Proof of Node Densities

W. Klein Wolterink, G. Heijenk, J.L. van den Berg

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Centre for Telematics and Information Technology (CTIT)
University of Twente, Enschede, The Netherlands

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Abstract

In this paper we present an analytical model accurately describing the forwarding behaviour of a multi-hop broadcast protocol. Our model covers the scenario in which a message is forwarded over a straight road and inter-node distances are distributed exponentially. Intermediate forwarders draw a small random delay before forwarding a message such as is done in flooding protocols to avoid the broadcast storm problem.

The analytical model presented in this chapter focuses on having a message forwarded a specific distance. For a given forwarding distance and a given node density our model analysis is able to capture the full distribution of *(i)* the end-to-end delay to have the message forwarded the entire distance, *(ii)* the required number of hops to have the message forwarded the entire distance, *(iii)* the position of each intermediate forwarder, *(iv)* the success probability of each hop, *(v)* the length of each hop, and *(vi)* the delay of each hop. The first three metrics are calculated assuming that the message is successfully forwarded the entire forwarding distance.

The model provides the results in terms of insightful, fast-to-evaluate closed-form expressions. The model has been validated by extensive simulations: modelling results stayed within typically 10%, depending on the source-to-sink distance and the node density.

1 Introduction

In this document we analytically model the behaviour of a multi-hop broadcast protocol. Specifically we consider a scenario in which nodes are spread out over a straight line with the source at one end and the sink at the other end. The source node initiates the forwarding by broadcasting an application message. The message has a geographically defined destination address which includes the position of the sink. All nodes apply the following forwarding rule: when a node receives a message for the first time, and the node is positioned closer to the sink than the previous sender, the node draws a forwarding delay that is exponentially distributed with mean T_d. If before the end of the delay the node receives the message from another node that is positioned closer to the sink than the node itself, then the node will cancel the scheduled rebroadcast.

Multi-hop broadcast protocols such as these, in which nodes have identically distributed forwarding delays, are often employed by delay tolerant flooding protocols. These are protocols that aim to deliver information to all nodes within a certain region but that do not have strict delay requirements. In vehicular networks such flooding protocols are used to disseminate non-safety local traffic information, such as the average speed on the road or dangerous road conditions [12] [14].

Although several studies exist on analytically modelling multi-hop forwarding in wireless networks, so far we have not found any models that use assumptions that apply to our scenario. Especially regarding the level of realism of modelling single-hop transmissions existing work is lacking, as often a fixed transmission range is assumed. In contrast, in this study we model the probability of a successful single-hop transmission as a function of the distance between sender and receiver. Moreover, whereas the focus of existing models is often limited to network connectivity, dissemination reliability, or end-to-end delay bounds, our model gives a full distribution of a number of performance metrics.

The contribution of this document is an analytical model that expresses the performance of a multi-hop broadcast protocol as presented above in terms of insightful and fast-to-evaluate formulas. Our model covers the scenario in which a message is forwarded a specific distance over a straight road, assuming exponentially distributed inter-node distances. In particular, for a given forwarding distance and transmission function, the model gives expressions of the following performance metrics:

1. the distribution of the end-to-end delay;
2. the distribution of the required number of hops;
3. the distribution of the position of each forwarder;
4. the success probability of each op;
5. the distribution of the length of each hop;
6. the distribution of the delay of each hop.

The model analysis applies to message that have successfully been forwarded for the entire forwarding distance only; the effect of message loss on the end-to-end metrics is left for future work. The model has been verified using extensive simulations. For the most relevant scenarios results typically stay within 10%; as node densities decrease and dissemination distances increase the model becomes less accurate.

We have split our model analysis into two parts: the first part shows how to express the behaviour of the first three hops of the forwarding protocol in an exact manner. Although this method can be applied for following hops as well, doing so becomes increasingly complex with each following hop. Based on the results of the first part we therefore show how to approximate the behaviour of the forwarding protocol for an arbitrary number of hops in the second part.

Before presenting our analytical model we first discuss some of the work that has been done previously on analytically modelling multi-hop forwarding in the next section.

2 Related work

Although there is a plethora of performance studies on multi-hop forwarding protocols in vehicular networks, practically all of these studies are simulation based. The available analytical studies mainly focus on network connectivity, i.e., the probability that a route exists between a source and a sink [18] [15] [17], or have assumptions that do not apply to our scenario. Below we briefly discuss some of the more relevant analytical studies on multi-hop forwarding. Their relevance to forwarding scenario considered in this report is discussed at the end of the section.

In [9] a scenario is considered in which a message is forwarded by means of broadcast transmissions over a straight line with fixed inter-node distances. When a car transmits a message, all nodes within a certain range from the sender have the same probability p $(0 \leq p \leq 1)$ of correctly receiving the message in absence of interference. Interference of transmissions may be taken into account and if so will result in a loss. Which node becomes the next forwarder depends on the dissemination strategy that is used: three such strategies are evaluated. Forwarding is performed in communication rounds with a constant forwarding delay between each round.

In [4] the end-to-end delay of an emergency message dissemination protocol is analytically calculated. A fixed transmission range and exponentially distributed inter-node distances are assumed. Nodes are assumed to have formed communication clusters with each cluster of nodes having a cluster head node. All forwarding is done by the head nodes, which makes it relatively easy to calculate the end-to-end delay. So far no standardisation on clustering has been performed however.

In [16] the required number of hops to disseminate a message from source to sink is analytically modelled. Nodes are spread out over a straight line with exponentially distributed inter-node distances, with a fixed transmission range. The node that lies furthest in the direction of the sink is assumed to forward the message, similar to how forwarding is done in distance-based forwarding. The model is quite accurate for high node densities and large distances but less so when densities are low and distances are short. Hop delays and end-to-end delays are not taken into account.

In [11] a straight road with exponentially distributed inter-node distances and a fixed transmission range are considered. Two forwarding strategies are considered. In the first each node that has received the message and that lies closer to the sink than the previous forwarding node will forward the message with a certain probability p $(0 \leq p \leq 1)$. The forwarding delay is considered constant for each hop. With the second strategy the forwarding delay is a function of the node's distance to the previous forwarder. The model gives bounds on the required number of hops to have a message forwarded a certain distance, as well as the end-to-end delay to have a message forwarded a certain distance.

For various reasons none of the studies described here can be applied to our forwarding scenario. Most importantly, all of the studies use strongly simplified assumptions regarding single-hop transmission. Inter-node distances are moreover fixed in [9], and none of the methods used in [9], [4] and [16] to determine the next forwarder apply to our forwarding protocol. Although the forwarding rules applied in [11] are quite similar to those in the forwarding protocol considered in our scenario, the lack of a realistic transmission model prevents the model from being used.

3 The system model

In this section we present our system model: an abstract representation of the forwarding scenario considered that forms the basis of our analysis in subsequent sections. We also specify the forwarding rules of the protocol and introduce definitions and notations that are used throughout this document. A complete overview of these is given in Table 1.

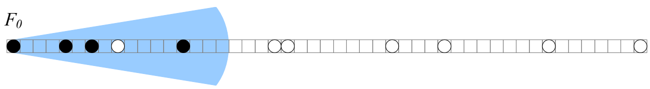
We model a road as a straight line with vehicles (henceforth referred to as nodes) placed on this line, with the source node and sink node at either end of the line. Inter-node distances are exponentially distributed with mean d_{IN} (in meters). Previous studies suggest that this distribution gives a good approximation of the inter-vehicle distance in case of free flowing traffic [6] [13] [5]. Due to the differences in scale between the speed with which information is usually routed through a network (meter per millisecond) and the speed with which nodes move (meter per second), we assume the network to be static for the duration of time that a message is being forwarded from source to sink. Nodes are therefore immobile.

Fig. 1 illustrates the system model. Nodes are numbered and referred to with X_i, $i = 0, 1, \ldots, 10$. Node X_0 acts as the source, node X_{10} acts as the sink. The first three hops are shown and with each hop the nodes that have received the message are coloured black. Initially only the source has the message.

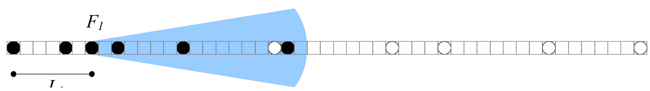
To facilitate our analysis we divide the road into equal-sized intervals: starting from the source the road is divided into intervals of length d_{int}, with the i^{th} interval referring to the range $\langle (i-1) \cdot d_{int}, i \cdot d_{int}]$ from the source. In our analysis the size of d_{int} is such that the probability of having more than one node in an interval becomes negligible. *For the remainder of our analysis an interval is therefore assumed to have either zero or one node(s).*

To model the propagation of a transmitted signal from a sender to a receiver we use a packet success ratio S_i that gives the success probability of a single-hop transmission as a function of the number of intervals i between sender and receiver. Each node thus has an independent probability S_i of receiving a transmission. The packet success rate S_i is non-zero over the range $[1, R]$, where R is the maximum number of intervals away from the sender at which the

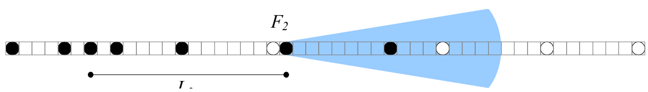
(a) Distances between the black nodes are exponentially distributed.



(b) The source broadcasts the message.



(c) Node C acts as the first forwarder and retransmits the message.



(d) Node F acts as the second forwarder and retransmits the message.

Figure 1: The 0^{th}, first, and second hop of an example scenario. The blue shape shows the maximum transmission distance R from the most recent forwarder. Black nodes have received the message.

receiver still has a non-zero probability of receiving the message. An abstraction such as S_i is commonly used to take into account fading effects that influence the reception of a signal. It ignores deterministic shadowing effects (e.g., due to an obstruction) however, since the signal reception probability is independent for each node and for each interval.

All delays related to transmitting and processing a signal (i.e., transmission delay, propagation delay, switching times, etc.) are assumed to be negligible.

The source node initiates the forwarding by broadcasting an application message. The message has a geographically defined destination address which includes the position of the sink. All nodes apply the following forwarding rule: when a node receives a message for the first time, and the node is positioned closer to the sink than the previous sender, the node draws a forwarding delay that is exponentially distributed with mean T_d. If before the end of the delay the node receives the message from another node that is positioned closer to the sink than the node itself, then the node will cancel the scheduled rebroadcast. In this way a message is progressively forwarded in the direction of the sink.

We use the following notation throughout the document. The n^{th} forwarder is the node that retransmits the message for the n^{th} time after the source's original transmission; F_n denotes the interval in which it is positioned. Although not a forwarder since it originates the message, the source node is referred to as the 0^{th} forwarder and is by definition positioned in interval 0, i.e., $F_0 = 0$. Because there can be at most one node in an interval two forwarders can never be in the same interval, i.e., $F_n > F_k$ for $n > k$. In Fig. 1 F_0, F_1, and F_2 are given.

The n^{th} hop refers to the transmission made by the n^{th} forwarder; the source's transmission is by definition hop 0. Fig. 1 shows the 0^{th}, first, and second hop. The hop length of the n^{th} hop L_n refers to the distance in intervals between the n^{th} forwarder and the $(n-1)^{th}$ forwarder, i.e., $L_n = F_n - F_{n-1}$. By definition $L_0 = 0$; Fig. 1 illustrates L_1 and L_2. The hop delay D_n of the n^{th} hop refers to the time between the moment the $n - 1^{th}$ forwarder transmits the message and the moment the n^{th} forwarder transmits the message.

In our model we focus on the progress that a message makes as it is forwarded through the network. Let N_i denote the number of hops required to have the message forwarded i intervals, i.e., to have it forwarded by a node that is positioned in interval i or beyond. The end-to-end delay to have the message forwarded i intervals is denoted E_i and is the sum of the delays of the required hops, given by $E_i = \sum_{n=1}^{N_i} D_i$.

Each time the message has been forwarded there will be a set of nodes that have all received the message and are all positioned closer to the sink node than the most recent forwarder. Since one of these nodes will become the next forwarder we call these nodes *candidate forwarders*. Let C_n be the number of candidate forwarders for the n^{th} hop, and let $C_{n,i}$ be the number of candidate forwarders for the n^{th} hop in interval i. In Fig. 1 nodes X_1, X_2, and X_4 are all first-hop candidate forwarders. The number of nodes in interval i that have not received the message from either the source or one of the $n-1$ previous forwarder, and have therefore not become n^{th}-hop candidate forwarder, is denoted $K_{n,i}$. In Fig. 1 node X_3 is the only such node. By definition it holds that the total number of nodes in interval i, V_i, is given by

$$V_i = C_{n,i} + K_{n,i}, \quad n = 1, 2, \ldots, \quad i = F_{n-1} + 1, F_{n-1} + 2, \ldots \tag{1}$$

Sometimes we are interested in the set of n^{th}-hop candidate forwarders that did not become the n^{th} forwarder. In Fig. 1 the set of first-hop candidate forwarders that did not become the first forwarder consists of nodes X_1 and X_4. Let $H_{n,i}$ denote the number of n^{th}-hop candidate forwarders in interval i, excluding the n^{th} forwarder itself, and let $G_{n,i}$ denote the number of n^{th}-hop forwarders in interval i. By definition it holds that

$$C_{n,i} = H_{n,i} + G_{n,i}, \quad n = 1, 2, \ldots, \quad i = F_{n-1} + 1, F_{n-1} + 2, \ldots \tag{2}$$

For each hop beyond the first hop the set of candidate forwarders consists of nodes that received the message for the first time from the most recent forwarder and of nodes that received it from some previous forwarder. The n^{th}-hop candidate forwarders that received the message for the first time from the $(n-1)^{\text{th}}$ forwarder are referred to as additional n^{th}-hop candidate forwarders. In Fig. 1c the set of additional second-hop candidate forwarders consists of nodes X_3 and X_6. The number of additional n^{th}-hop candidate forwarder is denoted A_n; the number of additional n^{th}-hop candidate forwarder in interval i is denoted $A_{n,i}$. By definition it holds that

$$A_{n,i} = C_{n,i} - C_{n-1,i}, \quad n = 1, 2, \ldots, \quad i = F_{n-1} + 1, F_{n-1} + 2, \ldots \tag{3}$$

with $C_{0,i} = 0$ by definition.

A_n	The number of additional n^{th}-hop candidate forwarders, i.e., the number of n^{th}-hop candidate forwarders i that first received the message from the $(n-1)^{\mathrm{th}}$ forwarder. It holds that $A_n = C_n - C_{n-1}$.
$A_{n,i}$	The number of additional n^{th}-hop candidate forwarders in interval i.
C_n	The number of n^{th}-hop candidate forwarders.
$C_{n,i}$	The number of n^{th}-hop candidate forwarders in interval i.
D_n	The hop delay (in seconds) of the n^{th} hop, i.e., the time between the moment the $(n-1)^{\mathrm{th}}$ forwarder forwards the message and the moment the n^{th} forwarder forwards the message.
E_i	The end-to-end delay (in seconds) to have the message forwarded by a node that is positioned in interval i or beyond.
F_n	The position (in intervals) of the n^{th} forwarder, i.e., the forwarder that retransmits the message for the n^{th} time after the source's transmission. The source is by definition position in interval 0, i.e., $F_0 = 0$.
$G_{n,i}$	The number of n^{th} forwarders in interval i.
$H_{n,i}$	The number of n^{th}-hop candidate forwarders in interval i, excluding the n^{th} forwarder.
$K_{n,i}$	The number of nodes in interval i that have not received the message from either the source or any of the first n forwarders. It holds that $K_{n,i} = V_{n,i} - C_{n,i}$.
L_n	The hop length (in intervals) of the n^{th} hop, defined as the distance between the $(n-1)^{\mathrm{th}}$ forwarder and the n^{th} forwarder, i.e., $L_n = F_n - F_{n-1}$.
N_i	The number of hops required to have the message forwarded by a node that is positioned in interval i or beyond.
S_i	The single-hop packet reception probability as a function of the distance (in intervals) between the sender and receiver.
R	The maximum transmission range in intervals.
T_d	The mean per-hop forwarding delay in seconds.
V_i	The number of nodes in interval i.

Table 1: Nomenclature used throughout this document.

4 Exact analysis of the first three hops

In this section we given an exact analysis of the system model for the first three hops. Although the method presented here can be applied for an arbitrary number of hops, it becomes increasingly complex with each hop however. We therefore determine the behaviour of the first three hops only. Based on the results of this section we then give a number of approximate methods in the next section that allow for fast calculation of hop metrics and end-to-end metrics for an arbitrary number of hops.

To determine the behaviour of a hop we require *(i)* the distribution of the number of candidate forwarders and *(ii)* how they are positioned. For the first two hops we specify both, allowing us to express the hop success probability, the position of the forwarder, the hop length, and the hop delay. For the third hop we specify how the candidate forwarders are positioned only, and for this reason only express the position of the forwarder and the hop length. For all hop metrics a full distribution is given.

We specify the behaviour of each hop separately. For each hop holds that we first determine the distribution of the candidate forwarders and then its hop metrics.

Throughout this section we clarify some of our modelling steps using (intermediate) results from an evaluation study that we performed. The set-up of this study is described in Section 6.1. Results include both analytical results and simulation results; analytical results are illustrated using solid lines while simulation results are illustrated using dashed lines.

4.1 First hop

4.1.1 Candidate forwarders

Since inter-node distances are distributed exponentially with mean d_{IN} m and intervals have a length of d_{IN} m, the distribution of V_i is given by the Poisson distribution with mean $\mathbb{E}(V_i)$ given by

$$\mathbb{E}(V_i) = \frac{d_{int}}{d_{IN}}, \quad i = 1, 2, \ldots \tag{4}$$

Note that the V_i's, $i = 1, 2, \ldots$, are independent.

Since a node in interval i has a probability S_i of becoming a first-hop candidate forwarder, and the number of nodes in interval i is Poisson distributed with mean $\mathbb{E}(V_i)$, the number of first-hop candidate forwarders in interval i is Poisson distributed with mean

$$\mathbb{E}(C_{1,i}) = \mathbb{E}(V_i) \cdot S_i, \quad i = 1, 2, \ldots, R, \tag{5}$$

with $\mathbb{E}(C_{1,i}) = 0$ for other values of i.

The total number of first-hop candidate forwarders is equal to the sum of first-hop candidate forwarders in the R intervals following the source. According to [10] the sum of a number of independent Poisson distributed random variables

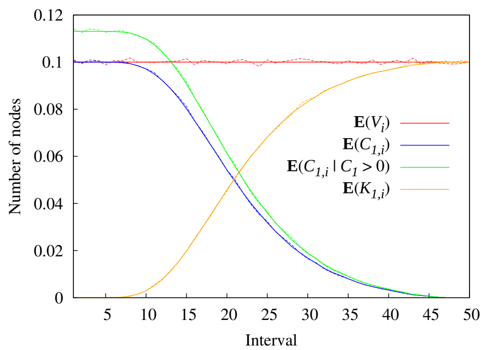


Figure 2: Expected values of V_i, $C_{1,i}$, $K_{1,i}$ for $d_{IN} = 50$ m and $d_{int} = 5$.

is also Poisson distributed, with its mean equal to the summed up means. Hence, C_1 has a Poisson distribution with mean

$$\mathbb{E}(C_1) = \sum_{i=1}^{R} \mathbb{E}(C_{1,i}) \tag{6}$$

$$= \sum_{i=1}^{R} S_i \mathbb{E}(V_i). \tag{7}$$

The probability of having at least one first-hop candidate forwarder is given by

$$\begin{aligned} \mathbb{P}(C_1 > 0) &= 1 - \mathbb{P}(C_1 = 0) \\ &= 1 - e^{-\sum_{i=1}^{R} S_i \mathbb{E}(V_i)}. \end{aligned} \tag{8}$$

Because we will need it later on, we determine here the distribution of the number of first-hop candidate forwarders, given that there is at least one first-hop candidate forwarder. It is calculated by normalising the distribution of C_1 with respect to $\mathbb{P}(C_1 > 0)$:

$$\begin{aligned} \mathbb{P}(C_1 = c_1 \mid C_1 > 0) &= \frac{\mathbb{P}(C_1 = c_1)}{\mathbb{P}(C_1 > 0)} \\ &= \frac{\frac{\lambda^{c_1}}{c_1!} e^{-\lambda}}{1 - e^{-\lambda}}, \quad \lambda = \sum_{i=1}^{R} S_i \mathbb{E}(V_i), \quad c_1 \in \mathbb{N}^+, \end{aligned} \tag{9}$$

with $\mathbb{P}(C_1 = c_1 \mid C_1 > 0) = 0$ for other values of c_1.

The expected number of first-hop candidate forwarders in interval i, given that there is at least one first-hop candidate forwarder is given by

$$\begin{aligned}
\mathbb{E}(C_{1,i} \mid C_1 > 0) &= \sum_{c_{1,i}=0}^{\infty} c_{1,i} \cdot \mathbb{P}(C_{1,i} = c_{1,i} \mid C_1 > 0) \\
&= \frac{1}{\mathbb{P}(C_1 > 0)} \cdot \sum_{c_{1,i}=1}^{\infty} c_{1,i} \cdot \mathbb{P}(C_1 > 0 \mid C_{1,i} = c_{1,i})\, \mathbb{P}(C_{1,i} = c_{1,i}) \\
&= \frac{\mathbb{E}(C_{1,i})}{\mathbb{P}(C_1 > 0)} \\
&= \frac{S_i \mathbb{E}(V_i)}{1 - e^{-\sum_{j=1}^{R} S_j \mathbb{E}(V_j)}},
\end{aligned} \tag{10}$$

with $\mathbb{E}(C_{1,i} \mid C_1 > 0) = 0$ for other values of i. Fig. 2 shows $\mathbb{E}(C_{1,i} \mid C_1 > 0)$ as a function of interval number i.

Finally, since the number of first-hop candidate forwarders in an interval is an independent Poisson process, the expected number of first-hop candidate forwarders, given that there is at least one first-hop candidate forwarder, is given by

$$\begin{aligned}
\mathbb{E}(C_1 \mid C_1 > 0) &= \sum_{i=1}^{R} \mathbb{E}(C_{1,i} \mid C_1 > 0) \\
&= \frac{\sum_{i=1}^{R} S_i \mathbb{E}(V_i)}{1 - e^{-\sum_{i=1}^{R} S_i \mathbb{E}(V_i)}},
\end{aligned} \tag{11}$$

4.1.2 Probability of success

The probability of success of the first hop is equal to the probability of having a first forwarder, i.e., of having at least one candidate forwarder for the first hop:

$$\mathbb{P}(\text{‘successful first hop’}) = \mathbb{P}(C_1 > 0), \tag{12}$$

with $\mathbb{P}(C_1 > 0)$ given by Eq. (8).

4.1.3 Position of the forwarder

For a given set of candidate forwarders, the candidate forwarder that has the shortest residual forwarding delay will become the next forwarder. Candidate forwarders draw their forwarding delay when they receive the message for the first time. Since the forwarding delay is distributed exponentially, and the exponential distribution is memoryless, the residual forwarding delay is i.i.d. with mean T_d for each candidate forwarder, regardless when the candidate forwarder first received the message. Thus, for a given set of candidate forwarders the probability of becoming the next forwarder is equal for all candidate forwarders.

Since the probability of becoming the next forwarder is equal for all candidate forwarders, the probability that the first forwarder will be located in interval i,

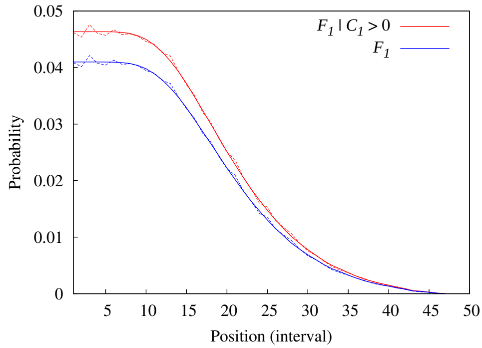


Figure 3: The distribution of the position of the first forwarder, for $d_{IN} = 50$ m and $d_{int} = 5$ m.

given that there is a first forwarder, is equal to the expected value of the number of candidate forwarders in interval i normalised over the expected total number of first-hop candidate forwarders, given that there is a first forwarder. As we prove in Appendix A.1.1 this is given by

$$\begin{aligned} \mathbb{P}(F_1 = i \mid C_1 > 0) &= \mathbb{E}\Big(\frac{C_{1,i}}{C_1} \mid C_1 > 0\Big) \\ &= \frac{\mathbb{E}(C_{1,i})}{\mathbb{E}(C_1)}, \quad i = 1, 2, \ldots, R. \end{aligned} \tag{13}$$

with $\mathbb{P}(F_1 = i \mid C_1 > 0) = 0$ for other values of i.

The probability that the first forwarder is positioned in interval i is equal to $\mathbb{P}(F_1 = i \mid C_1 > 0)$ multiplied with the probability that there is a first forwarder:

$$\mathbb{P}(F_1 = i) = \mathbb{P}(C_1 > 0) \cdot \mathbb{P}(F_1 = i \mid C_1 > 0), \quad i = 1, 2, \ldots, R, \tag{14}$$

with $\mathbb{P}(F_1 = i) = 0$ for other values of i and $\mathbb{P}(C_1 > 0)$ given by Eq. (8). Fig. 3 illustrates $\mathbb{P}(F_1 = i)$.

Given that there is first forwarder, the expected number of first forwarders in interval i is equal to the probability that the first forwarder is positioned in interval i, i.e.,

$$\mathbb{E}(G_{1,i} \mid C_1 > 0) = \mathbb{P}(F_1 = i \mid C_1 > 0), \quad i = 1, 2, \ldots \tag{15}$$

4.1.4 Hop length

Since the source is by definition positioned in interval 0, the distribution of the hop length of the first hop is equal to the distribution of the position of the first forwarder, given that there is a first forwarder, i.e.,

$$\mathbb{P}(L_1 = l_1) = \mathbb{P}(F_1 = l_1 \mid C_1 > 0), \quad l_1 = 1, 2, \ldots, R, \tag{16}$$

with $\mathbb{P}(L_1 = l_1) = 0$ for other values of l_1.

4.1.5 Hop delay

For each hop holds that the candidate forwarder that has the shortest forwarding delay will become the next forwarder. The hop delay is therefore distributed as the minimum residual forwarding delay of all the first-hop candidate forwarders. The first-hop candidate forwarders draw their forwarding delay when they first receive the message. Since the forwarding delay is distributed exponentially and the exponential distribution is memoryless, the forwarding delay of each candidate forwarder is identical, independent, and exponentially distributed with mean T_d. Given that there are c candidate forwarders, the hop delay is therefore distributed as the minimum value of c forwarding delays, that are each exponentially distributed with mean T_d. This minimum value is itself exponentially distributed with mean T_d/c. To calculate the distribution of the n^{th}-hop hop delay we therefore have to condition on the number of n^{th}-hop candidate forwarders, given that there is an n^{th} forwarder. The CDF of the hop delay of the first hop is thus given by

$$F_{D_1}(t) = 1 - \sum_{c_1=1}^{\infty} \mathbb{P}(C_1 = c_1 \mid C_1 > 0) \ \cdot \ e^{(c_1 \cdot \, t)/T_d}, \quad t > 0, \tag{17}$$

with $F_{D_1}(t) = 0$ for $t \leq 0$ and $\mathbb{P}(C_1 = c_1 \mid C_1 > 0)$ given by Eq. (9).

4.2 Second hop

4.2.1 Candidate forwarders

We first determine the expected number of second-hop candidate forwarders in an interval, given the position of the first forwarder, denoted $\mathbb{E}(C_{2,i} \mid F_1 = j)$, and then the distribution of the total number of second-hop candidate forwarders, given the position of the first forwarder, denoted $C_2 \mid F_1 = j$.

The set of second-hop candidate forwarders in an interval consists of remaining first-hop candidate forwarders (excluding the first forwarder itself) and additional second-hop candidate forwarders, i.e., $C_{2,i} = H_{1,i} + A_{2,i}$. We are interested in their expected values for a given position j of the first forwarder:

$$\mathbb{E}(C_{2,i} \mid F_1 = j) = \mathbb{E}(H_{1,i} \mid F_1 = j) + \mathbb{E}(A_{2,i} \mid F_1 = j), \quad i = j+1, j+2, \ldots \tag{18}$$

To determine $\mathbb{E}(C_{2,i} \mid F_1 = j)$ we first calculate $\mathbb{E}(H_{1,i} \mid F_1 = j)$, then $\mathbb{E}(A_{2,i} \mid F_1 = j)$.

To calculate the expected number of first-hop candidate forwarders in an interval, excluding the first forwarder itself and given that there is a first forwarder, we take the expected values of Eq. (2), condition on the existence of a first forwarder, and rearrange terms:

$$\mathbb{E}(H_{1,i} \mid C_1 > 0) = \mathbb{E}(C_{1,i} \mid C_1 > 0) - \mathbb{E}(G_{1,i} \mid C_1 > 0), \quad i = 1, 2, \ldots, R, \tag{19}$$

with $\mathbb{E}(H_{1,i} \mid C_1 > 0) = 0$ for other values of i, $\mathbb{E}(C_{1,i} \mid C_1 > 0)$ given by Eq. (10) and $\mathbb{E}(G_{1,i} \mid C_1 > 0)$ given by Eq. (15). Fig. 4 illustrates $\mathbb{E}(H_{1,i} \mid C_1 > 0)$.

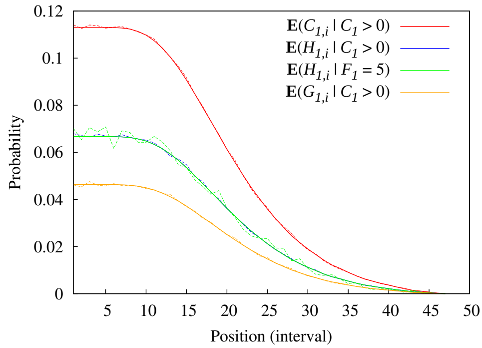


Figure 4: Expected values of $C_{1,i}$, $H_{1,i}$, and $G_{1,i}$ for $d_{IN} = 50$ m and $d_{int} = 5$ m. The analytical results of $\mathbb{E}(H_{1,i} \mid C_1 > 0)$ and $\mathbb{E}(H_{1,i} \mid F_1 = j)$ overlap.

Because of the complexities involved, and in order to keep our discussion focussed, the method to explicitly calculate $\mathbb{E}(H_{1,i} \mid F_1 = j)$ is given in Appendix A.1.2. However, both extensive simulations and numerical calculations for a wide range of parameters have shown that the expected number of first-hop candidate forwarders in interval i, excluding the first forwarder, given that there is a first forwarder, is independent of the actual location of the first forwarder, i.e.,

$$\mathbb{E}(H_{1,i} \mid F_1 = j) = \mathbb{E}(H_{1,i} \mid C_1 > 0), \quad i = 1, 2, \ldots \quad j = 1, \ldots, R. \tag{20}$$

Proof of Eq. (20) is given in Appendix A.1.3 in case of an ideal transmission model. Based on this proof, as well as on those results obtained by extensive simulations and numerical calculations, we conjecture that Eq. (20) also holds for the general case, i.e., for a non-ideal transmission model. As Eq. (20) is considerably faster than the method presented in Appendix A.1.2 we will use it to calculate $\mathbb{E}(H_{1,i} \mid F_1 = j)$ for the remainder of this document. The similarity of $\mathbb{E}(H_{1,i} \mid F_1 = j)$ and $\mathbb{E}(H_{1,i} \mid C_1 > 0)$ is illustrated in Fig. 4.

The number of additional second-hop candidate forwarders in interval i is defined as the number of nodes in interval i that received the message for the first time from the first forwarder. The number of nodes that did not receive the message from the source in interval i, denoted $K_{1,i}$, has a Poisson distribution with mean

$$\mathbb{E}(K_{1,i}) = \mathbb{E}(V_i) \cdot (1 - S_i), \quad i = 1, 2, \ldots, R, \tag{21}$$

with $\mathbb{E}(K_{1,i}) = 0$ for other values of i. The distribution of $K_{1,i}$ is independent of the distribution of the number of first-hop candidate forwarders. Fig. 2 shows $\mathbb{E}(K_{1,i})$.

Given that the first forwarder is positioned in interval j, the probability that a node in interval i successfully receives the message from the first forwarder

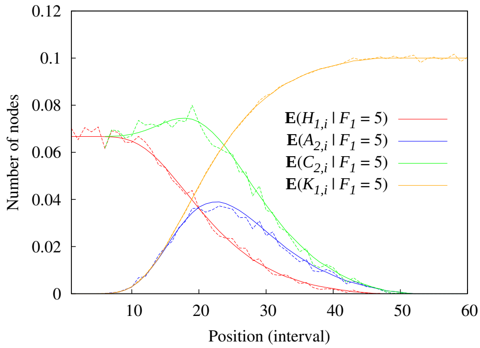


Figure 5: Expected values of $H_{1,i}$, $A_{2,i}$, and $C_{2,i}$ for $d_{IN} = 50$ and $d_{int} = 5$.

is given by S_{i-j}, $i > j$. $A_{2,i} \mid F_1 = j$ is thus Poisson distributed with mean $\mathbb{E}(A_{2,i} \mid F_1 = j)$ given by

$$\mathbb{E}(A_{2,i} \mid F_1 = j) = \mathbb{E}(K_{1,i})\; S_{i-j}, \quad i = j+1, \ldots, j+R, \tag{22}$$

with $\mathbb{E}(A_{2,i} \mid F_1 = j) = 0$ for other values of i, and with $\mathbb{E}(K_{1,i})$ given by Eq. (21). Combining Eq. (20) and Eq. (22) into Eq. (18) we thus have an expression to calculate $\mathbb{E}(C_{2,i} \mid F_1 = j)$. Fig. 5 illustrates $\mathbb{E}(A_{2,i} \mid F_1 = j)$.

Given that the first forwarder is positioned in interval j, the total number of second-hop candidate forwarders C_2 is made up out the number of remaining first-hop candidate forwarders in intervals $j+1$ through R, denoted $C_{1,j+1:R}$, plus the number of additional second-hop forwarders in intervals $j+1$ through $j+R$, i.e.,

$$\begin{aligned} \mathbb{P}(C_2 = c_2 \mid F_1 = j) = \quad & \mathbb{P}(C_{1,j+1:R} + A_2 = c_2 \mid F_1 = j) \\ = & \sum_{c_{1,j+1:R}=0}^{c_2} \mathbb{P}(C_{1,j+1:R} = c_{1,j+1:R} \mid F_1 = j) \cdot \\ & \mathbb{P}(A_2 = c_2 - c_{1,j+1:R} \mid F_1 = j), \\ & c_2 \in \mathbb{N}, \quad j = 1, 2, \ldots, R, \end{aligned} \tag{23}$$

with $\mathbb{P}(C_2 = c_2 \mid F_1 = j) = 0$ for other values of c_2, j. $\mathbb{P}(C_{1,j+1:R} = c_{1,j+1:R} \mid F_1 = j)$ is given by Eq. (90) in Appendix A.1.4. The distribution of $A_2 \mid F_1 = j$ is independent of the number of first-hop candidate forwarders; it is Poisson distributed with mean

$$\mathbb{E}(A_2 \mid F_1 = j) = \sum_{i=j+1}^{j+R} \mathbb{E}(A_{2,i} \mid F_1 = j), \quad j = 1, 2, \ldots, R. \tag{24}$$

The distribution of the number of second-hop candidate forwarders, given the position of the first forwarder and given that there is at least one second-hop candidate forwarder, is defined as

$$\mathbb{P}(C_2 = c_2 \mid F_1 = j \wedge C_2 > 0) = \frac{\mathbb{P}(C_2 = c_2 \mid F_1 = j)}{1 - \mathbb{P}(C_2 = 0 \mid F_1 = j)}, \quad j = 1, 2, \ldots, R. \tag{25}$$

Analogue to the first hop, see Eq. (10), the expected number of second-hop candidate forwarders in interval i, given the position of the first forwarder and given that there is at least one second-hop candidate forwarder, is given by

$$\begin{aligned} &\mathbb{E}(C_{2,i} \mid F_1 = j \wedge C_2 > 0) = \frac{\mathbb{E}(C_{2,i} \mid F_1 = j)}{1 - \mathbb{P}(C_2 = 0 \mid F_1 = j)}, \\ &j = 1, 2, \ldots, R, \quad i = j+1, j+2, \ldots, j+R, \end{aligned} \tag{26}$$

with $\mathbb{E}(C_{2,i} \mid F_1 = j \wedge C_2 > 0) = 0$ for other values of i, j.

Finally, because we will need it later on we express the expected number of nodes in an interval i following the first forwarder. Given that the first forwarder is positioned in interval j, the expected number of nodes in interval i is given by

$$\begin{aligned} \mathbb{E}(V_i \mid F_1 = j) &= \mathbb{E}(H_{1,i} + K_{1,i} \mid F_1 = j) \\ &= \mathbb{E}(H_{1,i} \mid F_1 = j) + \mathbb{E}(K_{1,i} \mid F_1 = j), \quad i = j+1, j+2, \ldots \end{aligned} \tag{27}$$

since $H_{1,i}$ and $K_{1,i}$ are independent, with $\mathbb{E}(H_{1,i} \mid F_1 = j)$ given by Eq. (20) and $\mathbb{E}(K_{1,i} \mid F_1 = j)$ given by Eq. (21).

4.2.2 Probability of success

The probability of success of the second hop is equal to the probability of having at least one second-hop candidate forwarder. For a given position of the first forwarder it is therefore given by

$$\mathbb{P}(\text{`successful second hop'} \mid F_1 = i) = \mathbb{P}(C_2 > 0 \mid F_1 = i), \quad i = 1, 2, \ldots, R, \tag{28}$$

with $C_2 > 0 \mid F_1 = i$ distributed according to Eq. (23). To determine the general probability of a successful second hop we take the position of the first forwarder into account:

$$\mathbb{P}(\text{`successful second hop'}) = \sum_{i=1}^{R} \mathbb{P}(F_1 = i) \cdot \mathbb{P}(C_2 > 0 \mid F_1 = i). \tag{29}$$

Finally, the probability that the first two hops are successful is given by

$$\mathbb{P}(\text{`two successful hops'}) = \mathbb{P}(\text{`successful first hop'}) \cdot \mathbb{P}(\text{`successful second hop'}), \tag{30}$$

with $\mathbb{P}(\text{`successful first hop'})$ given by Eq. (12).

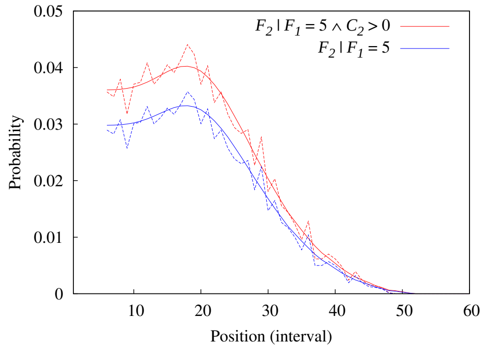


Figure 6: The distribution of the position of the second forwarder for a given position of the first forwarder, for $d_{IN} = 50$ m and $d_{int} = 5$ m.

4.2.3 Position of the forwarder

We determine the distribution of the position of the second forwarder in a manner analogue to how the position of the first forwarder is calculated in Eq. (13):

$$\begin{aligned}\mathbb{P}(F_2 = i \mid F_1 = j \wedge C_2 > 0) &= \mathbb{E}\Big(\frac{C_{2,i}}{C_2} \mid F_1 = j \wedge C_2 > 0\Big)\\ &\approx \frac{\mathbb{E}(C_{2,i} \mid F_1 = j)}{\mathbb{E}(C_2 \mid F_1 = j)},\\ & j = 1, 2, \ldots, R, \quad i = j+1, \ldots, j+R, \qquad (31)\end{aligned}$$

with $\mathbb{P}(F_2 = i \mid F_1 = j \wedge C_2 > 0) = 0$ for other values of i. $\mathbb{E}(C_2 \mid F_1 = j)$ can easily be derived from Eq. (23). Note that Eq. (31) is an approximation because $\mathbb{E}\big(\frac{C_{2,i}}{C_2} \mid F_1 = j \wedge C_2 > 0\big) = \frac{\mathbb{E}(C_{2,i} \mid F_1 = j)}{\mathbb{E}(C_2 \mid F_1 = j)}$ only holds if both $C_{2,i} \mid F_1 = j \wedge C_2 > 0$ and $C_2 \mid F_1 = j \wedge C_2 > 0$ are Poisson distributed, which is not the case: both variables have a shifted Poisson distribution. For practical purposes the margin of error introduced by this approximation is negligible however, as will be shown in Section **??**. Fig. 6 illustrates $\mathbb{P}(F_2 = i \mid F_1 = j \wedge C_2 > 0)$.

$\mathbb{P}(F_2 = i \mid F_1 = j)$ is calculated by multiplying $\mathbb{P}(F_2 = i \mid F_1 = j \wedge C_2 > 0)$ with the probability that there is a second forwarder:

$$\begin{aligned}&\mathbb{P}(F_2 = i \mid F_1 = j) = \mathbb{P}(C_2 > 0 \mid F_1 = j) \cdot \mathbb{P}(F_2 = i \mid F_1 = j \wedge C_2 > 0),\\ &j = 1, 2, \ldots, R, \quad i = j+1, \ldots, j+R, \qquad (32)\end{aligned}$$

with $\mathbb{P}(F_2 = i \mid F_1 = j) = 0$ for other values of i, $C_2 > 0 \mid F_1 = j$ distributed according to Eq. (23). Fig. 6 illustrates $\mathbb{P}(F_2 = i \mid F_1 = j)$ with respect to $\mathbb{P}(F_2 = i \mid F_1 = j \wedge C_2 > 0)$.

The probability that the second forwarder is in interval i, given that there is a first forwarder but irrespective of its position, is denoted $\mathbb{P}(F_2 = i \mid C_2 > 0)$.

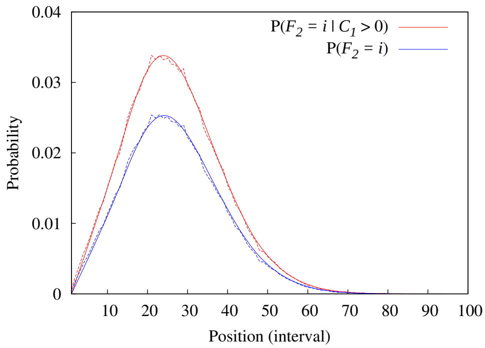


Figure 7: The distribution of the position of the forwarder for $d_{IN} = 50$ m and $d_{int} = 5$ m.

It is calculated by conditioning on the position of the first forwarder, given that there is a first forwarder:

$$\mathbb{P}(F_2 = i \mid C_2 > 0) = \sum_{j=1}^{R} \mathbb{P}(F_1 = j \mid C_1 > 0) \cdot \mathbb{P}(F_2 = i \mid F_1 = j \wedge C_2 > 0),$$
$$i = 2, 3, \ldots, 2R, \tag{33}$$

with $\mathbb{P}(F_2 = i \mid C_2 > 0) = 0$ for other values of i. Fig. 7 illustrates $\mathbb{P}(F_2 = i \mid C_2 > 0)$.

The probability that the second forwarder is in interval i is denoted $\mathbb{P}(F_2 = i)$ and is calculated by conditioning on the possible positions of the first forwarder:

$$\mathbb{P}(F_2 = i) = \sum_{j=1}^{R} \mathbb{P}(F_1 = j) \cdot \mathbb{P}(F_2 = i \mid F_1 = j), \quad i = 2, 3, \ldots, 2R, \tag{34}$$

with $\mathbb{P}(F_2 = i) = 0$ for other values of i, $\mathbb{P}(F_1 = j)$ given by Eq. (14), and $\mathbb{P}(F_2 = i \mid F_1 = j)$ given by Eq. (32). Fig. 7 illustrates $\mathbb{P}(F_2 = i)$.

Finally, given that there is a second forwarder and given that the first forwarder is positioned in interval j, the expected number of second forwarders in interval i is given by

$$\mathbb{E}(G_{2,i} \mid F_1 = j \wedge C_2 > 0) = \mathbb{P}(F_2 = i \mid F_1 = j \wedge C_2 > 0)$$
$$j = 1, 2, \ldots, R, \quad i = j + 1, \ldots, j + R, \tag{35}$$

with $\mathbb{E}(G_{2,i} \mid F_1 = j \wedge C_2 > 0) = 0$ for other values of i, j.

4.2.4 Hop length

The distribution of the hop length of the second hop is calculated with respect to the position of the first forwarder. For a given position of the first forwarder the distribution of L_2 is given by

$$\mathbb{P}(L_2 = l_2 \mid F_1 = j) = \mathbb{P}(F_2 = j + l_2 \mid F_1 = j \wedge C_2 > 0), \quad l_2, j = 1, 2, \ldots, R, \tag{36}$$

with $\mathbb{P}(L_2 = l_2 \mid F_1 = j) = 0$ for other values of l_2, j. For an arbitrary position of the first forwarder the distribution of L_2 is given by conditioning on the position of the first forwarder:

$$\mathbb{P}(L_2 = l_2) = \sum_{j=1}^{R} \mathbb{P}(F_1 = j \mid C_1 > 0)\, \mathbb{P}(F_2 = j + l_2 \mid F_1 = j \wedge C_2 > 0),$$
$$l_2, j = 1, 2, \ldots, R, \tag{37}$$

with $\mathbb{P}(L_2 = l_2 \mid F_1 = j) = 0$ for other values of l_2.

4.2.5 Hop delay

Analogue to how the hop delay of the first hop is determined, the distribution of the hop delay of the second hop, denoted D_2, is given by conditioning on the number of second-hop candidate forwarders. Given that the first forwarder is positioned in interval j, the CDF of the hop delay is given by

$$\mathbb{F}_{D_2 \mid F_1 = j}(t) = 1 - \sum_{c_2=1}^{\infty} \mathbb{P}(C_2 = c_2 \mid F_1 = j \wedge C_2 > 0) \cdot e^{(c_2 \cdot t)/T_d}, \quad t > 0, \tag{38}$$

with $\mathbb{F}_{D_2 \mid F_1 = j}(t) = 0$ for $t \leq 0$ and $\mathbb{P}(C_2 = c_2 \mid F_1 = j \wedge C_2 > 0)$ given by Eq. (25). For an arbitrary position of the first forwarder the distribution of D_2 is given by conditioning on the position of the first forwarder:

$$\mathbb{F}_{D_2}(t) = \sum_{j=1}^{R} \mathbb{P}(F_1 = j \mid C_1 > 0) \cdot \mathbb{F}_{D_2 \mid F_1 = j}(t), \quad t > 0, \tag{39}$$

with $\mathbb{F}_{D_2}(t) = 0$ for $t \leq 0$.

4.3 Third hop

4.3.1 Candidate forwarders

We determine the expected number of third-hop candidate forwarders in interval i, given that the first forwarder is positioned in interval j and the second forwarder is positioned in interval k, denoted $\mathbb{E}(C_{3,i} \mid F_1 = j \wedge F_2 = k)$.

The set of third-hop candidate forwarders in an interval consists of remaining second-hop candidate forwarders (excluding the second forwarder itself) and additional third-hop candidate forwarders, i.e., $C_{3,i} = H_{2,i} + A_{3,i}$. Taking the expected values, for given positions of the first two forwarders, we get

$$\mathbb{E}(C_{3,i} \mid F_1 = j \wedge F_2 = k) = \mathbb{E}(H_{2,i} \mid F_1 = j \wedge F_2 = k) + \mathbb{E}(A_{3,i} \mid F_1 = j \wedge F_2 = k),$$
$$j = 1, 2, \ldots, R, \quad k = j + 1, j + 2, \ldots, j + R, \quad i = k + 1, k + 2, \ldots, k + R. \tag{40}$$

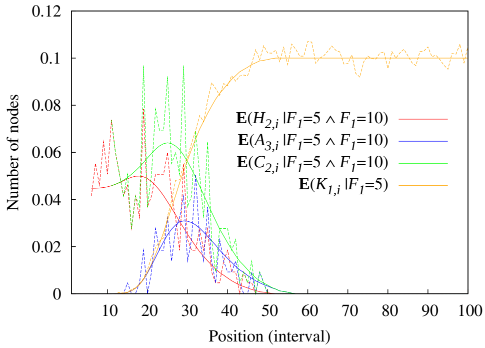


Figure 8: Expected values of $H_{2,i}$, $A_{3,i}$, $C_{3,i}$, and $K_{2,i}$ for $d_{IN} = 50$ m and $d_{int} = 5$ m.

Analogue to the second hop, see Eq. (20), we state

$$\mathbb{E}(H_{2,i} \mid F_1 = j \wedge F_2 = k) = \mathbb{E}(H_{2,i} \mid F_1 = j \wedge C_2 > 0),$$
$$j = 1, 2, \ldots, R, \quad i = j + 1, j + 2, \ldots, j + R, \tag{41}$$

with $\mathbb{E}(H_{2,i} \mid F_1 = j \wedge F_2 = k) = 0$ for other values of i, j. $\mathbb{E}(H_{2,i} \mid F_1 = j \wedge C_2 > 0)$ is by definition given as

$$\mathbb{E}(H_{2,i} \mid F_1 = j \wedge C_2 > 0) = \mathbb{E}(C_{2,i} \mid F_1 = j \wedge C_2 > 0) - \mathbb{E}(G_{2,i} \mid F_1 = j \wedge C_2 > 0),$$
$$j = 1, 2, \ldots, R, \quad i = j + 1, j + 2, \ldots, j + R, \tag{42}$$

with $\mathbb{E}(H_{2,i} \mid F_1 = j \wedge C_2 > 0) = 0$ for other values of i, j, $\mathbb{E}(C_{2,i} \mid F_1 = j \wedge C_2 > 0)$ given by Eq. (26), and $\mathbb{E}(G_{2,i} \mid F_1 = j \wedge C_2 > 0)$ given by Eq. (35).

The number of additional third-hop candidate forwarders in interval i is defined as the number of nodes in interval i that received the message for the first time from the second forwarder. The number of nodes in interval i that did not receive the message from the source and did not receive the message from the first forwarder, given that the first forwarder is positioned in interval j, has a Poisson distribution with mean

$$\mathbb{E}(K_{2,i} \mid F_1 = j) = \begin{cases} \mathbb{E}(K_{1,i}) \cdot (1 - S_{i-j}), & i = j + 1, \ldots, j + R, \\ \mathbb{E}(K_{1,i}), & i = j + R + 1, j + R + 2, \ldots, \end{cases} \tag{43}$$

with $\mathbb{E}(K_{2,i} \mid F_1 = j) = 0$ for other values of i. The distribution of $K_{2,i} \mid F_1 = j$ is independent of the distribution of the number of second-hop candidate forwarders.

Given that the second forwarder is positioned in interval k, the probability that a node in interval i successfully receives the message from the second forwarder is given by S_{i-k}, $i > k$. $A_{3,i} \mid F_1 = j \wedge F_2 = k$ is thus Poisson distributed with mean

$$\mathbb{E}(A_{3,i} \mid F_1 = j \wedge F_2 = k) = \mathbb{E}(K_{2,i} \mid F_1 = j)\, S_{i-k},$$
$$j = 1, 2, \ldots, R, \;\; k = j+1, j+2, \ldots, j+R, \;\; i = k+1, k+2, \ldots, k+R, \quad (44)$$

with $\mathbb{E}(A_{3,i} \mid F_1 = j \wedge F_2 = k) = 0$ for other values of i, j, k, and with $\mathbb{E}(K_{1,i} \mid F_1 = j)$ given by Eq. (21). Combining Eq. (41) and Eq. (42) we thus have an expression to calculate Eq. (40). Fig. 8 shows the expected values of $H_{2,i}$, $A_{3,i}$, $C_{3,i}$, and $K_{2,i}$.

The expected total number of third-hop candidate forwarders is given by

$$\mathbb{E}(C_3 \mid F_1 = j \wedge F_2 = k) = \sum_{i=k+1}^{k+R} \mathbb{E}(C_{3,i} \mid F_1 = j \wedge F_2 = k),$$
$$j = 1, 2, \ldots, R, \;\; k = j+1, j+2, \ldots, j+R, \quad (45)$$

with $\mathbb{E}(C_3 \mid F_1 = j \wedge F_2 = k) = 0$ for other values of j, k.

Lastly, because we will need it later on we express the expected number of nodes in an interval i following the second forwarder. Given that the first forwarder is positioned in interval j and the second forwarder is positioned in interval k, the expected number of nodes in interval i is given by

$$\mathbb{E}(V_i \mid F_1 = j \wedge F_2 = k) = \mathbb{E}(H_{1,i} \mid F_1 = j \wedge F_2 = k) + \mathbb{E}(K_{1,i} \mid F_1 = j \wedge F_2 = k),$$
$$i = j+1, j+2, \ldots, \quad (46)$$

with $\mathbb{E}(H_{2,i} \mid F_1 = j \wedge F_2 = k)$ given by Eq. (41) and $\mathbb{E}(K_{2,i} \mid F_1 = j \wedge F_2 = k)$ given by Eq. (43).

4.3.2 Position of the forwarder

Analogue to the first and second hop the position of the third forwarder, given that there is a third forwarder and given the position of the first two forwarders, is approximated by

$$\mathbb{P}(F_3 = i \mid F_1 = j \wedge F_2 = k \wedge C_3 > 0) = \mathbb{E}(\frac{C_{3,i}}{C_3} \mid C_3 > 0)$$
$$\approx \frac{\mathbb{E}(C_{3,i})}{\mathbb{E}(C_3)},$$
$$j = 1, 2, \ldots, R, \;\; k = j+1, j+2, \ldots, j+R, \;\; i = k+1, k+2, \ldots, k+R. \quad (47)$$

When it is given that there is a third forwarder, the distribution of its position is calculated by conditioning on the position of the first two forwarder:

$$\mathbb{P}(F_3 = i \mid C_3 > 0) = \sum_{j=1}^{R} \sum_{k=j+1}^{j+R} \mathbb{P}(F_1 = j \mid C_1 > 0) \cdot \mathbb{P}(F_2 = i \mid F_1 = j \wedge C_2 > 0) \cdot$$
$$\mathbb{P}(F_3 = i \mid F_1 = j \wedge F_2 = k \wedge C_3 > 0),$$
$$i = 3, 4, \ldots, 3 \cdot R, \quad (48)$$

with $\mathbb{P}(F_3 = i \mid C_3 > 0) = 0$ for other values of i. Fig. 9 illustrates $\mathbb{P}(F_3 = i \mid C_3 > 0)$.

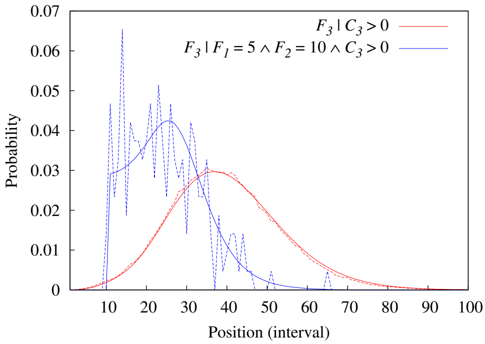


Figure 9: The distribution of the position of the third forwarder for $d_{IN} = 50$ m and $d_{int} = 5$ m.

4.3.3 Hop length

The hop length of the third hop is calculated with respect to the position of the second forwarder. For a given position of the first two forwarders the distribution of L_3 is given by

$$\mathbb{P}(L_3 = l_3 \mid F_1 = j \wedge F_2 = k) = \mathbb{P}(F_3 = k + l_3 \mid F_1 = j \wedge F_2 = k \wedge C_3 > 0),$$
$$l_3 = 1, 2, \ldots, R, \tag{49}$$

with $\mathbb{P}(L_3 = l_3 \mid F_1 = j \wedge F_2 = k) = 0$ for other values of l_3, j, k. For arbitrary positions of the first two forwarders the distribution of L_3 is given by conditioning on the position of the first two forwarders:

$$\mathbb{P}(L_3 = l_3) = \sum_{j=1}^{R} \sum_{k=j+1}^{j+R} \mathbb{P}(F_1 = j \mid C_1 > 0)\, \mathbb{P}(F_2 = k \mid F_1 = j \wedge C_2 > 0) \cdot \mathbb{P}(L_3 = l_3 \mid F_1 = j \wedge F_2 = k)$$
$$l_3 = 1, 2, \ldots, R, \tag{50}$$

with $\mathbb{P}(L_3 = l_3) = 0$ for other values of l_3.

5 Approximate analysis of following hops

Based on the exact analysis of the first three hops we give approximate methods in this section to determine the behaviour of the forwarding model for an arbitrary number of hops. These methods are of limited complexity, allowing for fast evaluation.

5.1 Probability of success

The probability of success of a hop is approximated for the third and following hops by assuming that it does not change beyond the first hop, i.e.,

$$\mathbb{P}(\text{`successful } n^{\text{th}} \text{ hop'}) = \mathbb{P}(\text{`Successful second hop'}), \quad n = 3, 4, \ldots, \tag{51}$$

with $\mathbb{P}$('successful second hop') given by Eq. (29). The probability of having n successful hops is then approximated by

$$\mathbb{P}(\text{`}n \text{ successful hops'}) = \prod_{k=1}^{n} \mathbb{P}(\text{`successful } k^{\text{th}} \text{ hop'}). \tag{52}$$

5.2 Hop length

We approximate the distribution of the hop length of the fourth and following hops with the distribution of the hop length of the third hop. As we will find from the numerical results in Section 6.2, the distribution of the hop length of the n^{th} is significantly influenced by the hop lengths of previous hops. We therefore condition the distribution of the hop length of the n^{th} hop on the hop length of the $(n-1)^{\text{th}}$ hop, i.e.,

$$F_{L_n \mid L_{n-1}=l_{n-1}} \sim F_{L_3 \mid L_2=l_{n-1}}, \quad n = 4, 5, \ldots, \tag{53}$$

with the distribution of $L_3 \mid L_2 = l_{n-1}$ given by

$$\begin{aligned} &\mathbb{P}(L_3 = l_3 \mid L_2 = l_2) = \\ &\sum_{i=1}^{R} \mathbb{P}(F_1 = i \mid C_1 > 0) \cdot \mathbb{P}(F_3 = i + l_2 + l_3 \mid F_1 = i \wedge F_2 = i + l_2 \wedge C_3 > 0), \\ &l_3 = 1, 2, \ldots, R, \quad l_2 = 1, 2, \ldots, R, \end{aligned} \tag{54}$$

with $\mathbb{P}(F_1 = i \mid C_1 > 0)$ given by Eq. (13) and
$\mathbb{P}(F_3 = i + l_2 + l_3 \mid F_1 = i \wedge F_2 = i + l_2 \wedge C_3 > 0)$ given by Eq. (47).

5.3 Position of the forwarder

We approximate the distribution of the position of the n^{th} forwarder ($n > 3$) in a recursive manner, assuming that there is such a forwarder, by taking into account the length of previous hops.

We have assumed in Eq. (53) that the hop length of each hop beyond the third hop is distributed identically to the hop length of the third hop, for a given hop length of the previous hop. The position of the n^{th} forwarder, given that there is an n^{th} forwarder, can thus be approximated by conditioning on the position of the $(n-2)^{\text{th}}$ forwarder and the length of the $(n-1)^{\text{th}}$ hop, i.e.,

$$\begin{aligned}
\mathbb{P}(F_n = i \mid C_n > 0) \approx & \sum_{j=n-2}^{(n-2)R} \mathbb{P}(F_{n-2} = j \mid C_{n-2} > 0) \cdot \\
& \sum_{l_{n-1}=1}^{R} \mathbb{P}(F_{n-1} = j + l_{n-1} \mid F_{n-2} = j \wedge C_{n-1} > 0) \cdot \\
& \mathbb{P}(L_n = i - j - l_{n-1} \mid L_{n-1} = l_{n-1}), \\
n = 4, 5, \ldots, \quad i = n, n+1, \ldots, n \cdot R, &
\end{aligned} \tag{55}$$

with $\mathbb{P}(F_n = i \mid C_n > 0) = 0$ for other values of i.
$\mathbb{P}(F_{n-1} = j + l_{n-1} \mid F_{n-2} = j \wedge C_{n-1} > 0)$ denotes the probability that the $(n-1)^{\text{th}}$ forwarder is positioned in interval $j + l_{n-1}$, given that the $(n-2)^{\text{th}}$ forwarder is positioned in interval j and given that there is an $(n-1)^{\text{th}}$ forwarder. It is given by

$$\begin{aligned}
& \mathbb{P}(F_n = i \mid F_{n-1} = j \wedge C_n > 0) = \\
& \sum_{k=n-2}^{(n-2)R} \mathbb{P}(F_{n-2} = k \mid C_{n-2} > 0) \cdot \mathbb{P}(L_n = i - j \mid L_{n-1} = j - k), \\
& n = 3, 4, \ldots, \quad i = n, n+1, \ldots, n \cdot R,
\end{aligned} \tag{56}$$

with $\mathbb{P}(F_n = i \mid F_{n-1} = j \wedge C_n > 0) = 0$ for other values of i.

5.4 Required number of hops

We determine the distribution of the required number of hops to have the message forwarded by a node at or beyond position i, denoted N_i. We make use of the fact that the probability that *at most* n hops are required to have the message forwarded by a node at or beyond position i is equal to the probability that the n^{th} forwarder is at or beyond position i, i.e.,

$$\begin{aligned}
\mathbb{P}(N_i \le n) &= \mathbb{P}(F_n \ge i \mid C_n > 0) \\
&= 1 - P(F_n < i \mid C_n > 0), \\
& i = 1, 2, \ldots, \quad n = 1, 2, \ldots
\end{aligned} \tag{57}$$

where $\mathbb{P}(F_n < i \mid C_n > 0)$ is equal to the sum of the probabilities that the forwarder is at position $j = 1, \ldots, i-1$, given by

$$\begin{aligned}
P(F_n < i \mid C_n > 0) = & \sum_{j=1}^{i-1} \mathbb{P}(F_n = j \mid C_n > 0), \\
& n = 1, 2, \ldots, \quad i = 1, \ldots, n \cdot R,
\end{aligned} \tag{58}$$

with $\mathbb{P}(F_n = j \mid C_n > 0)$ given by Eq. (55).

5.5 Hop delay

We approximate the n^{th}-hop hop delay D_n for $n > 2$.

As we will find from the numerical results in Section 6.2 the distribution of the hop delay changes little after the first two hops. We therefore approximate the hop delay distribution of the n^{th} hop with the hop delay distribution of the second hop:

$$F_{D_n}(\cdot) \sim F_{D_2}(\cdot), \quad n = 3, 4, \dots, \tag{59}$$

with $F_{D_2}(\cdot)$ given by Eq. (39). Likewise, for a given length of the $(n-1)^{\text{th}}$ hop l_{n-1} we approximate the hop delay distribution of the n^{th} hop by

$$F_{D_n \mid L_{n-1}=l_{n-1}}(\cdot) \sim F_{D_2 \mid F_1=l_{n-1}}(\cdot), \quad n = 3, 4, \dots, \tag{60}$$

5.6 End-to-end delay

The end-to-end delay to have a message forwarded at least i intervals is a convolution of the required number of hops to have the message forwarded at least i intervals and the delay per hop. We therefore first determine the end-to-end delay to have the message forwarded at least i intervals in n hops, denoted $F_{E_i \mid N_i=n}$ and then condition on the required number of hops to have a message forwarded i intervals. For the first two hops the end-to-end delay is exact; for following hops the end-to-end delay is approximated.

The distribution of the end-to-end delay for a single hop is independent of the value of i and is equal to the distribution of the hop delay of the first hop, i.e.,

$$F_{E_i \mid N_i=1}(t) = F_{D_1}(t), \quad t > 0 \tag{61}$$

with $F_{D_1}(t)$ given by Eq. (17).

The distribution of the end-to-end delay to have a message forwarded i intervals in exactly two hops is given by conditioning on the position of the first forwarder and the delay of the first hop, given that the second forwarder is position at interval i or beyond. Normalising F_1 with respect to the fact that it must be within R intervals of interval i but not at or beyond interval i we get

$$F_{E_i \mid N_i=2}(t) = \sum_{j=\max(1,i-R)}^{\min(R,i-1)} \frac{\mathbb{P}(F_1 = j \mid C_1 > 0)}{\sum_{k=\max(1,i-R)}^{\min(R,i-1)} \mathbb{P}(F_1 = k \mid C_1 > 0)} \cdot \int_{t_1=0}^{t} f_{D_1}(t_1) \cdot \mathbb{F}_{D_2 \mid F_1=j}(t-t_1)\, \mathrm{d}t_1, \quad t > 0, \tag{62}$$

with $\mathbb{P}(F_1 = j \mid C_1 > 0)$ given by Eq. (13), $f_{D_1}(t_1)$ can easily be derived from Eq. (17), and $\mathbb{F}_{D_2 \mid F_1=j}(t-t_1)$ given by Eq. (38).

If $n > 2$ hops are needed to have the message forwarded i intervals then the average hop length l is given by i/n. To approximate the distribution of the end-to-end delay we condition on the hop delay of the first hop, and approximate the end-to-end delay of the remaining $n-1$ hops as a convolution of $n-1$ independent hop delays distributed according to $F_{D_n \mid F_1=l}(t)$. The distribution of $n-1$ independent exponential hop delays with identical means is given by the Erlang distribution [10]. $F_{E_i \mid N_i=n}(t)$ is thus given by

$$F_{E_i \mid N_i=n}(t) \approx \int_{t_1=0}^{t} f_{D_1}(t_1) \cdot \Big(1 - \sum_{k=0}^{n-2} \frac{e^{-\lambda(t-t_1)}}{k!} (\lambda(t-t_1))^k\Big) \,\mathrm{d}t_1,$$
$$t > 0, \;\; \lambda = \mathbb{E}(C_2 \mid F_1 = l \wedge C_2 > 0)/T_d, \tag{63}$$

with $\mathbb{E}(C_2 \mid F_1 = l \wedge C_2 > 0)$ derived from Eq. (25). Note that by assuming that each hop is of average length and consecutive hops are independent of each other we ignore any dependencies between consecutive hop lengths. The effect that this has on the accuracy of our model is discussed in detail in Section 6.2.

Finally, to calculate the end-to-end delay to have a message forwarded i intervals we condition on the required number of hops, such that the end-to-end delay is given by

$$F_{E_i}(t) = \sum_{n=1}^{\infty} \mathbb{P}(N_i = n) \cdot F_{E_i \mid N_i=n}(t), \;\; t > 0, \;\; i \in \mathbb{N}^+. \tag{64}$$

6 Performance evaluation

Having analysed the forwarding protocol in an analytical manner in the previous sections, in this section we present the set-up and results of an evaluation study to assess *(i)* how the forwarding protocol described in Section 3 performs for varying network parameters, and *(ii)* how well our analysis presented in the previous section is able to capture its behaviour. We have done so by evaluating various forwarding scenarios with different network parameters, both by means of simulation and by means of our analysis. We discuss the performance of the forwarding protocol using the results of the simulation study and discuss the accuracy of our analysis by comparing the results of the simulation study and our analysis. Below we first describe the scenario and the set-up of our simulation study in Section 6.1 and then discuss the results in Section 6.2.

6.1 Experimental set-up

Nodes are positioned over a straight line of 3000 m with the source at one end and the message destination at the other end. The inter-node spacing is exponentially distributed with mean d_{IN} set to 10, 25, and 50 m. With each experiment a message is initially broadcasted by the source and forwarded towards the message destination by the remaining nodes, following the forwarding rules specified in Section 3 with the mean forwarding $T_d = 1$ s. To gain statistically significant results each experiment has been repeated at least 70,000 times with different random seeds.

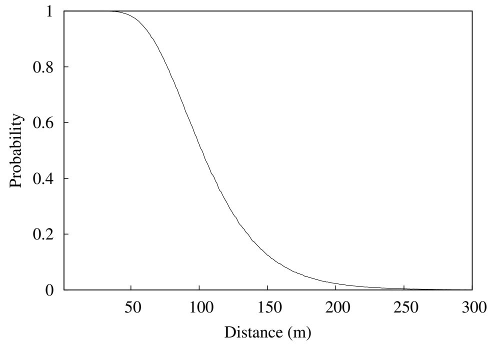


Figure 10: The packet reception curve S_i.

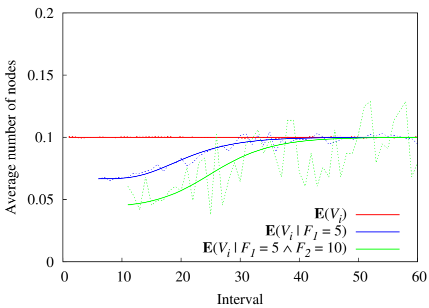
Experiments have been performed using the OMNET++ network simulator v4.1 [2] and using a self-modified version of the MiXiM framework v2.1 [1] to model the communication architecture. To model the behaviour of the 802.11p protocol as accurately as possible we have altered the IEEE 802.11 medium access module in such a way that all parameters follow the 802.11p specification [3]. The available 802.11 MiXiM physical layer was adapted to include bit error rates (BER) and packet error rates (PER) for all transmission bit rates used in our experiments. The centre frequency was set to 5.9 MHz and IEEE 802.11 access category (AC) 0 was used. We use the log-normal shadowing model [8] for signal propagation with the path loss exponent is set to 3.5 and the standard deviation to 6. Transmission power was set to 4 mW. To keep the influence of packet collisions due to hidden nodes as low as possible the packet sizes are kept small (only the headers are included) at 160 bits.

Our model analysis requires the packet reception rate S_i as input. Using the above simulations settings we have measured the packet reception probabilities at intervals of one meter for $R = 300$ m, for a single node that broadcasted a packet ten thousand times without any interfering network traffic. The resulting packet reception curve S_i can be seen in Fig. 10. The packet reception probability at the edge of the packet reception curve is less than 0.1 %, i.e., $S_R < 0.01$.

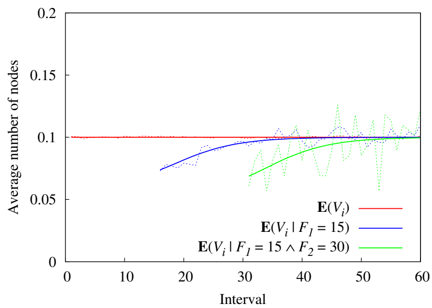
Note that it is also possible to model S_i as a function of transmission power, propagation effects, BER, PER, and forward error correction; see for example [7].

6.2 Results

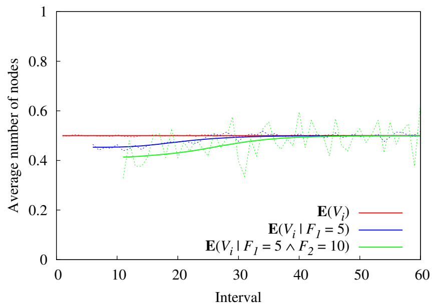
Our discussion of the results is split into two parts. We first show how the behaviour of a hop depends on the lengths of all previous hops, and how this affects performance. Then we discuss the results of our evaluation study.



(a) Two short hops ($F_1 = 5, F_2 = 10$), $d_{IN} = 50$ m..



(b) Two long hops ($F_1 = 20, F_2 = 40$), $d_{IN} = 50$ m..



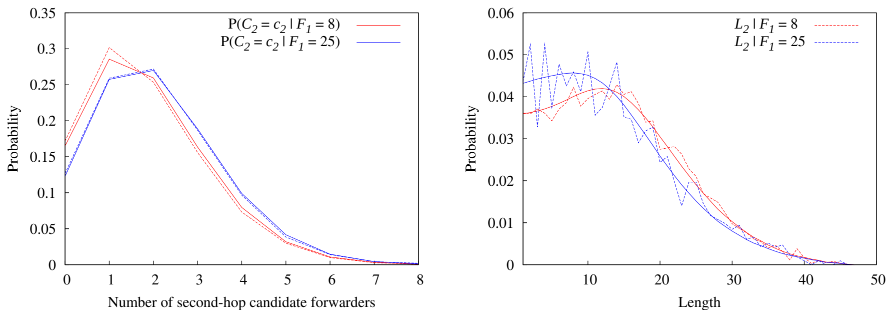
(c) Two short hops ($F_1 = 5, F_2 = 10$), $d_{IN} = 10$ m.

Figure 11: Average number of nodes in an interval following the source, the first forwarder, and the second forwarder, for different values of d_{IN} and different lengths of the first two hops.

6.2.1 Dependencies between consecutive hops

The behaviour of a hop depends on the behaviour of previous hops, especially regarding the lengths of those previous hop lengths. We explain why this is the case below and show how it affects performance. We conclude that to accurately analyse the behaviour of multiple hops, the length of each intermediate hop must be taken into account.

Each node that has received the message from a previous forwarder (or the source) has an equal probability of becoming the next forwarder. An interval that contains a more than average number of nodes is therefore more likely to 'produce' a forwarder than an interval that contains a less than average number of nodes and, consequently, an interval that does not produce a forwarder is more likely to have a less than average number of nodes. This effect is illustrated in Fig. 11, which shows the average number of nodes in an interval



(a) Number of candidate forwarders.

(b) Hop length.

Figure 12: The distribution of the hop length of the second hop L_2 and the number of second-hop candidate forwarders C_2, following a short first hop ($F_1 = 8$) and a long first hop ($F_1 = 25$), for $d_{IN} = 50$ m and $d_{int} = 5$ m.

$\mathbb{E}(V_i)$, the average number of nodes in an interval following the first forwarder $\mathbb{E}(V_i \mid F_1 = j)$, and the average number of nodes in an interval following the second forwarder $\mathbb{E}(V_i \mid F_1 = j \wedge F_2 = k)$. The figure also shows how the decrease in the average number of nodes in an interval is determined by the packet reception curve S_i: intervals that have a low probability of receiving the message from the previous forwarder are less likely to produce candidate forwarders, and are therefore less affected. The decrease in the average number of nodes in an interval surrounding the forwarder is stronger for low node densities, since the impact of the position of a single node (the forwarder) is stronger when the total number of nodes is less. This can be seen when comparing Fig. 11a and Fig. 11c. The effect furthermore adds up for consecutive hops and is stronger when forwarders are positioned close together, i.e., when hop lengths are short. This can be seen when comparing Fig. 11a and Fig. 11b.

The effect that this decrease in the number of nodes in an interval surrounding the forwarder has on performance is significant, since the number of nodes per interval in the intervals following the previous forwarders determines the number of candidate forwarders per interval as well as the total number of candidate forwarders. Fig. 12a shows the distribution of the number of second-hop candidate forwarders following a short first hop and a long first hop. It can be seen that on average there are more second-hop candidate forwarders following a long hop, and that the probability of success of the second hop (i.e., of having at least one second-hop candidate forwarder) is higher following a long hop. This dependency holds for each hop, i.e., the number of candidate forwarders following a long hop is on average always higher than the number of candidate forwarders following a short hop. A hop following one or more long hops therefore has a larger probability of being successful and will have a shorter hop delay.

The distribution of the hop length is also affected. Because the number of nodes in an interval directly surrounding the previous forwarder is less following one or more short hops, the number of candidate forwarders in an interval directly surrounding the previous forwarder is also less. As a result the probability that a candidate forwarder that is positioned further away from the forwarder becomes the next forwarder increases. Hop lengths are thus on average longer following one or more short hops. This has been illustrated in Fig. 12b for the second hop, showing the distribution of the hop length following a short first hop and a long first hop.

In conclusion, for all of the performance metrics discussed here it holds that to determine the performance of the n^{th} hop in an exact manner the length of all previous $n-1$ hops must be taken into account. Due to the complexities involved in doing so this is infeasible however, as we have argued in previous sections. In our analysis we therefore approximate the behaviour of the n^{th} hop by taking into account the length of the $(n-1)^{\text{th}}$ hop only or, in case of the end-to-end delay, by assuming that all $n-1$ preceding hops are of identical length. We discuss how well these approximations are able to described the behaviour of the forwarding protocol in the following.

6.2.2 Protocol performance

We use the *Kolmogorov-Smirnov* (K-S) statistic to express the difference between two distributions. The K-S statistic K for two distributions $F_1(x), F_2(x)$ is equal to the largest distance between the CDFs, given by

$$K = \max\{|F_1(x) - F_2(x)|\} \quad \forall\, x. \tag{65}$$

We discuss the following performance metrics in the same order in which we have presented them in Section 5: *(i)* the probability of success of each hop, *(ii)* the distribution of the hop length of each hop, *(iii)* the distribution of the position of each forwarder, *(iv)* the distribution of the number of hops to have a message forwarded i intervals, *(v)* the distribution of the hop delay of each hop, and *(vi)* the distribution of the end-to-end delay to have a message forwarded i intervals.

Performance metrics have been evaluated up to the tenth hop and for distances up to 1000 m, and are all included in Table 2, showing the K-S statistics of the resulting distributions. On average 16 hops are needed to have the message forwarded a 1000 m. For clarity of illustration the figures show only distributions of the first five hops and for distances up to 500 m. For all shown results $d_{int} = 1$ m unless specified otherwise. The solid lines represent analytical results, the dashed lines represent simulation results. In case of average values confidence intervals are less than 1 %.

In general we see that the accuracy of our model analysis is very high and that, excepting the end-to-end delay, all our analytical results stay within 0.1 of the simulation results. For the end-to-end delay results stay within 0.1 for high node densities and for forwarding scenarios in which the message is forwarded on average eight times or less.

Inaccuracies in our model analysis are mainly caused by *(i)* the fact that we ignore some effects caused by packet losses, such as the retransmission of messages, *(ii)* the fact that we ignore dependencies between consecutive hops following the third hop. The former holds for high-density scenarios in particular; the latter for low-density scenarios. Because of these conflicting effects we will sometimes see that results are most accurate for a medium-density scenario with $d_{IN} = 25$ m, as in such a scenario both effects have the least impact.

Fig. **??** shows the *hop success probability* of the first ten hops. As there are less nodes on the road the probability that a message gets lost increases: whereas the probability of having a message forwarded ten times is 1 for $d_{IN} = 10$ m, it is almost 0 for $d_{IN} = 50$ m. It can be seen in the figure that, regarding the hop success probability, results of the model simulation and the model analysis stay within 0.03.

Fig. 14 shows the distribution of the *hop length* of the first four hops for three values of d_{IN}. Regarding the model analysis only the distributions of the hop length of the first three hops are shown, since in our analysis we assume that the distribution of the hop length of the fourth hop and following hops is identical to the distribution of the hop length of the third hop.

Because each candidate forwarder has an equal probability of becoming the next forwarder, the distribution of the hop length is mainly determined by the shape of the packet reception curve S_i and the distribution of the nodes following the most recent forwarder. Because of the dependency between hops that was discussed in the previous section, hops become increasingly longer with each hop. After the first few hop however the distribution of the hop length converges however, such that the distribution of the hop length of the fourth hop is quite similar to the distribution of the hop length of the third hop.

As d_{IN} increases the dependency between successive hops increases as well, and the lengthening of successive hops becomes more pronounced. The effect is still limited however and the average length of a hop changes but little as d_{IN} is varied.

It can be seen in the figures as well as in Table 2 that, regarding the distribution of the hop length, results of the model simulation and the model analysis stay within 0.02. This confirms our assumption made in Eq. (53) that, for the purpose of our analysis and for the range of parameters tested here, the distribution of the hop length of the fourth hop (and of following hops) is identical to the distribution of the hop length of the third hop.

Fig. 15 shows the distribution of the *position of the forwarder* for the first five hops and for three values of d_{IN}. Since the impact of d_{IN} is limited on the distribution of the hop length, its impact on the position of a forwarder is similarly limited: the distribution of the position of the n^{th} forwarder does not change much as d_{IN} is varied.

In our model analysis the distribution of the position of the first three forwarders is calculated exactly (given the model assumptions); it can be seen in the figures as well as in Table 2 that, regarding the distribution of the position of the first three forwarders, results of the model simulation and the model analysis stay within 0.026. Our exact approach is thus very accurate, irrespective of the value of d_{IN}. Deviations are mainly caused by the effects of transmission errors that have not been taken into account.

The position of following forwarders is approximated in our model; it can be seen in the figures as well as in Table 2 that, regarding the distribution of the position of the fourth and following forwarders, results of the model simulation and the model analysis stay within 0.5 for the tenth forwarder. Results become less for each following hop because in our approximation we do not take into account the hop lengths of all preceding hops, but only the length of the most recent hop.

Fig. 16 shows the distribution of the *required number of hops* to have the message forwarded i intervals for three values of d_{IN}. Similar to the distributions of the hop length and the position of the forwarder, and for similar reasons, the distribution of the number of hops changes little as d_{IN} is varied. Fig. 19, which shows the average required number of hops as a function of d_{IN} and i, furthermore illustrate that the average required number of hops grows linearly as i increases.

It can be seen in the figures as well as in Table 2 that, regarding the distribution of the number of hops required to have the message forwarded i intervals, results of the model simulation and the model analysis stay within 0.1 for $d_{IN} = 50$ m, and generally become increasingly accurate as d_{IN} and i decrease. Inaccuracies are mainly caused by the fact that we do not take dependencies between successive hops into account in full.

Fig. 17 shows the distribution of the *hop delay* of the first four hops for three values of d_{IN}. Of the model analysis only the distribution of the hop delay of the first two hops are shown since in our analysis we assume that the distribution of the hop delay of the third hop (and of following hops) is identical to the hop delay of the second hop.

It can be seen that as d_{IN} decreases the average hop delay decreases, due to the increase in the number of candidate forwarders per hop. Although there is a small difference between the hop delay distribution of the first and the second hop, the differences between distribution of following hosp is negligible.

It can be seen in the figures as well as in Table 2 that, regarding the distribution of the hop delay, results of the model simulation and the model analysis stay within 0.02. This confirms our assumption that, for the purpose of our analysis, the distribution of the hop delay of the third hop (and of following hops) is identical to the distribution of the hop delay of the second hop.

Fig. 18 shows the distribution of the *end-to-end delay* to have the message forwarded i intervals for three values of d_{IN}. Fig. 20 moreover shows the average end-to-end delay for varying values of d_{IN} and i. It can be seen that the end-to-end delay increases linearly as i increases, and less than linearly as d_{IN} increases.

It can be seen in the figures as well as in Table 2 that, regarding the distribution of the end-to-end delay to have the message forwarded i intervals, results of the model simulation and the model analysis stay within 0.10 for $d_{IN} = 10$ m and within 0.19 for lower densities. For distances up to 500 m, which require on average eight hops to have to have the message forwarded this far, all results

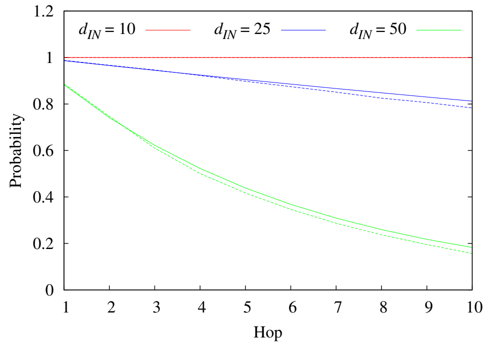
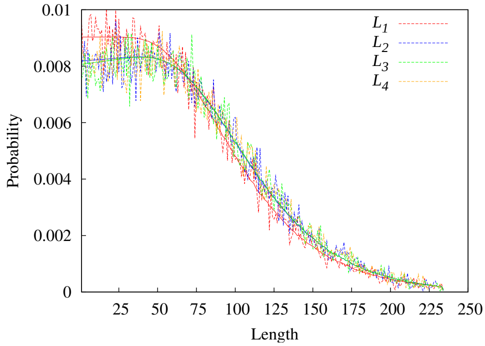
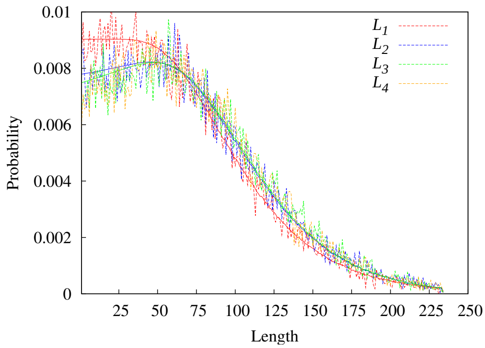


Figure 13: The probability of having an n^{th} hop.

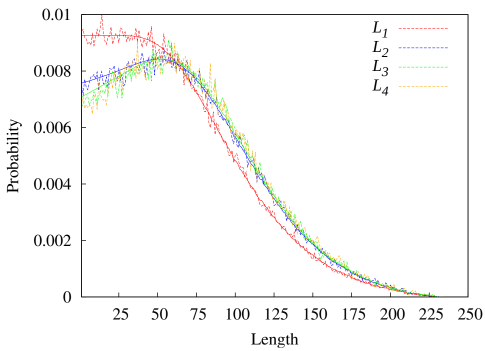
stay within 0.10. Regarding the average end-to-end delay results stay within 1 % for $d_{IN} = 10, 25$ m, but become less accurate as d_{IN} decreases.



(a) For $d_{IN} = 10$ m.

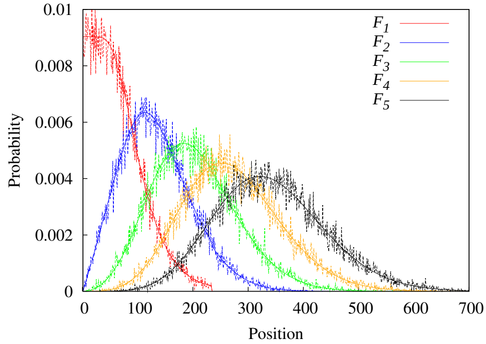


(b) For $d_{IN} = 25$ m.

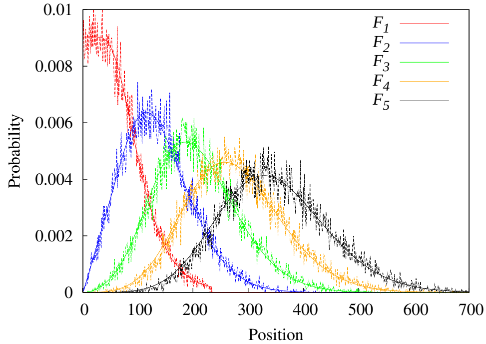


(c) For $d_{IN} = 50$ m.

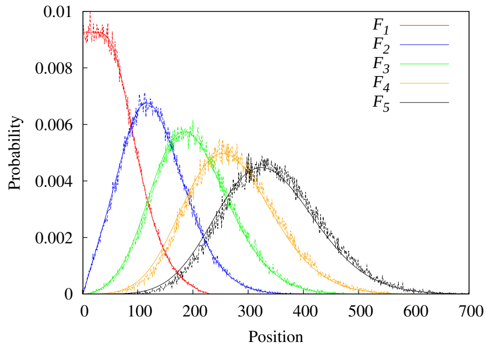
Figure 14: The distribution of the length of the first hop for varying value of d_{IN}. Of the analytical results only the first three hops are shown.



(a) For $d_{IN} = 10$ m.

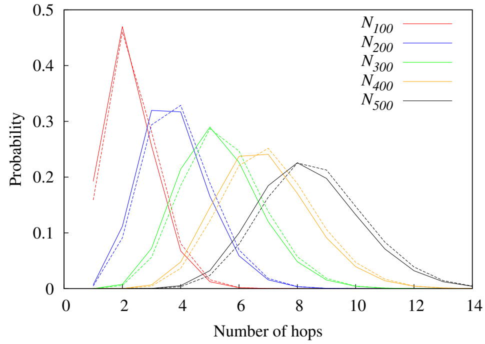


(b) For $d_{IN} = 25$ m.

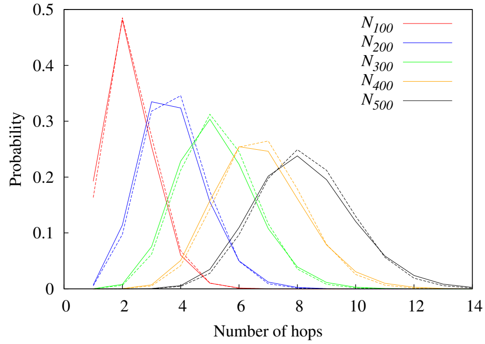


(c) For $d_{IN} = 50$ m.

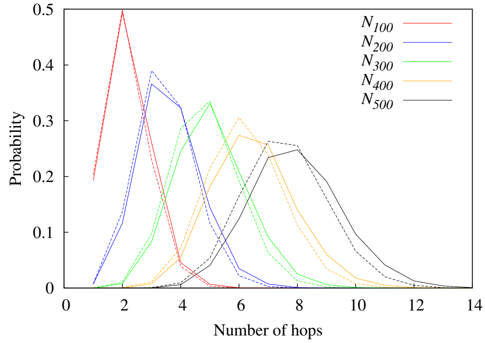
Figure 15: The position of the first five forwarder for $d_{IN} = 50$.



(a) For $d_{IN} = 10$ m.

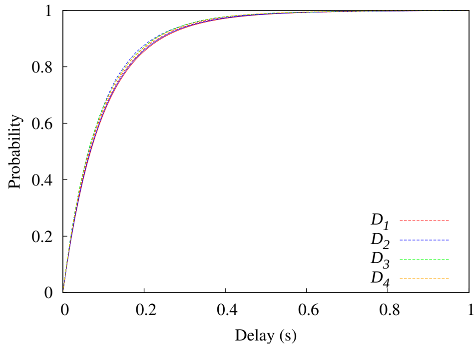


(b) For $d_{IN} = 25$ m.

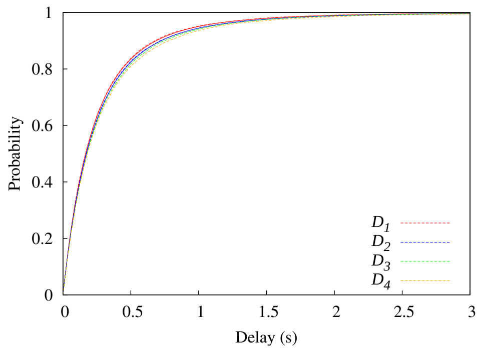


(c) For $d_{IN} = 50$ m.

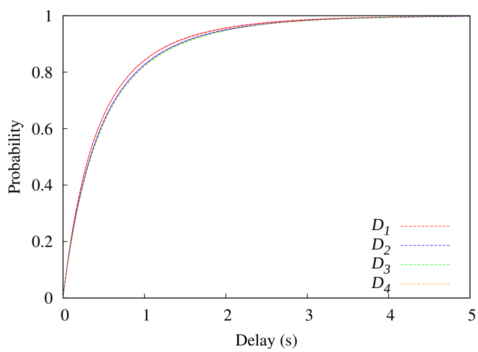
Figure 16: The required number of hops to have the sink receive the message for source-to-sink distances of 100, 200, 300, 400, and 500 m, for $d_{IN} = 10$.



(a) For $d_{IN} = 10$ m.

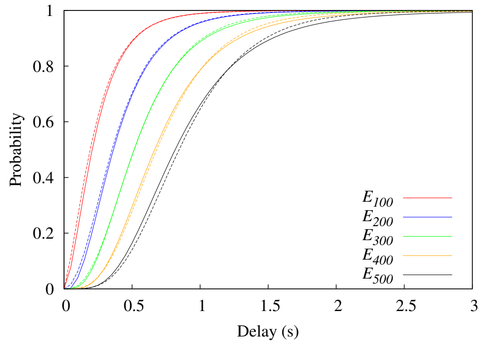


(b) For $d_{IN} = 25$ m.

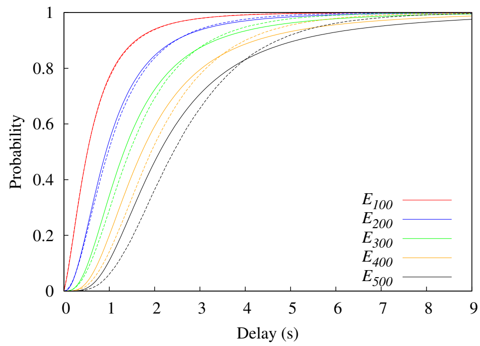


(c) For $d_{IN} = 50$ m.

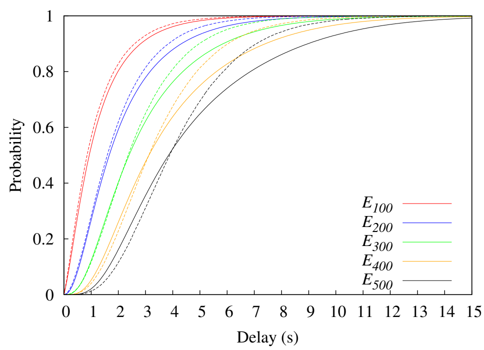
Figure 17: The distribution of the hop delay of the first four hops for $d_{IN} = 50$.Of the analytical results only the first two hops are shown.



(a) For $d_{IN} = 10$ m.

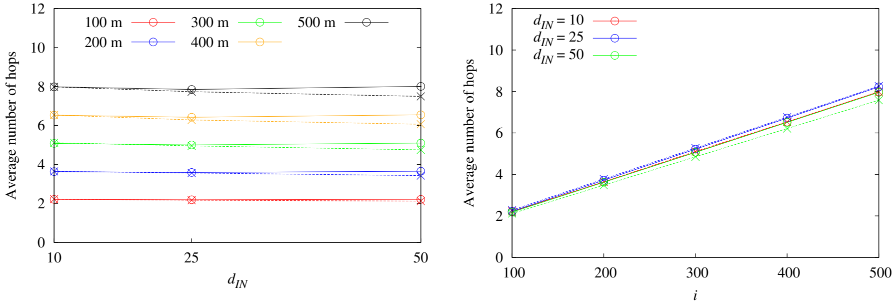


(b) For $d_{IN} = 25$ m.



(c) For $d_{IN} = 50$ m.

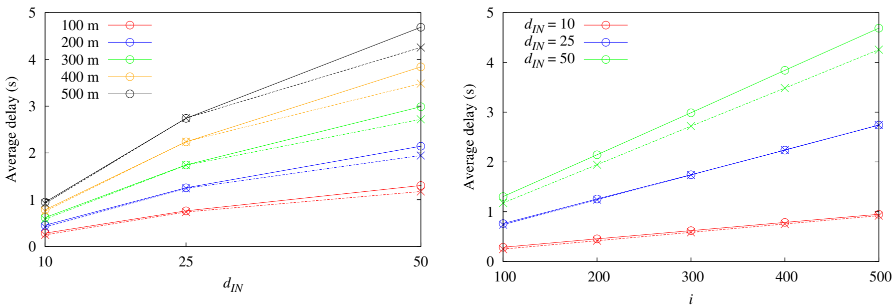
Figure 18: The distribution of the end-to-end delay to have the message forwarded by a node at or beyond 100, 200, 300, 400, and 500 m.



(a) For varying values of i.

(b) For varying values of d_{IN}.

Figure 19: The average required number of hops.



(c) For varying values of i.

(d) For varying values of d_{IN}.

Figure 20: The average end-to-end delay.

Hop	1	2	3	4	5	6	7	8	9	10
d_{IN}	Position of forwarder									
10	0.008	0.018	0.026	0.032	0.038	0.040	0.041	0.044	0.046	0.047
25	0.011	0.020	0.026	0.028	0.29	0.030	0.029	0.030	0.029	0.029
50	0.008	0.012	0.011	0.013	0.016	0.021	0.025	0.028	0.031	0.033
d_{IN}	Hop length									
10	0.008	0.016	0.015	0.013	0.016	0.016	0.015	0.016	0.017	0.019
25	0.011	0.016	0.016	0.013	0.015	0.016	0.015	0.013	0.014	0.014
50	0.008	0.018	0.014	0.018	0.016	0.016	0.018	0.016	0.014	0.017
d_{IN}	Hop delay									
10	0.008	0.005	0.010	0.010	0.013	0.011	0.015	0.013	0.013	0.010
25	0.005	0.006	0.016	0.016	0.016	0.018	0.018	0.020	0.020	0.016
50	0.003	0.007	0.008	0.013	0.015	0.013	0.009	0.015	0.012	0.012

Distance	100	200	300	400	500	600	700	800	900	1000
d_{IN}	Required number of hops									
10	0.043	0.047	0.045	0.041	0.041	0.036	0.035	0.032	0.034	0.031
25	0.026	0.040	0.032	0.036	0.028	0.030	0.026	0.022	0.022	0.020
50	0.041	0.045	0.055	0.065	0.070	0.079	0.074	0.082	0.094	0.082
d_{IN}	End-to-end delay									
10	0.045	0.026	0.010	0.018	0.034	0.050	0.065	0.078	0.089	0.099
25	0.006	0.029	0.054	0.082	0.104	0.124	0.144	0.159	0.176	0.189
50	0.036	0.036	0.052	0.071	0.087	0.103	0.117	0.131	0.144	0.154

Table 2: K-S statistics when comparing the results of the model analysis with the results of the model simulation, calculated using Eq. (65). Distances in meters.

7 Conclusions

In this report we have shown how to analytically model a multi-hop broadcast protocol that uses exponentially distributed forwarding delays, assuming exponentially distributed inter-node distances and a realistic transmission model. Our analysis is able to express the performance of the forwarding protocol in analytical expressions that allow for easy and fast evaluation of the protocol's performance, and that provide for an increased insight in the protocol's behaviour. Our analysis gives an exact description of the behaviour of the first three hops and approximates the behaviour of following hops.

For a given node density and a given transmission model the model is able to capture the full distribution of *(i)* the end-to-end delay to forward a message a specific distance, *(ii)* the required number of hops to forward a message a specific distance, *(iii)* the position of each intermediate forwarder, *(iv)* the length of each hop, *(v)* the delay of each hop, and *(vi)* the success probability of each hop. Verification of our model analysis by means of extensive simulations, for forwarding distances that require on average up to 16 hops, showed that our analysis is very accurate: all results of our model analysis stay within 0.10 of the simulated results for forwarding distances that require on average up to 8 hops, and within 0.19 for distances that require on average up to 16 hops. Especially regarding the required number of hops to disseminate the message accuracy is high, with results staying within 0.08 for distances that require on average up to 16 hops. Results are most accurate for high-density scenarios.

A main insight provided by our model is the interdependency that exists between consecutive hops, especially regarding the lengths of preceding hops. This interdependency has a significant effect on performance as it influences the success probability, length, and delay of each hop, especially in low-density scenarios. This interdependency applies for all multi-hop forwarding in scenarios in which traffic is free flowing and must therefore always be taken into account to accurately describe the behaviour of a forwarding protocol.

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A Distribution of candidate forwarders

A.1 First-hop calculations

A.1.1 Calculating $\mathbb{E}(\frac{C_{1,i}}{C_1} \mid C_1 > 0)$

Let X substitute $C_{1,i}$ and let Y be $C_1 - C_{1,i}$. X is Poisson distributed with mean $\lambda = S_i\mathbb{E}(V_i)$ and Y is Poisson distributed with mean $\mu = \sum_{\substack{i=1 \\ i\neq j}}^{R} S_i\mathbb{E}(V_i)$. Then

$$\begin{aligned}
\mathbb{E}(\frac{C_{1,i}}{C_1} \mid C_1 > 0) &= \sum_{k=0}^{\infty}\sum_{j=0}^{\infty} \frac{k}{k+j} \cdot \mathbb{P}(X = k \wedge Y = j \mid X + Y > 0) \\
&= \sum_{k=0}^{\infty}\sum_{j=0}^{\infty} \frac{k}{k+j} \cdot \frac{\mathbb{P}(X = k \wedge Y = j \wedge X + Y > 0)}{\mathbb{P}(X + Y > 0)} \\
&= \frac{1}{\mathbb{P}(X + Y > 0)} \cdot \sum_{k=0}^{\infty}\sum_{j=0}^{\infty} \frac{k}{k+j} \cdot \mathbb{P}(X = k \wedge Y = j \wedge X + Y > 0) \\
&= \frac{1}{\mathbb{P}(X + Y > 0)} \cdot \sum_{k=1}^{\infty}\sum_{j=0}^{\infty} \frac{k}{k+j} \cdot \mathbb{P}(X = k \wedge Y = j) \\
&= \frac{1}{1 - e^{-(\lambda+\mu)}} \cdot \sum_{k=1}^{\infty}\sum_{j=0}^{\infty} \frac{k}{k+j} \cdot \frac{\lambda^k}{k!}e^{-\lambda} \cdot \frac{\mu^j}{j!}e^{-\mu} \\
&= \frac{\lambda e^{-\lambda}e^{-\mu}}{1 - e^{-(\lambda+\mu)}} \cdot \sum_{k=1}^{\infty}\sum_{j=0}^{\infty} \frac{1}{k+j} \cdot \frac{\lambda^{k-1}}{(k-1)!} \cdot \frac{\mu^j}{j!} \\
&= \frac{\lambda e^{-\lambda}e^{-\mu}}{1 - e^{-(\lambda+\mu)}} \cdot \sum_{k=0}^{\infty}\sum_{j=0}^{\infty} \frac{1}{k+j+1} \cdot \frac{\lambda^{k}}{k!} \cdot \frac{\mu^j}{j!} \\
&= \frac{\lambda\mathbb{E}(\frac{1}{X+Y+1})}{1 - e^{-(\lambda+\mu)}}
\end{aligned} \tag{66}$$

According to [?] (see Eq. (32) for $Q = X + Y, a = 1$), if $X + Y$ is Poisson distributed with mean $\lambda + \mu$ the term $\mathbb{E}(\frac{1}{X+Y+1})$ can be written as $\frac{1-e^{-(\lambda+\mu)}}{\lambda+\mu}$, so we get

$$\begin{aligned}
\mathbb{E}(\frac{C_{1,i}}{C_1} \mid C_1 > 0) &= \frac{\lambda}{\lambda + \mu} \\
&= \frac{\mathbb{E}(C_{1,i})}{\mathbb{E}(C_1)}, \quad i = 1, 2, \dots, R.
\end{aligned} \tag{67}$$

A.1.2 Calculating $\mathbb{E}(H_{1,i} \mid F_1 = j)$

This appendix is meant to support Chapter **??**. We show how to calculate the expected number of first-hop candidate forwarders in interval i, given that the first forwarder is positioned in interval j, denoted $\mathbb{E}(C_{1,i} \mid F_1 = j)$.

To calculate $\mathbb{E}(H_{1,i} \mid F_1 = j)$ we first condition on the number of first-hop candidate forwarders, given that the first forwarder is in interval i:

$$\mathbb{E}(H_{1,i} \mid F_1 = j) = \sum_{c_1=1}^{R} \mathbb{P}(C_1 = c_1 \mid F_1 = j)\, \mathbb{E}(H_{1,i} \mid F_1 = j \wedge C_1 = c_1) \quad (68)$$

for $i = 1, \ldots, R$, $j = 1, \ldots, R$, with $\mathbb{P}(C_1 = c_1 \mid F_1 = j)$ being the probability of having c_1 first-hop candidate forwarders, given that the first forwarder is positioned in interval j, and $\mathbb{E}(H_{1,i} \mid F_1 = j \wedge C_1 = c_1)$ being the expected number of first-hop candidate forwarders in interval i, excluding the first forwarder itself, given that the first forwarder is positioned in interval j and given that there are exactly c_1 first-hop candidate forwarders.

According to Bayes' theorem $\mathbb{P}(C_1 = c_1 \mid F_1 = j)$ is given by

$$\mathbb{P}(C_1 = c_1 \mid F_1 = i) = \mathbb{P}(C_1 = c_1)\, \frac{\mathbb{P}(F_1 = i \mid C_1 = c_1)}{\mathbb{P}(F_1 = i)}, \quad i = 1, \ldots, R. \quad (69)$$

$\mathbb{P}(F_1 = i \mid C_1 = c_1)$ is calculated in a manner similar to Eq. (13), given that there are exactly c_1 first-hop candidate forwarders:

$$\mathbb{P}(F_1 = i \mid C_1 = c) = \frac{\mathbb{E}(C_{1,i} \mid C_1 = c_1)}{\mathbb{E}(C_1 \mid C_1 = c_1)}, \quad i = 1, \ldots, R. \quad (70)$$

Although $\mathbb{E}(C_1 \mid C_1 = c_1)$ is clearly equal to c_1, we can also write it as a summation of the expected number of first-hop candidate forwarders of all the intervals in which there is a non-zero probability of receiving the source's transmission:

$$\mathbb{E}(C_1 \mid C_1 = c_1) = \sum_{i=1}^{R} \mathbb{E}(C_{1,i} \mid C_1 = c_1). \quad (71)$$

$\mathbb{E}(C_{1,i} \mid C_1 = c_1)$ is given by

$$\mathbb{E}(C_{1,i} \mid C_1 = c_1) = \frac{\mathbb{E}(C_{1,i})}{\mathbb{P}(C_1 = c_1)}, \quad i = 1, \ldots, R, \quad (72)$$

with $\mathbb{E}(C_{1,i})$ given by Eq. (5) and C_1 being Poisson distributed with mean $\mathbb{E}(C_1)$, see Eq. (6). Since $\mathbb{P}(C_1 = c_1)$ is a constant it holds that

$$\begin{aligned} \mathbb{P}(F_1 = i \mid C_1 = c) &= \frac{\mathbb{E}(C_{1,i} \mid C_1 = c_1)}{\mathbb{E}(C_1 \mid C_1 = c_1)} \\ &= \frac{\mathbb{E}(C_{1,i})}{\sum_{j=1}^{R} \mathbb{E}(C_{1,j})} \\ &= \mathbb{P}(F_1 = i \mid C_1 > 0), \quad i = 1, \ldots, R, \end{aligned} \quad (73)$$

with $\mathbb{P}(F_1 = i \mid C_1 > 0)$ given by Eq. (13).

To calculate $\mathbb{E}(H_{1,i} \mid F_1 = j \wedge C_1 = c_1)$ we condition on the number of first-hop candidate forwarders in interval i, given that the first forwarder is positioned in interval j and there are exactly c_1 first-hop candidate forwarders:

$$\begin{aligned}
&\mathbb{E}(H_i \mid F_1 = j \wedge C_1 = c_1) = \\
&\sum_{c_{1,i}=1}^{c_1} \mathbb{P}(C_{1,i} = c_{1,i} \mid F_1 = j \wedge C_1 = c_1) \cdot \mathbb{E}(H_i \mid F_1 = j \wedge C_{1,i} = c_{1,i}), \\
&i = 1, \ldots, R, \quad j = 1, \ldots, R,
\end{aligned} \tag{74}$$

with $\mathbb{P}(C_{1,i} = c_{1,i} \mid F_1 = j \wedge C_1 = c_1)$ being the probability of having $c_{1,i}$ first-hop candidate forwarders in interval i, given that the first forwarder is positioned in interval j and given that there are exactly c_1 first-hop candidate forwarders, and with $\mathbb{E}(H_{1,i} \mid F_1 = j \wedge C_{1,i} = c_{1,i})$ being the expected number of first-hop candidate forwarders in interval i, excluding the first forwarder, given that the first forwarder is positioned in interval j and given that there are exactly c_1 first-hop candidate forwarders. It should be clear that $H_{1,i} = c_{1,i}$ for $i \neq j$ and $H_{1,i} = c_{1,i} - 1$ for $i = j$. Using Bayes' theorem $\mathbb{P}(C_{1,i} = c_{1,i} \mid F_1 = j \wedge C_1 = c_1)$ can be rewritten as

$$\begin{aligned}
&\mathbb{P}(C_{1,i} = c_{1,i} \mid C_1 = c_1 \wedge F_1 = j) = \\
&\mathbb{P}(C_1 = c_1 \wedge F_1 = j \mid C_{1,i} = c_{1,i}) \, \frac{\mathbb{P}(C_{1,i} = c_{1,i})}{\mathbb{P}(F_1 = j \wedge C_1 = c_1)}, \\
&i = 1, \ldots, R, \quad j = 1, \ldots, R,
\end{aligned} \tag{75}$$

with $\mathbb{P}(C_1 = c_1 \wedge F_1 = j \mid C_{1,i} = c_{1,i})$ being the probability that the first forwarder is positioned in interval j and there are exactly c_1 first-hop candidate forwarders, given that there are exactly $c_{1,i}$ first-hop candidate forwarders in interval i, $\mathbb{P}(F_1 = i \wedge C_1 = c_1)$ being the probability that the first forwarder is positioned in interval j and there are exactly c_1 first-hop candidate forwarders, and $\mathbb{P}(C_{1,i} = c_{1,i})$ having a Poisson distribution with mean $\mathbb{E}(C_{1,i})$ given by Eq. (5). $\mathbb{P}(F_1 = i \wedge C_1 = c_1)$ is given by

$$\mathbb{P}(F_1 = i \wedge C_1 = c_1) = \mathbb{P}(F_1 = i \mid C_1 = c_1) \, \mathbb{P}(C_1 = c_1), \quad i = 1, \ldots, R, \tag{76}$$

with both right-hand terms known. $\mathbb{P}(C_1 = c_1 \wedge F_1 = j \mid C_{1,i} = c_{1,i})$ can be written as

$$\begin{aligned}
&\mathbb{P}(F_1 = i \wedge C_1 = c_1 \mid C_{1,j} = c_{1,j}) = \\
&\mathbb{P}(F_1 = i \mid C_1 = c_1 \wedge C_{1,j} = c_{1,j}) \, \mathbb{P}(C_1 = c_1 \mid C_{1,j} = c_{1,j}), \\
&i = 1, \ldots, R, \quad j = 1, \ldots, R,
\end{aligned} \tag{77}$$

with $\mathbb{P}(F_1 = i \mid C_1 = c_1 \wedge C_{1,j} = c_{1,j})$ being the probability that the first forwarder is positioned in interval i, given that there are exactly c_1 first-hop candidate forwarders and given that there are exactly $c_{1,j}$ candidate forwarders in interval j, and with $\mathbb{P}(C_1 = c_1 \mid C_{1,i} = c_{1,i})$ being the probability that there are exactly c_1 first-hop candidate forwarders, given that there are exactly $c_{1,i}$ first-hop candidate forwarders in interval i. $\mathbb{P}(F_1 = i \mid C_1 = c_1 \wedge C_{1,j} = c_{1,j})$ can be calculated by conditioning on the number of first-hop candidate forwarders in interval i:

$$\mathbb{P}(F_1 = i \mid C_1 = c_1 \wedge C_{1,j} = c_{1,j}) = \\ \sum_{c_{1,i}=1}^{c_1 - c_{1,j}} \mathbb{P}(C_{1,i} = c_{1,i} \mid C_1 = c_1 \wedge C_{1,j} = c_{1,j}) \cdot \\ \mathbb{P}(F_1 = i \mid C_1 = c_1 \wedge C_{1,i} = c_{1,i} \wedge C_{1,j} = c_{1,j}), \quad i = 1, \ldots, R, \tag{78}$$

with $\mathbb{P}(C_{1,i} = c_{1,i} \mid C_1 = c_1 \wedge C_{1,j} = c_{1,j})$ being the probability of having exactly $c_{1,i}$ first-hop candidate forwarders in interval i, given that there are exactly c_1 first-hop candidate forwarders and that there are exactly $c_{1,j}$ first-hop candidate forwarders in interval j, and $\mathbb{P}(F_1 = i \mid C_1 = c_1 \wedge C_{1,i} = c_{1,i} \wedge C_{1,j} = c_{1,j})$ being the probability that the first forwarder is positioned in interval i, given that there are exactly c_1 first-hop candidate forwarders, exactly $c_{1,j}$ first-hop candidate forwarders in interval j, and exactly $c_{1,i}$ first-hop candidate forwarders in interval i. Since the probability of becoming the next forwarder is equal for all candidate forwarders $\mathbb{P}(F_1 = i \mid C_1 = c_1 \wedge C_{1,i} = c_{1,i} \wedge C_{1,j} = c_{1,j})$ is calculated by dividing the number of first-hop candidate forwarders in interval i by the total number of first-hop candidate forwarders:

$$\mathbb{P}(F_1 = i \mid C_1 = c_1 \wedge C_{1,i} = c_{1,i} \wedge C_{1,j} = c_{1,j}) = \frac{c_{1,i}}{c_1}, \\ i = 1, \ldots, R, \quad j = 1, \ldots, R. \tag{79}$$

$\mathbb{P}(C_{1,i} = c_{1,i} \mid C_1 = c_1 \wedge C_{1,j} = c_{1,j})$ is calculated by multiplying the probability that there are exactly $c_{1,i}$ first-hop candidate forwarders in interval i with the probability of having $c_1 - c_{1,i} - c_{1,j}$ first-hop candidate forwarders in the remaining $R - 2$ intervals excluding intervals i and j, divided by the probability that there are exactly $c_1 - c_{1,j}$ first-hop candidate forwarders in the $R - 1$ intervals excluding interval j. The number of first-hop candidate forwarders in interval k is Poisson distributed with mean $\mathbb{E}(C_{1,k}) = S_k \cdot \mathbb{E}(V_k)$, see Eq. (5). According to [10] the summation of a number of Poisson distributed variables is also Poisson distributed with the mean equal to the summed up means. The number of first-hop candidate forwarders in n intervals is therefore Poisson distributed with the mean equal to the expected number of first-hop candidate forwarders in those intervals. $\mathbb{P}(C_{1,i} = c_{1,i} \mid C_1 = c_1 \wedge C_{1,j} = c_{1,j})$ is thus given by

$$\mathbb{P}(C_{1,i} = c_{1,i} \mid C_1 = c_1 \wedge C_{1,j} = c_{1,j}) = \frac{\frac{(S_i \cdot \mathbb{E}(V_i))^{c_{1,i}}}{c_{1,i}!} e^{-S_i \cdot \mathbb{E}(V_i)} \frac{\hat{\lambda}^{c_1 - c_{1,i} - c_{1,j}}}{(c_1 - c_{1,i} - c_{1,j})!} e^{-\hat{\lambda}}}{\frac{\check{\lambda}^{(c_1 - c_{1,j})}}{(c_1 - c_{1,j})!} e^{-\check{\lambda}}},$$

$$\hat{\lambda} = \sum_{\substack{k=1 \\ k \neq i \\ k \neq j}}^{R} S_k \cdot \mathbb{E}(V_k), \quad \check{\lambda} = \sum_{\substack{k=1 \\ k \neq j}}^{R} S_k \cdot \mathbb{E}(V_k), \quad i = 1, \ldots, R, \quad j = 1, \ldots, R. \tag{80}$$

$\mathbb{P}(C_1 = c_1 \mid C_{1,i} = c_{1,i})$ in Eq. (77) is equal to the probability of having exactly $c_1 - c_{1,i}$ first-hop candidate forwarders in the $R-1$ intervals excluding interval i, given by

$$\mathbb{P}(C_1 = c_1 \mid C_{1,i} = c_{1,i}) = \frac{\hat{\lambda}^{(c_1 - c_{1,i})}}{(c_1 - c_{1,i})!} e^{-\hat{\lambda}},$$

$$\hat{\lambda} = \sum_{\substack{k=1 \\ k \neq i}}^{R} S_k \lambda, \quad i = 1, \ldots, R. \tag{81}$$

Using Eq. (68)-(81) we now have a complete expression for calculating $\mathbb{E}(H_{1,i} \mid F_1 = j)$.

A.1.3 Proof for $\mathbb{E}(H_{1,i} \mid F_1 = j) = \mathbb{E}(H_{1,i} \mid C_1 > 0)$

We prove Eq. (20) in case of an ideal transmission model, i.e., $S_i = 1$ for $i = 1, 2, \ldots, R$ and $S_i = 0$ otherwise.

Proof. We write out $\mathbb{E}(H_{1,i} \mid C_1 > 0)$ and $\mathbb{E}(H_{1,i} \mid F_1 = j)$ in full and show that they are equal.

Assuming the simple transmission model and substituting $\mathbb{E}(V_{1,i})$ by λ, writing out $\mathbb{E}(H_{1,i} \mid C_1 > 0)$ in full gives

$$\begin{aligned}
\mathbb{E}(H_{1,i} \mid C_1 > 0) &= \mathbb{E}(C_{1,i} \mid C_1 > 0) - \mathbb{E}(G_{1,i} \mid C_1 > 0) \\
&= \frac{\mathbb{E}(C_{1,i})}{1 - \mathbb{P}(C_1 = 0)} - \mathbb{P}(F_1 = i \mid C_1 > 0) \\
&= \frac{\lambda}{1 - e^{-R\lambda}} - \frac{\mathbb{E}(C_{1,i} \mid C_1 > 0)}{\mathbb{E}(C_1 \mid C_1 > 0)} \\
&= \frac{\lambda}{1 - e^{-R\lambda}} - \frac{\lambda}{R\lambda} \\
&= \frac{\lambda}{1 - e^{-R\lambda}} - \frac{1}{R}, \quad i = 1, 2, \ldots, R,
\end{aligned} \tag{82}$$

with $\mathbb{E}(H_{1,i} \mid C_1 > 0) = 0$ for $i < 0$ and $\mathbb{E}(H_{1,i} \mid C_1 > 0) = 0$ for $i > R$.

Writing out $\mathbb{E}(H_{1,i} \mid F_1 = j)$ in full gives

$$\mathbb{E}(H_{1,i} \mid F_1 = j) = \sum_{c_1=1}^{\infty} \Bigg(\mathbb{P}(C_1 = c_1 \mid F_1 = j)\, \mathbb{E}(H_{1,i} \mid F_1 = j \wedge C_1 = c_1) \Bigg)$$

$$= \sum_{c_1=1}^{\infty} \Bigg(\mathbb{P}(C_1 = c_1)\, \frac{\mathbb{P}(F_1 = j \mid C_1 = c_1)}{\mathbb{P}(F_1 = j)}\, \mathbb{E}(H_{1,i} \mid F_1 = j \wedge C_1 = c_1) \Bigg)$$

$$= \sum_{c_1=1}^{\infty} \Bigg(\mathbb{P}(C_1 = c_1)\, \frac{\frac{\mathbb{E}(C_{1,j})}{\mathbb{E}(C_1)}}{\mathbb{P}(C_1 > 0)\frac{\mathbb{E}(C_{1,j})}{\mathbb{E}(C_1)}} \cdot$$

$$\sum_{c_{1,i}=1}^{c_1} \Bigg(\mathbb{P}(C_{1,i} = c_{1,i} \mid F_1 = j \wedge C_1 = c_1)\, \mathbb{E}(H_i \mid F_1 = j \wedge C_{1,i} = c_{1,i}) \Bigg) \Bigg)$$

$$= \sum_{c_1=1}^{\infty} \Bigg(\mathbb{P}(C_1 = c_1)\, \frac{\frac{\mathbb{E}(C_{1,j})}{\mathbb{E}(C_1)}}{\mathbb{P}(C_1 > 0)\frac{\mathbb{E}(C_{1,j})}{\mathbb{E}(C_1)}} \cdot$$

$$\sum_{c_{1,i}=1}^{c_1} \Bigg(\mathbb{P}(C_1 = c_1 \wedge F_1 = j \mid C_{1,i} = c_{1,i})\, \frac{\mathbb{P}(C_{1,i} = c_{1,i})}{\mathbb{P}(F_1 = j \wedge C_1 = c_1)} \cdot$$

$$\mathbb{E}(H_i \mid F_1 = j \wedge C_{1,i} = c_{1,i}) \Bigg) \Bigg)$$

$$= \sum_{c_1=1}^{\infty} \Bigg(\mathbb{P}(C_1 = c_1)\, \frac{\frac{\mathbb{E}(C_{1,j})}{\mathbb{E}(C_1)}}{\mathbb{P}(C_1 > 0)\frac{\mathbb{E}(C_{1,j})}{\mathbb{E}(C_1)}} \cdot$$

$$\sum_{c_{1,i}=1}^{c_1} \Bigg(\mathbb{P}(F_1 = j \mid C_1 = c_1 \wedge C_{1,i} = c_{1,i})\, \mathbb{P}(C_1 = c_1 \mid C_{1,i} = c_{1,i}) \cdot$$

$$\frac{\mathbb{P}(C_{1,i} = c_{1,i})}{\mathbb{P}(F_1 = j \mid C_1 = c_1)\, \mathbb{P}(C_1 = c_1)}\, \mathbb{E}(H_i \mid F_1 = j \wedge C_{1,i} = c_{1,i}) \Bigg) \Bigg)$$

$$= \sum_{c_1=1}^{\infty} \Bigg(\mathbb{P}(C_1 = c_1)\, \frac{\frac{\mathbb{E}(C_{1,j})}{\mathbb{E}(C_1)}}{\mathbb{P}(C_1 > 0)\frac{\mathbb{E}(C_{1,j})}{\mathbb{E}(C_1)}} \cdot$$

$$\sum_{c_{1,i}=1}^{c_1} \Bigg(\sum_{c_{1,j}=1}^{c_1 - c_{1,i}} \Bigg(\mathbb{P}(C_{1,j} = c_{1,j} \mid C_1 = c_1 \wedge C_{1,i} = c_{1,i}) \cdot$$

$$\mathbb{P}(F_1 = j \mid C_1 = c_1 \wedge C_{1,j} = c_{1,j} \wedge C_{1,i} = c_{1,i}) \Bigg)\, \mathbb{P}(C_1 = c_1 \mid C_{1,i} = c_{1,i}) \cdot$$

$$\frac{\mathbb{P}(C_{1,i} = c_{1,i})}{\mathbb{P}(F_1 = j \mid C_1 = c_1)\, \mathbb{P}(C_1 = c_1)}\, \mathbb{E}(H_i \mid F_1 = j \wedge C_{1,i} = c_{1,i}) \Bigg) \Bigg)$$

$$= \sum_{c_1=1}^{\infty} \Bigg(\mathbb{P}(C_1 = c_1) \, \frac{\frac{\mathbb{E}(C_{1,j})}{\mathbb{E}(C_1)}}{\mathbb{P}(C_1 > 0) \frac{\mathbb{E}(C_{1,j})}{\mathbb{E}(C_1)}} \cdot$$

$$\sum_{c_{1,i}=1}^{c_1} \Bigg(\sum_{c_{1,j}=1}^{c_1 - c_{1,i}} \Bigg(\mathbb{P}(C_{1,j} = c_{1,j} \mid C_1 = c_1 \wedge C_{1,i} = c_{1,i}) \cdot$$

$$\frac{c_{1,j}}{c_1} \Bigg) \, \mathbb{P}(C_1 = c_1 \mid C_{1,i} = c_{1,i}) \cdot$$

$$\frac{\mathbb{P}(C_{1,i} = c_{1,i})}{\mathbb{P}(F_1 = j \mid C_1 = c_1) \, \mathbb{P}(C_1 = c_1)} \, \mathbb{E}(H_i \mid F_1 = j \wedge C_{1,i} = c_{1,i}) \Bigg) \Bigg),$$

$$i = 1, 2, \ldots, R, \;\; j = 1, 2, \ldots, R, \qquad (83)$$

with $\mathbb{E}(H_{1,i} \mid F_1 = j) = 0$ for $i < 0$ and $\mathbb{E}(H_{1,i} \mid F_1 = j) = 0$ for $i > R$.

Assuming the simple transmission model, substituting $\mathbb{E}(V_{1,i})$ by λ, and (for now) ignoring the case $i = j$, the equation above can rewritten as follows

$$\mathbb{E}(H_{1,i} \mid F_1 = j) = \sum_{c_1=1}^{\infty} \Bigg(\mathbb{P}(C_1 = c_1) \, \frac{\frac{\mathbb{E}(C_{1,j})}{\mathbb{E}(C_1)}}{\mathbb{P}(C_1 > 0) \frac{\mathbb{E}(C_{1,j})}{\mathbb{E}(C_1)}} \cdot$$

$$\sum_{c_{1,i}=1}^{c_1} \Bigg(\sum_{c_{1,j}=1}^{c_1 - c_{1,i}} \Bigg(\mathbb{P}(C_{1,j} = c_{1,j} \mid C_1 = c_1 \wedge C_{1,i} = c_{1,i}) \, \cdot \frac{c_{1,j}}{c_1} \Bigg) \cdot$$

$$\mathbb{P}(C_1 = c_1 \mid C_{1,i} = c_{1,i}) \, \frac{\mathbb{P}(C_{1,i} = c_{1,i})}{\mathbb{P}(F_1 = j \mid C_1 = c_1) \, \mathbb{P}(C_1 = c_1)} \, c_{1,i} \Bigg) \Bigg)$$

$$= \sum_{c_1=1}^{\infty} \Bigg(\frac{(R\lambda)^{c_1}}{c_1!} e^{-R\lambda} \, \frac{\frac{\lambda}{R\lambda}}{(1 - e^{-R\lambda}) \frac{\lambda}{R\lambda}} \cdot$$

$$\sum_{c_{1,i}=1}^{c_1} \Bigg(\sum_{c_{1,j}=1}^{c_1 - c_{1,i}} \Bigg(\frac{\frac{\lambda^{c_{1,j}}}{c_{1,j}!} e^{-\lambda} \cdot \frac{(R-2)\lambda^{c_1 - c_{1,j} - c_{1,i}}}{(c_1 - c_{1,j} - c_{1,i})!} e^{-(R-2)\lambda}}{\frac{(R-1)\lambda^{c_1 - c_{1,i}}}{(c_1 - c_{1,i})!} e^{-(R-1)\lambda}} \, \cdot \frac{c_{1,j}}{c_1} \Bigg) \cdot$$

$$\frac{((R-1)\lambda)^{c_1 - c_{1,i}}}{(c_1 - c_{1,i})!} e^{-(R-1)\lambda} \, \frac{\frac{\lambda^{c_{1,i}}}{c_{1,i}!} e^{-\lambda}}{\frac{1}{R} \frac{(R\lambda)^{c_1}}{c_1!} e^{-R\lambda}} \, c_{1,i} \Bigg) \Bigg),$$

$$i = 1, 2, \ldots, R, \;\; j = 1, 2, \ldots, R, \;\; i \neq j, \qquad (84)$$

with $\mathbb{E}(H_{1,i} \mid F_1 = j) = 0$ for $i < 0$ and $\mathbb{E}(H_{1,i} \mid F_1 = j) = 0$ for $i > R$. Renaming c_1 to c, $c_{1,i}$ to k, and $c_{1,j}$ to n, and still assuming $i \neq j$, $\mathbb{E}(H_{1,i} \mid F_1 =$

j) can be written as

$$\mathbb{E}(H_{1,i} \mid F_1 = j) = \frac{1}{1 - e^{-R\lambda}} \cdot$$
$$\sum_{c=1}^{\infty} \left(\frac{(R\lambda)^c}{c!} e^{-R\lambda} \sum_{k=1}^{c} \left(\sum_{n=1}^{c-k} \left(\frac{\frac{\lambda^n}{n!} e^{-\lambda} \frac{((R-2)\lambda)^{c-n-k}}{(c-n-k)!} e^{-(R-2)\lambda}}{\frac{((R-1)\lambda)^{c-k}}{(c-k)!} e^{-(R-1)\lambda}} \cdot \frac{n}{c} \cdot \right.\right.\right.$$
$$\left.\left.\left. \frac{((R-1)\lambda)^{c-k}}{(c-k)!} e^{-(R-1)\lambda} \cdot \frac{\frac{\lambda^k}{k!} e^{-\lambda}}{\frac{1}{R} \frac{(R\lambda)^c}{c!} e^{-R\lambda}} \cdot k \right)\right)\right)$$
$$= \frac{R}{1 - e^{-R\lambda}} \sum_{c=1}^{\infty} \left(\sum_{k=1}^{c} \left(\sum_{n=1}^{c-k} \left(\frac{\lambda^n}{n!} e^{-\lambda} \cdot \frac{((R-2)\lambda)^{c-n-k}}{(c-n-k)!} e^{-(R-2)\lambda} \cdot \frac{n}{c} \cdot \frac{\lambda^k}{k!} e^{-\lambda} \cdot k \right)\right)\right),$$
$$i = 1, 2, \ldots, R, \quad j = 1, 2, \ldots, R, \quad i \neq j, \tag{85}$$

with $\mathbb{E}(H_{1,i} \mid F_1 = j) = 0$ for $i < 0$ and $\mathbb{E}(H_{1,i} \mid F_1 = j) = 0$ for $i > R$. Rearranging summations followed by a number of substitutions gives us

$$\mathbb{E}(H_{1,i} \mid F_1 = j) =$$
$$\frac{R}{1 - e^{-R\lambda}} \sum_{k=1}^{\infty} \left(\sum_{n=1}^{\infty} \left(\sum_{c=0}^{\infty} \left(\frac{\lambda^n}{n!} e^{-\lambda} \cdot \frac{((R-2)\lambda)^{c-n-k}}{(c-n-k)!} e^{-(R-2)\lambda} \cdot \frac{n}{c} \cdot \frac{\lambda^k}{k!} e^{-\lambda} \cdot k \right)\right)\right)$$
$$= \frac{R}{1 - e^{-R\lambda}} \sum_{k=1}^{\infty} \left(\sum_{n=1}^{\infty} \left(\sum_{c=0}^{\infty} \left(\frac{\lambda^n}{n!} e^{-\lambda} \cdot \frac{((R-2)\lambda)^{c}}{c!} e^{-(R-2)\lambda} \cdot \frac{n}{c+k+n} \cdot \frac{\lambda^k}{k!} e^{-\lambda} \cdot k \right)\right)\right)$$
$$= \frac{\lambda^2 R}{1 - e^{-R\lambda}} \sum_{k=1}^{\infty} \left(\sum_{n=1}^{\infty} \left(\sum_{c=0}^{\infty} \left(\frac{\lambda^n}{n!} e^{-\lambda} \cdot \frac{((R-2)\lambda)^{c}}{c!} e^{-(R-2)\lambda} \cdot \frac{\lambda^k}{k!} e^{-\lambda} \cdot \frac{1}{c+k+n+2} \right)\right)\right)$$
$$= \frac{\lambda^2 R}{1 - e^{-R\lambda}} \mathbb{E}\left(\frac{1}{X+Y+Z+2}\right), \quad i = 1, 2, \ldots, R, \quad j = 1, 2, \ldots, R, \quad i \neq j, \tag{86}$$

with $\mathbb{E}(H_{1,i} \mid F_1 = j) = 0$ for $i < 0$, $\mathbb{E}(H_{1,i} \mid F_1 = j) = 0$ for $i > R$, X being Poisson distributed with mean λ, Y being Poisson distributed with mean λ, and Z being Poisson distributed with mean $(R-2)\lambda$. Let $Q = X + Y + Z$, then Q is Poisson distributed with mean $R\lambda$. It has been shown in [?] that

$$\mathbb{E}\left(\frac{1}{Q+a}\right) = \frac{(a-1)!(-1)^{a-1}}{\mu^a} \left(1 - e^{-\mu} \sum_{r=1}^{a-1} \frac{(-\mu)^r}{r!}\right), \quad a \in \mathbb{N}^+ \tag{87}$$

with Q being Poisson distributed with mean μ. Thus we can state

$$\mathbb{E}\left(\frac{1}{X+Y+Z+2}\right) = \frac{R\lambda - (1 - e^{-R\lambda})}{(R\lambda)^2}, \tag{88}$$

and

$$
\begin{aligned}
\mathbb{E}(H_{1,i} \mid F_1 = j) &= \frac{\lambda^2 R}{1-e^{-R\lambda}} \mathbb{E}(\frac{1}{X+Y+Z+2}) \\
&= \frac{\lambda^2 R}{1-e^{-R\lambda}} \cdot \frac{R\lambda - (1-e^{-R\lambda})}{(R\lambda)^2} \\
&= \frac{R\lambda - (1-e^{-R\lambda})}{R(1-e^{-R\lambda})} \\
&= \frac{\lambda}{1-e^{-R\lambda}} - \frac{1}{R} \\
&= \mathbb{E}(H_{1,i} \mid C_1 > 0), \\
&\quad i = 1, 2, \ldots, R, \;\; j = 1, 2, \ldots, R, \;\; i \neq j.
\end{aligned} \tag{89}
$$

A.1.4 Calculating the distribution of $C_{1,j+1:R} \mid F_1 = j$

Although the number of first-hop candidate forwarders in interval i is Poisson distributed, this does no longer hold when it is given that the first forwarder is positioned in interval j. We therefore given a step-by-step breakdown of how to calculate the distribution of $C_{1,j+1:R}$. We start by conditioning on the number of first-hop candidate forwarders, given that the first forwarder is positioned in interval j:

$$
\begin{aligned}
\mathbb{P}(C_{1,j+1:R} = c_{1,j+1:R} \mid F_1 = j) = \sum_{c_1=1}^{\infty} &\mathbb{P}(C_1 = c_1 \mid F_1 = j) \cdot \\
&\mathbb{P}(C_{1,j+1:R} = c_{1,j+1:R} \mid F_1 = j \wedge C_1 = c_1), \\
&c_{1,j+1:R} = 0, 1, 2, \ldots, \quad j = 1, 2, \ldots, R,
\end{aligned} \tag{90}
$$

with $\mathbb{P}(C_1 = c_1 \mid F_1 = j)$ given by Eq. (69), and $\mathbb{P}(C_{1,j+1:R} = c_{1,j+1:R} \mid F_1 = j \wedge C_1 = c_1)$ being the probability that there are $c_{1,j+1:R}$ remaining candidate forwarders, given that the first forwarder is positioned in interval j and given that there are c_1 first-hop candidate forwarders.

$\mathbb{P}(C_{1,j+1:R} = c_{1,j+1:R} \mid F_1 = j \wedge C_1 = c_1)$ can be rewritten as

$$
\begin{aligned}
&\mathbb{P}(C_{1,j+1:R} = c_{1,j+1:R} \mid F_1 = j \wedge C_1 = c_1) = \\
&\mathbb{P}(F_1 = j \wedge C_1 = c_1 \mid C_{1,j+1:R} = c_{1,j+1:R}) \cdot \frac{\mathbb{P}(C_{1,j+1:R} = c_{1,j+1:R})}{\mathbb{P}(F_1 = j \wedge C_1 = c_1)}, \quad (91) \\
&c_{1,j+1:R} = 0, 1, 2, \ldots, \quad j = 1, 2, \ldots, R, \quad (92)
\end{aligned}
$$

with $\mathbb{P}(F_1 = j \wedge C_1 = c_1 \mid C_{1,j+1:R} = c_{1,j+1:R})$ being the probability that the first forwarder is positioned in interval j and there are c_1 first-hop candidate forwarders, given that there are $c_{1,j+1:R}$ remaining first-hop candidate forwarders, $\mathbb{P}(C_{1,j+1:R} = c_{1,j+1:R})$ being the probability that there are $c_{1,j+1:R}$ remaining first-hop candidate forwarders, and with $\mathbb{P}(F_1 = j \wedge C_1 = c_1)$ given by Eq. (76).

$\mathbb{P}(F_1 = j \wedge C_1 = c_1 \mid C_{1,j+1:R} = c_{1,j+1:R})$ can in turn be rewritten to

$$\begin{aligned}
&\mathbb{P}(F_1 = j \wedge C_1 = c_1 \mid C_{1,j+1:R} = c_{1,j+1:R}) = \\
&\mathbb{P}(F_1 = j \mid C_1 = c_1 \wedge C_{1,j+1:R} = c_{1,j+1:R}) \cdot \mathbb{P}(C_1 = c_1 \mid C_{1,j+1:R} = c_{1,j+1:R}), \\
&j = 1, 2, \ldots, R, \;\; c_1 = 0, 1, 2, \ldots, \;\; c_{1,j+1:R} = 0, 1, 2, \ldots, \qquad (93)
\end{aligned}$$

with $\mathbb{P}(F_1 = j \mid C_1 = c_1 \wedge C_{1,j+1:R} = c_{1,j+1:R})$ being the probability that the first forwarder is positioned in interval i, given that there are c_1 first-hop candidate forwarders and $c_{1,j+1:R}$ remaining first-hop candidate forwarders, and $\mathbb{P}(C_1 = c_1 \mid C_{1,j+1:R} = c_{1,j+1:R})$ being the probability that there are c_1 first-hop candidate forwarders, given that there are $c_{1,j+1:R}$ remaining first-hop candidate forwarders.

As was already stated in the beginning of this section the number of first-hop candidate forwarders in interval i is Poisson distributed with mean $\mathbb{E}(C_{1,i})$, given by Eq. (5), so $\mathbb{P}(C_{1,j+1:R} = c_{1,j+1:R})$ in Eq. (91) is Poisson distributed with mean $\sum_{i=j+1}^{R} \mathbb{E}(C_{1,i})$.

$\mathbb{P}(F_1 = j \mid C_1 = c_1 \wedge C_{1,j+1:R} = c_{1,j+1:R})$ can be calculated by conditioning on the number of first-hop candidate forwarders in interval i, given that there are c_1 first-hop candidate forwarders and $c_{1,j+1:R}$ remaining first-hop candidate forwarders:

$$\begin{aligned}
&\mathbb{P}(F_1 = j \mid C_1 = c_1 \wedge C_{1,j+1:R} = c_{1,j+1:R}) = \\
&\sum_{c_{1,i}=1}^{c_1 - c_{1,j+1:R}} \mathbb{P}(C_{1,j} = c_{1,j} \mid C_1 = c_1 \wedge C_{1,j+1:R} = c_{1,j+1:R}) \, \mathbb{P}(F_1 = i \mid C_{1,i} = c_{1,i} \wedge C_{=} c_1), \\
&j = 1, 2, \ldots, R, \;\; c_1 = 0, 1, 2, \ldots, \;\; c_{1,j+1:R} = 0, 1, 2, \ldots, \qquad (94)
\end{aligned}$$

with $\mathbb{P}(C_{1,j} = c_{1,j} \mid C_1 = c_1 \wedge C_{1,j+1:R} = c_{1,j+1:R})$ being the probability that there are $c_{1,j}$ first-hop candidate forwarders in interval j, given that there are in total c_1 first-hop candidate forwarders and $c_{1,j+1:R}$ remaining first-hop candidate forwarders, and $\mathbb{P}(F_1 = i \mid C_{1,i} = c_{1,i} \wedge C_{=} c_1)$ being the probability that the first forwarder is positioned in interval i, given that there are $c_{1,i}$ first-hop candidate forwarders in interval i and c_1 first-hop candidate forwarders.

Since each candidate forwarder has an equal probability of becoming the next forwarder, $\mathbb{P}(F_1 = i \mid C_{1,i} = c_{1,i} \wedge C_{=} c_1)$ is given by $\frac{c_{1,j}}{c_1}$.

$\mathbb{P}(C_{1,j} = c_{1,j} \mid C_1 = c_1 \wedge C_{1,j+1:R} = c_{1,j+1:R})$ is given by

$$\begin{aligned}
&\mathbb{P}(C_{1,j} = c_{1,j} \mid C_1 = c_1 \wedge C_{1,j+1:R} = c_{1,j+1:R}) = \\
&\frac{\mathbb{P}(C_{1,j} = c_{1,j}) \, \mathbb{P}(C_{1,1:j-1} = c_{1,1:j-1})}{\mathbb{P}(C_{1,1:j} = c_{1,1:j})}, \\
&j = 1, 2, \ldots, R, \;\; c_1 = 0, 1, 2, \ldots, \;\; c_{1,j+1:R} = 0, 1, 2, \ldots, \qquad (95)
\end{aligned}$$

with $\mathbb{P}(C_{1,j} = c_{1,j})$ being the probability that there are $c_{1,j}$ first-hop candidate forwarders in interval j, $\mathbb{P}(C_{1,1:j-1} = c_{1,1:j-1})$ being the probability that there are $c_{1,1:j-1}$ first-hop candidate forwarders in intervals 1 through $j-1$, and

$\mathbb{P}(C_{1,1:j} = c_{1,1:j})$ being the probability that there are $c_{1,1:j}$ first-hop candidate forwarders in intervals 1 through j. Again the number of first-hop candidate forwarders in interval i is Poisson distributed with mean $\mathbb{E}(C_{1,i})$, given by Eq. (5), so $\mathbb{P}(C_{1,j} = c_{1,j})$ is Poisson distributed with mean $\mathbb{E}(C_{1,j})$, $\mathbb{P}(C_{1,1:j-1} = c_{1,1:j-1})$ is Poisson distributed with mean $\sum_{i=1}^{j-1} \mathbb{E}(C_{1,i})$, and $\mathbb{P}(C_{1,1:j} = c_{1,1:j})$ is Poisson distributed with mean $\sum_{i=1}^{j} \mathbb{E}(C_{1,i})$.