Method and electronic device for global optimization to exact solutions of convex semi-infinite problems

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(54) Title: METHOD AND ELECTRONIC DEVICE FOR GLOBAL OPTIMIZATION TO EXACT SOLUTIONS OF CONVEX SEMI-INFINITE PROBLEMS

(57) Abstract: Embodiments herein provide a method for determining approximation of Semi-Infinite optimization problem (SIP) to identify outcome of the SIP using a sampler device. Inputs are received from user and defining infinite number of constraints of the SIP making the SIP intractable. A global maximization problem is determined by reformulating the finite dimensional approximation of SIP into a finite dimension problem with finite constraints. An optimal value of the finite dimensional approximation of SIP is obtained based on the global maximization problem using a global optimization technique. A plurality of outcomes of the finite dimensional approximation of SIP is exacted based on the optimal value of the finite dimensional approximation. One of the major advantages of this approach is that it admits a plug-and-play module, where one can use any global optimization method to obtain the optimal value.

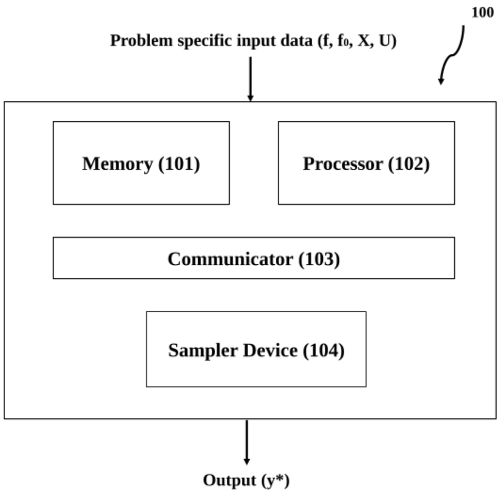

FIG. 1

FORM 2

The Patent Act 1970

(39 of 1970)

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The Patent Rules, 2005

COMPLETE SPECIFICATION

(SEE SECTION 10 AND RULE 13)

TITLE OF THE INVENTION

"METHOD AND ELECTRONIC DEVICE FOR GLOBAL OPTIMIZATION TO EXACT SOLUTIONS OF CONVEX SEMI-INFINITE PROBLEMS"

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The following specification describes the method:

FIELD OF INVENTION

[0001] The embodiments disclosed herein generally relate to semi-infinite optimization approach, and more particularly, to a method and an electronic device for determining approximation of Semi-Infinite optimization problem (SIP) to identify outcome of the SIP using a sampler device. The present application is based on, and claims priority from an Indian Application Number **202121061723** filed on **30th December, 2021** the disclosure of which is hereby incorporated by reference herein.

BACKGROUND

[0002] In general, a central-cutting-plane method for semi-infinite mechanism describes a class of linear semi-infinite problems. The central-cutting-plane method is developed for solving the convex problem which iteratively proceeds to the optimum by constructing a cutting plane through the center of a polyhedral approximation to the optimum. This generates a sequence of primal feasible points whose limit points satisfy the Kuhn—Tucker conditions of the problem. Additionally, a simple, effective rule is presented for dropping prior cuts, an easily calculated bound on the objective function, and a rate of convergence. The corresponding method has the property of being able to generate the cut from any violated or tight constraint without affecting convergence or convergence rate of the central-cutting-plane method. Thus, to insure convergence, it is not required to find the most violated (or nearly most violated) constraint as required by the alternating mechanism.

[0003] Further, ‘Discretization in semi-infinite mechanism’, describes a discretization method to solve a semi-infinite problem. The discretization method involves a coarse grid that is successively refined to arrive at a reasonably good approximate solution to the original semi-infinite problem by solving a relaxed problem at each step. In ‘Uncertain convex mechanism: randomized solutions and confidence levels’, is a ‘randomized’ or 'scenario' approach based on constraint sampling for dealing with uncertainty in the optimization. A constrained optimization problem with only a finite set of N constraints is solved. The constraints are chosen uniformly at random among the possible constraint instances of the uncertain problem. It is shown that the resulting randomized solution fails to satisfy only a small portion of the original constraints provided that a sufficient number of samples are drawn. Furthermore, 'An inexact primal-dual mechanism for semi-infinite functioning, describes an inexact primal-dual method for semi-infinite functioning, where the dual variable is updated by creating a new proximal function for non-negative measures.

OBJECT OF INVENTION

[0004] The principal object of the embodiments herein is to provide a method and an electronic device for determining approximation of Semi-Infinite optimization problem (SIP) to identify outcome of the SIP using a sampler device.

SUMMARY

[0005] Accordingly the embodiment herein is to provide a method for determining a approximation of Semi-Infinite optimization problem (SIP) to identify outcome of the SIP using a sampler device. The method includes receiving inputs from a user. The method further includes defining infinite number of constraints of the SIP and the infinite number of constraints of the SIP makes the SIP intractable. Furthermore, the method includes determining a global approximation of the SIP into a finite dimension problem using a global optimization technique. Finally, the method includes exacting a plurality of outcomes of the finite dimensional approximation of SIP based on the optimal value of the finite dimensional approximation.

[0006] In an embodiment, the finite dimensional approximation of SIP is a convex Semi-Infinite Problem (SIP).

[0007] In an embodiment, the SIP problem comprises finite set of decision variables and infinite number of constraints.

[0008] In an embodiment, the optimal value of the convex SIP problem is equal to an approximate global optimum the convex SIP problem.

[0009] In an embodiment, reformulating the convex SIP problem into approximate with the finite number of constraints of the convex SIP problem comprises determining deterministic and randomized value for low dimension and high dimension respectively of a constraint index, wherein a Markov chain is initialized randomly.

[0010] In an embodiment, the global optimization technique is a simulated annealing-based technique used for the convex SIP problem to obtain the optimal value of the convex SIP problem.

[0011] In an embodiment, the global optimization technique includes determining a set of initial guesses (G) associated with the simulated annealing-based method of the global optimization scheme and examining, by the sampler device, a structural property of the initial guesses (G) in simulated annealing-based technique of global optimization scheme.

[0012] Accordingly the embodiment herein is to provide an electronic device for determining approximation of the SIP to determine the outcome of the SIP using the sampler device. The electronic device includes a memory, a processor and a sampler device for receiving inputs from the user. An infinite number of constraints of the SIP are defined and the number of constraints of the SIP makes the SIP intractable. A global maximization problem is determined by reformulating the finite dimensional approximation of SIP into a finite dimension problem with finite constraints. An optimal value of the finite dimensional approximation is obtained based on the global maximization problem using a global optimization technique. Finally a plurality of outcomes of the finite dimensional approximation of the SIP are exacted based on the optimal value of the finite dimensional approximation.

[0013] These and other aspects of the embodiments herein will be better appreciated and understood when considered in conjunction with the following description and the accompanying drawings. It should be understood, however, that the following descriptions, while indicating preferred embodiments and numerous specific details thereof, are given by way of illustration and not of limitation. Many changes and modifications may be made within the scope of the embodiments herein without departing from the spirit thereof, and the embodiments herein include all such modifications.

BRIEF DESCRIPTION OF FIGURES

[0014] This method is illustrated in the accompanying drawings, throughout which like reference letters indicate corresponding parts in the various figures. The embodiments herein will be better understood from the following description with reference to the drawings, in which:

[0015] FIG. 1 is a block diagram of an electronic device determining approximation of Semi-Infinite optimization problem (SIP) to determine outcome of the SIP, according to the embodiments as disclosed herein;

[0016] FIG. 2 illustrating a flow diagram of the sampler device performing global optimization to exact solutions of convex SIP, according to the embodiments as disclosed herein;

[0017] FIG. 3 is a flow chart illustrating a method of sampling for control via simulated annealing for global optimization to exact solutions of the convex SIPs for communication module, according to the embodiments as disclosed herein;

[0018] FIG. 4A illustrating a scenario of a histogram of the optimal value for global optimization to exact solutions of the convex SIP, according to the embodiments as disclosed herein;

[0019] FIG. 4B illustrating a scenario of time evolution of the optimal value for global optimization to exact the solutions of convex SIPs, according to the embodiments as disclosed herein;

[0020] FIG. 5A illustrates a scenario of a plot of the absolute error between a number of iterations and a true optimal value for global optimization to exact the solutions of convex SIPs, according to the embodiments as disclosed herein;

[0021] FIG. 5B illustrates a scenario of a plot of the absolute error between the number of samples and a true optimal value for global optimization to exact solutions of the convex SIPs, according to the embodiments as disclosed herein; and

[0022] FIG. 6 illustrates the flow diagram for determining approximation of SIP to determine a outcome of the SIP using the sampler device, according to the embodiments as disclosed herein.

DETAILED DESCRIPTION OF INVENTION

[0023] The embodiments herein and the various features and advantageous details thereof are explained more fully with reference to the non-limiting embodiments that are illustrated in the accompanying drawings and detailed in the following description. Descriptions of well-known components and processing techniques are omitted so as to not unnecessarily obscure the embodiments herein. Also, the various embodiments described herein are not necessarily mutually exclusive, as some embodiments can be combined with one or more other embodiments to form new embodiments. The term "or" as used herein, refers to a non-exclusive or, unless otherwise indicated. The examples used herein are intended merely to facilitate an understanding of ways in which the embodiments herein can be practiced and to further enable those skilled in the art to practice the embodiments herein. Accordingly, the examples should not be construed as limiting the scope of the embodiments herein.

[0024] As is traditional in the field, embodiments may be described and illustrated in terms of blocks which carry out a described function or functions. These blocks, which may be referred to herein as managers, units, modules, hardware components or the like, are physically implemented by analog and/or digital circuits such as logic gates, integrated circuits, microprocessors, microcontrollers, memory circuits, passive electronic components, active electronic components, optical components, hardwired circuits and the like, and may optionally be driven by firmware and software. The circuits may, for example, be embodied in one or more semiconductor chips, or on substrate supports such as printed circuit boards and the like. The circuits constituting a block may be implemented by dedicated hardware, or by a processor (e.g., one or more programmed microprocessors and associated circuitry), or by a combination of dedicated hardware to perform some functions of the block and a processor to perform other functions of the block. Each block of the embodiments may be physically separated into two or more interacting and discrete blocks without departing from the scope of the disclosure. Likewise, the blocks of the embodiments may be physically combined into more complex blocks without departing from the scope of the disclosure.

[0025] Accordingly the embodiment herein is to provide a method for determining approximation of Semi-Infinite optimization problem (SIP) to determine outcome of the SIP using a sampler device. The method includes receiving inputs from a user. The method further includes defining infinite number of constraints of the SIP and the infinite number of constraints of the SIP makes the SIP intractable. Furthermore, the method includes determining a global approximation of the SIP into a finite dimension problem using a global optimization technique. Finally, the method includes exacting a plurality of outcomes of the finite dimensional approximation of SIP based on the optimal value of the finite dimensional approximation.

[0026] The proposed method, provides global optimization to exact solutions of convex SIPs. The proposed method provides an approach that ascertains the optimal value of a wide class of convex SIPs as accurately as possible within limitations of the resources at one’s disposal. Further, the proposed method including SIP can be equivalently written as a finite-dimensional global maximization problem in a sense that the optimal value of the latter is identical to that of a former. One of major advantage of this approach is that the approach admits a plug-and-play module, where one can use any global optimization mechanism to obtain the optimal value.

[0027] In conventional system use of a discretization approach for solving semi-infinite optimization problems is considered. Convergence rate of error between solution of the SIP and solution of the discretized function depending on the discretization mesh-size. The convergence rate depends on whether the minimizer is strict of order one or two and on whether the discretization includes boundary points of the index set in specific way. The discretization approach is used for ordinary and for generalized semi-infinite problem. Unlike the conventional systems, the proposed method discloses global optimization to exact the solution of the convex SIP and for ascertaining the optimal value of the wide class of convex semi-infinite problems (SIPs) as accurately as possible within the limitations of the resources.

[0028] In some conventional systems, a randomized approach is considered for dealing with uncertainty in optimization, based on constraint sampling. The constrained optimization problem resulting by taking into account only a finite set of N constraints, chosen randomly among the possible constraint instances of the uncertain problem. The resulting randomized solution fails to satisfy only a small portion of the original constraints, provided that a sufficient number of samples is drawn, providing efficient and explicit bound on the measure of the original constraints that are possibly violated by the randomized solution. Unlike the conventional systems, the proposed method global optimization is considered to exact the solutions of the convex SIPs. The global optimization method ascertains the optimal value of the wide class of convex SIP problems as accurately as possible within the limitations of the resources. The SIP is converted to a finite dimensional global maximization problem in the sense that the optimal value of the latter is identical to that of the former.

[0029] Further, ‘a conventional new exchange method for convex semi-infinite problem’ is an exchange method for solving convex semi-infinite problems was proposed. An adding-dropping rule that only keeps those active constraints with positive Lagrange multipliers was used in the proposed mechanism. Unlike the conventional system, the proposed method under some conditions, the adding-dropping rule guarantees that the proposed method results in an approximate optimal solution for the convex semi-infinite problem in a finite number of iterations.

[0030] Further, in conventional CoMirror mechanism with random constraint sampling for convex SIP describes CoMirror mechanism with inexact cut generation is used to create the SIP-CoM mechanism for solving SIPs. Two specific random constraint sampling schemes are presented to approximately solve the cut generation problem for generic SIP.

[0031] Referring now to the drawings, and more particularly to FIGS. 2 through 6, where similar reference characters denote corresponding features consistently throughout the figures, there are shown preferred embodiments.

[0032] FIG. 1 is a block diagram of an electronic device (100) determining approximation of Semi-Infinite optimization problem (SIP) to determine outcome of the SIP, according to the embodiments as disclosed herein. Referring to FIG. 1, the electronic device (100) includes a memory (101), processor (102), communicator (103) and sampler device (104).

[0033] In an embodiment, the electronic device (100) includes the electronic circuit specific to the standard that enables wired or wireless communication. The electronic device (100) is configured for communicating internally between internal hardware components and with external devices via one or more networks. The electronic device (100) may be but not limited to a laptop, a palmtop, a desktop, a mobile phone, a smart phone, Personal Digital Assistant (PDA), a tablet, a wearable device, an Internet of Things (IoT) device, a virtual reality device, a foldable device, a flexible device, a display device and an immersive system.

[0034] In an embodiment, the processor (102) may include one or a plurality of processors. The one or the plurality of processors (102) may be a general-purpose processor, such as a Central Processing Unit (CPU), an Application Processor (AP), or the like, a graphics-only processing unit such as a Graphics Processing Unit (GPU), a Visual Processing Unit (VPU), and/or an AI-dedicated processor such as a Neural Processing Unit (NPU). The processor (102) may include multiple cores and is configured to execute the instructions stored in the memory (101). The processor (102) is configured to execute instructions stored in the memory (101) and to perform various processes.

[0035] In an embodiment, the communicator (103) includes the electronic circuit specific to the standard that enables wired or wireless communication. The communicator (103) is configured to communicate internally between internal hardware components of the electronic device (101) and with external devices via one or more networks.

[0036] The memory (101) can include non-volatile storage elements. Examples of such non-volatile storage elements may include magnetic hard discs, optical discs, floppy discs, flash memories, or forms of electrically programmable memories (EPROM) or electrically erasable and programmable (EEPROM) memories. In addition, the memory (101) may, in some examples, be considered a non-transitory storage medium. The term “non-transitory” may indicate that the storage medium is not embodied in a carrier wave or a propagated signal. However, the term “non-transitory” should not be interpreted that the memory (101) is non-movable. In some examples, the memory (101) is configured to store larger amounts of information. In certain examples, a non-transitory storage medium may store data that can, over time, change (e.g., in Random Access Memory (RAM) or cache). The memory (101) stores information related to but not limited to simulator applications to be deployed or controlled or configured. The memory (101) can store proto file, service configuration parameters, service procedures, information about plurality of attributes received from the user, performance statistics of the service procedures.

[0037] In an embodiment, the sampler device (104) is configured to determine the approximation of SIP to determine outcome of the SIP. The sampler device is configured to receive inputs from user, defines an infinite number of constraints of the SIP and the infinite number of constraints of the SIP makes the SIP intractable. An optimal value of the finite dimensional approximation of the SIP based on the global maximization problem using a global optimization technique. The detailed description is described herein.

[0038] FIG. 2 illustrating a flow diagram of the sampler device controller performing global optimization to exact solutions of convex SIP, according to the embodiments as disclosed herein.

[0039] Referring to FIG. 2, the flow diagram includes the sampler device (104) that receives the input data. The input data can be but not limited to optimization problem. In an embodiment, the input for example can be but not limited to (f, fo, X, U). The input is feed to the electronic device (100) including the processor (102), communicator (103) and the sampler device

(104). A number of finite constraints are specified for the input received from the sampler device (104). The optimization problem comprises a finite set of decision variables and infinitely many constraints. A convex SIP problem contains the objective function and the constraint functions are convex in the decision variable. The sampler device (104) will determine the optimal constraints to solve the finite-dimensional relaxed optimization problem. The output will be a y* that is an optimal value.

[0040] The internal components of the sampler device include following steps:

- Input will be applied to the electronic device
- The electronic device will specify the number of finite constraints
- The electronic device will find the optimal finite constraints and
- At the final stage, the system will solve the finite-dimensional relaxed optimization problem.
- The output will be a y* which is optimal value.

[0041] One of the primary features of the approach is that the approach admits a plug-n-play module. The plug-n-play can use any global optimization function to recover the optimal value of the original convex SIP. The plug-n-play character of the approach includes a simulated annealing based mechanism (requiring a quantum of memory (101) bounded by a linear function of the dimension of the decision variable) to solve the convex SIP, if solved using the simulated annealing based method, converges to the actual optimal value in probability provided certain mild assumptions are satisfied.

[0042] The proposed method also provides simulated annealing based techniques, such as the one treated in the method, which work well even when the dimension of the constraint index set is high. This is because such techniques employ.

Consider the following SIP,

$$\text{minimize}_x \ f_0(x)$$

$$\text{Subject to} \begin{cases} f(x,u) \leq 0 \; for \; all \; u \in U, \\ x \in X, \end{cases} \qquad (1.1)$$

[0043] With the following data:

((1.1)-a) – The domain $X \subset R^n$ is a closed and convex set with non-empty interior.

((1.1)-b) – The admissible set is defined as $\{x \in X\} \mid f(x, u) \leq 0$ for all $u \in U\}$ and is assumed to have a non-empty interior, i.e. there exists a point $\bar{x} \in X$ such that $f(\bar{x},u) < 0$ for every $u \in U$.

((1.1)-c) – The objective function $X \ni x \rightarrow f_0(x) \in R$ is convex and continuous in x.

((1.1)-d) – The constraint function $X \times U \ni (x,u) \rightarrow f(x,u) \in R$ is continuous in both the variables and is convex in x for each fixed u.

((1.1)-e) – The set U is the constraint index set, and its assumed to be a compact set $U \subset R^k$.

[0044] The value of (1.1) is denoted by $y*$. The family $\{f(x, u) \leq 0 \mid u \in U\}$ of constraints is often known as a semi-infinite constraint since the constraints may contain uncountable many inequality constraints depending on the set U. Consequently, the optimization problem (1.1) consists of a finite set of decision variables and infinitely many constraints. A convex SIP is one in which the objective function and the constraint functions are convex in the decision variable as in ((1.1)-a) and ((1.1)-d).

[0045] To ascertain the optimal value of a convex SIP under fixed memory requirement, the method proposes an equivalent finite-dimensional reformulation of the convex SIP. The reformulation leads to a global maximization problem whose optimal value is equal to the optimal value of the original problem. One of the primary features of the approach is that the method admits a plug-n-play module and one can use any global optimization method to recover the optimal value of the original convex SIP.

[0001] The proposed method provides advantages and improvements over existing methods are as follows –

- The approach has a plug-n-play module where any tailor-made deterministic/randomized method can be used to solve the original SIP.
- The approach has a constant memory requirement that depends on the dimension of the optimization variables.\
- Attributes such as convexity of the constraint index set play no role in the selection of the global optimization method. Consequently, a specific method (e.g., deterministic and randomized for low and high dimensions, respectively, of the constraint index set) can be chosen effectively keeping in mind the application at hand.

[0046] To ascertain the optimal value of a convex SIP under fixed memory requirement, the method proposes an equivalent finite-dimensional reformulation of the convex SIP. The reformulation leads to a global maximization problem whose optimal value is equal to the optimal value of the original problem. One of the primary features of the proposed method is that the method admits a plug-n-play module wherein one can use any global optimization mechanism to recover the optimal value of the original convex SIP. The SIP cannot be solved reasonably well by means of the standard techniques and well-known methods, even though the underlying problem is convex. This is because an infinite number of constraints must be satisfied simultaneously, one for each $u \in U$, this feature makes an SIP computationally intractable as stated in (1.1).

[0047] Attributes such as convexity of the constraint index set play no role in the selection of the global optimization methods. Consequently, a specific mechanism (e.g., deterministic and randomized for low and high dimensions, respectively, of the constraint index set) can be chosen effectively keeping in mind the application at hand.

[0048] A SIP cannot be solved reasonably well by means of the standard techniques and well-known methods, even though the underlying problem is convex. This is because an infinite number of constraints must be satisfied simultaneously, one for each $u \in U$; this feature makes an SIP computationally intractable.

[0049] In an embodiment, the method demonstrates that the SIP can be equivalently reformulated as a suitable finite-dimensional maximization problem, so that a global optimization scheme can be utilized to obtain the optimal value of (1.1). As an example, in simulated annealing based technique for convex SIP, proposes a simulated annealing based mechanism to solve a convex SIP. The method identifies a set of points and recovers the optimal value of the original SIP. Furthermore, advantages of the proposed method in more detail are reported, especially in the high-dimensions through both qualitative and numerical observations.

[0050] The SIP can be recast as a suitable finite-dimensional maximization problem such that the optimal values of both the problems are identical. Referring to equation (1.1) and it is associated data that 'n' is the dimension of the decision variable. The method defines a function 'G' on the Cartesian product U^n of the constraint index set by –

$U^n \ni (u_1, u_2, \ldots\ldots. u_n) \to G(u_1, u_2, \ldots\ldots. u_n) := inf_{x \in X} \{f_0(x) \mid f(x, u_i) \leq$ for $i=1,2,\ldots.,n\} \in R.$ (2.1)

[0051] The values of the function G are obtained by solving a finite-dimensional convex optimization problem, and fast and accurate mechanism are known art. The function G will play a central role in the proposed method, and a structural property of G will be examined in simulated annealing based technique.

[0052] Note that the constraint set $\{x \in X \mid f(x, u_i) \leq 0$ for all $u \in U\}$ in equation (1.1) and will refer to the former as a *relaxed admissible set*. Then the SIP in equation (1.1) with a relaxed admissible set is relaxed optimization problem associated with equation (1.1).

[0053] Under the preceding premise, there exists an n-tuple ($\bar{u}_1$, $\bar{u}_2$,……,$\bar{u}_n$)∈ U^n such that the optimal value of equation (1.1) is equal to G ($\bar{u}_1$, $\bar{u}_2$,……,$\bar{u}_n$).

[0054] **Main Results**: Consider the SIP defined in equation (1.1) along with its associated data ((1.1)-a-((1.1)-e). if (u_1*,u_2*,……,u_n*) ∈ U^n is an optimizer of the maximization problem,

$$\sup_{(u_1,\ldots\ldots,u_n) \in U^n} G(u_1,u_2,\ldots\ldots,u_n),$$

Then G(u_1*,u_2*,……,u_n*) = y*.

[0055] In the sense that $\underline{y}$*=y*. Below the method provides the calculating the value of (1.1) as accurately as possible by treating as its surrogate, and searching in a directed fashion for the optimizers (u_1^0,u_2^0,……,u_n^0).

[0056] The existence of the optimizers in the maximization problem (2.6) is guaranteed from equation (2.2) and from the data of the problem (1.1). However it is difficult to assert the uniqueness of the optimizers.

[0057] Simulated annealing based method for convex semi-infinite problem is disclosed herein. The method illustrates the plug-and-play nature of the approach established in the above section. The proposed method proposes a simulated annealing based technique 1 to solve a convex SIP and demonstrate that the optimal value of converges in probability to the optimal value of (1.1). A similar technique is used to solve the SIP model for tardiness production planning based on simulated annealing is introduced. The constraint functions in the form of penalties are to be added to the objective function in order to obtain an unconstrained SIP, following which the unconstrained optimization problem was solved in the decision variable by means of a simulated annealing mechanism.

[0058] Technique 1 is beneficial in applications where the SIP is an intermediate step and the decisions are taken based on the value of the SIP.

Examples of such applications include portfolio optimization and intelligent investment decision making based on profit-loss margins.

[0059] Technique 1: Simulated annealing based technique for convex semi-infinite is as follows –

Data: exploration set $\mathcal{O} \subset \mathfrak{U}^n$, threshold number of iterations τ, temperature
parameter C>0
Initialize: constraint points $(u^0_{1,} \ldots\ldots, u^0_n \in \mathfrak{U}^n, y^0 = G(u^0_{1,} \ldots\ldots, u^0_n)$, initial
guess for optimal value $y_{max} = y^0$
1.while $N \leq \tau$, do
2. Sample, $(u^N_{1,} \ldots\ldots, u^N_n)$ from the distribution $R((u^{N-1}_{1,} \ldots\ldots, u^{N-1}_n).)$
3. Set $y^N \leftarrow G(u^N_{1,} \ldots\ldots, u^N_n)$
4. Sample η from distribution unit (0.5, 0.5)
5. Set $T_N \leftarrow \frac{C}{\log(N+1)}$
6. if $y^N > y^0$ then
7. | Set $y^0 \leftarrow y^N$
8. else if $C^{(y^N - y^0))/T_N} \geq \eta$ then
9. | Set $u^N_{i,} \leftarrow u^{N-1}_i$ for all $i \in \{1, \ldots n\}$
10. if $y^0 > y_{max}$ then
11. | $y_{max} \leftarrow y^0$
12. Set N← N+1
13. end

[0060] In an embodiment, consider the equation (1.1) along with its associated data ((1.1)-a)– ((1.1)-e). Pick $C > 0$ and the sequence $(T_N)_{N \leq 1}$ given by $T_N := \frac{C}{\log N+1}$. Suppose that Assumption (3.2) and the hypotheses of Lemma (3.6) hold. Let $(u^N_1, u^N_2, \ldots, u^N_n)_{N \leq 1}$ be a Markov chain generated by technique 1 with the selection Markov kernel R(.,.) and the cooling scheme

$(T_N)_{N\leq 1}$. Then the sequence of function values $(G(u_1^N,\ldots..,u_n^N))_{N\leq 1}$. Converges in probability to y*.

[0061] Technique 1 exploits the reformulation to obtain an approximate global optimum for (1.1). It uses the well-known simulated annealing method for this task. Here is an informal description of Technique 1 to aid writing pseudo codes:

- Initiate the technique using an initial guess of n points $(u_i^0)^n i=1 \in U^n$. These points correspond to the initial states of the Markov chain on U^n. When propagated in accordance to the accept reject rule for simulated annealing, the chain $(u_1^N, u_2^N,\ldots..,u_n^N)_{N\leq 1}$ converges in probability to a maximizer of G.
- For the initial guess, $G(u_1^0, u_2^0,\ldots..,u_n^0)$ is calculated and stored in y_{max} i.e. the current value of G(.) and the global optimum is initialized as y_{max}. at each iteration N, a candidate subset $U^N \subset U^n$ is obtained where the elements U^N are chosen according to $R((u_1^{N-1}, u_2^{N-1},\ldots..,u_n^{N-1}),.)$ (step 2).
- Based on this candidate subset Un, the corresponding value $y^N = G(u_1^N,,\ldots..,u_n^N)$ is calculated (step 3).
- To decide whether the candidate subset should be accepted or not, for each iteration N we generate a random number n and the annealing temperature T_n (step 4-5).
- Step 6-9 in technique 1 are performed using the random variable η and the annealing temperature T_N being generated in the preceding step. Accordingly the candidate subset is accepted or rejected.
- Finally when the stopping condition is triggered, y_{max} gives the global optimum value of (1.1) is probability.
- Here, the variable y_{max} is used to store the highest optimal value encountered by Technique 1 until the N^{th} iteration since we seek the

highest optimal value possible irrespective of the current state of the Markov chains.

[0062] FIG. 3 is a flow chart illustrating a method of sampling for control via simulated annealing for global optimization to exact solutions of convex semi-infinite problems for communication module, according to the embodiments as disclosed herein.

[0063] Referring to the FIG. 3 considering the proposed method, illustrates the method of sampling control via simulated annealing. Followings are the steps –

1. Start (301)
2. Exploration set 0 ⊂ U; iteration thresholdΓ; iteration counter N=0; Temperature parameter C>0 (302).
3. Initial guesses (u_1^0,…,u_n^0) ϵ U^n; y^0 = G(u_1^0,…,u_n^0); y_{max} =y^0 (303).
4. Sample (u_1^N,…,u_n^N) from the distribution R((u_1^{N-1},…,u_n^{N-1}),.) (304).
5. Set y^N<- G(u_1^N,…,u_n^N) sample $\eta$from Unit (0.5,0.5) set Tn <- $\frac{C}{\log(N+1)}$ (305).
6. y^N>y0
7. if the step 6 is (YES) then it will set y^0<- y^N(set y^0<- y^N is directly the step 8) (306)
8. if the step 6 in (NO) then it will be $e^{(yn-y0)/TN} \geq \eta$ (307)
9. if the step 8 is (YES) then it will be set $u_i^N \longleftarrow u_i^{N-1} \forall$ i ϵ [1,n] (308)
10. if the step 8 is (NO) then it will be y^0>y_{max} (309)
11. if the step 8 is (YES) then it will be $y_{max} \longleftarrow y^0$ (310)
12. if the step 8 is (NO) then it will be N≤ τ ($y_{max} \longleftarrow y^0$ is directly given to this step)
13. if the N≤ τ is (NO), (311)
14. the step will be y_{max} (312).
15. stop (313).

[0064] FIG. 4A illustrating the scenario of histogram of the optimal value for global optimization to exact solutions of convex SIP, according to the embodiments as disclosed herein. Referring to FIG. 4A, the histogram shows number of visits to the optimal value. The optimal value obtained by solving the convex SIP (4.2) using Technique 1 is identical to the optimal value of (4.1), which is 0. For the example picked the N==5*10^3, C=1 and exploration parameter α was 0.05. Here n=2 since this is the number of decision variables (4.2) and the dimension of constraint index set k=15. Further, initialize the Markov chain randomly according to Unit (-1, 1).

[0065] FIG. 4B illustrating the scenario of time evolution of the optimal value for global optimization to exact solutions of convex SIPs, according to the embodiments as disclosed herein.

[0066] FIG. 4A and FIG. 4B shows the histogram plots well as a time evolution plot of the optimal value obtained using Technique1. At iteration N = 700 approximately, the optimal value $y*$ =-2.71×10^{-3} is approximately equal to the optimal value, as shown in the FIG. 4(B). This is also evident from the histogram FIG. 4(A), which suggests that the concentration of Markov chains around zero is the highest and supports the proposed claim. Of course, the error between the optimal value obtained using method 1 and the true optimal value increases to 0 as the number of iterations N-> +∞.

[0067] FIG. 5A illustrates the scenario of plot of the absolute error between number of iterations and a true optimal value for global optimization to exact solutions of convex SIP, according to the embodiments as disclosed herein.

[0068] FIG. 5B illustrates the scenario of plot of the absolute error between the number of samples and a true optimal value for global optimization to exact solutions of convex SIP, according to the embodiments as disclosed herein.

[0069] FIG. 6 illustrates a flow diagram for determining approximation of SIP to determine outcome of the SIP using sampler device (104), according to the embodiments as disclosed herein.

[0070] At step 601, inputs are received from the user.

[0071] At step 602, infinite number of constraints of the SIP, the infinite number of constraints of the SIP makes the SIP intractable.

[0072] At step 603, a global maximization problem is determined by reformulating the finite dimensional approximation of SIP into a finite dimension problem with finite constraints.

[0073] At step 604, an optimal value of the finite dimensional approximation of SIP based on the global maximization problem is obtained using a global optimization technique.

[0074] At step 605, a plurality of outcomes of the finite dimensional approximation of SIP based on the optimal value of the finite dimensional approximation.

[0075] **Numerical examples from the literature**: Examples from the SIPAMPL test problem database. The test problem database for SIPAMPL has a collection of SIPs taken from various conventional methods and systems. Table1 shows a comparison of the optimal values obtained via Technique1 with respect to the actual optimal values of some of the problems taken from the SIPAMPL database.

Problem	Dim (x)	Dim($\mathcal{U}$)	Actual optimal value	Obtained optimal value
Anderson1	3	2	-1/3	-0.3333
coopeM	2	1	1	1.0
Leon13	2	1	0.236068	0.236068
Liu1	2	1	-4.65385	-4.65385
Liu2	2	1	-3.3635	-3.3635
Kortanek2	2	2	0.686291501	0.685877

Still	2	1	1	1.0
Watson2	2	1	0.194466	0.194466

Table 1. Optimal value comparison for a few problems from SIPAMPL database.

[0076] The foregoing description of the specific embodiments will so fully reveal the general nature of the embodiments herein that others can, by applying current knowledge, readily modify and/or adapt for various applications such specific embodiments without departing from the generic concept, and, therefore, such adaptations and modifications should and are intended to be comprehended within the meaning and range of equivalents of the disclosed embodiments. It is to be understood that the phraseology or terminology employed herein is for the purpose of description and not of limitation. Therefore, while the embodiments herein have been described in terms of preferred embodiments, those skilled in the art will recognize that the embodiments herein can be practiced with modification within the spirit and scope of the embodiments as described herein.

CLAIMS

We Claim:

1. A method for determining approximation of Semi-Infinite optimization problem (SIP) to identify outcome of the SIP using a sampler device, the method comprises;

receiving, by the sampler device, inputs from a user;

defining, by the sampler device, infinite number of variables and constraints of the finite dimensional approximation of SIP, wherein the infinite number of constraints of the SIP makes the SIP intractable.;

determining, by the sampler device, a global maximization problem by reformulating the finite dimensional approximation of SIP into a finite dimension problem with finite constraints;

obtaining, by the sampling optimizer, an optimal value of the finite dimensional approximation of SIP based on the global maximization problem using a global optimization technique; and

exacting, by the sampler device, a plurality of outcomes of the finite dimensional approximation of SIP based on the optimal value of the finite dimensional approximation.

2. The method as claimed in claim 1, wherein the finite dimensional approximation of SIP is a convex Semi-Infinite Problem (SIP).

3. The method as claimed in claim 1, wherein the SIP problem comprises finite set of decision variables and infinite number of constraints.

4. The method as claimed in claim 1, wherein the optimal value of the convex SIP problem is equal to an approximate global optimum of the infinite constraints of the convex SIP problem.

5. The method as claimed in claim 1, wherein reformulating the convex SIP problem into theapproximate with the finite number of constraints finite dimension problem with the finite constraints of the convex SIP problem comprises determining deterministic and randomized value for low

dimension and high dimension respectively of a constraint index, wherein a Markov chain is initialized randomly.

6. The method as claimed in claim 1, wherein the global optimization technique is a simulated annealing basedannealing-based technique used for the convex SIP problem to obtain the optimal value of the convex SIP problem.

7. The method as claimed in claim 5, the global optimization technique comprises;

determining, by the sampler device, a set of initial guesses (G) associated with the simulated annealing basedannealing-based method of the global optimization scheme; and

examining, by the sampler device, a structural property of the initial guesses (G) in simulated annealing basedannealing-based technique of global optimization scheme.

8. The method as claimed in claim 5, wherein the simulated annealing basedannealing-based method for convex SIP comprises:

identifying, by the sampler device, finitedevice, finite number of constraints from the infinite number of constraints; and

recovering, by the sampler device, the approximate global optimum value of the convex SIP problem.

9. An electronic device (100) for determining approximation of Semi-Infinite optimization Problem (SIP) to determine outcome of the SIP, the electronic device comprising:

a memory (101);

a processor (102);

a sampler device (104), operably connected to the memory and the processor, configured to:

receive inputs from a user;

define infinite number of variables and constraints of the finite dimensional approximation of SIP, wherein the infinite number of constraints of the SIP makes the SIP intractable.;

determine a global maximization problem by reformulating the finite dimensional approximation of SIP into a finite dimension problem with finite constraints;

obtain an optimal value of the finite dimensional approximation of SIP based on the global maximization problem using a global optimization technique; and

exact a plurality of outcomes of the finite dimensional approximation of SIP based on the optimal value of the finite dimensional approximation.

10. The electronic device (100) as claimed in claim 1, wherein the finite dimensional approximation of SIP is a convex Semi-Infinite Problem (SIP).

11. The electronic device (100) as claimed in claim 1, wherein the SIP problem comprises finite set of decision variables and infinite number of constraints.

12. The electronic device (100) as claimed in claim 1, wherein the optimal value of the convex SIP problem is equal to an approximate global optimum of the infinite constraints of the convex SIP problem.

13. The electronic device (100) as claimed in claim 1, wherein reformulating the convex SIP problem into theapproximate with the finite number of constraints finite dimension problem with the finite constraints of the convex SIP problem comprises determining deterministic and randomized value for low dimension and high dimension respectively of a constraint index, wherein a Markov chain is initialized randomly.

14. The electronic device (100) as claimed in claim 1, wherein the global optimization technique is a simulated annealing basedannealing-based technique used for the convex SIP problem to obtain the optimal value of the convex SIP problem.

15. The electronic device (100) as claimed in claim 5, the global optimization technique comprises;

determining, by the sampler device, a set of initial guesses (G) associated with the simulated annealing basedannealing-based method of the global optimization scheme;

examining, by the sampler device, a structural property of the initial guesses (G) in simulated annealing basedannealing-based technique of global optimization scheme.

16. The electronic device (100) as claimed in claim 5, wherein the simulated annealing basedannealing-based method for convex SIP comprises:

identifying, by the sampler device, finitedevice, finite number of constraints from the infinite number of constraints; and

recovering, by the sampler device, the approximate global optimum value of the convex SIP problem.

Dated as 15th December, 2022

Signatures:

Name of the Signatory: Arun Kishore Narasani

Patent Agent IN/PA 1049

ABSTRACT

"METHOD AND ELECTRONIC DEVICE FOR GLOBAL OPTIMIZATION TO EXACT SOLUTIONS OF CONVEX SEMI-INFINITE PROBLEMS"

Embodiments herein provide a method for determining approximation of Semi-Infinite optimization problem (SIP) to identify outcome of the SIP using a sampler device. Inputs are received from user and defining infinite number of constraints of the SIP making the SIP intractable. A global maximization problem is determined by reformulating the finite dimensional approximation of SIP into a finite dimension problem with finite constraints. An optimal value of the finite dimensional approximation of SIP is obtained based on the global maximization problem using a global optimization technique. A plurality of outcomes of the finite dimensional approximation of SIP is exacted based on the optimal value of the finite dimensional approximation. One of the major advantages of this approach is that it admits a plug-and-play module, where one can use any global optimization method to obtain the optimal value.

FIG. 1

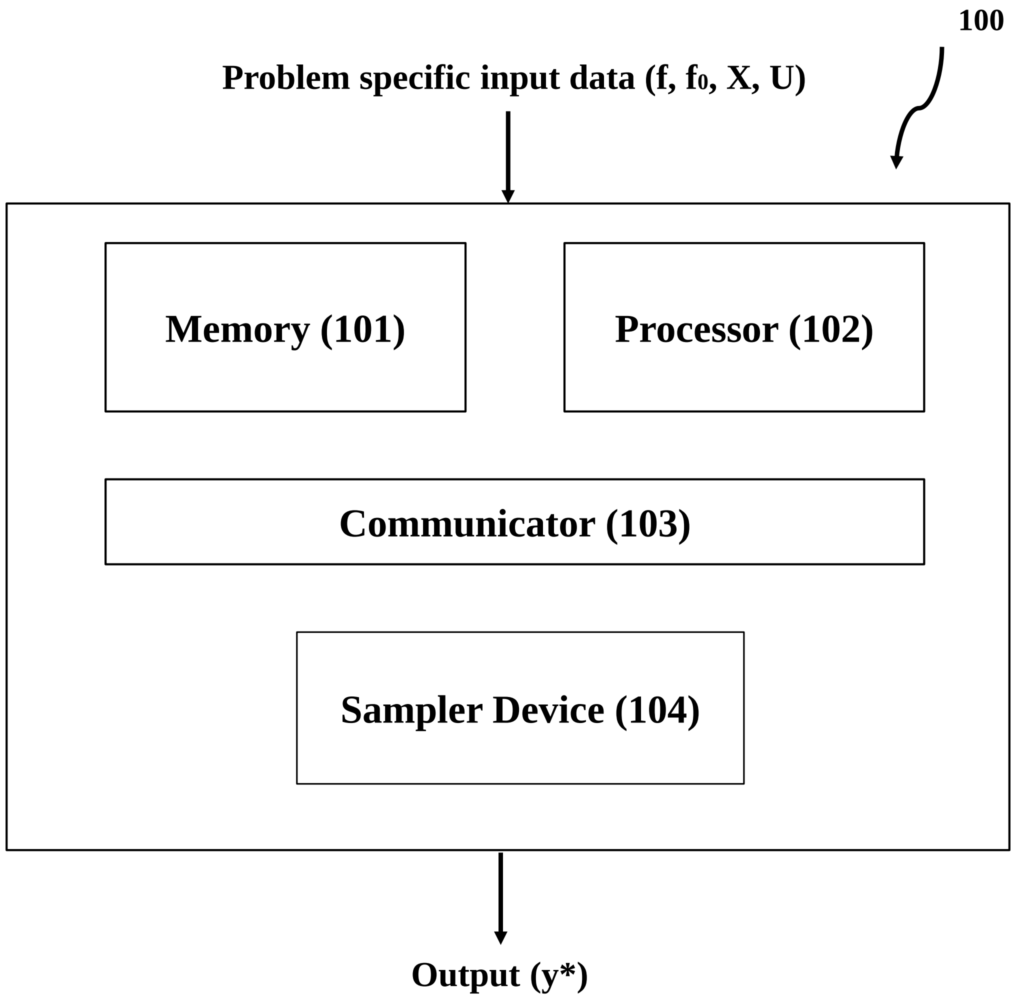

FIG. 1

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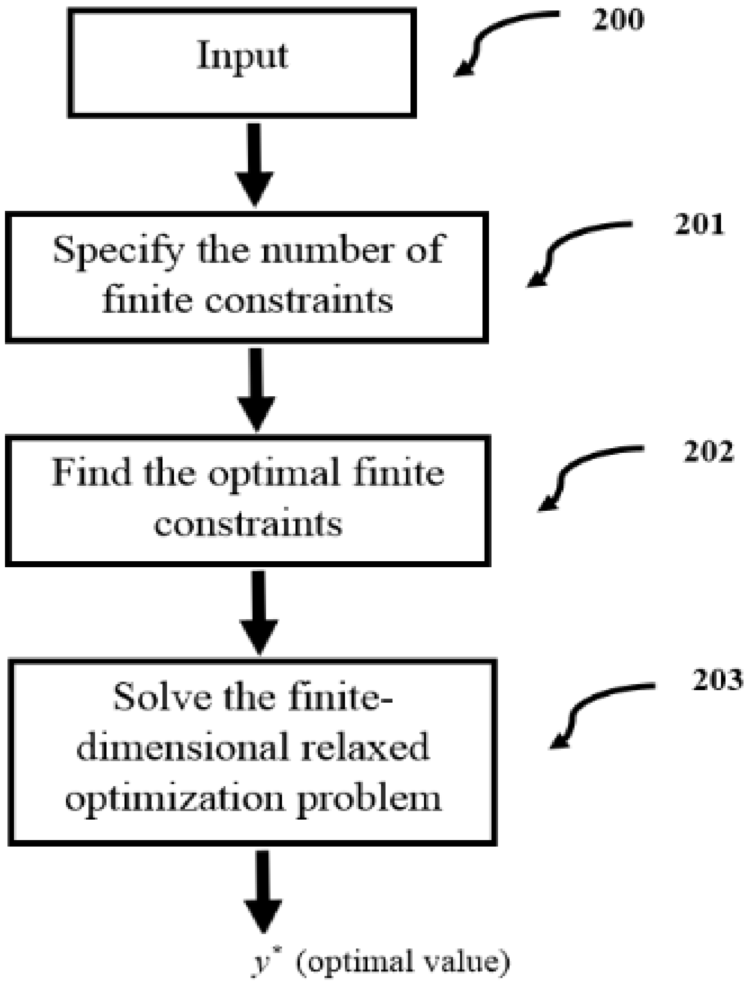

FIG. 2

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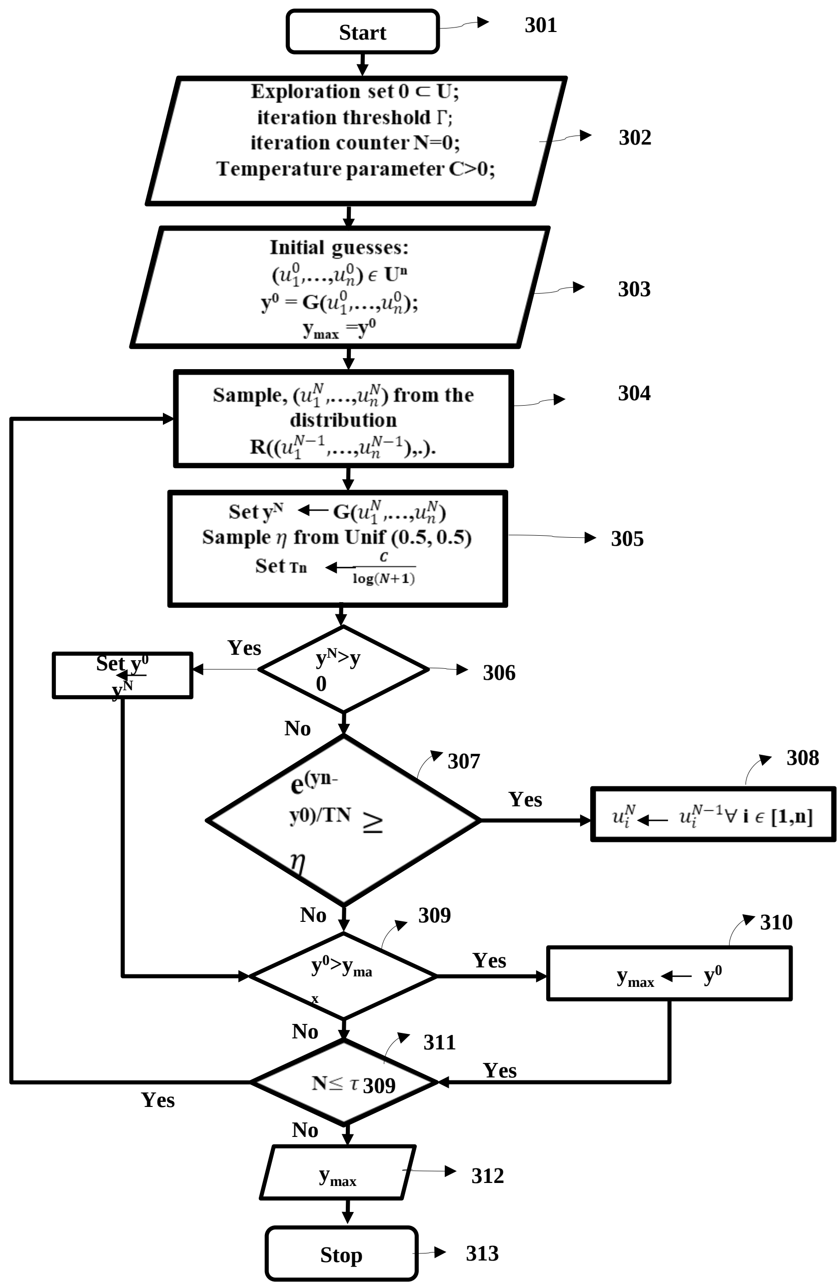

FIG. 3

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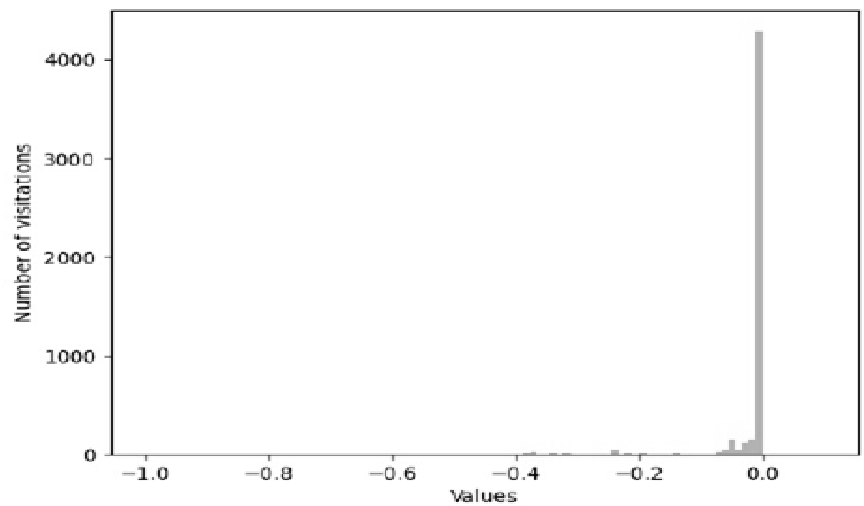

FIG. 4(A)

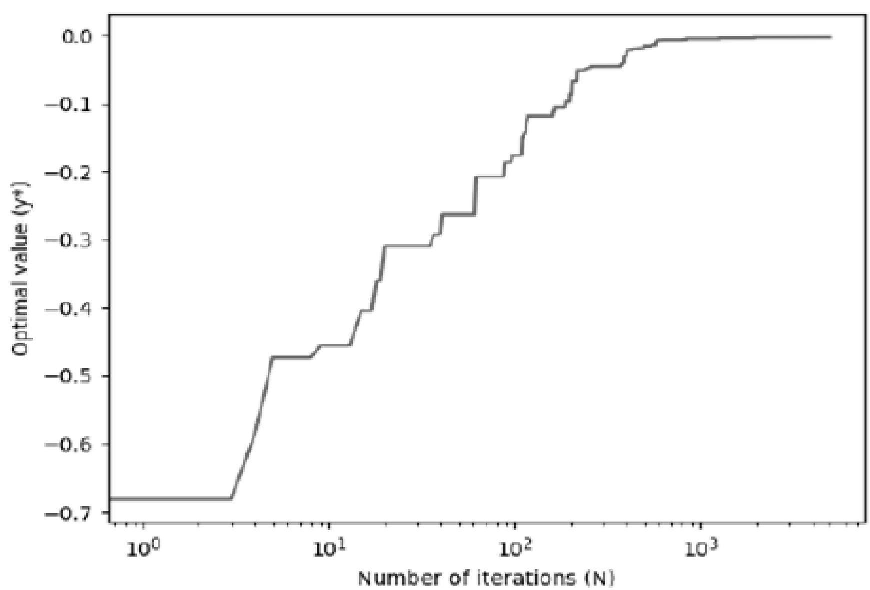

FIG. 4(B)

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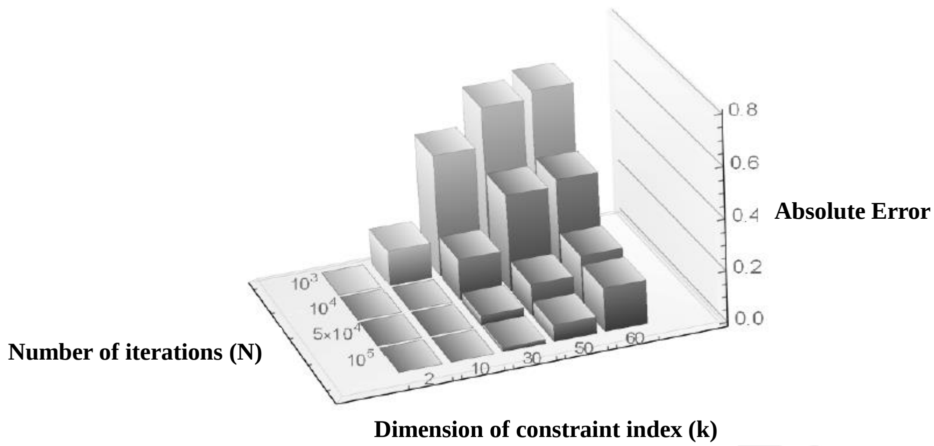

FIG. 5A

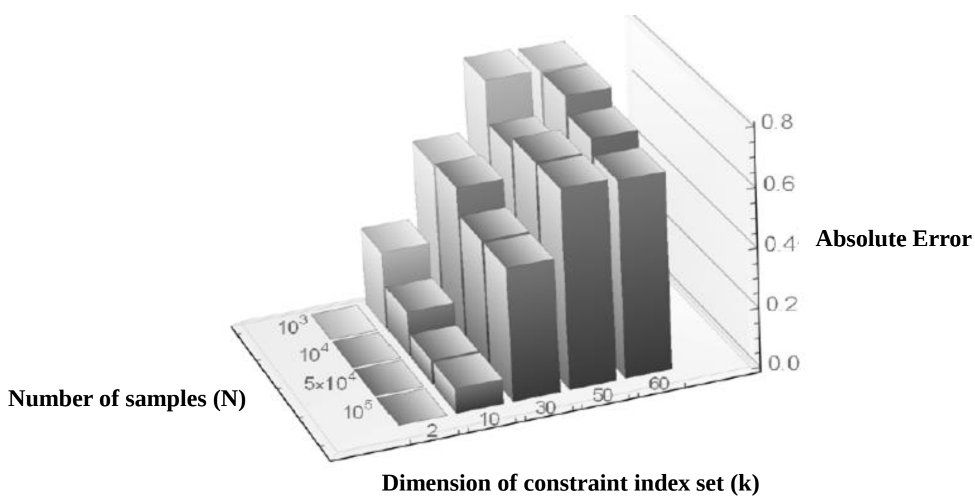

FIG. 5B

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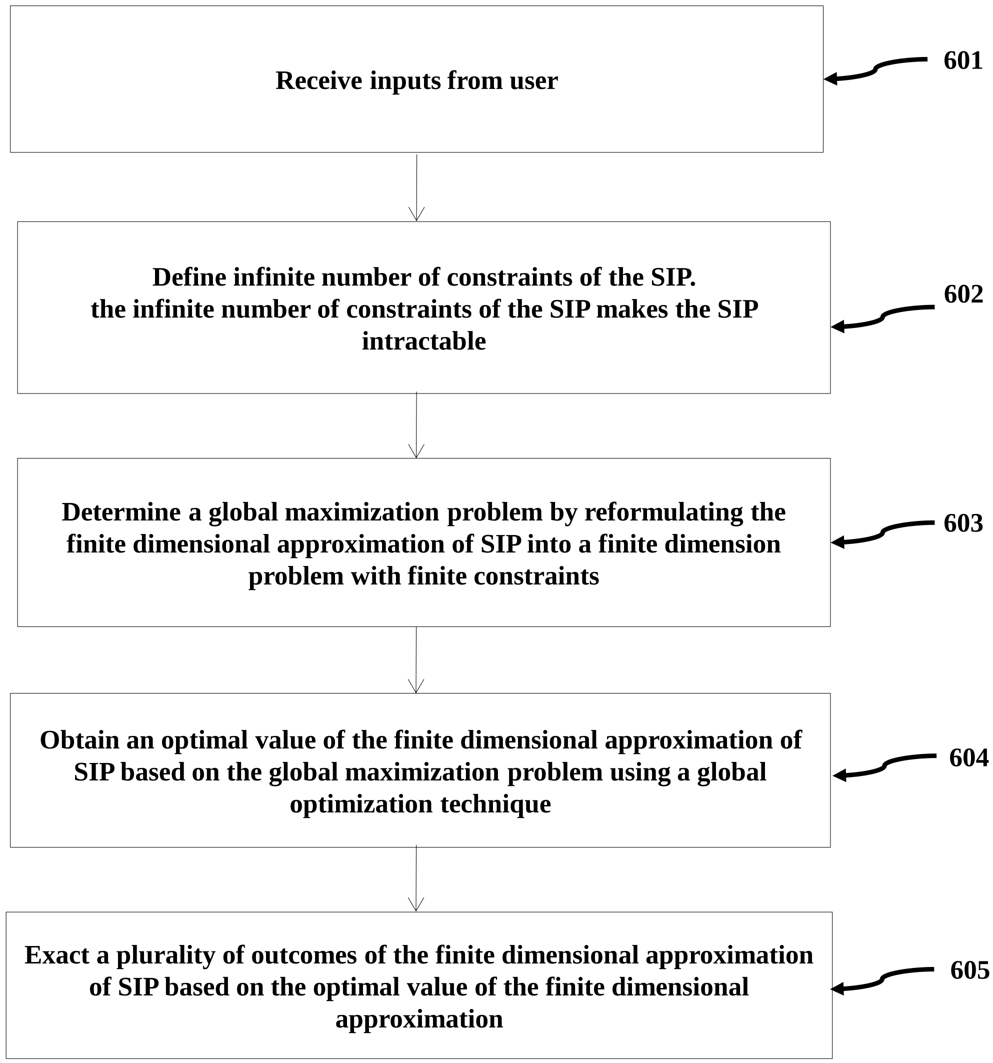

FIG. 6

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