.4 (superconducting), 19.5, 20.4, 21.9, 23.2, 26.3, 28.7, 31.1, 33.5 and 36.3 μA, **(b)** 16 (superconducting) till 32 μA in steps of 1 μA. In **(b)** each simulated point was averaged for 1 μs with low pass filtering from 300 MHz. The grey lines mark the points of the noise simulation shown in figure 7.

Around 40 μV the washer resonance of the signal washer leads to steeper characteristics and in the simulation to a higher noise. The resonance of the effective SQUID inductance with the capacitance of the Josephson junctions can be found at voltages of 50 to 60 μV. The irregular structures at voltages

around 25 μV observed in the measurement are not reproduced by our model and are thus of currently unknown origin.

A measurement on the sensitivity was done in a two-stage SQUID setup, meaning the SQUID was voltage-biased and read out via another SQUID. Noise measurements were performed in the stable points of the two-stage configuration, the minimum noise yielded 1 μΦ$_0$/√Hz at a temperature of 0.6 K [1]. The exact working point of this sensitivity measurement is unknown due to the two-stage SQUID configuration. Our guess is in the area qualitatively shown by a dashed circle in figure 6a, close to the washer resonance of the signal washer. We conducted a simulation on the noise of the model in the range of this point. The calculated flux noise ASD, estimated as described above, is shown in figure 7 and, considering all uncertainties, shows a good agreement with the experimental value.
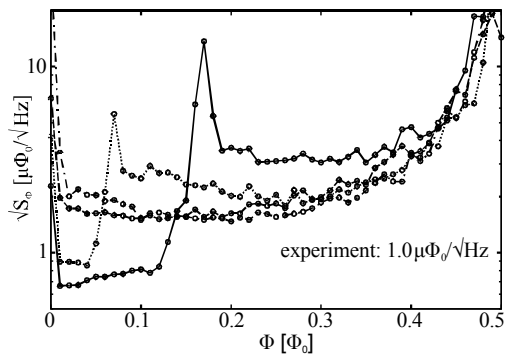

Figure 7. Simulated flux noise ASD at 0.6 K for the bias currents 24 (solid), 25 (pointed), 26 (point-dashed) and 27 (dashed) μA and all flux values also shown in figure 6b, there marked as grey lines. Each simulated point was averaged for 10μs with low pass filtering from 100MHz.

6. Conclusions

We modeled and simulated two designs of dc-SQUID sensors including parasitic elements mainly introduced by the washer structures. Several features of the measured characteristics of both designs could be reproduced in good quality. A hysteresis due to a reduction of the effective SQUID inductance could be seen in measurement and simulation of the dc-SQUID with integrated flux transformer. For the dc-SQUID in a parallel washer configuration we conducted a simulation on the sensitivity which is in good agreement with the experimental value at a temperature of 0.6 K.

The simulations show that the implemented model is able to reproduce the behavior of SQUID sensors with integrated input coil in a good way. Combined with more detailed estimations of inductances and capacitances in the complete structures this would form a powerful design tool.

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