A perceptually relevant model for aliasing in the triplet stripe filter CCD image sensor

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Abstract

In this paper we present a signal-theoretical model of the aliasing in the triplet stripe filter CCD image sensor. In this sensor, the colour filters are placed in a repetitive pattern of three columns on the monochrome sensor. Therefore the projected scene is sampled at every three columns and aliasing can result. We use the signal-theoretical model to predict the visibility of the aliasing.

1 Introduction

Colour cameras with one CCD (Charge Coupled Device) image sensor are widely used. A CCD image sensor consists of a matrix of light sensitive pixels (see Fig. 1). This device is a monochrome image sensor.

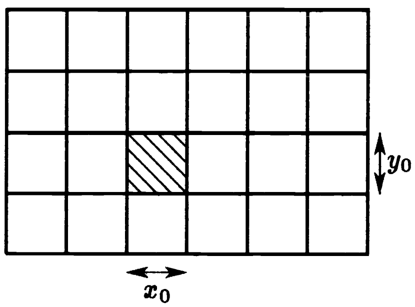

Figure 1: A front view of a CCD image sensor, which is an array of light sensitive pixels

To use it as a colour image sensor, colour filters are placed on top of the light sensitive pixels. In this paper we will consider a special placement of the colour filters, namely the triplet stripe filter. The colour filters are placed as stripes on top of the columns in a repetative pattern of three columns (see Fig. 2).

The colour filters sample the light, which is projected on the sensor. Suppose the sensor has red, green and blue colour filters. Since the green and blue colour filters do not transmit red light, a red scene gives an output from columns one, four, seven, etc. of the CCD. If high spatial frequencies are not suppressed by optical filters, aliasing results. An Optical Low-Pass filter (OLP), which is not ideal, is used to reduce the high frequency content (see Fig. 3). The aliasing is visible as blockiness and colour errors.

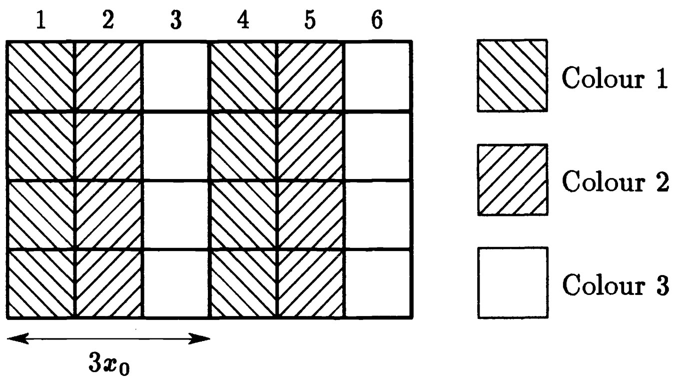

Figure 2: The colour filters on top of the CCD image sensor

In this paper we will give a quantitative model of the visible aliasing as a function of the choice of the three colour filters. This model can easily be adapted for other types of colour sensors.

2 Perceptually independent colour signals

A colour stimulus is defined by its spectral energy distribution $s(\lambda)$, where λ is the wavelength of the light. It is generally believed that there are three classes of light sensitive elements in the human visual system[1], i.e. the human vision is trichromatic. From this trichromacy it follows that the appearance of a colour stimulus is described by three independent signals, e.g. red, green and blue. Suppose we have two colour stimuli with different spectral energy distributions. If the values of the colour signals are equal, the stimuli cannot be distinguished.

A good choice of the colour signals enables us to give quantitative figures for the visibility of the aliasing. Each colour signal should describe an independent signal in the human visual system. Often the appearance of a colour stimulus is described in terms of *brightness*, *saturation* and *hue*. However, for mathematical simplicity we demand that the chosen colour signals depend linearly on the spectral energy distribution. The NTSC signal L (luminance) is closely related to the brightness. The other two signals should not depend on the luminance of the colour, but describe the saturation and the hue of the colour stimulus. We choose the NTSC signals Q and I. It must be noted that the signals Q and I are not perceptually independent, but they are the best choice given the constraint of linearity.

The relation between the signal L, Q and I and the spectral energy distribution $s(\lambda)$ of a colour stimulus is given by

$$\begin{aligned} L &= \int s(\lambda) \cdot \bar{l}(\lambda) d\lambda \\ Q &= \int s(\lambda) \cdot \bar{q}(\lambda) d\lambda \\ I &= \int s(\lambda) \cdot \bar{i}(\lambda) d\lambda, \end{aligned} \tag{1}$$

where $\bar{l}(\lambda)$, $\bar{q}(\lambda)$ and $\bar{i}(\lambda)$ are the *colour matching functions*.

In the American television system, the signals L, Q and I are defined as (see[1], page 4.12-4.16)

$$\begin{aligned} L &= 0.30E_R + 0.59E_G + 0.11E_B \\ Q &= 0.21E_R - 0.52E_G + 0.31E_B \\ I &= 0.60E_R - 0.28E_G - 0.32E_B, \end{aligned} \tag{2}$$

where the signals E_R, E_G and E_B are the red, green and blue signals (based on the FCC primaries). In the European television system (based on the EBU primaries), we must use

$$\begin{aligned} L &= 0.22R + 0.71G + 0.07B \\ Q &= 0.14R - 0.49G + 0.30B \\ I &= 0.41R - 0.17G - 0.24B. \end{aligned} \tag{3}$$

3 A model of the triplet sensor

Our (linear) model consists of two parts. The first part describes the sampling at each physical pixel. The second part of the model describes the resampling due to the colour filters. Since the resampling is in the horizontal direction only, the second part of the model describes only one line of the image sensor.

The colour filters are modelled in two steps. From perception it is known that for a true colour reproduction the colour filters must be equal to a linear combination of the colour matching functions $\bar{l}(\lambda)$, $\bar{q}(\lambda)$ and $\bar{i}(\lambda)$. Let $c_i(\lambda)$ denote the transmission of colour filter i (i=1,2,3), then we have

$$c_i(\lambda) \quad = \quad m_{i1} \cdot \bar{l}(\lambda) + m_{i2} \cdot \bar{q}(\lambda) + m_{i3} \cdot \bar{i}(\lambda), \tag{4}$$

where m_{ij} (j=1,2,3) are constants, which describe this colour filter. The colour signal K_i, the integrated light transmitted by colour filter i, equals

$$\begin{aligned} K_i &= \int s(\lambda) \cdot c_i(\lambda) d\lambda \\ &= \int s(\lambda) \left(m_{i1} \cdot \bar{l}(\lambda) + m_{i2} \cdot \bar{q}(\lambda) + m_{i3} \cdot \bar{i}(\lambda)\right) d\lambda \\ &= m_{i1} \cdot L + m_{i2} \cdot Q + m_{i3} \cdot I. \end{aligned} \tag{5}$$

Eq. (6) shows that the signal K_i can be derived in two steps. First we derive the signals L, Q and I, and then take a linear combination to obtain K_i. The first step is described in the next section and the second step can be found in section 3.2. Section 3.3 gives an analysis of the aliasing due to the colour filters.

3.1 The sampling at each physical pixel

Figure 4 shows the first part of the model. As a notational convention we use square brackets for discrete signals. The signals $L[k,m]$, $Q[k,m]$ and $I[k,m]$, which are the values of L, Q and I at position $[k,m]$ on the sensor, are derived in this part of the model. Each point (x,y) in the scene has a spectral energy distribution, characterised by $s(x,y,\lambda)$. The lens, the OLP and the integration by each monochrome pixel are modelled as a spatial filter H with transfer function $H(\omega_x, \omega_y)$. After filtering the scene is sampled at each pixel at a horizontal distance x_0 and a vertical distance y_0. The signals L, Q and I are obtained using the colour matching functions $\bar{l}(\lambda)$, $\bar{q}(\lambda)$ and $\bar{i}(\lambda)$. Since the signals L, Q and I do not depend on the choice of the colour filters, we will not analyse this part of the model.

3.2 The resampling due to the colour filters

Figure 5 shows the second part of the model. We want our camera to produce the signals L, Q and I. However, due to aliasing the signals $\tilde{L}$, $\tilde{Q}$ and $\tilde{I}$ are produced. The matrix $M=\{m_{ij}\}$ describes the choice of the colour filters (see Eq. (6)). In the resampling block the signal $K_1[k]$ is sampled at k=0, 3, 6, $\cdots$, the signal $K_2[k]$ at k=1, 4, 7, $\cdots$, and $K_3[k]$ at k=2, 5, 8, $\cdots$. The matrix M^{-1}, the inverse of M, denotes an electronic 3x3 matrix.

Figure 6 shows a model of the resampling block from Figure 5. Each colour signal $K_i[k]$ is resampled at every third position ($\downarrow$ 3) and expanded by inserting two zeros ($\uparrow$ 3). The resulting sampling distance is $3x_0$. The phase of the resampling, however, is different for all three colour signals ($e^{\pm jn\theta}$, where θ denotes the normalised frequency in radians and n=0,1,2).

3.3 The aliasing due to the colour filters

In this section we will show that the aliasing can be described by an alias cross-over matrix A.

The Discrete Fourier Transform (DFT) of a signal $x[k]$ is denoted by $X\left(e^{j\theta}\right)$. A column vector of discrete signals $x_i[k]$ (i=1,2,3) is denoted by $\underline{\mathbf{x}}[k]$ and the vector with elements $X_i\left(e^{j\theta}\right)$ (i=1,2,3) is denoted by $\underline{\mathbf{X}}\left(e^{j\theta}\right)$.

The DFT of the signals $\tilde{K}_i[k]$ equals[2] (see Fig. 6)

$$\begin{aligned}
\underline{\tilde{\mathbf{K}}}\left(e^{j\theta}\right) &= \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3e^{j\theta} & 0 \\ 0 & 0 & 3e^{2j\theta} \end{bmatrix} \frac{1}{3} \sum_{i=-1}^{1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{-j(\theta+\frac{2\pi i}{3})} & 0 \\ 0 & 0 & e^{-2j(\theta+\frac{2\pi i}{3})} \end{bmatrix} \underline{\mathbf{K}}\left(e^{j(\theta+\frac{2\pi i}{3})}\right) \\
&= \sum_{i=-1}^{1} D^i \cdot \underline{\mathbf{K}}\left(e^{j(\theta+\frac{2\pi i}{3})}\right), \qquad (6)
\end{aligned}$$

where the diagonal matrix D equals

$$D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{-j\frac{2\pi}{3}} & 0 \\ 0 & 0 & e^{j\frac{2\pi}{3}} \end{bmatrix}. \qquad (7)$$

Note that the matrix D^{-1} is the complex conjugate of D.

Let $\underline{\mathbf{L}}$ and $\underline{\tilde{\mathbf{L}}}$ denote the vectors

$$\underline{\mathbf{L}}[k] = \begin{pmatrix} L[k] \\ Q[k] \\ I[k] \end{pmatrix} \quad \text{and} \quad \underline{\tilde{\mathbf{L}}}[k] = \begin{pmatrix} \tilde{L}[k] \\ \tilde{Q}[k] \\ \tilde{I}[k] \end{pmatrix}. \qquad (8)$$

Using Eq. (6) and (see Fig. 5)

$$\underline{\mathbf{K}}[k] = M \cdot \underline{\mathbf{L}}[k], \qquad \underline{\tilde{\mathbf{L}}}[k] = M^{-1} \cdot \underline{\tilde{\mathbf{K}}}[k], \qquad (9)$$

the DFT of $\tilde{\underline{\mathbf{L}}}[k]$ equals

$$\tilde{\underline{\mathbf{L}}}\left(e^{j\theta}\right) = \underline{\mathbf{L}}\left(e^{j\theta}\right) + A \cdot \underline{\mathbf{L}}\left(e^{j(\theta+\frac{2\pi}{3})}\right) + A^* \cdot \underline{\mathbf{L}}\left(e^{j(\theta-\frac{2\pi}{3})}\right), \tag{10}$$

where

$$A = M^{-1} D M. \tag{11}$$

The matrix $A = \{a_{ij}\}$, is the *alias cross-over matrix.* Given our choice of the perceptually independent signals, A depends only on the choice of the colour filters.

4 The appearance of the aliasing

In the last section we showed (Eq. (10)) that the signal $\tilde{\underline{\mathbf{L}}}[k]$ equals $\underline{\mathbf{L}}[k]$ plus alias terms. In this section we will describe each of the alias terms. The DFT of $\tilde{L}[k]$ equals

$$\begin{aligned}\tilde{L}\left(e^{j\theta}\right) = {} & L\left(e^{j\theta}\right) + \\ & a_{11} \cdot L\left(e^{j(\theta+\frac{2\pi}{3})}\right) + a_{12} \cdot Q\left(e^{j(\theta+\frac{2\pi}{3})}\right) + a_{13} \cdot I\left(e^{j(\theta+\frac{2\pi}{3})}\right) + \\ & a_{11}^* \cdot L\left(e^{j(\theta-\frac{2\pi}{3})}\right) + a_{12}^* \cdot Q\left(e^{j(\theta-\frac{2\pi}{3})}\right) + a_{13}^* \cdot I\left(e^{j(\theta-\frac{2\pi}{3})}\right).\end{aligned} \tag{12}$$

Thus the amount of aliasing in $\tilde{L}[k]$ is determined by the alias factors a_{1j} (j=1,2,3). The amount of aliasing in $\tilde{Q}[k]$ and $\tilde{I}[k]$ is given by a_{2j} and a_{3j}, respectively.

We simulated a theoretical sensor with only one alias term, i.e. a_{ij} is non-zero for only one pair $(i, j) = (i_0, j_0)$. Table 1 gives the appearance of each alias term. The source signal is the signal which causes the visible aliasing and the output signal is the signal where the aliasing is found.

Table 1: The appearance of an alias term

Alias factor	Source signal	Output signal	Visual impression
a_{11}	L	$\tilde{L}$	Blockiness
a_{12}	Q	$\tilde{L}$	Noise
a_{13}	I	$\tilde{L}$	Noise, or blockiness in red-saturated areas
a_{2j}	L,Q,I	$\tilde{Q}$	Violet-green colour errors
a_{3j}	L,Q,I	$\tilde{I}$	Orange-blue colour errors

Table 1 shows that the coefficient a_{11} determines the amount of blockiness in the picture. The coefficients a_{2j} and a_{3j} determine the amount of colour errors. The colour-to-luminance aliasing (a_{12}, a_{13}) is mostly visible as an unstructured artefact. These alias terms can be wrongly identified as CCD noise, like shot-noise.

5 Threshold of visibility

We simulated again a theoretical sensor with only one alias term. We selected a critical test picture from a set of scanned pictures. We adjusted the absolute value of the alias factor, such that the aliasing was just visible. In this simulation we did not use an Optical Low-Pass filter.
Let $|A|^2$ denote a matrix with elements $\{|a_{ij}|^2\}$ $(i,j = 1,2,3)$. Table 2 shows the squared absolute value of each alias factor at the visibility threshold. The values of these visibility thresholds will depend on the

Table 2: The visibility threshold of the alias factors

		Source signal		
	$\|a_{ij}\|^2$	L	Q	I
Output signal	$\tilde{L}$	0.02	0.50	0.05
	$\tilde{Q}$	0.02	8.00	4.00
	$\tilde{I}$	0.02	1.00	4.00

picture being used, but since we selected a critical picture, the thresholds for other pictures are expected to be larger.

Table 2 shows that the first column contains the lowest alias factors, i.e. aliasing caused by the luminance signal $L[k]$ is very visible. The aliasing caused by non-zero a_{22}, a_{23}, a_{32} and a_{33} is almost not visible. This implies that the colour errors are given by a_{21} and a_{31} (and are due to the luminance signal $L[k]$). Netravali shows (see [3], page 226) that the signals L, Q and I give a good energy compaction. The L signal contains the most energy and the Q signal contains the least energy. This gives an expanation of low values in the first column and the high values in column two (except for a_{32}).
The photographs in the figures 7 to 9 show the alias terms given in the first column of Table 2. The modulus of the corresponding alias factor a_{ij} equals one.

Further experiments have shown that the thresholds are almost independent. Therefore we conclude that the L, Q and I are indeed perceptually independent colour signals.

6 The alias cross-over matrix for two practical sensors

In this section we give two examples. In each example we calculate the alias cross-over matrix for a particular sensor. We then compare each alias factor to the visibility threshold. Finally, we compare the two sensors.

First, we analyse a sensor with red, green and blue colour filters which are based on the FCC standard[1]. Therefore the matrix M_{rgb} equals

$$M_{rgb} = \begin{bmatrix} 1.00 & 0.62 & 0.96 \\ 1.00 & -0.65 & -0.27 \\ 1.00 & 1.69 & -1.10 \end{bmatrix}. \tag{13}$$

The second example shows a yellow/green/cyan sensor with matrix

$$M_{ygc} = \begin{bmatrix} 2.00 & -0.23 & 0.68 \\ 1.00 & -0.65 & -0.27 \\ 2.00 & 1.05 & -1.37 \end{bmatrix}. \quad (14)$$

Figure 10 and 11 show simulations of both sensors. The OLP is simulated by a $\frac{1}{3}[1\ 1\ 1]$ digital filter.

The squared absolute value of the elements of the alias cross-over matrix for the sensors equals

$$|A_{rgb}|^2 = \begin{bmatrix} 0.17 & 0.32 & 0.18 \\ 0.62 & 0.12 & 0.27 \\ 0.80 & 0.70 & 0.18 \end{bmatrix}, \quad |A_{ygc}|^2 = \begin{bmatrix} 0.16 & 0.04 & 0.10 \\ 2.50 & 0.35 & 0.43 \\ 3.21 & 0.36 & 0.07 \end{bmatrix}. \quad (15)$$

For both sensors the coefficients $|a_{21}|^2$ and $|a_{31}|^2$ are larger than the threshold, so in both cases we expect visible colour errors. The coefficient $|a_{11}|^2$ is also above threshold, so we also expect to see some blockiness. According to Eq. (15), the model predicts that the yellow/green/cyan sensor produces more visible colour errors than the red/green/blue sensor. The blockiness of both sensors is about the same.

The colour errors in Figure 10 and Figure 11 agree well with the model predictions.

7 Conclusions

We have constructed a model, which gives a good prediction of the visible aliasing due to the choice of the colour filters. The model is based on the description of a colour stimulus by perceptual independent signals L, Q and I. The model shows that the aliasing consists of nine alias terms.

We obtained a threshold of visibility for each alias terms. It appears that the most visible aliasing is due to changes in the luminance of the recorded scene. In fact, all visible colour errors are due to changes in the luminance. Our experiments confirm that the NTSC signals L, Q and I are perceptually independent colour signals and lead to useful percepts.

We looked at two existing colour sensors, the red/green/blue sensor and the yellow/green/cyan sensor. It appeared that both sensors produce visible colour errors which is predicted by the model.
The red/green/blue sensor produces less visible colour errors than the yellow/green/cyan sensor, whereas the blockiness is about the same.

8 References

1. K. Blair Benson, *Television Engineering Handbook,* MacGraw-Hill, Inc., New York, 1986.

2. A.W.M. van den Enden and N.A.M. Verhoeckx, *Discrete-Time Signal Processing: An Introduction,* Prentice-Hall, Englewood Cliffs, New Jersey, 1989.

3. A.N. Netravali and B.G. Haskell, *Digital Pictures; Representation and Compression,* Plenum Press, New York, 1988, p. 226.

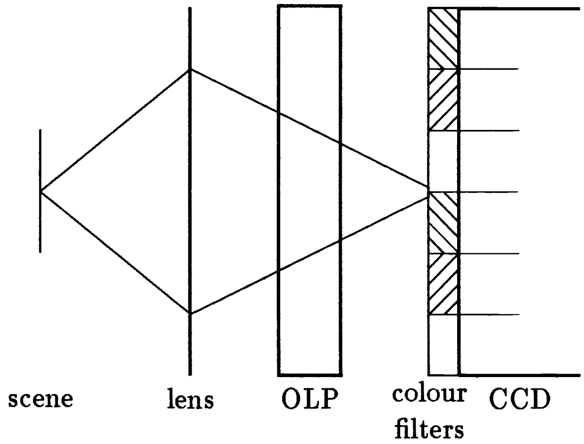

Figure 3: A top view of a CCD based colour camera

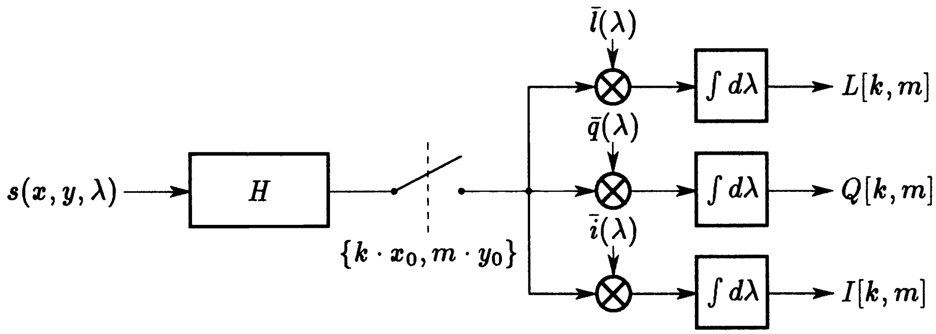

Figure 4: A model of the sampling at each physical pixel

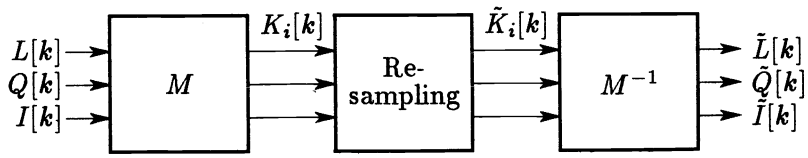

Figure 5: A model of the resampling due to the colour filters

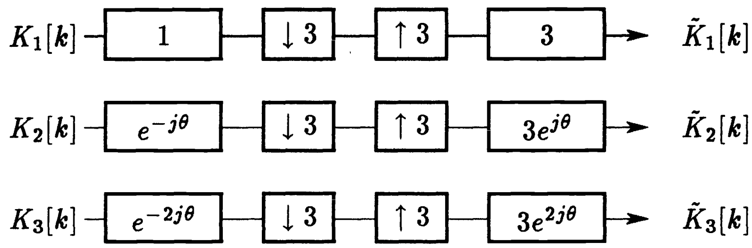

Figure 6: A model of the resampling block of Figure 5

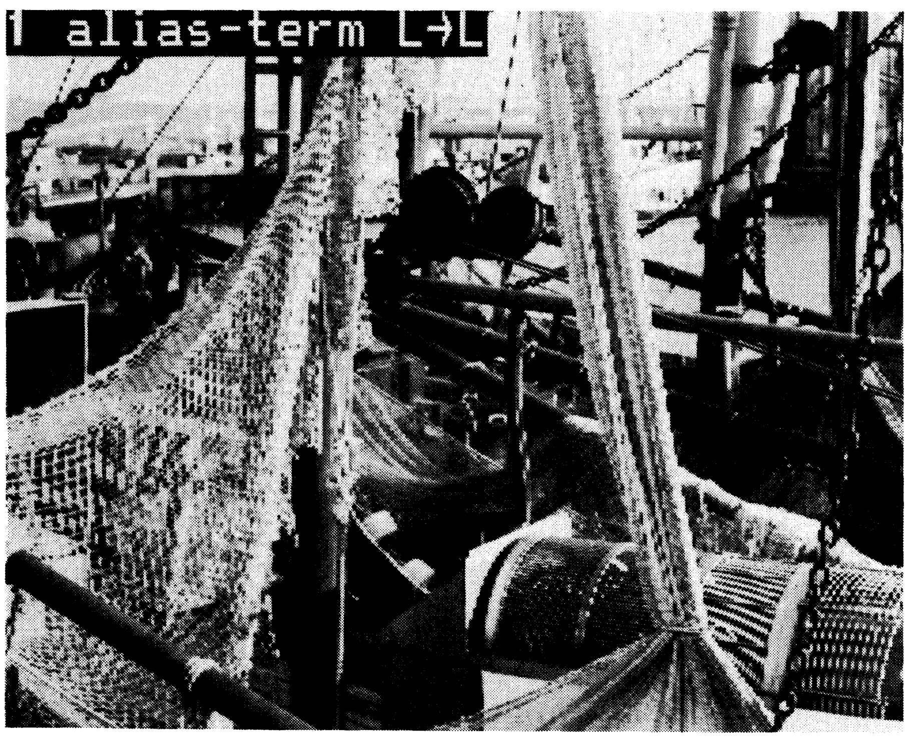

Figure 7: The L to L alias factor

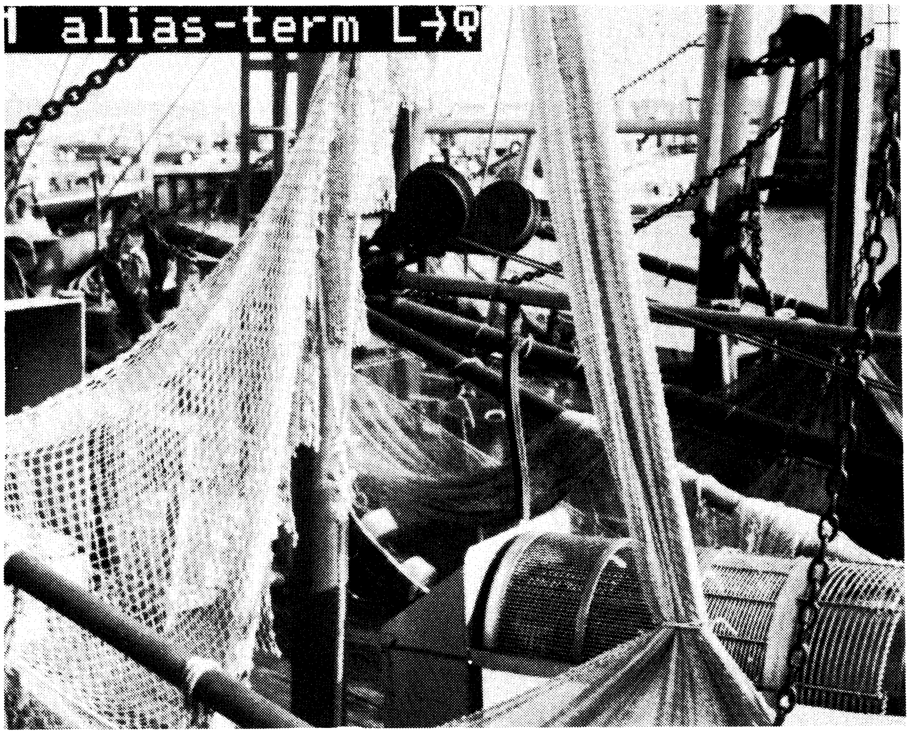

Figure 8: The L to Q alias factor

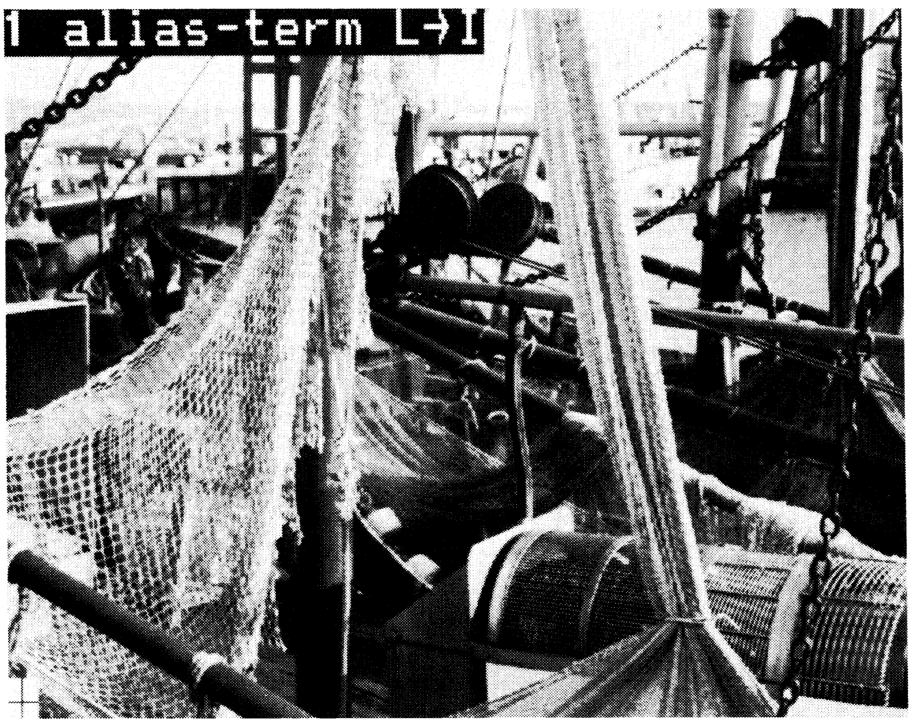

Figure 9: The L to I alias factor

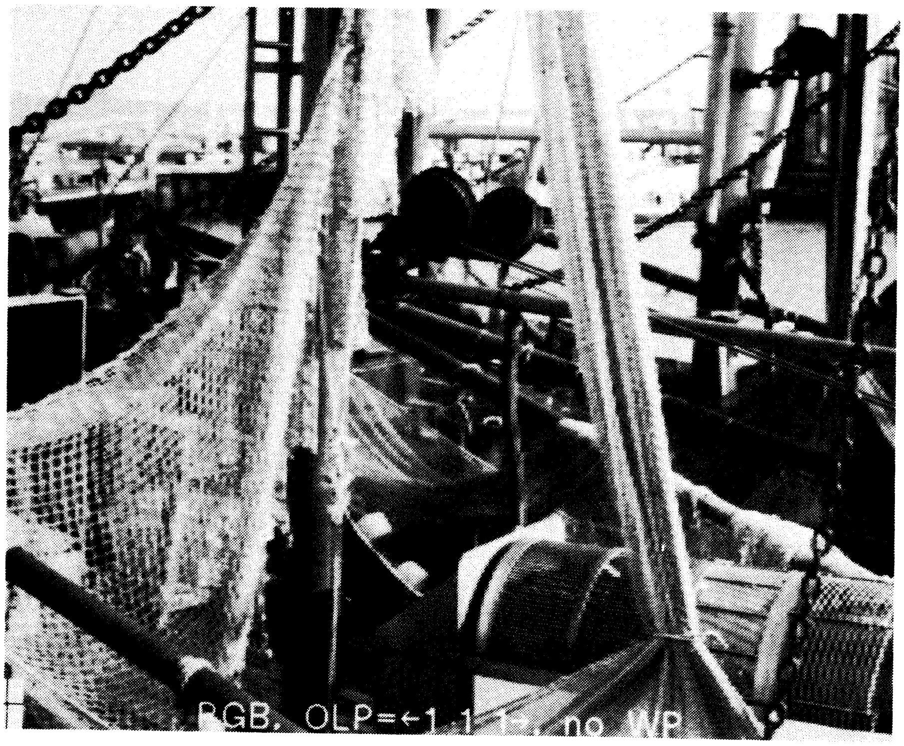

Figure 10: The rgb-sensor with an [1 1 1]-OLP

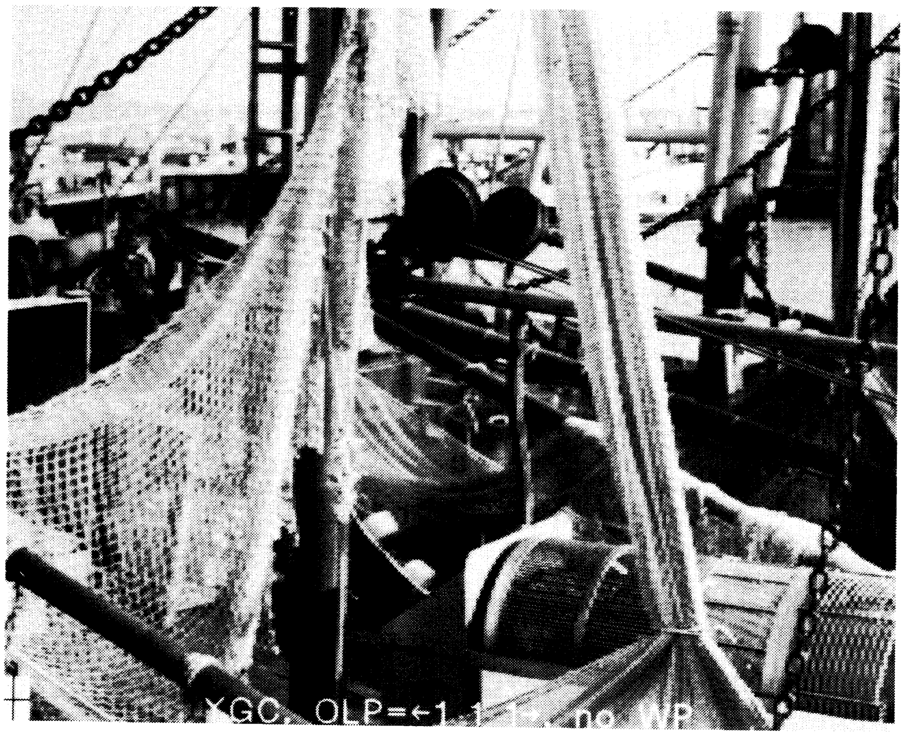

Figure 11: The ygc-sensor with an [1 1 1]-OLP