Conditions for Rank Reversal in Supplier Selection

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Abstract

Supplier selection is the process of selecting the best bid among a number of bids submitted by different suppliers. The bids are all evaluated against a set of predetermined criteria. If there are more criteria, the bids receive a score on each criterion. The bid with the best total score over all criteria wins.

A common and sometimes even mandatory scoring rule per criterion is to assign the maximum score to the best bid on that criterion and to relate all other scores on that criterion to that best bid. This is a form of relative scoring, which is well known to open the way to *rank reversal*, the phenomenon that the ranking between two bidders is reversed. This may happen when, for example, an additional bid is taken into account, or when one of the original bids is removed.

In supplier selection this is especially harmful when the rank reversal involves the winning bid. In this paper we investigate under which conditions rank reversal with regard to the winning bid can occur.

Key words: Purchasing, multiple criteria, supplier selection, decision analysis, rank reversal

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1 Introduction

Supplier selection is the process of selecting the best bid among a number of bids submitted by different suppliers. The supplier with the best bid is awarded a contract. The selection mechanism usually considers either price only or several criteria simultaneously. In the latter case the most frequently used method is Weighted Factor Score (WFS): all criteria are evaluated separately and this evaluation results in a score on each criterion. Scores on individual criteria are multiplied by their weights and added, the contract being awarded to the highest total score (Sykes, 2012).

A scoring rule is a function that is used to assign a numerical value to different levels of performance on a criterion offered by the bids. Many different scoring rules are available and used (de Boer, Labro, & Morlacchi, 2001; Dini, Pacini, & Valletti, 2009).

A major distinction can be made between scoring rules that are dependent and scoring rules that are not dependent on the performance level on that criterion offered in other submitted bids. Scoring rules that are dependent are called relative scoring rules.

Within the class of relative scoring rules again many variants exist: the scores may be dependent on the best bid, on the average bid, on the median bid or on a combination of these e.g. the best and the worst bid. And within these different possibilities more variants exist depending on the way the scores relate to that reference bid (e.g. linear or nonlinear). Here we consider the two most frequently used rules: one scoring rule used for a criterion that is to be minimized (such as price) and one for a criterion that has to be maximized (such as quality). In both cases the scoring rules assign the maximum score to the best bid on that criterion and to relate all other scores on that criterion to that best bid.

Some forms of relative scoring, including the ones we discuss, are well known to open the way to *rank reversal* (Dini, Pacini, & Valletti, 2009) which is the phenomenon that the ranking between two bidders is reversed. This may happen when, for example, an additional bid is taken into account, or when one of the original bids is removed.

In supplier selection this is especially harmful when the rank reversal involves the winning bid. In this paper we investigate under which conditions rank reversal with regard to the winning bid can occur.

A commonly used relative scoring rule for a criterion to be minimized (such as price) is:

$$Score_{(i)} = Score_{max} * Price_{best} / Price_{(i)}$$

Price$_{(i)}$ and Score$_{(i)}$ represent de price and the score of bid i respectively, Score$_{max}$ is the highest possible score and Price$_{best}$ is the best submitted bid. This scoring rule is relative as it includes the best submitted price, which may be another price than the one submitted by bidder i.

Consider the following example with price and quality as criteria and the relative scoring rule for price mentioned before with highest possible score 10. Highest total score wins the contract.

	Original situation					**New situation**			
	Price	Price Score	Quality	Total		Price	Price Score	Quality	Total
Bid 1	€ 20.000	10	6.5	**16.5**		€ 20.000	6	6.5	12.5
Bid 2	€ 25.000	8	8	16		€ 25.000	4.8	8	**12.8**
Bid 3	n.a.					€ 12.000	10	2	12

Table 1: example rank reversal. Best price shown underlined. Winner shown in bold.

In the example the ranking between bid 1 and bid 2 is reversed with the introduction of bid 3, even though bid 3 is non-competitive overall (it is in fact the worst of the three bids). This effect is caused by the relative character of the price scoring rule, as the price scores had to be recalculated when the best submitted price changed from €20.000 to €12.000.

Rank reversal is highly undesirable, as it implies non-competitive bids may influence the outcome. The existence of the possibility for rank reversal implies that the decision by a non-competitive supplier to enter or not to enter the bidding competition may affect the evaluation by the buyer and hence the winner. Consider the difference in the evaluation of price between bids 1 and 2 in the example: in the original situation this difference is (10-8=) 2, in the new situation this difference is (6-4.8=) 1.2, which can hardly be considered as consistent behavior or even professional buying practice.

It is worth noting that a relative scoring rule does not necessarily imply rank reversal may occur. It is the interplay between the scoring rule and the bids received that determines if rank reversal can occur. This paper investigates under what conditions rank reversal may occur and under what conditions rank reversal cannot occur.

The outline of this paper is as follows. In Section 2 we sketch several approaches used in practice on the issue of relative scoring. We review related literature in Section 3. Thereafter, in Section 4, we start our analysis. Section 5 considers the case of including another bid with scoring relative to the maximum bid, and in Section 6 we consider bid inclusion with scoring relative to the minimum bid. Section 7 discusses our results.

2 Practice

In practice we see various approaches to the issue of relative scoring ranging from legal prohibition to frequent use and even mandatory application.

In the Public Contracts Code, which regulates public procurement in Portugal, relative scoring is prohibited (Mateus, Ferreira, & Carreira, 2010). Similarly, several organizations including the city of Rotterdam and the Dutch National Police have banned relative scoring that may cause rank reversal.

Banning or prohibiting is considered necessary as relative scoring frequently occurs in practice. Hoogeveen (2014) reports 66.8% of all multi criteria public tenders in the Netherlands involve some form of relative scoring. Similar figures are reported in Sweden (Bergman & Lundberg, 2013) and the UK (Sykes, 2012). Especially for price the relative scoring formulas seem to be popular (Dini, Pacini, & Valletti, 2009).

We did not come across any data on why relative scoring is used, but from our own experience we quote as reasons mentioned frequently:

- it can always be applied,
- it is simple,
- you do not have to know the market very well,
- it does not matter if it turns out prices you get are much different from the ones you expected.

To some extent these reasons could be associated with issues like laziness, or ease of use or not knowing enough of the market. But in fact there may be situations where it is very hard to predict (even approximately) the prices the market will offer.

The other end of the spectrum are the situations and organizations where the use of relative scoring is even mandatory or required. In this regard we mention public procurement in Belgium (FOD Personeel en Organisatie, 2014) and the UN and World Bank procurement handbooks (the United Nations, 2013; The World Bank, 2014). Apparently these organizations have not yet realized the

potential problems associated with this scoring rule. This is remarkable to say the least as all are active in Public Procurement for which the Uncitral code is a kind of "model law". And the Uncitral code argues against relative scoring (UNCITRAL, 2016). For Belgium it is remarkable as Portugal, one of their companions in the EU with identical public procurement directives, outlawed the same practice that is mandatory in Belgium.

3 Literature review

The subject of supplier selection in procurement has received attention in many publications (Ho, Xu, & Dey, 2010; de Boer, Labro, & Morlacchi, 2001; Saen, 2006; Shyur & Shih, 2006).

Most of these papers deal with a situation in which all criteria have been scored and we need to combine the scores on all individual criteria into one overall score to determine the winner (de Boer, 2007). Various approaches are put forward ranging from simple weighing (Dimitri, 2013) to complex and advanced multi criteria decision making methods (e.g. (de Boer, van der Wegen, & Telgen, 1998; Amid, Ghodsypour, & O'Brien, 2010)). Basically a debate on scoring in points or translating all performance information into monetary terms also belongs to this category (Bergman & Lundberg, 2013).

Much less has been published in purchasing literature on the scoring of individual criteria. And even if this topic is addressed the discussion is usually about the scale to be used (e.g. how to measure sustainability e.g. (Sarkis & Dhavale, 2014; Parikka-Alhola & Nissinen, 2012); or how to measure quality performance (Saen, 2006).

What we are interested in is how to assign a score on an individual criterion to a performance offered in a bid. It is not so much the measurement that is at stake, but how to translate the measurement in a score. In purchasing literature there is scattered evidence of the recognition of this issue (de Boer, Linthorst, Schotanus, & Telgen, 2006; Dini, Pacini, & Valletti, 2009; Telgen & Schotanus, 2010a; Smith, 2010).

However purchasing is not the only application area for techniques dealing with multiple criteria analysis or MCDM. In the vast MCDM literature (for an overview see (Chai, Liu, & Ngai, 2013)) several papers address scoring on one criterion (Mateus, Ferreira, & Carreira, 2010; Telgen & Schotanus, 2010a; Telgen & Schotanus, 2010b). Some research is focused on the phenomenon of rank reversal (Stilger, Siderius, & van Raaij, 2015), also from a philosophical point of view (Saaty & Sagir, 2009). Relative scoring is acknowledged and mentioned as one of the possible reasons for rank reversal

(Kuiper, 2009), but very few papers make an effort to analyze relative scoring formulas, their characteristics and their consequences.

There are many possible relative scoring rules: the scores may be dependent on the best bid, on the average bid, on the median bid or on a combination of these e.g. the best and the worst bid. And within these different possibilities more variants exist depending on the way the scores relate to that reference bid (e.g. linear or nonlinear).

Some papers that deal with one form of relative scoring: Chen suggests a logarithmic transformation to avoid the possibility of rank reversal (Chen, 2008).
But to our knowledge there has not been any attention for the fact that relative scoring does not always imply that rank reversal is possible and for the conditions under which it is and it is not possible. This is the gap this paper aims to fill.

4 Analysis

Throughout this analysis we consider a situation with several bids being evaluated on two criteria, one of which is relative. The total score used to rank the bids is the sum of the scores on the two criteria. The scores on the non-relative criterion are determined independently, are not relative and consequently will not change by inclusion or exclusion of a non-competitive bid.
The consideration of only two criteria is no restriction on the analysis as we may assume without loss of generality that the score on the second (non- relative) criterion is calculated as the weighted sum of any number of criteria scores.

The scores on the relative criterion depend on the best submitted bid - that is, the bid with the best performance on the relative criterion - and therefore the scores will change as the best submitted bid changes. This happens when the best submitted bid is withdrawn or when an additional bid is accepted that is better on the relative criterion than the best submitted bid. In both these conditions the scores on the relative criterion have to be recalculated for all bids which can, but not necessarily will, lead to rank reversal.

If inclusion or exclusion of a non-competitive bid does not lead to a new best submitted bid, the scores on the relative criterion will not change. This implies, as the scores on the non-relative criterion will not change either, that the total scores of all original bids will not change compared to the first evaluation. Rank reversal therefore cannot occur.

Lemma 1: Rank reversal can only occur if the best offer on the relative criterion has changed.

When the best submitted bid – again in terms of its performance on the relative criterion - does change, the scores on the relative criterion of all bids have to be recalculated. The result may be that the *difference* between the scores of two bids increases or decreases. Rank reversal occurs when an increase in score difference offsets the difference on the non-relative criterion. Alternatively, rank reversal occurs when a decreased score difference no longer offsets the difference on the non-relative criterion.

Recalculating the scores of two bids on a relative criterion can increase or decrease the score difference between the bids. However, if a bid is better on both criteria in the first evaluation, it will be better on both criteria in the second evaluation – after inclusion or exclusion of a non-competitive bidder -- and it will be ranked higher both times. Then rank reversal cannot occur.

Lemma 2: If the original winner had the best performance on all criteria, inclusion or exclusion of a non-competitive bidder will not lead to rank reversal between the winner and any other bid. (but note numbers 2 and 3 e.g. may reverse!)

4.1 Excluding bids

In adding a non-competitive bid we have the freedom to select bid characteristics to satisfy the requirements. In removing a bid we do not have this freedom: we can only remove one of the existing bids. If the removed bid is not the best on the relative criterion, nothing will change, neither in the scores of the other bids nor in the ranking of the bids. Removing a bid from the competition may only result in different scores and hence possibly lead to rank reversal if the bid removed is best on the relative criterion (Lemma 1).

Then we can distinguish two situations: the bid removed is either non-competitive (not the overall winner) or it is the overall winner. If the bid removed is non-competitive we have to check to see if the overall winner remains the overall winner. If the bid removed is the overall winner we have to check to see if the formerly second place bidder is now the winner.

In both cases recalculating scores on the relative criterion and consequently on the total score will reveal the consequences of the removal in a simple way. There is no practical reason to provide a description of the situations in which rank reversal may occur or cannot occur.

4.2 Including additional bids

So we focus on adding a bid and derive formulas for the characteristics of the bid we have to add in order to achieve rank reversal involving the winning bid (rank reversal may also occur in other places of the ranking: we will ignore those).

Rank reversal is only interesting to study when the bid additionally included is non-competitive. Obviously, when a new bid enters that is superior to the current bids, the ranking will change to the new bid being ranked number 1. However, when a non-competitive bid enters, ideally the ranking between the original competitive bids remains the same, as the (competitive) bids themselves do not change either.

Here we consider the two most frequently used rules: one scoring rule used for a criterion that is to be minimized (such as price) and one for a criterion that has to be maximized (such as quality). In both cases the scoring rules assign the maximum score to the best bid on that criterion and relate all other scores on that criterion to that best bid.

For a criterion to be maximized (we will refer to this criterion as quality) we work with the formula:

$Score_{(i)} = Score_{max} * (x_i/\bar{x})$, with $\bar{x}$ the maximum bid.

For a criterion to be minimized (we will refer to this criterion as price) we work with the formula as used in the introductory example:

$Score_{(i)} = Score_{max} * Price_{best} / Price_{(i)}$

4.3 Notation

Consider $n+1$ bidders labeled from 0 till n. Bidder 0 is the initial winner, before inclusion of a new non-competitive bidder. Assume the bid of each bidder is evaluated on two criteria. Bidder i offers a bid with performance x_i, $x_i \geq 0$, on the first criterion with relative score, and performance y_i, $0 \leq y_i \leq 1$, on the second criterion with non-relative score. Lemma 2 implies that if a bidder has a better bid on one criterion than another bidder, its bid on the other criterion should be worse to allow for rank reversal.

5 Including another bid with maximization

With scoring relative to the maximum bid, a performance x_i on the first criterion results in a partial score on the first criterion of $x_i/\bar{x}$, with $\bar{x}$ the maximum bid. Performance y_i on the second criterion

gives a partial score of y_i for the second criterion. Using weights λ, $0 < \lambda < 1$, for the first criterion and $1 - \lambda$ for the second criterion, and $\alpha = \frac{\lambda}{1-\lambda}$, the weighted (normalized) score of bidder i equals $\alpha x_i/\bar{x} + y_i$.

Let bidder t be the non-competitive bidder that is to be included. As argued above, rank reversal may only occur if this bidder has the best performance on the relative criterion. That is, it's bid x_t on the first criterion should be the best one and hence (due to maximization) the largest one, $x_t > \bar{x}$. We first investigate which bids of the included bidder t result in rank reversal involving the winner.

Proposition 1   The tender is won by another bidder k with $x_k < x_0$ if and only if we can add a bidder t such that

   a. $x_t > \bar{x}$,
   b. $x_t > \alpha(x_0 - x_k)/(y_k - y_0)$, and
   c. $x_t < \alpha x_k/(\alpha + y_t - y_k)$, if $\alpha + y_t - y_k > 0$.

Proof

A new winner occurs if and only if:

(i)   the relative scores change,

(ii)   there is a bidder k with a better total score than the initial winner 0, and

(iii)   the bid of bidder t is non-competitive.

 We will show that these three statements hold if and only if conditions a, b and c are satisfied.

Condition a ensures that the relative scores will change, which is equivalent to statement (i). (See Lemma 1.)

Further, suppose we had a bidder k with $x_k \geq x_0$. Since bidder 0 is the initial winner, its initial score is larger than the score of bidder k: $\frac{\alpha x_0}{\bar{x}} + y_0 > \frac{\alpha x_k}{\bar{x}} + y_k$. Together with $x_t > \bar{x}$, this results in

$$y_0 - y_k > \alpha \frac{x_k - x_0}{\bar{x}} > \alpha \frac{x_k - x_0}{x_t}.$$

But then $\alpha x_0/x_t + y_0 > \alpha x_k/x_t + y_k$; after entrance of the extra bidder, the score of bidder k is still smaller than the score of bidder 0. So under the assumption $x_k \geq x_0$ we cannot satisfy (ii). Thus for a new winner $x_k < x_0$ is needed, which implies $x_0 - x_k > 0$, and $y_k - y_0 > 0$. Combining this with condition b gives

$$y_k - y_0 > \alpha(x_0 - x_k)/x_t \iff \alpha x_k/x_t + y_k > \alpha x_0/x_t + y_0,$$

bidder k has a better score than bidder 0. This shows condition b is equivalent to item (ii).

Finally, if $\alpha + y_t - y_k > 0$, then by condition c

$$\alpha + y_t - y_k < \alpha x_k / x_t \iff \alpha x_t / x_t + y_t < \alpha x_k / x_t + y_k,$$

the extra bidder t has a smaller score than bidder k, and cannot win. If $\alpha + y_t - y_k \leq 0$, then

$$\alpha x_k / x_t > 0 \geq \alpha + y_t - y_k \iff \alpha x_k / x_t + y_k > \alpha x_t / x_t + y_t.$$

Also in this case bidder t has a smaller score than bidder k. This proves the equivalence of condition c and item (iii), which concludes the proof. □

We may now derive the following conditions on candidate winner k.

Theorem 2   Rank reversal involving the winner is possible if and only if there is a bidder k with $x_k < x_0$, and if $\alpha - y_k > 0$ then also $x_k / (\alpha - y_k) > x_0 / (\alpha - y_0)$.

Proof

Assume without loss of generality that $y_t = 0$. According to Proposition 1, the bid of bidder t has to satisfy the conditions a, b, and c if $\alpha - y_k > 0$. This is possible if and only if

$$\bar{x} < \alpha x_k / (\alpha - y_k),$$

and

$$\alpha(x_0 - x_k) / (y_k - y_0) < \alpha x_k / (\alpha - y_k).$$

Since bidder 0 is the initial winner, its initial score is larger than the initial score of bidder k:

$$\alpha \frac{x_0}{\bar{x}} + y_0 > \alpha \frac{x_k}{\bar{x}} + y_k \iff \alpha(x_0 - x_k) / (y_k - y_0) > \bar{x}.$$

But then $\bar{x} < \alpha(x_0 - x_k) / (y_k - y_0) < \alpha x_k / (\alpha - y_k)$, so only the condition

$$\alpha(x_0 - x_k) / (y_k - y_0) < \alpha x_k / (\alpha - y_k)$$

remains. Rearranging terms leads to the desired result.

If $\alpha - y_k \leq 0$, then the bid of bidder t has to satisfy the conditions a and b of Proposition 1. As argued above, condition b dominates condition a. If the bid of bidder t is large enough, then the lower bound on x_t in condition b is satisfied and there are no extra conditions needed on the bid of bidder k. □

Corollary 3   Rank reversal involving the winner cannot occur if for any bidder i

- $x_i > x_0$, or
- $x_i < x_0$, $\alpha - y_i > 0$, and $x_i / (\alpha - y_i) \leq x_0 / (\alpha - y_0)$.

These results follow immediately from Theorem 2.

To illustrate these results, consider the following example with three bidders, weight $\lambda = 0.46$ and hence $\alpha = 0.85$. The bids are shown in Table 5.

|	x_i	Partial score	y_i	Total score
Bid 0	48	0.96	0.83	**1.646**
Bid 1	45	0.90	0.88	1.645
Bid 2	<u>50</u>	1	0.74	1.59

Table 2: Example rank reversal at bid inclusion and maximization.

Best performance on first criterion shown underlined. Winner shown in bold.

Bidder 0 is the initial winner. Now consider the inclusion of an additional bidder t. The bid of this extra bidder should satisfy $x_t > 50$ for the relative scores to change (Proposition 1a). Bidder 1 may be a winner since $x_1 < x_0$ (Theorem 2). Because $\alpha - y_1 \le 0$, we need $x_t > 51$ (Proposition 1b) for $y_t = 0$. For illustration, if $x_t = 52$ and $y_t = 0$ then the scores are as follows.

	x_i	Partial score	y_i	Total score
Bid 0	48	0.923	0.83	1.615
Bid 1	45	0.865	0.88	**1.616**
Bid 2	50	0.962	0.74	1.557
Bid t	<u>52</u>	1	0	0.85

Bidder 1 is the new winner after inclusion of the non-competitive bidder t. Figure 1 shows how the winning bidder depends on the bid of the additional bidder t.
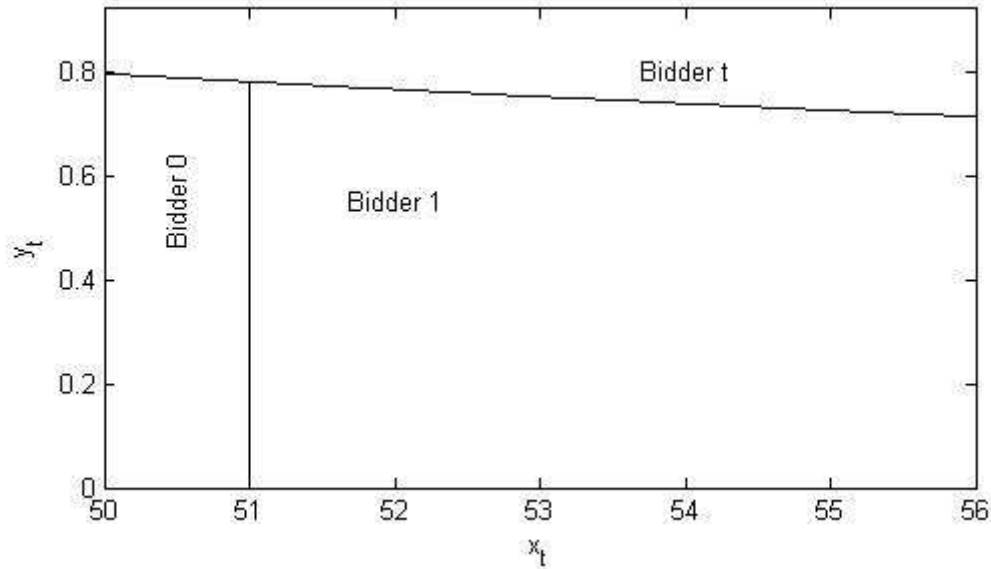
Figure 1 Bid inclusion and maximization.
This figure shows the winning bidder for different bids (x_t, y_t) of the included bidder t.

6 Including another bid with minimization

With scoring relative to the minimum bid, a performance x_i on the first criterion results in a partial score of $\underline{x}/x_i$, with $\underline{x}$ the minimum bid on the first criterion. The weighted (normalized) score of bidder i now equals $\alpha \underline{x}/x_i + y_i$.

As argued before, rank reversal may only occur if bidder t has the best performance on the relative criterion. That is, it's bid x_t on the first criterion should be the smallest one, $x_t < \underline{x}$. We investigate which bids of the included bidder t result in rank reversal with regard to the winning bid.

Proposition 4 The tender is won by another bidder k with $x_k > x_0$ if and only if we can add a bidder t such that

    a. $x_t < \underline{x}$,

    b. $x_t < (y_k - y_0)/\left(\frac{\alpha}{x_0} - \frac{\alpha}{x_k}\right)$, and

    c. $x_t > \frac{x_k}{\alpha}(\alpha + y_t - y_k)$, if $\alpha + y_t - y_k > 0$.

Proof

A new winner occurs if and only if:

(i) the relative scores change,

(ii) there is a bidder k with a larger score than the initial winner 0, and

(iii) the bid of bidder t is non-competitive.

We show that these requirements hold if and only if the conditions a, b and c are met.

Condition a ensures that the relative scores may change, which equals item (i).

Further, suppose $x_k \leq x_0$ for bidder k. Since bidder 0 is the initial winner, its initial score is larger than the score of bidder k: $\alpha \underline{x}/x_0 + y_0 > \alpha \underline{x}/x_k + y_k$. Together with $\underline{x} > x_t$, this results in

$$y_0 - y_k > \alpha \left(\frac{\underline{x}}{x_k} - \frac{\underline{x}}{x_0} \right) > \alpha \left(\frac{x_t}{x_k} - \frac{x_t}{x_0} \right).$$

But then $\alpha x_t/x_0 + y_0 > \alpha x_t/x_k + y_k$; after entrance of the extra bidder, the score of bidder k is still smaller than the score of bidder 0. So, under the assumption $x_k \leq x_0$, item (ii) cannot be satisfied. Thus for a new winner the condition $x_k > x_0$ is needed, which implies $x_k - x_0 > 0$, and $y_k - y_0 > 0$. Combining this with condition b gives

$$y_k - y_0 > \alpha \left(\frac{x_t}{x_0} - \frac{x_t}{x_k} \right) \Leftrightarrow \alpha x_t/x_k + y_k > \alpha x_t/x_0 + y_0,$$

bidder k has a larger score than bidder 0. This shows the equivalence of condition b and item (ii).

Finally, if $\alpha + y_t - y_k > 0$, then by condition c

$$\alpha + y_t - y_k < \alpha x_t/x_k \Leftrightarrow \alpha x_t/x_k + y_k > \alpha x_t/x_t + y_t,$$

the extra bidder t has a smaller score than bidder k, and cannot win. If $\alpha + y_t - y_k \leq 0$, then

$$\alpha x_t/x_k > 0 \geq \alpha + y_t - y_k \Leftrightarrow \alpha x_t/x_k + y_k > \alpha x_t/x_t + y_t.$$

Also in this case bidder t has a smaller score than bidder k. This proves that condition c is equal to item (iii). The proof is concluded. □

Using this, we may derive the following conditions on candidate winner k.

Theorem 5   Rank reversal involving the winner is possible if and only if there is a bidder k with $x_k > x_0$, and if $\alpha - y_k > 0$ then also $x_k(\alpha - y_k) < x_0(\alpha - y_0)$.

Proof

Assume without loss of generality that $y_t = 0$. According to Proposition 4, the bid of bidder t has to satisfy the conditions a, b and c. This is possible if and only if, combining a and c,

$$\frac{x_k}{\alpha}(\alpha + y_t - y_k) < \underline{x},$$

and by combining b and c,

$$\frac{x_k}{\alpha}(\alpha + y_t - y_k) < (y_k - y_0)/\left(\frac{\alpha}{x_0} - \frac{\alpha}{x_k}\right).$$

Since bidder 0 is the initial winner, its initial score is larger than the initial score of bidder k:

$$\alpha\frac{\underline{x}}{x_0} + y_0 > \alpha\frac{\underline{x}}{x_k} + y_k \Leftrightarrow \underline{x} > (y_k - y_0)/\left(\frac{\alpha}{x_0} - \frac{\alpha}{x_k}\right).$$

But then $\frac{x_k}{\alpha}(\alpha + y_t - y_k) < (y_k - y_0)/\left(\frac{\alpha}{x_0} - \frac{\alpha}{x_k}\right) < \underline{x}$ so only the condition

$$\frac{x_k}{\alpha}(\alpha + y_t - y_k) < (y_k - y_0)/\left(\frac{\alpha}{x_0} - \frac{\alpha}{x_k}\right)$$

remains. Rearranging terms leads to the desired result.

If $\alpha - y_k \leq 0$, then the bid of bidder t has to satisfy the conditions a and b of Proposition 4. As argued above, condition b. dominates condition a. If the bid of bidder t is small enough, then the upper bound for x_t in condition b is satisfied, and there are no extra conditions needed on the bid of bidder k. □

Corollary 6   Rank reversal involving the winner cannot occur if for any bidder i

- $x_i < x_0$, or
- $x_i > x_0$, $\alpha - y_i > 0$, and $x_i(\alpha - y_i) \geq x_0(\alpha - y_0)$.

These results follow immediately from Theorem 5.

To illustrate the results, consider the following example with three bidders and weight $\lambda = 1/2$, so $\alpha = 1$.

	x_i	Partial score	y_i	Total score
Bid 0	50	0.80	0.90	**1.70**
Bid 1	<u>40</u>	1.00	0.69	1.69
Bid 2	60	0.67	0.95	1.62

Table 6: Example rank reversal at bid inclusion and minimization.

Best performance on first criterion shown underlined. Winner shown in bold.

Bidder 0 is the initial winner. Now consider the inclusion of an extra bidder t. The bid of this extra bidder should satisfy $x_t < \underline{x} = 40$; then the relative scores change (Proposition 4a.). Since $x_1 < x_0$, bidder 1 cannot be a winner (Theorem 5). Bidder 2 can be a winner since $x_2 > x_0$ and $\frac{x_2}{1-y_2} = 3 < \frac{x_0}{1-y_0} = 5$ (Theorem 5, and Figure 2). For this, we need $60(0.05 + y_t) < x_t$, and $x_t < 15$ (Proposition 4c. and b.). Figure 3 shows how the winning bidder depends on the bid of the extra bidder t.
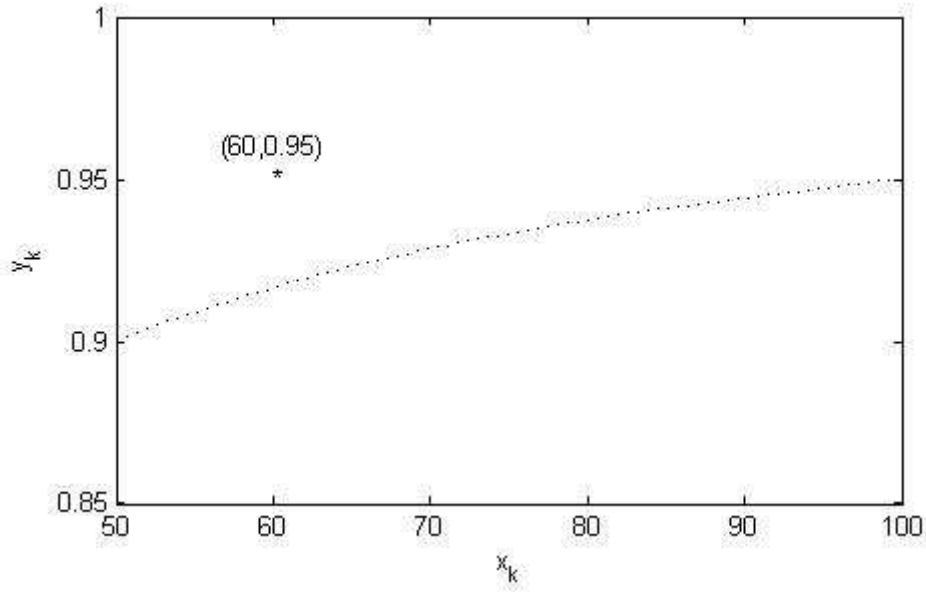
Figure 2 According to Theorem 5, rank reversal in the example is possible if $x_k > 50$, and $y_k > 1 - 5/x_k$ for some bidder k. The latter inequality is satisfied for bids above the dotted line, so for the bid (60,0.95) of bidder 2.
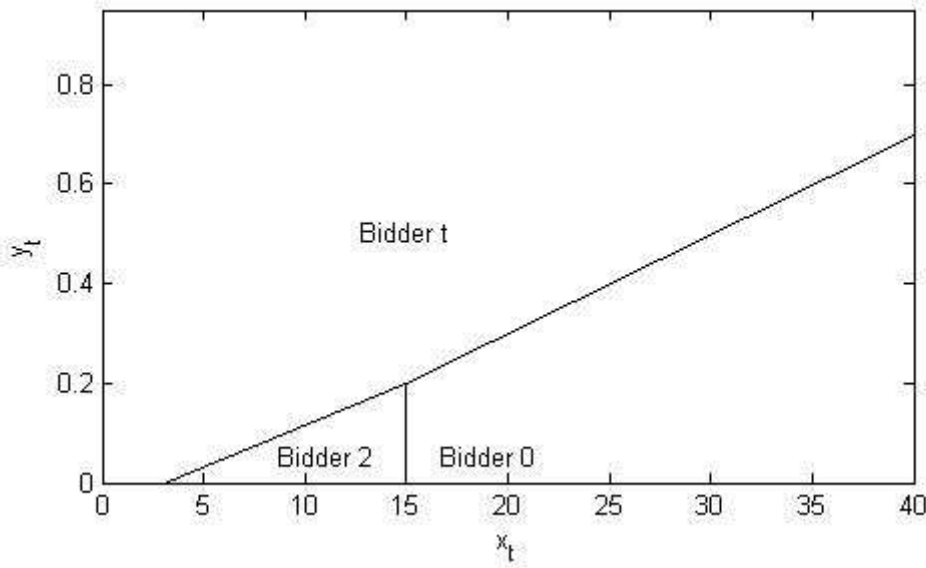

Figure 3 Bid inclusion and minimization.
This figure shows the winning bidder for different bids (x_t, y_t) of included bidder t.

7 Discussion

Rank reversal may occur when a supplier selection mechanism is used that includes a relative scoring rule. Rank reversal is undesirable, as it implies even non-competitive bids may influence the outcome of a tender, leading to non-optimal results, arbitrariness and possible bid rigging.

Of course the best way to avoid rank reversal from happening is not to use relative scoring. But relative scoring remains popular and is in some jurisdictions even mandatory.

The evaluation of tenders that involve relative scoring usually proceed and finish without problems. The scores are calculated and the winner is determined. Very seldom someone asks: "what would have happened to the end result if supplier X would not have participated or an additional supplier Y would have participated?". Asking such a question is not in the interest of the buying organization (it can only cause problems if it would yield another winner) and the participating suppliers lack the information (on all bids) to do the calculations.

And if buying organizations are aware of the possible problems associated with relative scoring they probably also know that rank reversal is not always possible (only if the conditions given in this paper are satisfied). That knowledge may lead to the excuse that in their specific case it will not appear. Here we gave exact rules to determine if the excuse holds.

The analysis in this paper does not cover all relative scoring rules. Similar analyses of other scoring rules are possible. Also we took into account relative scoring on only one criterion. If more criteria are scored in a relative way, obviously it will be more likely that additional bids cause rank reversal. Also it will be clear that the conditions to be derived in a similar way, will be more complex.

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