Memorandum 1875 (September 2008). ISSN 1874-4850. Available from: http://www.math.utwente.nl/publications
Department of Applied Mathematics, University of Twente, Enschede, The Netherlands

Modelling in environments without numbers

Roxana Grigoraş, University of Bremen, Germany
Cornelis Hoede, University of Twente, The Netherlands

Introduction

In order to study how students are handling modelling situations, we address the type of tasks without an obvious mathematical character. The mathematical elements are somehow hidden and are to be elaborated by the students, if their solving strategy goes in that direction. The main reason why we elected such a kind of task is that we wanted to prevent students from concentrating on calculations, but to challenge them getting involved in argumentation processes.
In mathematics classrooms students can face such situations, when the task they are given to work on does apparently not fit into any of the subfields they know or that were previously dealt with. Therefore they are not able to decide which method could be appropriate for finding a solution.
Moreover, it is not often the case that students are asked to work on mathematical tasks where there is nothing to be calculated. Also for this reason, it is even questionable whether this kind of tasks belongs to mathematics. „How do the students tackle these problems?“ and „to which extent do they use mathematics?“ are challenging questions which indubitably deserve attention and study.
The overall goal of our study is to reconceptualise the term 'mathematising' in the context of classroom tasks, by empirical means, looking at what students see as being mathematics.

Research questions

The questions addressed by our study are:
1. How are 'environments without numbers' mathematically worked out by the students?
2. What does mathematising mean for students? What is mathematising in tasks where mathematics is not 'obvious'?
3. How does the need of mathematising occur/develop?
4. How does the contextual situation influence the notion of mathematising?

Theoretical background

Mathematising is the process taking place while modelling a real-life situation, i.e. solving a word-problem, by mathematical means. Modelling can be viewed as linking the two sides of mathematics, namely its grounding in aspects of reality, and the development of abstract formal structures (Greer, 1997). Modelling asks for certain cognitive demands, being determined by competencies like designing and applying problem solving strategies, arguing or representing, but it involves also communication skills, as well as real life knowledge (Blum and Borromeo-Ferri, 2007, Kaiser, 2006).
According to Freudenthal (1968), mathematising is the human activity consisting in organising matters from reality or mathematical matters, and represents therefore the core goal of mathematics education.
Treffers (1987) formulated in an educational context the idea of two ways of mathematising, originating from Freudenthal, distinguishing 'horizontal' and 'vertical' mathematisation. In the horizontal mathematisation, mathematical tools are promoted and used to organise and solve a real-life problem. Vertical mathematisation, in turn, supposes re-organisations and operations done by students within the mathematical system itself. Adopting Freudenthal's (1991) formulation, horizontally mathematising means to go from the real world to the world of symbols, whereas vertically mathematising means to move within the symbols' world.

Maria van den Heuvel-Panhuizen (2003) studied the didactical use of models, i.e. realistic contexts as determining characteristic of RME (Realistic Mathematics Education). Models are seen in RME context as representations of problem situations, reflecting essential aspects of mathematical concepts and structures that are relevant for the problem situation, but which can have various manifestations. Modelling is directly associated with mathematising. We see mathematising as the *activity* of observing, structuring and interpreting the world by means of mathematical models.
As Lambert[1] states, the patterns and structures students can find in their exploring are indicating the mathematical character of the modelling tasks: „... we mathematise our created real model of the observed situation – by (re-)identification of patterns and structures – and obtained so a mathematical model of the situation“. For that reason, finding/identifying patterns is considered a main mathematical activity in our research, and therefore part of mathematising.
Considering their structuring character, fundamental ideas in mathematics (Schweiger, 2006) play a big role in constructing mathematical knowledge. Fundamental ideas have their roots in „spiral learning“ concept and strategy of learning, introduced by Jerome Bruner. According to this, concepts are introduced in some early stage, in a rather intuitive way, then they are later on revisited and connected to other knowledge, and so on, to some higher level of abstraction and understanding. Spiral learning is possible due to the fundamental ideas. Sometimes they occur in a rather hidden way, implicitly or explicitly, but whenever they are to be found, it indicates the mathematical character of an activity. When fundamental ideas are related to the description of the given task, then it is mathematising.

Experiments so far

Experiments (among others, 'Africa', 'TSP' (Travelling Salesman Problem) tasks) with 12-14 aged children were performed in school-environment. Extended descriptions of these original tasks can be found in Grigoraş and Hoede (2008) and Grigoraş and Halverscheid (2008), respectively. These are tasks containing modelling aspects and were given to students with no special modelling experience, to be solved in groups of three - four members. About 90 minutes were at their disposal. Video-tapes and their transcripts of students' discussions were used as material for analysis.
It was looked for elements indicating *mathematical* activities and argumentation was identified as an important one. There have been no specific expectations from children, they were free to 'produce' whatever answers they felt like. Therefore, there was minimal intervention from teacher's side, only when asked for. A somehow similar approach in modelling real-life situations with no evident mathematical character, but with younger children and focusing on conjecturing is also treated by Mousoulides and English (2008).

Methodology

As future experiment, we will be giving the same task, but formulated in different ways, according to three alternative work-situations. That will presumably determine students to tackle the given task differently.

„In Africa there are territories consisting of three areas. On one side of a river, there is an area with grass and trees. On the other side, there are two areas, one only with grass, and one only with trees. These two areas are separated by mountains.

There are seven species of animals: antelopes, crocodiles, gnus, elephants, lions, monkeys and panthers. Grass and trees grow depending on the rain fall, grass serves as food for antelopes and gnus, trees give food to elephants and monkeys, lions feed on antelopes and gnus, panthers feed on

---
[1] www.math.uni-sb.de/ag/lambert/MuM_Vorlesung3Folien.pdf (accessed on 30-05-2008)

monkeys and crocodiles feed on gnus whenever these pass the river. Gnus pass the river if there is not enough grass on the side they are on. Elephants can also pass the river if they want to get to another area with trees, without being threatened by crocodiles.

a) In which areas do you think the species will be living? Consider a drawing of the situation and indicate where grass, trees, as well as each species is to be found.

bi) Give two or three properties of the animals.

bii) Group the animals according to the chosen properties.

ci) Suppose that in the area where there are grass and trees, it stops raining for a period of some months. Investigate what happens in the three areas with all the species.

cii) Investigate what happens in the three areas with all the species, if it will not rain for some months in the area where only grass grows.

ciii) Investigate what happens in the three areas with all the species, if it will not rain for some months in the area where only trees grow.“

The three alternative formulations of the task are:

1. Work out the problem and look for patterns (structures) and describe them.

2. Approach the assignment mathematically and look for mathematical patterns (structures), and thoroughly describe them.

3. You have been given a map and a box with 'animals'. Use these to solve the problem. Look for patterns (structures).

The experiments will be performed after the summer-holiday, in the university environment, with approximately ten groups of 6th class students consisting of three teams each made of three children, different children for each of the three working-situations. A group of children will have to deal with the problem just once, meaning to treat it in only one of the three alternative situations previously described. The students will be video-taped while working on the tasks and their transcripts will be studied.

Data analysis

The aim of the research is identifying, classifying and studying items which indicate mathematisation elements, as well as what do students consider and believe to be mathematics.
The means to measure the outcome can be, for example, of linguistic nature, words (or gestures) that make it possible to conclude that mathematising took place. These words can hint at horizontal modelling, i.e. mathematising (cf. van den Heuvel-Panhuizen, 2003), or at vertical modelling. A list of these words for these three activities could be made up.
Once the sets of words or gestures are fixed, the assessment can take place in a systematic way. If e.g. no words related to vertical mathematising were used, then that goal was not reached.
We will presumably have students who 'just' do horizontal mathematisation, since vertical mathematising involves higher mathematical activities, levels and presuppose quite some experience.

Horizontal mathematisation	**Vertical mathematisation**
- plural: *lion-lions* (set idea) - words: *larger*, *smaller*, *more*, *less*, *fewer* (ordering idea) - *same* as word (equality) - *many – few* (concept of number) - *represents* corresponds to relations	- number - set - ordering - 'equal' sign - symbols: L, A, G, P, ... - formulae - *relation* as word - other words referring to essentially mathematical concepts

Table 1: Words indicating various mathematisation types

Students perceive 'things' and create, see, imagine relationships between them, for example between lions and antelopes. What is the relationship they see between them, how do they relate them to other things? Every time a kind of graph structure comes up in the mind. We can call that a 'pattern'. The children are told that 'patterns are relationships.'
In formulation 2 we tell students that giving a mathematical representation is also a pattern (real world – mathematical world relationships), (Grigoraş and Halverscheid, 2008), like *lions–L*. And even that between representations there are also patterns (mathematical world- mathematical world relationships).

In the following figure, we can see how horizontal and vertical mathematisation are to be found in our 'Africa' task. We build up a 'ladder' starting with a vertical line representing a (r-r)-relationship (lions-antelopes). Then come two horizontal links, (r-m)-relationships. Choosing L and A for representation is a horizontal mathematisation, cf. (Freudenthal, 1991). Then comes a vertical link again, a (m-m)-relationship between L and A, representing the (r-r)-relationship in the m-world. From m-world we can go to m*-world, e.g. by going to |L| and |A|, so from the set idea to the cardinality idea. From m*-world we can go to m**-world with two horizontal and one vertical link again, depending on what we do it for (for instance, we study the variation in time of the number of lions and antelopes).
If we interchange horizontal and vertical, i.e. put the 'ladder', say, against a wall, then the feet, horizontally connected, are in the real world, the transformations are then vertical and lead to the first step of the 'ladder', that is mathematical. All the other steps are mathematical and lead higher and higher up into the mathematical world.

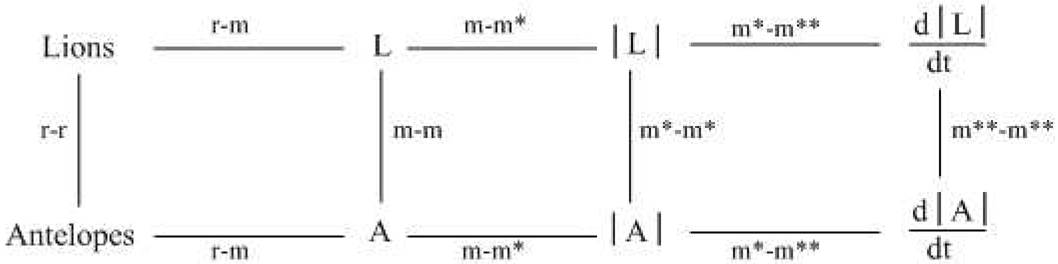

Figure 1: Freudenthal's horizontal and vertical mathematisation for our case-study

This description gives a strict correspondence with Freudenthal's (1991) and Treffers' (1987) discussion of mathematisation.

Discussion

Of course, it is most probable that students in our experiment will make only the first step. At higher age, mathematics students should be able to go further 'up the ladder'.

The 'ladder' description is given as it corresponds with the theories of Freudenthal and Treffers. The latter author stressed the difference between "modelling OF" and "modelling FOR". The next step up the ladder always has a specific aim. In our case in the first step the aim is to chose mathematical symbols and relate them in correspondence with relationships seen in the real world. The exploration of r-r relationships in the real world is seen as part of horizontal mathematisation.

We would like to argue that seeing those r-r relationships is (literally) parallel to seeing the m-m relationships, or the following relationships "up the ladder". Therefore seeing those r-r relationships by students might also be called vertical mathematisation. One should not forget that also the „real world“ is just an image in the mind.

References

Blum W., Borromeo-Ferri R., *Mathematical modelling: Can it be taught and learnt?*, Presented during 'The future of Mathematics Education in Europe', 17 Dec 2007, Lisbon, Portugal.

Freudenthal, H. (1968). *Why to teach mathematics as to be useful?*, Educational Studies in Mathematics, 1, p. 3-8.

Freudenthal, H. (1991). *Revisiting Mathematics Education. China Lectures*, Kluwer Academic Publishers, Dordrecht, The Netherlands.

Greer, B. (1997). *Modelling reality in mathematics classrooms: the case of word problems*, Learning and Instructions, Vol. 7, No. 4, p. 293-307, Elsevier Science Ltd, Great Britain.

Grigoraş, R., Hoede, C. (2008). *Reasoning indicators – a case study*, 42 Jahrestagung der Gesellschaft fuer Didaktik der Mathematik, Budapest, Hungary.

Grigoraş, R., Halverscheid, S. (2008). *Modelling the travelling salesman problem: relationship between the world of mathematics and the rest of the world*, In Proceedings of the 32nd Conference of the International Group for the Psychology of Mathematics Education, Morelia, Mexico: PME.

Kaiser, G. (2006). *Modelling and modelling competencies in school,* In C. P. Haines, P. Galbraith, W. Blum, & S. Khan (Eds.), Mathematical Modelling (ICTMA 12): Education, Engineering and Economics, p. 110-119. Chichester: Horwood Publishing.

Mousoulides, N.G., English, L.D. (2008). *Modelling with data in cypriot and australian primary classrooms*, presented at the 32nd Conference of the International Group for the Psychology of Mathematics Education, Morelia, Mexico: PME.

Schweiger, F. (2006). *Fundamental ideas. A bridge between mathematics and mathematical education*, in Maasz, J. and Schloeglmann, W. (Eds.), New Mathematics Education Research and Practice, Sense Publishers.

Treffers, A. (1987). *Three Dimensions. A Model of Goal and Theory Description in Mathematics Instruction – The Wiskobas Project*, Reidel Publishing Company, Dordrecht, The Netherlands.

van den Heuvel-Panhuizen, M. (2003). *The didactical use of models in realistic mathematics education: an example from a longitudinal trajectory on percentage*, Educational Studies in Mathematics, 54, p. 9-35, Kluwer Academic Publishers, The Netherlands.