REGULATION AND INPUT DISTURBANCE SUPPRESSION FOR PORT-CONTROLLED HAMILTONIAN SYSTEMS [1]

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Abstract: In this paper the output feedback regulation problem for port-controlled Hamiltonian systems (PCHS) is addressed. Following the *nonlinear output regulation* theory, the regulator which solves the problem is given by a parallel connection of two subcontrollers: an internal model unit and a regulator to stabilize the extended system composed by the plant and the internal model unit. The main idea is to use the PCHS theory in order to design that stabilizer controller: as in many cases the plant to be addressed is indeed a mechanical/electric system, and it is very easy to think about it as a PCHS, the paper shows the conditions to fulfill in order to design the internal model unit as a PCHS, allowing to use the powerful energy-shaping theory in order to stabilize the extended system. Moreover the same techniques are used to design an internal model based controller able to globally solve a problem of input disturbance suppression.

Keywords: Hamiltonian systems, nonlinear output regulation, internal model, dumping injection, input disturbance suppression

1. INTRODUCTION

The problem of controlling the output of a system in order to achieve asymptotic rejection of some undesired disturbances is a central problem in control theory. The classical solution developed for the output feedback regulation of general nonlinear systems in (Byrnes *et al.*, 1997*b*), (Byrnes *et al.*, 1997*a*) shows that the regulator which solves the problem is given by a parallel connection of two subcontrollers: an internal model unit and a stabilizer controller to stabilize the extended system composed by the plant and the internal model unit.

This paper is devoted to investigate some elegant technique to design this stabilizer controller, considering general port-controlled Hamiltonian systems (with dissipation) (PCHS) and taking advantage of their peculiar properties. In (Maschke and van der Schaft, 1992), PCHS were introduced as a generalization of Hamiltonian systems, described by Hamiltonian's canonical equations which may represent general physical systems (i.e. mechanical, electric and electro-mechanical systems, nonholomic systems and their combinations). As in many cases the plant to address is indeed a mechanical/electric system, and it is very easy to think about it as a PCHS, the main idea is to use the PCHS peculiar properties and the classical passivity-based stabilization theory in order to design the stabilizer controller to use with the internal model unit in the output regulation framework.

This paper shows the conditions to fulfill in order to design an internal model unit as a PCHS, allowing to use the powerful energy-shaping theory in order to stabilize the extended system that is still a PCHS as

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interconnection of PCHS systems.
The paper is organized as follows. In section 2 we briefly recall the classical theory regarding the nonlinear output feedback regulation, introducing some results that will be useful in the following. In section 3 a local solution for a regulation problem of PCHS is presented; moreover it is shown that a local solution for a tracking problem is also available. In section 4, previous results are used to design a regulator to solve globally the same output feedback regulation problem for PCHS. Moreover, in section 5, an input disturbance suppression problem is considered. In the last section 6, some simulation results regarding a permanent magnet synchronous motor affected by some voltage disturbance are shown to confirm the effectiveness of the design.

2. INTRODUCTION TO OUTPUT FEEDBACK NONLINEAR REGULATION

In order to introduce the main contribution of this paper, it is necessary to briefly recall the main theory regarding the nonlinear output regulation (see (Byrnes *et al.*, 1997*b*)); to that aim, consider a nonlinear system described by differential equations of the form

$$\begin{cases} \dot{x} = f(x,u,w) \\ \dot{w} = s(w) \\ e = y = h(x,w) \end{cases} \qquad (1)$$

with state $x \in \mathcal{X} \subset \mathbb{R}^n$ and control input $u \in \mathbb{R}^m$. In a regulation problem, the output $e = y = h(x,w) \in \mathbb{R}^m$ is the output of the plant affected by some exogenous disturbances $w \in \mathcal{W} \subset \mathbb{R}^s$; we assume $f(x,u,w)$, $h(x,w)$ and $s(w)$ to be C^k functions (for some large k) of their arguments, and also $f(0,0,0) = 0$, $h(0,0) = 0$, and $s(0) = 0$. The second equation in (1) describes an autonomous system (*exosystem*) defined in a neighborhood $\mathcal{W}$ of the origin of $\mathbb{R}^s$. Moreover the exosystem have to satisfy a basic assumption:
Hypothesis H1. $w = 0$ is a stable equilibrium for the exosystem, and there exists a neighborhood $\hat{\mathcal{W}} \subset \mathcal{W}$ of the origin with the property that each initial condition $w(0) \in \hat{\mathcal{W}}$ is Poisson stable.
Remark. This hypothesis implies that the matrix S which characterizes the linear approximation of the system a the equilibrium $w = 0$, has all its eigenvalues on the imaginary axis.
Remark. In order to introduce the main result in (Byrnes *et al.*, 1997*b*), let also recall the definition of immersion of a system into another: consider two autonomous systems described by:

$$\begin{cases} \dot{x} = f(x) \\ y = h(x) \end{cases} \qquad \text{and} \qquad \begin{cases} \dot{\tilde{x}} = \tilde{f}(\tilde{x}) \\ y = \tilde{h}(\tilde{x}) \end{cases}$$

defined in different state space $\mathcal{X}$ and $\tilde{\mathcal{X}}$ but with the same output space $\mathcal{Y} = \mathbb{R}^m$. Assuming $f(0) = 0$, $h(0) = 0$, $\tilde{f}(0) = 0$ and $\tilde{h}(0) = 0$, let indicate the two systems as $\{\mathcal{X}, f, h\}$ and $\{\tilde{\mathcal{X}}, \tilde{f}, \tilde{h}\}$ respectively.
System $\{\mathcal{X}, f, h\}$ is said to be immersed in system $\{\tilde{\mathcal{X}}, \tilde{f}, \tilde{h}\}$ if there exists a C^k map $\tau : \mathcal{X} \to \tilde{\mathcal{X}}$, with $k \geq 1$, $\tau(0) = 0$ such that $h(x) \neq h(z) \Rightarrow \tilde{h}(\tau(x)) \neq \tilde{h}(\tau(z))$ and $\forall x \in \mathcal{X}$

$$\begin{cases} \dfrac{\partial \tau}{\partial x} f(x) = \tilde{f}(\tau(x)) \\ h(x) = \tilde{h}(\tau(x)) \end{cases}$$

The general problem to deal with is to find a dynamic output feedback control law to obtain a locally (globally) asymptotically stable closed-loop system in which, the response of the regulated output asymptotically converges to 0 as time tends to ∞.
To state the main proposition about nonlinear error feedback regulation, for convenience set:

$$A = \left[\frac{\partial f}{\partial x}\right]_{(0,0,0)} \quad B = \left[\frac{\partial f}{\partial u}\right]_{(0,0,0)} \quad C = \left[\frac{\partial h}{\partial x}\right]_{(0,0)} \qquad (2)$$

Proposition 1: *Assume Hypothesis H1. The error feedback regulation problem for system (1) is solvable iff there exists* $C^k (k \geq 2)$ *mappings* $x = \pi(w)$, *with* $\pi(0) = 0$, *and* $u = c(w)$, *with* $c(0) = 0$, *both defined in a neighborhood* $\mathcal{W}^\circ \subset \mathcal{W}$ *of the origin, satisfying the conditions (regulator equations(see (Byrnes* et al., *1997*b*)))*

$$\frac{\partial \pi}{\partial w} s(w) = f(\pi(w), c(w), w)$$
$$h(\pi(w), w) = 0$$

and the autonomous system $\{\hat{\mathcal{W}}, s(w), c(w)\}$ *is immersed into a system* $\dot{\xi}_1 = \phi(\xi_1)$, $u = \gamma(\xi_1)$ *with* $\phi(0) = 0$ *and* $\gamma(0) = 0$ *by an immersion* C^k *map* $\tau(w)$.
Moreover calling $\Phi = [\frac{\partial \phi}{\partial \xi_1}]_{\xi_1=0}$ *and* $\Gamma = [\frac{\partial \gamma}{\partial \xi_1}]_{\xi_1=0}$ *the pair*

$$\begin{bmatrix} A & 0 \\ NC & \Phi \end{bmatrix}, \begin{bmatrix} B \\ 0 \end{bmatrix}$$

has to be stabilizable for some matrix N *and the pair*

$$\begin{bmatrix} C & 0 \end{bmatrix}, \begin{bmatrix} A & B\Gamma \\ 0 & \Phi \end{bmatrix}$$

has to be detectable.
A controller that solves the problem of error feedback regulation could be seen as a parallel connection of two subcontrollers (see fig.1): the role of the internal model subsystem is to render invariant the manifold identified by $\{x = \pi(w), \xi_1 = \tau(w), w\}$: regulator equations assure that it's a zero error manifold as on that manifold the control input is exactly $\gamma(\xi_1) = \gamma(\tau(w)) = c(w)$; the role of the stabilizer controller is to stabilize in first approximation the system composed by the plant and the internal model unit; in other words, the role of that stabilizer unit is to make locally (globally) exponentially attractive the same (zero error) manifold.
The main idea is to use the PCHS theory in order to design such stabilizer controller: the principal issue to deal with is to describe the internal model unit in the PCHS framework.

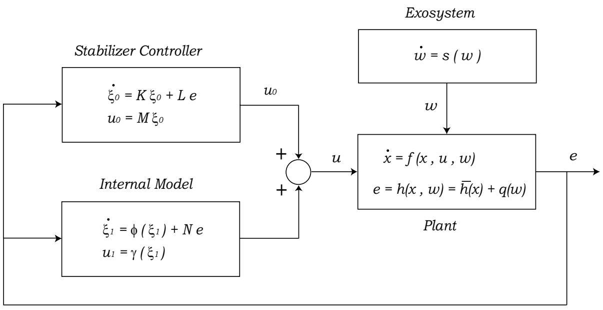

Fig. 1. Controller scheme

3. LOCAL SOLUTION: REGULATION AND TRACKING WITH HAMILTONIAN INTERNAL MODEL UNIT

To illustrate the conditions to fulfill in order to design the internal model unit as a port-controlled Hamiltonian system, let now consider a local output regulation problem; this simple case is interesting as it shows how is possible to solve locally also a nonlinear output tracking problem when the output of the plant is required to track a not-known exogenous signal.
To consider both regulation and tracking problem, let assume a plant described by

$$\begin{cases} \dot{x} = f(x, w, u) \\ e = h(x, w) = \bar{h}(x) + q(w) \end{cases} \qquad (3)$$

where $x(t) \in \mathbb{R}^n$ is the plant state, $u(t) \in \mathbb{R}^m$ is the control input, $e(t) \in \mathbb{R}^p$ the error signal representing the regulated output of the plant or the tracking error and $w(t) \in \mathbb{R}^q$ the exogenous signal representing the disturbance or the reference input. Let assume that $w(t)$ is generated by an autonomous exosystem (satisfying Hypothesis 1) described by

$$\dot{w}(t) = Sw(t) \qquad w(0) = w_0 \qquad (4)$$

where the (perfectly known) matrix S is defined by

$$S = \text{diag}\{S_0, S_1, \ldots, S_k\} \qquad (5)$$

with $S_0 = 0$, and

$$S_i = \begin{bmatrix} 0 & \omega_i \\ -\omega_i & 0 \end{bmatrix} \qquad \omega_i > 0 \qquad i = 1, \ldots, k \qquad (6)$$

Proposition 2: *It is possible to design for system (3) an internal model unit described as a PCHS and, moreover, the output regulation (or tracking) problem for system (3) is locally solvable if (defining A, B and C as in (2)) the pair (A, B) is stabilizable, the pair (A, C) is detectable and there exist a mappings $x = \pi(w)$ and $u = c(w)$, with $\pi(0) = 0$ and $c(0) = 0$, satisfying the conditions*

$$\begin{aligned} \frac{\partial \pi}{\partial w} Sw &= f(\pi(w), c(w), w) \\ 0 &= h(\pi(w), w) \end{aligned} \qquad (7)$$

with $c(w)$ polynomial of the form

$$c(w) = \sum_{l=1}^{s} \psi_l w^{[l]} \qquad (8)$$

where $w^{[1]} = w = [w_1, w_2, \ldots, w_q]^T$ with $q = 2k+1$, and for $l = 1, 2, 3, \ldots, s$

$$\begin{aligned} w^{[l]} = [&w_1^l, w_1^{l-1} w_2, \ldots, w_1^{l-1} w_q, \\ &w_1^{l-2} w_2^2, w_1^{l-2} w_2 w_3, \ldots, w_1^{l-2} w_2 w_q, \ldots, w_q^l]^T \end{aligned} \qquad (9)$$

Proof: Main proposition in (Huang, 2001) states that conditions (8), (9) are equivalent to require the existence of exists some set of r real numbers $a_0, a_1, \ldots, a_{r-1}$ such that

$$L_{Sw}^r c(w) = a_0 c(w) + a_1 L_{Sw} c(w) + \cdots + a_{r-1} L_{Sw}^{r-1} c(w) \qquad (10)$$

Moreover (see (Huang, 2001)) conditions (8) and (9) assure the existence of $\hat{\omega}_0 = 0$ and $\hat{\omega}_1, \ldots, \hat{\omega}_{n_k} \in \Omega$ where $r = 2n_k + 1$ and $\Omega = \{l_1\omega_1 + \cdots + l_k\omega_k \geq 0\,,\, l_1, \ldots, l_k = 0, \pm 1, \pm 2, \ldots\}$, such that

$$\lambda \prod_{l=1}^{n_k} (\lambda^2 + \hat{\omega}_l^2) = \lambda^r - a_0 - a_1\lambda - \cdots - a_{r-1}\lambda^{r-1} \qquad (11)$$

From (11) we immediately found out that $a_i = 0$ for $i = 2n$ $n = 0, 1, 2, \ldots$
Condition (10) implies that $\{\mathcal{W}, Sw, c(w)\}$ is immersed by a map $\bar{\tau}(w)$ into the linear observable system (see (Byrnes *et al.*, 1997*a*)) defined by

$$\begin{cases} \dot{\xi} = \Phi\xi \\ u = \Gamma\xi \end{cases} \qquad (12)$$

with $\Phi = \text{diag}(\tilde{\Phi}, \ldots, \tilde{\Phi})\,,\ \ \Gamma = \text{diag}(\tilde{\Gamma}, \ldots, \tilde{\Gamma})$ where

$$\tilde{\Phi} = \begin{pmatrix} 0 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 0 & 1 \\ 0 & a_1 & 0 & a_3 & \cdots & a_{r-2} & 0 \end{pmatrix} \qquad \tilde{\Gamma} = \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \end{pmatrix}$$

It is easy to realize that system (12) is equivalent, and therefore immersed, by means of a simple linear transformation, to the linear system $\dot{z} = \Psi z\ ,\ u = \Lambda z$ with

$$\begin{aligned} \Psi &= \text{diag}(\tilde{\Psi}_0, \tilde{\Psi}_1, \ldots, \tilde{\Psi}_{n_k}) \\ \bar{\Lambda} &= \text{diag}(\tilde{\Lambda}, \ldots, \tilde{\Lambda}) \end{aligned}$$

where $\tilde{\Lambda} = \begin{pmatrix} 1 & 0 & 1 & 0 & \cdots & 1 & 0 \end{pmatrix}$, $\tilde{\Psi}_0 = 0$, and

$$\tilde{\Psi}_i = \begin{pmatrix} 0 & \hat{\omega}_i \\ -\hat{\omega}_i & o \end{pmatrix} \qquad i = 1, 2, \ldots, n_k$$

The linear transformation is defined by $z = T\xi$ with $T^{-1} = \text{diag}(\tilde{T}, \ldots, \tilde{T})$ where

$$\tilde{T} = \begin{bmatrix} \Lambda^{\mathrm{T}} & \Psi^{\mathrm{T}}\Lambda^{\mathrm{T}} & \cdots & {\Psi^{\mathrm{T}}}^{r-1}\Lambda^{\mathrm{T}} \end{bmatrix}$$

To design a suitable internal model unit we have to choose a matrix N such that the pair (Ψ, N) is stabilizable, for instance $N = \text{diag}(\tilde{N}, \ldots, \tilde{N})$ with

$$\tilde{N} = \begin{pmatrix} 1 & 0 & 1 & 0 & \cdots & 0 & 1 \end{pmatrix}^{\mathrm{T}} = \Lambda^{\mathrm{T}}$$

Now we can conclude that the regulation problem can be solved (see (Byrnes *et al.*, 1997*a*)), and moreover the internal model unit can be written as a PCHS:

$$\begin{cases} \dot{z} = J_{mi}\dfrac{\partial H_{mi}}{\partial z} + g_{mi}e \\ u_{mi} = g_{mi}^{\mathrm{T}}\dfrac{\partial H_{mi}}{\partial z} \end{cases}$$

where $J_{mi} = \Psi$ is a skew-symmetric matrix, the Hamiltonian function is defined as $H_{mi} = \frac{1}{2}z^{\mathrm{T}}z$ and $g_{mi} = N = \Lambda^{\mathrm{T}}$. □

An immediate consequence of the previous statement is that if our system satisfies all conditions of *Proposition 2*, and moreover the unforced plant can be described as a PCHS, i.e.

$$\begin{cases} \dot{x} = f(x,u,0) = [J(x) - R(x)]\dfrac{\partial H}{\partial x} + g(x)u \\ e = h(x,0) = \bar{h}(x) = g^{\mathrm{T}}(x)\dfrac{\partial H}{\partial x} \end{cases} \tag{13}$$

where $x \in \mathcal{X} \subset \mathbb{R}^n$, $u \in \mathcal{U} \subset \mathbb{R}^m$, $e \in \mathcal{Y} \equiv \mathcal{U}^*$, being $\mathcal{U}^*$ the dual space of $\mathcal{U}$, $H : \mathcal{X} \to \mathbb{R}$ is the energy function and $J(\cdot) = -J^{\mathrm{T}}(\cdot)$, $R(\cdot) = R^{\mathrm{T}}(\cdot)$, then, to complete the design of a controller that solves locally the output regulation and tracking problem, we have only to study a stabilizer controller for the PCHS described by

$$\begin{bmatrix}\dot{x}\\ \dot{z}\end{bmatrix} = (\begin{bmatrix} J(x) & g(x)g_{mi}^{\mathrm{T}} \\ -g_{mi}g(x)^{\mathrm{T}} & J_{mi}\end{bmatrix} - \begin{bmatrix} R(x) & 0 \\ 0 & 0\end{bmatrix}) \begin{bmatrix}\dfrac{\partial H(x)}{\partial x} & \dfrac{\partial H_{mi}}{\partial z}\end{bmatrix}^T + \begin{bmatrix} g(x) & 0 \\ 0 & g_{mi}\end{bmatrix}\begin{bmatrix}u_{st}\\ 0\end{bmatrix}$$
$$\begin{bmatrix} e \\ u_{mi}\end{bmatrix} = \begin{bmatrix} g(x)^{\mathrm{T}} & 0 \\ 0 & g_{mi}^{\mathrm{T}}\end{bmatrix}\begin{bmatrix}\dfrac{\partial H(x)}{\partial x} & \dfrac{\partial H_{mi}}{\partial z}\end{bmatrix}^T \tag{14}$$

with u_{st} as new control input.

Remark. Note that, as we need to design a stabilizer controller for the connection between the unforced plant and the internal model unit in order to obtain a local solution (i.e. to stabilize the unforced plant in first approximation), it is possible to consider both regulation and tracking problems; in fact in a tracking problem we only need that the real output of the plant $\bar{h}(x) = y$ is dual to the input signal acting on the system u, and that is always fulfilled considering Hamiltonian systems.

4. GLOBAL SOLUTION: REGULATION WITH AN HAMILTONIAN INTERNAL MODEL UNIT

Thanks to result stated in *Proposition 2*, it is now possible to extend our considerations and propose how to solve the global problem of nonlinear output feedback regulation considering a port-controlled Hamiltonian system subject to some exogenous disturbance:

$$\begin{cases} \dot{x} = f(x,w,u) = [J(x,w) + \\ \quad - R(x,w)]\dfrac{\partial H(x,w)}{\partial x} + g(x,w)u \\ e = y = h(x,w) = g(x,w)^{\mathrm{T}}\dfrac{\partial H(x,w)}{\partial x} \\ \dot{w} = Sw \end{cases} \tag{15}$$

where the exosystem is defined as in (4), (5) and (6).

Proposition 3: *Defined A, B, C as in (2), assume that the pair (A,B) is stabilizable, the pair (A,C) is detectable and there exist a mappings $x = \pi(w)$ and $u = c(w)$, with $\pi(0) = 0$ and $c(0) = 0$, satisfying the conditions (7) with $c(w)$ polynomial of the form (8), (9). If the unforced system $\dot{x} = f(x,0,0)$ is stable, then it's possible to design an output feedback controller able to assure the output going globally and asymptotically to zero.*

Proof: As *Proposition 2* holds, we are able to design an Hamiltonian internal model unit, and to write the whole system (plant+internal model unit+exosystem) as a PCHS of the form:

$$\begin{bmatrix}\dot{x}\\ \dot{z}\\ \dot{w}\end{bmatrix} = (\begin{bmatrix} J(x,w) & g(x,w)g_{mi}^{\mathrm{T}} & 0 \\ -g_{mi}g(x,w)^{\mathrm{T}} & J_{mi} & 0 \\ 0 & 0 & S\end{bmatrix} + \\ - \begin{bmatrix} R(x,w) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0\end{bmatrix}) \begin{bmatrix}\dfrac{\partial H(x,w)}{\partial x} \\ \dfrac{\partial H_{mi}(z)}{\partial z} \\ \dfrac{\partial H_w(w)}{\partial w}\end{bmatrix} + \begin{bmatrix} g(x,w) \\ 0 \\ 0\end{bmatrix} u_{st}$$
$$y = \begin{bmatrix} g(x,w)^{\mathrm{T}} & 0 & 0\end{bmatrix}\begin{bmatrix}\dfrac{\partial H(x,w)}{\partial x} & \dfrac{\partial H(z)_{mi}}{\partial z} & \dfrac{\partial H_w(w)}{\partial w}\end{bmatrix}^T \tag{16}$$

with total Hamiltonian defined by $H_{tot} = H + H_{mi} + H_w = H + \frac{1}{2}z^{\mathrm{T}}z + \frac{1}{2}w^{\mathrm{T}}w \geq 0$.

From *Proposition 1* we know that the internal model unit assures that there exists an invariant zero-output manifold for system (16), namely $[\pi(w)\ \ \tau(w)\ \ w]^T$ where $\tau(w) = T\hat{\tau}(w)$ and $\hat{\tau}(w)$ is the immersion map to define system (12).

Showing that it is possible to make this manifold globally attractive by a simple dumping injection ($u_{st} = -y$) will end the proof.

Let consider the time derivative of the total Hamiltonian of (16):

$$\frac{dH_{tot}}{dt} = -y^2 - \frac{\partial^{\mathrm{T}} H}{\partial x}R(x,w)\frac{\partial H}{\partial x} \leq 0 \tag{17}$$

LaSalle invariant principle guarantees that system (16) tends to the largest invariant manifold compatible with $\dot{H}_{tot} = 0$.

We have simply to show that there's only one manifold assuring $y(t) = 0$ and at the same time $\frac{\partial^{\mathrm{T}} H}{\partial x}R(x,w)\frac{\partial H}{\partial x} = 0$, and that this manifold is just the one above defined (namely $[\pi(w)\ \tau(w)\ w]^T$.

Indeed $\pi(w)$ and $c(w)$ are mappings providing zero output for system (15), then it is the unique manifold providing $y(t) = 0$; moreover, as $x = \pi(w)$ is a controlled, invariant, zero output manifold for system (15), then, on that manifold, applying the right control input $c(w)$ we have

$$\frac{dH}{dt}_{|x=\pi(w)} = -c(w)^{\mathrm{T}}y(t) + \\ - \frac{\partial^{\mathrm{T}} H}{\partial x}R(\pi(w),w)\frac{\partial H}{\partial x}_{|x=\pi(w)} \leq 0$$

As the trajectories of the system, on that manifold, are characterized by quasi-periodical energy values (there exists a time $\bar{T}$ such that $H_{tot}(k\bar{T}) = H_{tot}((k+1)\bar{T})$ for $k \in Z$), system must satisfies $\dot{H} = 0$, and then

$\frac{\partial^{\mathrm{T}} H}{\partial x} R(\pi(w), w) \frac{\partial H}{\partial x}_{|x=\pi(w)} = 0.$

It's easy to realize from equation (17), that proposition 3 is proved by LaSalle invariant principle, as $[\pi(w) \quad \tau(w) \quad w]^T$ is the only manifold providing $y(t) = 0$ and, on that manifold $\frac{\partial^{\mathrm{T}} H}{\partial x} R(\pi(w), w)$ $\frac{\partial H}{\partial x}_{|x=\pi(w)} = 0$. □

It is worth to note that (15) unfortunately does not allow to study tracking problems, as output map $h(x, w)$ (dual to the input u) is the real output of the plant and not the tracking error.

For tracking problem, the system to deal with should be

$$\begin{cases} \dot{x} = f(x,u) = [J(x) - R(x)]\dfrac{\partial H(x)}{\partial x} + g(x)u \\ \dot{w} = Sw = S\,\dfrac{\partial H_w(w)}{\partial w} \\ e = y + \bar{q}(w) = g^{\mathrm{T}}(x)\dfrac{\partial H(x)}{\partial x} + q^{\mathrm{T}}(w)\dfrac{\partial H_w(w)}{\partial w} \end{cases} \tag{18}$$

It easy to realize that, as input (u) and the output we want to control to zero (e) are not dual, system (18) doesn't fit in the PCHS framework.

Then, while a local study pointed out the possibility of tracking locally unknown trajectories, the tracking problem is still a big issue for a global characterization of the problem.

5. INPUT DISTURBANCES SUPPRESSION

To complete our discussion, let discuss about another important issue in the output regulation framework; consider the case of an unknown exogenous disturbance acting on the control input channel: we want to globally regulate the output of the plant in despite of the presence of that input disturbance. In order to present the main result, let consider a port-controlled Hamiltonian system of the form:

$$\begin{cases} \dot{x} = f(x,u,w) = (J(x) - R(x))\dfrac{\partial H(x)}{\partial x} + \\ \quad + g(x)u - g(x)q(w) \\ y = h(x) = g^{\mathrm{T}}(x)\dfrac{\partial H(x)}{\partial x} \\ \dot{w} = Sw \end{cases} \tag{19}$$

where the exosystem is still defined as in (4), (5) and (6).

Proposition 4: *Defining A, B, C as in (2), assume that the pair (A, B) is stabilizable, the pair (A, C) is detectable and there exist a mappings $x = \pi(w)$ and $u = c(w) = q(w)$, with $\pi(0) = 0$ and $c(0) = 0$, satisfying the conditions (7) with $c(w) = q(w)$ polynomial of the form (8), (9). If the unforced system $\dot{x} = f(x, 0, 0)$ is stable, then it's possible to design an output feedback controller able to assure the output going globally and asymptotically to zero.*

Proof: As *Proposition 2* holds, we are able to design an Hamiltonian internal model unit, and to write the whole system (plant+internal model unit+exosystem) as:

$$\begin{bmatrix} \dot{x} \\ \dot{z} \\ \dot{w} \end{bmatrix} = \left(\begin{bmatrix} J(x) & g(x)g_{mi}^{\mathrm{T}} & 0 \\ -g_{mi}g(x)^{\mathrm{T}} & J_{mi} & 0 \\ 0 & 0 & S \end{bmatrix} - \begin{bmatrix} R(x) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right) \begin{bmatrix} \dfrac{\partial H(x)}{\partial x} & \dfrac{\partial H_{mi}(z)}{\partial z} & \dfrac{\partial H_w(w)}{\partial w} \end{bmatrix}^T + \begin{bmatrix} g(x) & 0 & 0 \end{bmatrix}^T u_{st} + \begin{bmatrix} -g(x)q(w) & 0 & 0 \end{bmatrix}^T$$

$$y = \begin{bmatrix} g(x)^{\mathrm{T}} & 0 & 0 \end{bmatrix} \begin{bmatrix} \dfrac{\partial H(x)}{\partial x} & \dfrac{\partial H(z)_{mi}}{\partial z} & \dfrac{\partial H_w(w)}{\partial w} \end{bmatrix}^T \tag{20}$$

with $H_{mi} = \frac{1}{2} z^{\mathrm{T}} z$, $H_w = \frac{1}{2} w^{\mathrm{T}} w$, $g_{mi} = \Lambda^{\mathrm{T}}$ and $J_{mi} = \Psi$ as defined in section 3.

As system $\{W, Sw, c(w)\}$ is immersed into the linear observable system $\dot{z} = \Psi z$, $u = \Lambda z$ by a nonlinear map defined by $\tau(w) = T\bar{\tau}(w)$, we can state the following:

$$\begin{aligned} \dot{\tau}(w) &= \frac{\partial \tau}{\partial w} S w = \Psi \tau(w) \\ c(w) &= q(w) = \Lambda \tau(w) \end{aligned}$$

Defining a new coordinate as $\chi = z - \tau(w)$ and time-deriving we obtain:

$$\begin{aligned} \dot{\chi} &= \Psi z - \Psi \tau(w) - \Lambda^{\mathrm{T}} g^{\mathrm{T}}(x) \frac{\partial H}{\partial x} \\ &= \Psi \chi - \Lambda^{\mathrm{T}} g^{\mathrm{T}}(x) \frac{\partial H}{\partial x} = \Psi \frac{\partial H_\chi}{\partial \chi} - g_{mi} g^{\mathrm{T}}(x) \frac{\partial H}{\partial x} \end{aligned}$$

with $H_\chi = \frac{1}{2} \chi^{\mathrm{T}} \chi$.

Taking in account that $\Lambda\chi = \Lambda z - \Lambda\tau(w) = g_{mi}^{\mathrm{T}} \frac{\partial H_{mi}}{\partial z} - q(w)$, we could rewrite system (20) as a PCHS:

$$\begin{bmatrix} \dot{x} \\ \dot{\chi} \\ \dot{w} \end{bmatrix} = \left(\begin{bmatrix} J(x) & g(x)g_{mi}^{\mathrm{T}} & 0 \\ -g_{mi}g(x)^{\mathrm{T}} & J_{mi} & 0 \\ 0 & 0 & S \end{bmatrix} - \begin{bmatrix} R(x) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right) \begin{bmatrix} \dfrac{\partial H(x)}{\partial x} & \dfrac{\partial H_\chi(\chi)}{\partial \chi} & \dfrac{\partial H_w(w)}{\partial w} \end{bmatrix}^T + \begin{bmatrix} g(x) \\ 0 \\ 0 \end{bmatrix} u_{st}$$

$$y = \begin{bmatrix} g(x)^{\mathrm{T}} & 0 & 0 \end{bmatrix} \begin{bmatrix} \dfrac{\partial H(x)}{\partial x} & \dfrac{\partial H(z)_\chi}{\partial \chi} & \dfrac{\partial H_w(w)}{\partial w} \end{bmatrix}^T \tag{21}$$

As system (21) is similar to (16) with $H_{tot} = H + H_\chi + H_w = H + \frac{1}{2}\chi^{\mathrm{T}}\chi + \frac{1}{2} w^{\mathrm{T}} w$ and with an invariant zero output manifold now defined by $[\pi(w) \;\; 0 \;\; w]^T$ the proof can be completed following the one stated in section 4.

6. PERMANENT MAGNET SYNCHRONOUS MOTOR EXAMPLE

In this section, in order to point out the physical effectiveness of the input disturbance suppression result, we show some simple simulation results regarding a well known electro-mechanical problem: we want to stabilize a permanent magnet synchronous motor around its equilibrium point robustly in despite of some voltage disturbances occurring to the control inputs.

A permanent magnet synchronous motor (in a rotating reference, i.e. the dq frame) can be written as a port-controlled Hamiltonian system with dissipation (see

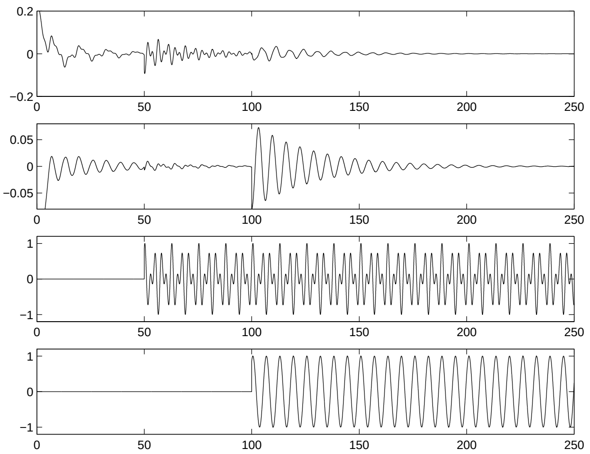

Fig. 2. From upper plot to lower: tracking error $\tilde{x}_1(t)$, $\tilde{x}_2(t)$ and input voltage disturbances $\delta_1(t)$ and $\delta_2(t)$

(van der Schaft, 1999), (Ortega *et al.*, 1999)) for the state vector

$$x = M \begin{bmatrix} i_d \\ iq \\ \omega \end{bmatrix}, \quad M = \begin{bmatrix} L_d & 0 & 0 \\ 0 & L_q & 0 \\ 0 & 0 & \dfrac{j}{n_p} \end{bmatrix}$$

where i_d and i_q are the currents, ω the angular velocity, L_d, L_q the stator inductances, j the inertia momentum and n_p the number of pole pairs. The Hamiltonian is defined by $H(x) = \frac{1}{2} x^{\mathrm{T}} M^{-1} x$ while $J(x)$, $R(x)$ and $g(x)$ are determined as

$$J(x) = \begin{bmatrix} 0 & L_0 x_3 & 0 \\ -L_0 x_3 & 0 & -\Phi_{q0} \\ 0 & \Phi_{q0} & 0 \end{bmatrix}$$

$$R(x) = \begin{bmatrix} R_s & 0 & 0 \\ 0 & R_s & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad g(x) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -\dfrac{1}{n_p} \end{bmatrix}$$

with R_s the stator winding resistance, Φ_{q0} a constant term due to interaction of the permanent magnet and the magnetic material in the stator, and $L_0 = L_d n_p / j$. Inputs are the stator voltages $(v_d, v_q)^{\mathrm{T}}$ and the load torque.

Considering a constant load torque τ_l, it is easy to realize that there exists an equilibrium point described by $\bar{x}_1$, $\bar{x}_2$ and $\bar{x}_3$; the whole system can be rewritten in the new (error) coordinates $\tilde{x} = (\tilde{x}_1 = x_1 - \bar{x}_1\,, \tilde{x}_2 = x_2 - \bar{x}_2\,, \tilde{x}_3 = x_3 - \bar{x}_3)^{\mathrm{T}}$ as

$$\begin{cases} \dot{\tilde{x}} = (\tilde{J}(\tilde{x}) - R(\tilde{x}))\dfrac{\partial H(\tilde{x})}{\partial \tilde{x}} + g[v_d \; v_q]^{\mathrm{T}} \\ \quad + \; g[\delta_1(t) \; \delta_2(t)]^{\mathrm{T}} \\ \tilde{y} = g^{\mathrm{T}} \dfrac{\partial H(\tilde{x})}{\partial \tilde{x}} \end{cases} \tag{22}$$

where $\tilde{J}(\tilde{x})$ is defined by

$$\tilde{J}(\tilde{x}) = \begin{bmatrix} \dfrac{\bar{x}_2 L_d L_0 \tilde{x}_3}{L_q \tilde{x}_1} & L_0(\tilde{x}_3 + \bar{x}_3) & 0 \\ -L_0(\tilde{x}_3 + \bar{x}_3) & -\dfrac{\bar{x}_1 L_0 L_q \tilde{x}_3}{L_d \tilde{x}_2} & -\Phi_{q0} \\ 0 & \Phi_{q0} & 0 \end{bmatrix}$$

and $\delta_1(t) = \sin(\Omega_1 t)\sin(\Omega_2 t)$, $\delta_2(t) = \sin(\Omega_3 t)$ are two sinusoidal disturbances acting on the voltage inputs. It is immediate to check that system (22) satisfies all conditions imposed in *Proposition 4*; we simulate the behavior of system (22) considering $\Omega_1 = 0.5\,rad/sec$, $\Omega_2 = 1\,rad/sec$, $\Omega_3 = 2\,rad/sec$, connected with an internal model unit and a dumping injection designed following the procedure introduced in section 5. Fig.2 shows the tracking errors $\tilde{x}_1(t)$ and $\tilde{x}_2(t)$ and the input disturbances $\delta_1(t)$ and $\delta_2(t)$.

7. CONCLUSIONS

In this paper the output feedback regulation problem for port-controlled Hamiltonian systems (PCHS) is discussed. Main results, stated in section 3, for a local solution of the problem, and in section 4, for a global solution, show the conditions to fulfill in order to design, following the classical *nonlinear output regulation* theory, a regulator which solves the problem as a parallel connection of two subcontrollers, both conserving the PCHS structure: an internal model unit and a stabilizer controller to stabilize the extended system composed by the plant and the internal model unit that is still a PCHS.

Moreover, in section 5, the same techniques are used to design an internal model based controller able to globally solve a problem of input disturbance suppression, i.e. to globally stabilize a system affected by an unknown exogenous input through the input channel. In section 6, some simulation results regarding a permanent magnet synchronous motor affected by some voltage disturbance are shown to confirm the effectiveness of the design.

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