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Ibrahim, Rabha W.; Altulea, Dania

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Controlled homeodynamic concept using a conformable calculus in artificial biological systems

Rabha W. Ibrahim[a,b,*], Dania Altulea[c]

[a] Informetrics Research Group, Ton Duc Thang University, Ho Chi Minh City, Vietnam
[b] Faculty of Mathematics & Statistics, Ton Duc Thang University, Ho Chi Minh City, Vietnam
[c] Faculty of Science, University of Groningen, the Netherlands

A R T I C L E I N F O

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A B S T R A C T

Homeodynamic system (HS) in the biological studies (from the Greek homoios (similar) and Dynamis (energy)) designates the accommodating instruments of stabilizing and repairing of the fundamental reliability and functional efficiency of living schemes. In this effort, we employ the concept of conformable calculus (CC) to generalize the homeodynamic system. The generalization requires a controller in the system to preserve the variables robustly regulated, oscillated, and synchronized variables at a certain set point. Here, we show how the selectivity of the CC makes differences in the behavior of an oscillation and the other properties of HS.

1. Introduction

Control theory ascends from the conceptualization and simplification of scheme approaches intended at refining the permanence, robustness, and stability of practical systems. There are two types of architecture in biological cells; In the first one, the controller is present the controller together with the process, which are both functioning in the cell (Fig. 1); and in the second type, the whole process is the cell itself while the controller is performed on a computer (Fig. 2). In traditional set-up systems, the controller acts on the process output ψ with a wanted value μ, and, depending on the error between these two, calculates the contribution to be employed to the process to eventually reduce the inconsistency between ψ and μ. Furthermore, when the presentation, consistency, and robustness of definite hardware mechanisms cannot be upgraded additionally by recovering categorization or hardware scheme, negative feedback control is particularly beneficial. These techniques are used widely in artificial biology systems [1–4]. These systems work together in the living cells to perform various functions ranging from energy production, to drug transport and metabolism and so on. The ability to modify living creatures has many beneficial outcomes. For an instance, researchers have been able to utilize microorganisms to produce bio-fuels using gene editing tools. On the other hand, water, soil, and industrial services can also influence artificial biology e.g emergence of certain bacterial species that can damage herbicides [5,6]. The effect of integral control in oscillatory and chaotic reaction kinetic networks has been studied in Thorsen et al. [7].

Homeodynamics [8](the concept of life as self-cloning) describes the requirements that any model of a complex system must fulfill. This concept is being presented to study the limitation of regulatory behavior, rate of stability (not far from the equilibrium), and rate-oscillation (or noise information). This study aims to utilize a controller that responds to kinetic energy systems and focuses on the regulation of the bio-systems [7]. This concept was first expressed by John von Neumann [9]. He considered a systematic rule set to varying a singular spatial configuration tasks with reproducing itself. Information obtained from the investigation showed that self-reproduction gives a self-description. The self-reproduction training shows that the quantities of communicating materials are adequate to initiate a new cell cycle after a definite phase of alteration to new conditions. This concept is suitable to study or modify artificial biological systems.

Artificial biology system (ABS) combines traditional engineering methods with competing models like homogenous sections, functional systems, and computer schemes. In artificial biology, modeling functions as an instrument to calculate how a network will perform after it improved in normal techniques. Easy simulations are commonly employed to analyze and recognize parameters. In addition, they are selected over complex simulations as long as the capability to repeat the detected performance of the system is not

* Corresponding author at: Faculty of Mathematics & Statistics, Ton Duc Thang University, Ho Chi Minh City, Vietnam.

E-mail addresses: rabhaibrahim@tdtu.edu.vn (R.W. Ibrahim), d.altulea@student.rug.nl (D. Altulea).

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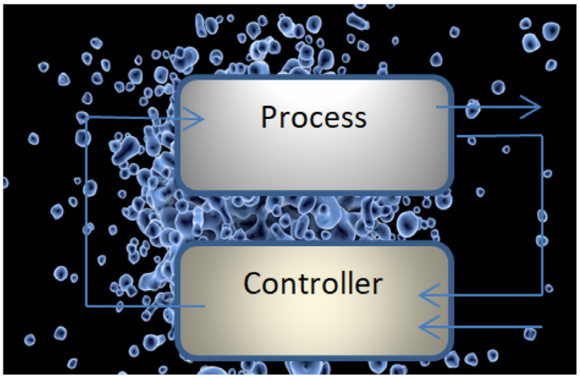

Fig. 1. The process and the controller are both functioning in the cell.

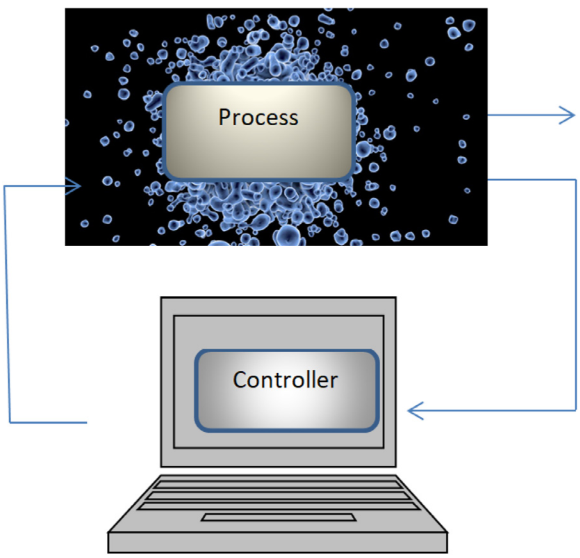

Fig. 2. The process is the cell itself while the controller is a tool in a computer.

altered. Therefore, many systems utilize different assumptions for simplicity. For example, the main assumption in some networks is that the molecular classes are consistently distributed in the interior of the cell. This hypothesis removes the diffusion amount from the model, because there are no attentiveness gradients (diffusion differential equation) and replace it with a controller to do the ideal job of the system. Information tells us that this assumption may become unacceptable in certain states. When simulations fail to repeat detected behaviors, some of the assumptions must be re-examined [10]. Recently, fractional dynamic systems have been studied because of their efficiency in applications (see [11–16]).

The controller must lead a trajectory across a state space, and it gives no data about the construction or dynamical characteristics of the state space other than the present position of the trajectory showed and the distance to the goal region. In this effort, we develop ABS with a controller by using the concept of conformable calculus. This branch of calculus is defined by combining the idea of a control system and the conformable calculus. Thus, such a concept does not only study the past information, but seeks the future expectation for the detected behaviors of ABS.

2. Related researches

Conformable calculus in biological systems is a relatively new and rich area in the current field of biological systems. This concept is utilized to refine, generate, control, and modify many biological systems. The authors in Seadawy et al. [17] created the traveling and solitary wave solutions of the fractional-order biological population system, including the time fractional Burgers equation, the Drinfel'd-Sokolov-Wilson equation, and three of shallow water wave equations. A conformable logistic differential equation counting together discrete and continuous time is attracting attention in Kartal and Gurcan [18]. By utilizing a theory of approximation type piecewise constant, a pre-processing technique which renovates a fractional-order differential equation into a difference operator is presented. Necessary and sufficient expressions for the local and global stability are indicated. The behaviors of conformable Lotka-Volterra predator-prey scheme is studied in Gürcan et al. [19]. They employed the piecewise constant approximation method to obtain the discrete version of the system, then, they considered stability, existence, and direction of Neimark-Sacker bifurcation of the positive equilibrium point of the discrete scheme. It was detected that the discrete system exposed much richer dynamic behaviors than its fractional-order form such as Neimark-Sacker bifurcation and chaos. The non-linear fractional KdV-Burgers equation in expressions of conformable derivative is reconstructed in its place of the Caputo fractional derivative and the series result of this case is also presented by using the residual power series technique in El-Ajou et al. [20]. The authors in Nazir et al. [21] employed a conformable model in malnutrition community. An improved type of the mathematical model for malnutrition community was developed. Stability of the established system was tested and an exploration of utilizing fixed-point theory on the existence and uniqueness of the system's results has been made [22–24].

3. Formulated system

In this section, we formulate the conformable artificial biological system (C-ABS).

3.1. Conformable calculus (CC)

Definition 1. (Conformable differential operator)
A differential operator $\mathcal{D}^{\beta}$, $\beta \in [0,1]$ is conformable if and only if $\mathcal{D}^0$ is the identity operator and $\mathcal{D}^1$ is the ordinary differential operator. Particularly, the operator is conformable if and only if a differential function $\chi(t)$ satisfies

$$\mathcal{D}^0\chi(t) = \chi(t) \quad and \quad \mathcal{D}^1\chi(t) = \frac{d}{dt}\chi(t) = \chi'(t).$$

In the theory of control systems, a proportional-differential controller for controlling resultant υ at time t with two tuning criteria has the setting

$$\upsilon(t) = \nu_p\Sigma(t) + \nu_d\frac{d}{dt}\Sigma(t), \tag{1}$$

where ν_p is the proportional gain, ν_d is the derivative gain, and Σ is the error between the formal variable and the actual variable. Based on (1), Anderson and Ulness [25] presented the common idea of CC. Note that, this CC is extended in Ibrahim and Jahangiri [26].

Definition 2. (A special class of conformable calculus)
For two continuous functions $\nu_0, \nu_1 : [0,1] \times \mathbb{R} \to (0,\infty)$ we attain

$$\mathcal{D}^{\beta}\chi(t) = \nu_1(\beta,t)\chi(t) + \nu_0(\beta,t)\chi'(t) \tag{2}$$

where

$$\lim_{\beta\to 0}\nu_1(\beta,t) = 1, \quad \lim_{\beta\to 1}\nu_1(\beta,t) = 0, \quad \nu_1(\beta,t) \neq 0, \ \forall t,$$
$$\beta \in (0,1),$$

and

$\lim_{\beta\to 0} \nu_0(\beta, t) = 0, \quad \lim_{\beta\to 1} \nu_0(\beta, t) = 1, \quad \nu_0(\beta, t) \neq 0, \ \forall t,$
$\beta \in (0, 1).$

Definition 3. The integral operator corresponding to $\mathcal{D}^\beta$ is given by the following equation:

$$\int \mathcal{D}^\beta \chi(t) d_\beta t = \chi(t) + k e_0(t, t_0), \tag{3}$$

where $k \in \mathbb{R}$, $d_\beta t = \frac{dt}{\nu_0(t)}$, $\nu \neq 0$ and

$$e_0(t, \kappa) = \exp\left(-\int_\kappa^t \frac{\nu_1(\beta, \varsigma)}{\nu_0(\beta, \varsigma))} d\varsigma \right). \tag{4}$$

Moreover, the definite integral of the derivative of χ over the interval $[a, b]$ is formulated as follows:

$$\int_a^t [\mathcal{D}^\beta \chi(\varsigma)] e_0(t, \varsigma) d_\beta \varsigma = \chi(t) - \chi(a) e_0(t, a).$$

In our investigation, we request one of the following formulas of ν_1 and ν_0:

$$\nu_1(\beta, t) = (1-\beta) t^\beta, \quad \nu_0(\beta, t) = \beta t^{1-\beta}, \quad t \in (0, \infty), \tag{5}$$

$$\nu_1(\beta, t) = (1-\beta)|t|^\beta, \quad \nu_0(\beta, t) = \beta |t|^{1-\beta}, \tag{6}$$

$$\nu_1(\beta, t) = \cos(\frac{\beta\pi}{2}) t^\beta, \quad \nu_0(\beta, t) = \sin(\frac{\beta\pi}{2}) t^{1-\beta}, \quad t \in (0, \infty) \tag{7}$$

$$\nu_1(\beta, t) = \cos(\frac{\beta\pi}{2}) |t|^\beta, \quad \nu_0(\beta, t) = \sin(\frac{\beta\pi}{2}) |t|^{1-\beta} \quad t \in \mathbb{R}\backslash\{0\} \tag{8}$$

or for $\rho_0, \rho_1 \in (0, \infty)$

$$\nu_0(\beta, t) = \beta \rho_0^{1-\beta}, \quad \nu_1(\beta, t) = (1-\beta) \rho_1^\beta. \tag{9}$$

Finally, the conformable inner product between two continuous functions χ and φ is given in the following formula:

$$< \chi, \varphi >= \int_a^b \chi(t) \varphi(t) e_0(b, t) d_\beta t.$$

3.2. Artificial biological system

An important technique of modeling ABS is that to designate it as a dynamical system consisting of molecular classes and responses. Every response is considered by the class that created it and a re-activity ratio, which is a function of the class applications. Whenever the re-activity ratios are defined, then the activities of the chemical re-activity system are detected through deterministic or stochastic simulations [10]. Presence of solution of ABS as a collection of differential equations allows analysis of existing numerical approaches from the well-known studies of nonlinear activities. Numerical methods are utilized to create time series paths of the class concentrations. It is often interesting to observe the fluctuations in a variable based on how the steady state principles are affected. In this case, a steady state analysis is utilized.

ABS modeling has been studied expansively, and many approaches were generated. Most of these deal with deterministic and stochastic approaches for periodic analysis. Few of these systems have a controller term. Connections among path mechanisms can often suitably designate ordinary differential equations. A simple model [27] as follows:

$$\frac{d\chi(t)}{dt} = \nu\phi(\chi), \tag{10}$$

where $\chi(t)$ indicates the concentrations of each molecule type during time t, ν (fixed constant for all states) represents the stoichiometry value of reaction networks and ϕ refers to the rate of change of the concentration of each type. When cellular noise significantly moves a path function, it has to reformulate in view of the mathematical analysis. This can be represented by Langevin chemical equations or chemical master equations. In this case, Eq. (10) must include a noise term [28]

$$\frac{d\chi(t)}{dt} = \nu\phi(\chi) + \sqrt{\nu\phi(\chi)}\eta(t), \tag{11}$$

where η represents to the white noise.

Both models are studied by utilizing two current categories of mathematical analysis: parametric analysis and bifurcation analysis. Parametric analysis supports the measurable modifications of path dynamics in response to perturbations of path parameters. This analysis is mostly used for recognizing critical path mechanisms. Therefore, there are different techniques to seek the behavior of the system, such as critical point theory, and fixed-point theory. Focusing on the second option, we employed the recent fixed-point theorem of self-mapping. This theorem covers the idea of homeodynamic concept.

Bifurcation analysis is utilized to control how qualitative possessions of a path depend on its parameters. Specifically, it aims to find the steady-state results of a scheme and their stability. Bifurcation is supposed to happen when there is a modification in either the number of steady state results or the stability of one or multiple solutions. Moreover, bifurcation analysis is employed to analyze the oscillator segment in the system, by calculating the effects of path parameters on the oscillation amplitude and occurrence.

A nonlinear ABS studied in Chen et al. [29] taking the formula

$$\frac{d\chi(t)}{dt} = \Phi(t, \chi), \tag{12}$$

where Φ is the nonlinear interaction function among classes within ABS satisfying Fig. 3.

By putting (12) in (2), we have the following conformable ABS (CABS)

$$\begin{aligned} \mathcal{D}^\beta \chi(t) &= \nu_1(\beta, t)\chi(t) + \nu_0(\beta, t)\chi'(t) \\ &= \nu_1(\beta, t)\chi(t) + \nu_0(\beta, t)\Phi(t, \chi), \end{aligned} \tag{13}$$

where $t \in J = [0, \mathfrak{T}]$, ν_0 and ν_1 defined in Definition 2 and χ, Φ: $J \times [0, \infty) \to [0, \infty)$ considered to be a continuous non-decreasing function such that $\chi(t) > 0$ for all $t \in (0, \infty)$ and $\Phi(0) = \chi(0) = 0$. Our aim is to study the existence and the uniqueness solution of Eq. (13). To make this request, we shall use self-mapping fixed point theorem [30].

Lemma 3.1. *Let $(X, \triangle)$ be a complete metric space and $\mathcal{Q} : X \to X$ a self-mapping satisfying the inequality*

$$\flat(\triangle(\mathcal{Q}(x), \mathcal{Q}(y))) \leq \flat(\triangle(x, y)) - \wp(\triangle(x, y)) \tag{14}$$

for all $x, y \in X$, where $\flat$, $\wp$: $[0, \infty) \to [0, \infty)$ are both continuous and non-decreasing functions with $\flat(0) = \wp(0) = 0$. Then $\mathcal{Q}$ indicates a unique fixed point.

Denote $X = \mathbb{R}$ and in view of (3) and (13), we define an operator Q: $X \to X$ as follows:

$$(Q\chi)(t) = \int (\nu_1(\beta, t)\chi(t) + \nu_0(\beta, t)\Phi(t, \chi(t))) d_\beta t + c e_0(t, t_0). \tag{15}$$

Since $\chi \in X$ then Q is a self-mapping. In addition, define a function $\mathfrak{S} : X^3 \to \mathbb{R}^+$ by

$$\mathfrak{S}(\chi_1, \chi_2, \chi_3) = \max\{|\chi_\imath - \chi_\jmath| : \imath, \jmath = 1, 2, 3, \imath \neq \jmath\}.$$

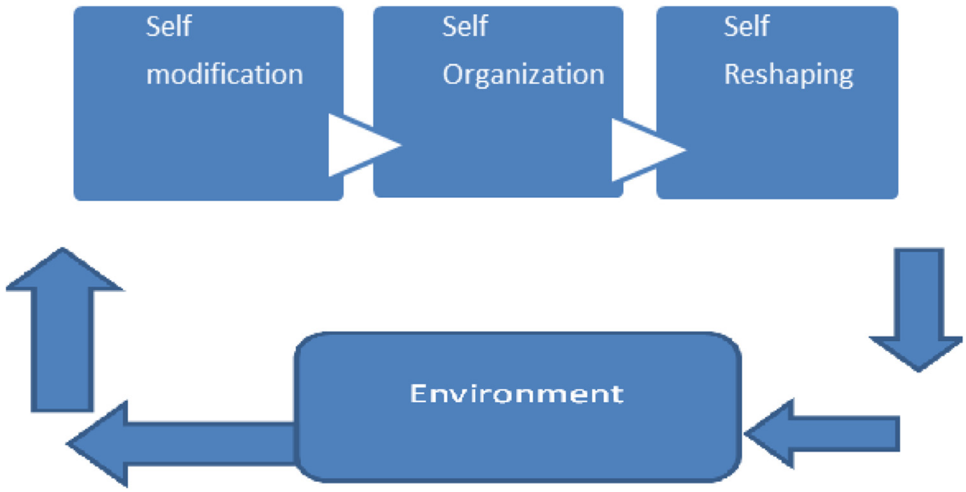

Fig. 3. ABS of three classes within a biological cell.

Obviously, $\mathfrak{S}(\chi_1, \chi_2, \chi_3) = 0$ for $\chi_1 = \chi_2 = \chi_3$; also, we have

$$\begin{aligned}
&\mathfrak{S}(\chi_1, \chi_1, \chi_i) + \mathfrak{S}(\chi_2, \chi_2, \chi_j) + \mathfrak{S}(\chi_3, \chi_3, \chi_k) \\
&= \max_{i=2,3}\{|\chi_1 - \chi_i|\} + \max_{j=1,3}\{|\chi_2 - \chi_j|\} + \max_{k=1,2}\{|\chi_3 - \chi_k|\} \\
&= \max\{|\chi_1 - \chi_2|, |\chi_1 - \chi_3\} + \max\{|\chi_2 - \chi_1|, |\chi_2 - \chi_3\} \\
&\quad + \max\{|\chi_3 - \chi_1|, |\chi_3 - \chi_2|\} \\
&= 2\max\{|\chi_1 - \chi_2|, |\chi_2 - \chi_3|, |\chi_3 - \chi_1|\} \\
&> \max\{|\chi_1 - \chi_2|, |\chi_2 - \chi_3|, |\chi_3 - \chi_1|\} \\
&= \max\{|\chi_\imath - \chi_\jmath| : \imath, \jmath = 1, 2, 3, \imath \neq \jmath\} \\
&= \mathfrak{S}(\chi_1, \chi_2, \chi_3).
\end{aligned} \tag{16}$$

Hence, the function $\mathfrak{S}(\chi_1, \chi_2, \chi_3)$ is a metric on the set X. This metric indicates the maximum measurement among the three classes of ABS within a biological cell. Note that this metric can extend to include other classes in the system's cell.

Theorem 3.2. *Consider the conformable Eq. (13). If*

$$|\Phi(t, \chi(t)) - \Phi(t, \eta(t))| < \ell|\chi(t) - \eta(t)|$$

for some positive constant $\ell < \frac{1-(1-\beta)\mathfrak{T}^\beta}{\beta\mathfrak{T}^{1-\beta}}$, $\mathfrak{T} < \infty$. *Then Q has a unique fixed point in X.*

Proof. Let the functions ν_0 and ν_1 be given by

$$\nu_1(\beta, t) = (1-\beta)t^\beta, \quad \nu_0(\beta, t) = \beta t^{1-\beta}, \quad t \in (0, \mathfrak{T}), \mathfrak{T} < \infty.$$

Note that, similar proof can be presented for other formulas. Then, by the assumption on ℓ, we have

$$\begin{aligned}
&\mathfrak{S}(Q\chi_1(t), Q\chi_2(t), Q\chi_3(t)) = \max\{|Q\chi_\imath(t) - Q\chi_\jmath(t)| : \imath, \jmath = 1, 2, 3, \imath \neq \jmath\} \\
&\le \max\{|\nu_1(\beta, t)\chi_\imath(t) + \nu_0(\beta, t)\Phi(\chi_\imath(\chi_\imath(t)) \\
&\quad -\left(\nu_1(\beta, t)\chi_\jmath(t) + \nu_0(\beta, t)\Phi(\chi_\jmath(\chi_\jmath(t))\right)|\frac{\mathfrak{T}^\beta}{\beta^2} : \imath, \jmath = 1, 2, 3, \imath \neq \jmath\} \\
&\le \max\{\left(\nu_1(\beta, t)|\chi_\imath - \chi_\jmath| + \nu_0(\beta, t)\ell|\chi_\imath - \chi_\jmath|\right)\frac{\mathfrak{T}^\beta}{\beta^2} : \imath, \jmath = 1, 2, 3, \imath \neq \jmath\} \\
&\le \max\{\left((1-\beta)\mathfrak{T}^\beta|\chi_\imath - \chi_\jmath| + \beta\mathfrak{T}^{1-\beta}\ell|\chi_\imath - \chi_\jmath|\right)\frac{\mathfrak{T}^\beta}{\beta^2} : \imath, \jmath = 1, 2, 3, \imath \neq \jmath\} \\
&= \max\{[(1-\beta)\mathfrak{T}^\beta + \beta\mathfrak{T}^{1-\beta}\ell]\frac{\mathfrak{T}^\beta}{\beta^2}|\chi_\imath - \chi_\jmath| : \imath, \jmath = 1, 2, 3, \imath \neq \jmath\} \\
&:= r\mathfrak{S}(\chi_1, \chi_2, \chi_3).
\end{aligned}$$

Since $[(1-\beta)\mathfrak{T}^\beta + \beta\mathfrak{T}^{1-\beta}\ell]\frac{\mathfrak{T}^\beta}{\beta^2} < 1 \Rightarrow [(1-\beta)\mathfrak{T}^\beta + \beta\mathfrak{T}^{1-\beta}\ell] < \frac{\beta^2}{\mathfrak{T}^\beta} < 1$. Hence, Q is bounded in the unit ball B_r of radius $0 < r < 1$. We proceed to discover more properties about the operator Q. Let $t, \tau \in (0, \mathfrak{T})$ such that $t > \tau$ then $\chi(t) > \chi(\tau)$ (increasing function). A computation yields that

$$\begin{aligned}
&\mathfrak{S}\big(Q\chi_1(t), Q\chi_2(t), Q\chi_3(t) - (Q\chi_1(\tau), Q\chi_2(\tau), Q\chi_3(\tau)\big)\\
&= \mathfrak{S}\Big(Q\big(\chi_1(t)-\chi_1(\tau)\big), Q\big(\chi_2(t)-\chi_2(\tau)\big), Q\big(\chi_3(t)-\chi_3(\tau)\big)\Big)\\
&= \mathfrak{S}\big(Q\chi_1(t-\tau), Q\chi_2(t-\tau), Q\chi_3(t-\tau)\big)\\
&\le \mathfrak{S}\big(Q\chi_1(t), Q\chi_2(t), Q\chi_3(t)\big)\\
&\le r\mathfrak{S}(\chi_1, \chi_2, \chi_3).
\end{aligned}$$

Thus, Q is equicontinuous on B_r. Moreover, by letting $\chi_l(t) - \eta_l(t) = \xi_l(t),\ l = 1, 2, 3,$ we attain that

$$\begin{aligned}
&\mathfrak{S}\Big(Q\big(\chi_1(t)-\eta_1(t)\big), Q\big(\chi_2(t)-\eta_2(t)\big), Q\big(\chi_3(t)-\eta_3(t)\big)\Big)\\
&= \mathfrak{S}\Big(Q\big(\xi_1(t)\big), Q\big(\xi_2(t)\big), Q\big(\xi_3(t)\big)\Big)\\
&\le \max\{|\nu_1(\beta,t)\xi_\imath(t) + \nu_0(\beta,t)\Phi(\xi_\imath(\chi_\imath(t))\\
&\quad -\big(\nu_1(\beta,t)\xi_\jmath(t) + \nu_0(\beta,t)\Phi(\xi_\jmath(\xi_\jmath(t))\big)|\frac{\mathfrak{T}^\beta}{\beta^2} : \imath, \jmath = 1,2,3, \imath \ne \jmath\}\\
&\le \max\{\nu_1(\beta,t)|\xi_\imath - \xi_\jmath|\frac{\mathfrak{T}^\beta}{\beta^2} + \nu_0\ell|\xi_\imath - \xi_\jmath|\frac{\mathfrak{T}^\beta}{\beta^2} : \imath, \jmath = 1,2,3, \imath \ne \jmath\}\\
&\le \max\{(1-\beta)\mathfrak{T}^\beta|\xi_\imath - \xi_\jmath|\frac{\mathfrak{T}^\beta}{\beta^2} + \beta\mathfrak{T}^{1-\beta}\ell|\xi_\imath - \xi_\jmath|\frac{\mathfrak{T}^\beta}{\beta^2} : \imath, \jmath = 1,2,3, \imath \ne \jmath\}\\
&= \max\{[(1-\beta)\mathfrak{T}^\beta + \beta\mathfrak{T}^{1-\beta}(\beta,t)\ell]\frac{\mathfrak{T}^\beta}{\beta^2}|\xi_\imath - \xi_\jmath| : \imath, \jmath = 1,2,3, \imath \ne \jmath\}\\
&= r\mathfrak{S}(\xi_1, \xi_2, \xi_3) \le r\mathfrak{S}(\chi_1, \chi_2, \chi_3).
\end{aligned}$$

Therefore, the operator Q is continuous in B_r. This implies that Q has a fixed point $Q\chi = \chi$.

Next we aim to satisfy inequality (14). Suppose that there are two continuous and non-decreasing functions $\flat$, $\wp$: $[0, \infty) \to [0, \infty)$ achieving the properties: $\flat(t), \wp(t) > 0$ for $t > 0$ and $\flat(0) = \wp(0) = 0$. Now, by putting $\flat(\rho) = \rho/r$ and $\wp(\rho) = \frac{\rho(1-r)}{r}$, then by the boundedness of Q and (16), we conclude that

$$\begin{aligned}
&\flat(\mathfrak{S}Q(\chi_1, \chi_1, \chi_i)) = \frac{\mathfrak{S}Q(\chi_1, \chi_1, \chi_i)}{r} \le \mathfrak{S}(\chi_1, \chi_2, \chi_3)\\
&\le \mathfrak{S}(\chi_1, \chi_1, \chi_i) + \mathfrak{S}(\chi_2, \chi_2, \chi_j) + \mathfrak{S}(\chi_3, \chi_3, \chi_k)\\
&= \flat(\mathfrak{S}(\chi_1, \chi_1, \chi_i)) - \wp(\mathfrak{S}(\chi_1, \chi_1, \chi_i)) + \mathfrak{S}(\chi_2, \chi_2, \chi_j) + \mathfrak{S}(\chi_3, \chi_3, \chi_k)\\
&\le \flat(\mathfrak{S}(\chi_1, \chi_1, \chi_i)) - \wp(\mathfrak{S}(\chi_1, \chi_1, \chi_i))\\
&\quad + \min\{\mathfrak{S}(\chi_2, \chi_2, Q\chi_2), \mathfrak{S}(\chi_2, \chi_2, Q\chi_1), \mathfrak{S}(\chi_1, \chi_1, Q\chi_1), \mathfrak{S}(\chi_1, \chi_1, Q\chi_2)\}.
\end{aligned}$$

Hence, we achieve the inequality (14). Then, in view of Lemma 3.1, we conclude that Q has a unique fixed point lying in B_r, $r < 1$. □

Example 3.3. Consider the following data: $t \in (0, 1]$, $\beta = 0.5$, $\Phi(w) = 0.5w$, and the conformable iterative equation

$$\begin{aligned}
\mathcal{D}^{0.5}\chi(t) &= \nu_1(\beta,t)\chi(t) + \nu_0(\beta,t)\chi'(t)\\
&= \nu_1(0.5,t)\chi(t) + 0.5\nu_0(0.5,t)(\chi(\chi(t)),
\end{aligned} \tag{17}$$

Clearly, $\ell = 0.5$. Then the Eq. (3.3) has a unique solution (see Theorem 3.2) in the unit ball B_r, where $r = [(1-\beta)\mathfrak{T}^\beta + \beta\mathfrak{T}^{1-\beta}\ell] = 0.75$. By using this method, one can generalize all types of differential equations to improve the solutions. In view of homeodynamic concept, the solution lies in the interval $\chi \in (0, 0.75]$.

4. Integral control

The integral control (IC) is based on the idea that a controller Φ adjusts a set of points which is consequent from the steady state assumption, $\mathcal{D}^\beta\chi(t) = 0$. The steady-state condition is not utilized yet when the controllers are oscillating. In other words, to develop the assets of IC throughout periodic oscillations, one can start with the formula of periodicity as follows:

For every cycle, χ is back at precisely the similar amount

$$\chi(t + \mathfrak{T}) = \chi(t), \quad \forall t \in J.$$

The transformation in χ cannot be expected to be nil, as in non-oscillatory systems (steady state situation), but the IC changes in χ over a period tends zero [31]

$$\int_t^{t+\mathfrak{T}} \chi'(\varsigma)d\varsigma = \chi(t+\mathfrak{T}) - \chi(t) = 0.$$

Analytically, the IC can be recognized as chaotic and periodic situations (in view of homeodynamic concept); therefore, the above integral satisfies

$$\int_t^{t+\mathfrak{T}} \chi'(\varsigma)d\varsigma = \chi(t+\mathfrak{T}) - \chi(t) = \epsilon, \quad \epsilon \ge 0,$$

where ϵ is limited value whenever $\mathfrak{T} \to \infty$. This will appear in the average of the controller, which is given as follows:

$$< \chi >_{\mathfrak{T}} := \frac{1}{\mathfrak{T}}\int_t^{t+\mathfrak{T}} \chi'(\varsigma)d\varsigma = \frac{\epsilon}{\mathfrak{T}}, \quad \epsilon \ge 0, \mathfrak{T} \to \infty.$$

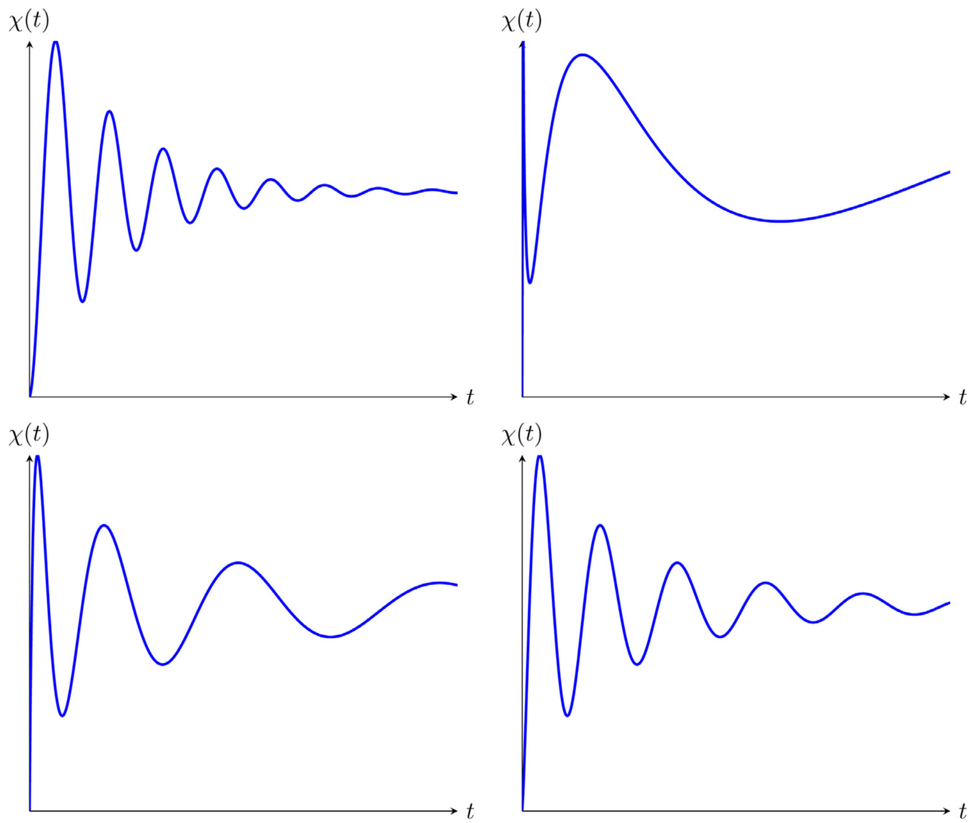

Fig. 4. Oscillatory controller of Eq. (18) for different value of β. From upper left, $\beta \approx 1$, $\beta = 0.2$, lower left $\beta = 0.5$ and $\beta = 0.75$.

By using the conformable derivative, we have

$$\begin{aligned}\int_t^{t+\mathfrak{T}} \mathcal{D}^{\beta}[\chi(\varsigma)]e_0(\varsigma,\varsigma+\mathfrak{T})d_{\beta}\varsigma &= \chi(t+\mathfrak{T}) - \chi(t)e_0(t,t+\mathfrak{T})\\ &= \chi(t+\mathfrak{T}) - \chi(t)\ \exp\left(-\int_t^{t+\mathfrak{T}} \frac{\nu_1(\beta,\varsigma)}{\nu_0(\beta,\varsigma))}d\varsigma\right)\\ &:= \varepsilon(\beta), \quad \varepsilon(\beta)\geq 0,\ \beta\in(0,1),\end{aligned} \tag{18}$$

with the conformable average

$$< \chi >^{\beta}_{\mathfrak{T}} = \frac{1}{\mathfrak{T}}\int_t^{t+\mathfrak{T}} \mathcal{D}^{\beta}[\chi(\varsigma)]e_0(\varsigma,\varsigma+\mathfrak{T})d_{\beta}\varsigma = \frac{\varepsilon(\beta)}{\mathfrak{T}},$$

where, for a system with a chaotic attractive, $\varepsilon(\beta)$ is bounded whenever, $\mathfrak{T}\to\infty$ (in preparation, $\mathfrak{T}$ does not require to attend to ∞; it is sufficient to be large enough). We conclude that a trajectory on a chaotic attractive (after transients) will forever move on the attractive, and thus $\chi(t+\mathfrak{T})$ cannot transfer advance away from $\chi(t)$ than the range of the attractive along the χ-axis in phase space. This shows that integral control delivers robust instruction even when the system performs chaotically. The above technique for calculating the average of a variable is appropriate as it functions well for periodic, stationary, and chaotic performance without much requirement for prior evidence about how the situation acts or the form, shape, and location of the attractive in phase space. For example, suppose that $\chi\in(0,\ 1]$, in terms of sin or cos, we have a periodic solution (in view of (18)) (Fig 4). The above construction of IC implies that unstable fixed points of (13) can become locally stable. But Eq. (13) has a unique fixed point, which is stable locally.

4.1. Optimal controller

We construct the modest control system based on Eq. (13). For a linear formula of $\Phi(\chi(t)) = \chi(t)$, Eq. (13) becomes

$$\begin{aligned}\mathcal{D}^{\beta}\chi(t) &:= \Upsilon(t)\\ &= \nu_1(\beta,t)\chi(t) + \nu_0(\beta,t)\Phi(\chi(t))\\ &= \nu_1(\beta,t)\chi(t) + \nu_0(\beta,t)\chi(t)\\ &= [(1-\beta)t^{\beta} + \beta t^{1-\beta}]\chi(t)\\ &= \frac{t^{-\beta}}{1+t^{-\beta}}\big((1-\beta)t^{2\beta}(1+t^{-\beta}) + (\beta t)(1+t^{-\beta}\big)\chi(t)\\ &:= \left(\frac{t^{-\beta}}{1+t^{-\beta}}\right)\chi_{\beta}(t),\end{aligned} \tag{19}$$

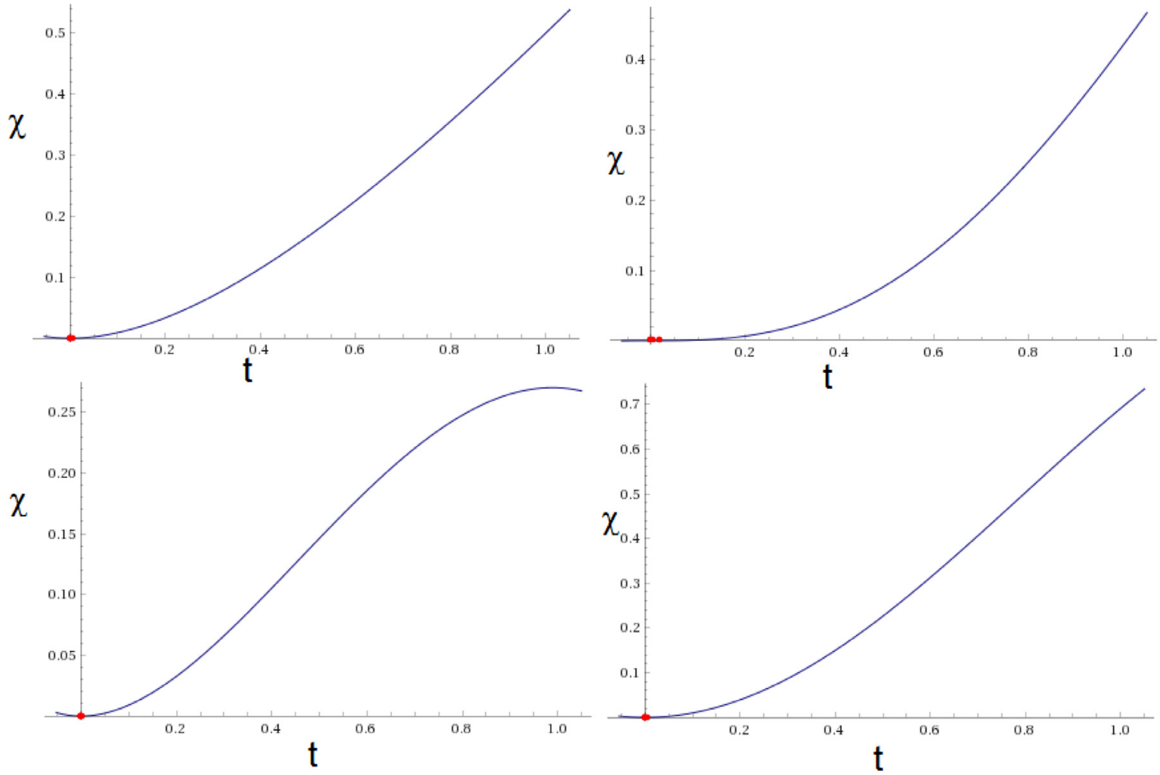

Fig. 5. Minimize points of $\|H\|_\infty$ for different types of $\chi(t)\in(0,1]$. From the upper left $\chi(t)=1, \cos(t), \sin(t), \cos(t)+sin(t)$. The minimization converges to the origin of the unit circle in time (t-axis) $t=0.0015, t=0.003, t=0.003, t=0.02$ respectively to obtain $\chi(t)$.

where Υ is the output of the system and $\chi_\beta(t)$ is the conformable controller of order β. When $\beta\to1$, we have a controller modeled in Wilson et al. [32]. This control models a simple cell, which we aim to minimize it. To complete the minimization, we define the following norm

$$\|H(\Upsilon)\|_\infty = \sup_t \bar{\sigma}(\Upsilon(t)),$$

where $\bar{\sigma}$ represents to the maximum singular value of Υ. The problem of the H controller is to select χ_β that makes the close-loop system internally stable, i.e. minimize the value $\|H\|_\infty$. With the help of Mathematica 11.2, we have minimized the norm for $\chi_\beta\in(0,1]$ (Fig. 5).

4.2. Harmonic and damped oscillation

A harmonic oscillator states the main law of thermodynamics as potential and kinetic energies without damage of entire energy. Yates [33] described this concept by a second order differential equation

$$\mu\frac{d^2\chi}{dt^2}+\kappa\chi(t)=0, \tag{20}$$

where μ is the mass, χ is the position (distance) and κ is a constant between stress and strain. Later, he suggested a damped oscillation equation taking the formula

$$\mu\frac{d^2\chi}{dt^2}+\delta\frac{d\chi}{dt}+\kappa\chi(t)=0, \tag{21}$$

where δ is the damped coefficient. To create a real situation, it is necessary to insert a pulse of energy. The first and second rules are fulfilled, and the periodic wave continues, as long as every cycle catches its individual shot of energy, as follows:

$$\mu\frac{d^2\chi}{dt^2}+\delta\frac{d\chi}{dt}+\kappa\chi(t)=\xi(\theta), \tag{22}$$

where $\xi(\theta)$ is a pulse of energy with a particular phase θ of the oscillation.

By using the definition of conformable operator (2), then the generalized of (22) becomes

$$\mu\mathcal{D}^{2\beta}\chi(t)+\delta\mathcal{D}^{\beta}\chi(t)+\kappa\chi(t)=\xi(\theta), \quad t\in J, \tag{23}$$

where $\mathcal{D}^{2\beta}=\mathcal{D}^{\beta}(\mathcal{D}^{\beta})$. The homogenious type of Eq. (23) has a connected auxiliary equation of taking the formula

$$\mu\Lambda^2+\delta\Lambda+\kappa=0, \tag{24}$$

and the general solution is elicited by one of the following formulas (Theorem 3.1 [25])

$$\begin{aligned} &\chi(t) = \Upsilon_1 e_{\Lambda_1}(t,t_0) + \Upsilon_2 e_{\Lambda_2}(t,t_0), \quad \Lambda_1 \neq \Lambda_2 \\ &\chi(t) = \Upsilon_1 e_{\Lambda_1}(t,t_0) + \Upsilon_2 e_{\Lambda_2}(t,t_0)\int_{t_0}^{t} d_\beta \varsigma, \quad \Lambda_1 = \Lambda_2 = \frac{-\delta}{2\mu} \\ &\chi(t) = \Upsilon_1 e_{\lambda_1}(t,t_0)\cos\left(\int_{t_0}^{t} \lambda_2\, d_\beta\varsigma\right) + \Upsilon_2 e_{\lambda_1}(t,t_0)\sin\left(\int_{t_0}^{t} \lambda_2\, d_\beta\varsigma\right), \quad \Lambda = \lambda_1 \pm i\lambda_2 \end{aligned} \tag{25}$$

where $d_\beta \varsigma = \frac{d\varsigma}{\nu_0(\varsigma)}$ and $e_x(t,t_0) = \exp\left(\int_{t_0}^{t} \frac{x-\nu_1(\beta,\varsigma)}{\nu_0(\beta,\varsigma))} d\varsigma\right)$.

4.3. Linear systems

Example 4.1. Consider the following data:

$$\mathcal{D}^{2\beta}\chi(t) + \chi(t) = 0, \quad t \in J, \tag{26}$$

where $\mu = \kappa = 1$ and $\mathcal{D}^{\beta}\chi(t) \simeq \xi(\theta)$ (this case indicates that each cycle brings its possess injection of energy and marks a cycle average, which is very importunate case in the homeodynamic system of the cell). Thus, the roots of $\Lambda^2 + 1 = 0$ are purely imagery $\Lambda_{1,2} = \pm i$. The general solution of (26) is taking the formula

$$\chi(t) = \Upsilon_1 e_0(t,t_0)\cos\left(\int_{t_0}^{t} d_\beta\varsigma\right) + \Upsilon_2 e_0(t,t_0)\sin\left(\int_{t_0}^{t} d_\beta\varsigma\right), \quad \Lambda = \pm i, \tag{27}$$

where $e_0(t, t_0)$ is given in (4). Now by letting

$$\nu_1(\beta,t) = (1-\beta)\rho_1^{\beta}, \quad \nu_0(\beta,t) = \beta\rho_0^{1-\beta}, \quad \rho_{0,1} \in (0,\infty),$$

we have the following details

$$\begin{aligned} &e_0(t,0) = \exp\left(\frac{-(1-\beta)\rho_1^{\beta}}{\beta\rho_0^{1-\beta}} t\right) \\ &d_\beta\varsigma = \frac{d\varsigma}{\beta\,\rho_0^{1-\beta}} \\ &\int_0^t d_\beta\varsigma = \frac{t}{\beta\,\rho_0^{1-\beta}} \end{aligned}$$

Hence, the exact solution is taking the formula as follows:

$$\begin{aligned} \chi(t) = &\ \Upsilon_1 \exp\Big((1-\beta^{-1})\rho_0^{\beta-1}\rho_1^{\beta t}\Big)\cos\left(\frac{t}{\beta\,\rho_0^{1-\beta}}\right) \\ &+ \Upsilon_2 \exp\Big((1-\beta^{-1})\rho_0^{\beta-1}\rho_1^{\beta t}\Big)\sin\left(\frac{t}{\beta\,\rho_0^{1-\beta}}\right). \end{aligned} \tag{28}$$

Obviously, when $\beta \to 1$, we have the ordinary case [33]. Figs. 6 and 7 show the oscillation behavior of the system (26).

Example 4.2. We have the following system:

$$\mathcal{D}^{2\beta}\chi(t) + 2\mathcal{D}^{\beta}\chi(t) + \chi(t) = 0, \quad t \in J, \tag{29}$$

where $\mu = 1, \delta = 2, \kappa = 1$ and $\mathcal{D}^{\beta}\chi(t) \simeq \xi(\theta)$. We indicate that $\Lambda_1 = \Lambda_2 = -1$. Hence, the general solution becomes (see Fig. 8)

$$\chi(t) = \Upsilon_1 e_{-1}(t,0) + \Upsilon_2 e_{-1}(t,0)\frac{t}{\beta\,\rho_0^{1-\beta}},$$

where $e_{-1}(t,0) = \exp\left(\int_0^t \frac{-1-\nu_1(\beta,\varsigma)}{\nu_0(\beta,\varsigma))} d\varsigma\right) = \exp\left(\frac{1-(1-\beta)\rho_1^{\beta}}{\beta\rho_0^{1-\beta}} t\right)$

Example 4.3. Suppose the following system:

$$\mathcal{D}^{2\beta}\chi(t) + 2\mathcal{D}^{\beta}\chi(t) + \frac{1}{2}\chi(t) = 0, \quad t \in J, \tag{30}$$

where $\mu = 1, \delta = 2, \kappa = 1/2$ and $\mathcal{D}^{\beta}\chi(t) \simeq \xi(\theta)$. We indicate that $\Lambda_1 = -1 - 1/\sqrt{2},\ \Lambda_2 = 1/\sqrt{2} - 1$. Consequently, we have the general solution (see Fig. 9)

$$\chi(t) = \Upsilon_1 e_{\Lambda_1}(t,0) + \Upsilon_2 e_{\Lambda_2}(t,0).$$

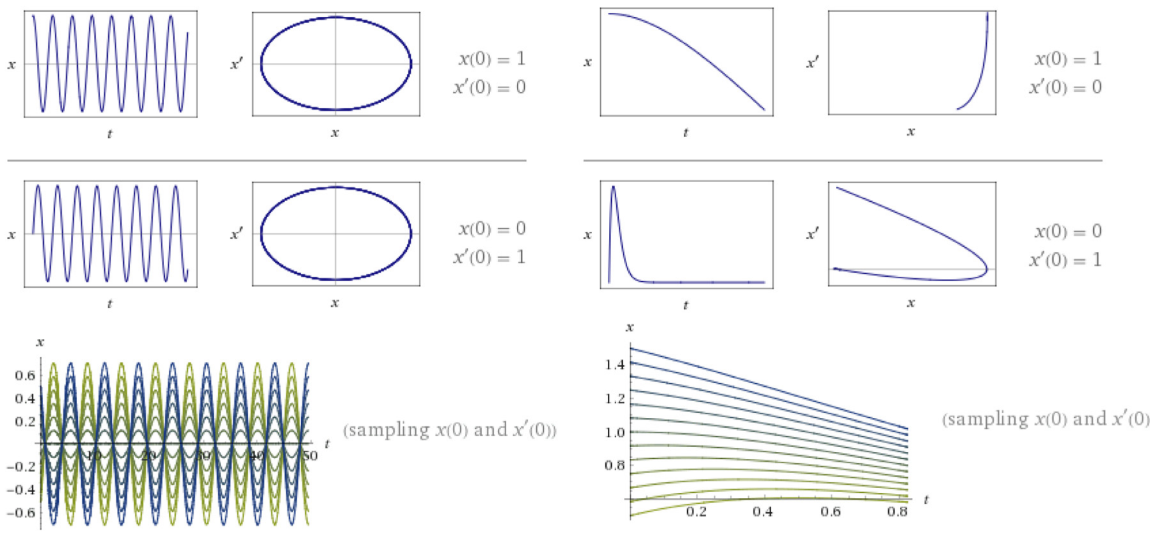

Fig. 6. Solutions of (26), when $\beta = 0.989 \to 1$ (left column) and for $\beta = 0.5$, $\rho_0 = \rho_1 = 1$ (right column).

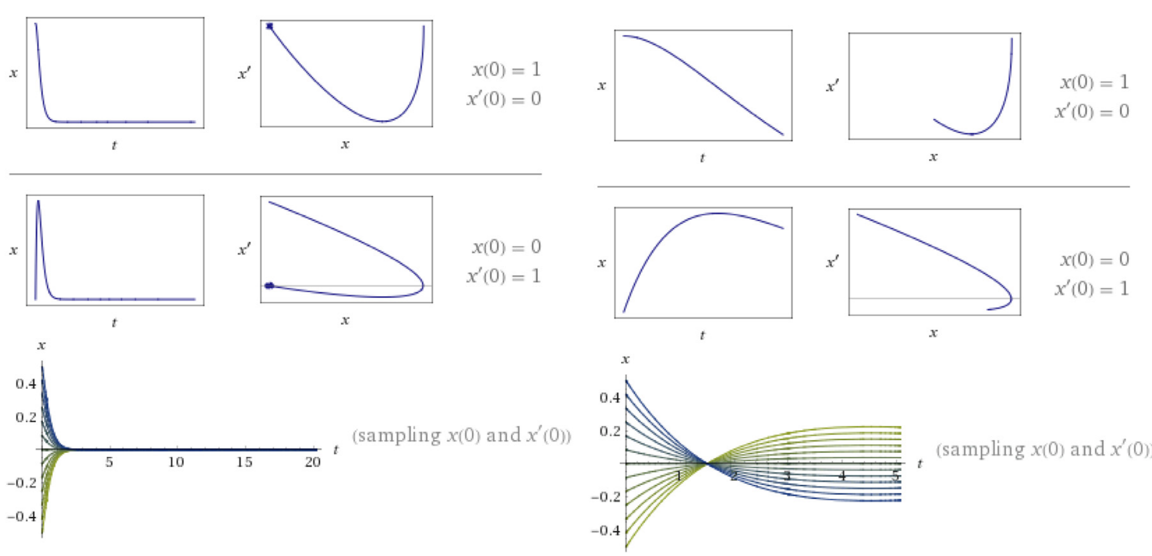

Fig. 7. Solutions of (26), when $\beta = 0.25$ (left column) and for $\beta = 0.75$, $\rho_0 = \rho_1 = 1$ (right column).

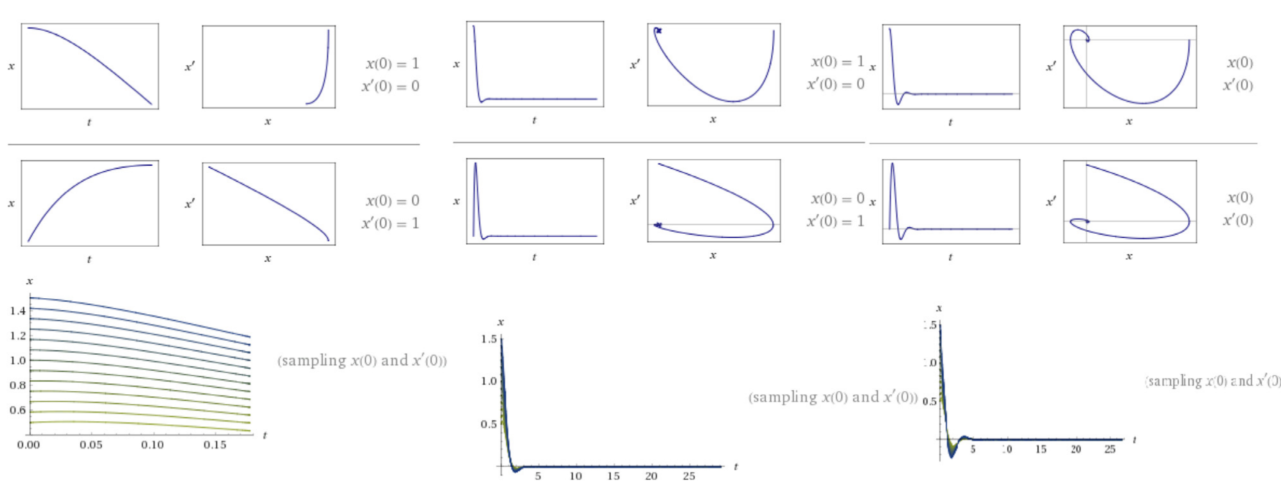

Fig. 8. Solutions of (29), when $\beta = 0.25$, $\beta = 0.75$, $\beta = 0.99$, $\rho_0 = \rho_1 = 1$ respectively.

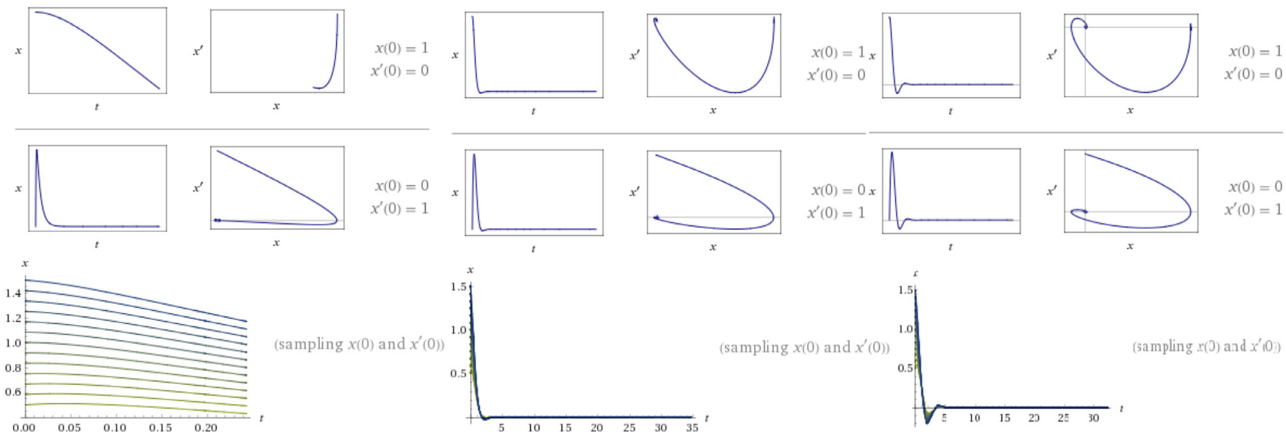

Fig. 9. Solutions of (30), when $\beta = 0.25$, $\beta = 0.75$, $\beta = 0.99$, $\rho_0 = \rho_1 = 1$ respectively.

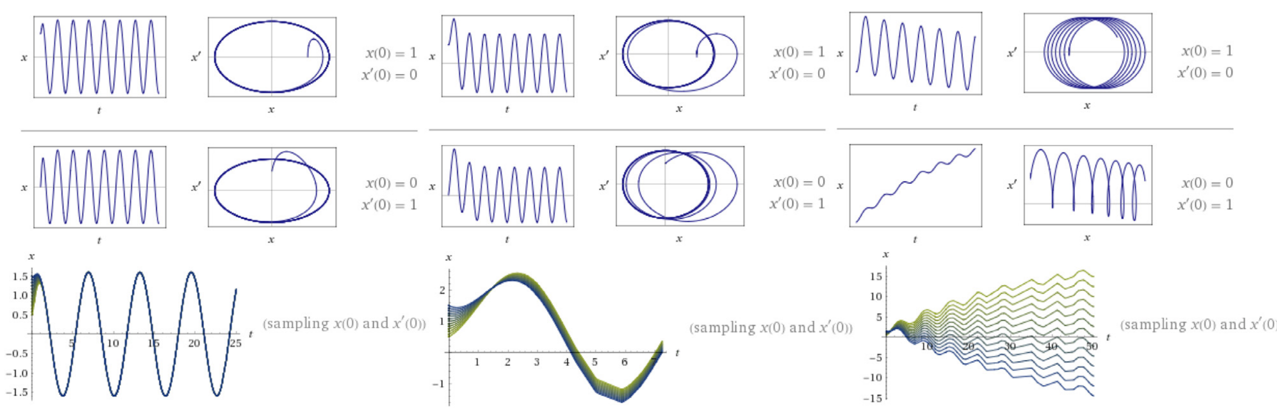

Fig. 10. Solutions of (31), when $\beta = 0.25$, $\beta = 0.75$, $\beta = 0.99$, $\rho_0 = \rho_1 = 1$ respectively.

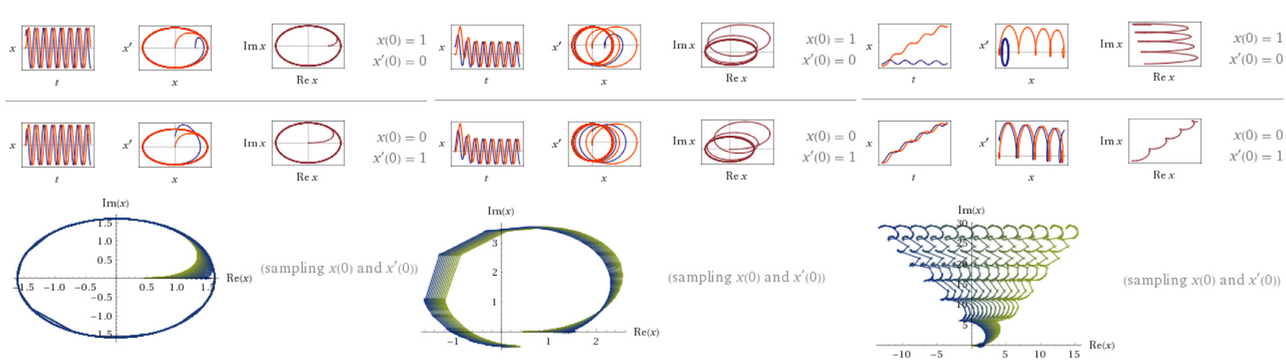

Fig. 11. Solutions of (32), when $\beta = 0.25$, $\beta = 0.75$, $\beta = 0.99$, $\rho_0 = \rho_1 = 1$ respectively.

4.4. Non-linear systems

Consider the following nonlinear system

$$\mathcal{D}^{2\beta}\chi(t) = cos(t), \quad t \in J. \tag{31}$$

The solutions can be found in Fig. 10 and formulated by

$$\beta = 0.25 \Rightarrow \chi(t) = c_1 e^{(-3t)} + c_2 e^{(-3t)} t - 2.22 \times 10^{-16} t\sin(t) + 0.96\sin(t) + (-8.88 \times 10^{-16} + 2.22 \times 10^{-16i}) t\cos(t) + 1.28\cos(t)$$
$$\beta = 0.75 \Rightarrow \chi(t) = c_1 e^{(-0.33t)} + c_2 e^{(-0.33t)} t + 0.96\sin(t) - 1.11 \times 10^{-16} t\cos(t) - 1.28\cos(t)$$
$$\beta = 0.99 \Rightarrow \chi(t) = c_1 e^{(-0.01t)} + c_2 e^{(-0.01t)} t + 2.22 \times 10^{-16} t\sin(t) + 0.02\sin(t) - 1.02\cos(t).$$

Assume the following nonlinear system

$$\mathcal{D}^{2\beta}\chi(t) = \exp(it), \quad t \in J. \tag{32}$$

The solutions can be seen in Fig. 11 and recognized by

$\beta = 0.25 \Rightarrow \chi(t) = (1.28 - 0.96i)e^t + c_1 e^{(-3t)} + c_2 e^{(-3t)} t$

$\beta = 0.75 \Rightarrow \chi(t) = -(1.28 + 0.96i)e^t + c_1 e^{(-0.33t)} + c_2 e^{(-0.33t)} t$

$\beta = 0.99 \Rightarrow \chi(t) = -(1.02 + 0.02i)e^t + c_1 e^(- 0.01t) + c_2 e^{(-0.01t)} t.$

Note that all the computations are brought by Mathematica 11.2

4.5. Comparison

There are different forms of conformable calculus depending on the purposes of the study (see [34–37]). Recently, Baleanu et al. [34] introduced a hybrid integral and differential operators based on the definition of the well known Caputo fractional derivative and integral as follows:

$$\begin{aligned} {}^{C}\mathcal{D}^{\beta}\chi(t) &= \frac{1}{\Gamma(1-\beta)}\int_0^t \Big(\nu_1(\beta,\tau)\chi(\tau) + \nu_0(\beta,\tau)\chi'(\tau)\Big)(t-\tau)^{-\beta} d\tau \\ &= \left(\frac{t^{-\beta}}{\Gamma(1-\beta)}\right) * \Big(\nu_1(\beta,t)\chi(t) + \nu_0(\beta,t)\chi'(t)\Big), \end{aligned} \tag{33}$$

where

$$\lim_{\beta\to 0} {}^{C}\mathcal{D}^{\beta}\chi(t) = \int_0^t \chi(\tau)d\tau, \quad \lim_{\beta\to 1} {}^{C}\mathcal{D}^{\beta}\chi(t) = \chi'(t).$$

Now by using (33), we have the generalized hybrid conformable equation of the form (13)

$$\begin{aligned} {}^{C}\mathcal{D}^{\beta}\chi(t) &= \frac{1}{\Gamma(1-\beta)}\int_0^t (\nu_1(\beta,\tau)\chi(\tau) + \nu_0(\beta,\tau)\Phi(\tau,\chi))(t-\tau)^{-\beta} d\tau \\ &= \left(\frac{1}{t^{\beta}\Gamma(1-\beta)}\right) * (\nu_1(\beta,t)\chi(t) + \nu_0(\beta,t)\Phi(t,\chi)), \end{aligned} \tag{34}$$

which is corresponding to the hybrid conformable integral

$$ {}^{C}\mathcal{I}^{\beta}\chi(t) = \int_0^t e_0(t,\tau)\frac{{}^{RL}\mathcal{D}^{1-\beta}\chi(\tau)}{\nu_0(\beta,\tau)}d\tau,$$

where the operator ${}^{RL}\mathcal{D}^{1-\beta}$ indicated the Riemann-Liouville differ-integrals operator. Moreover, by Proposition 2 in Baleanu et al. [34], the corresponding integral satisfies the relation

$$ {}^{C}\mathcal{I}^{\beta}\, {}^{C}\mathcal{D}^{\beta}\chi(t) = \chi(t) - e_0(t,\tau)\chi(0). \tag{35}$$

Note that the initial solution of (34) is $\chi(0) = 0$. Define an operator O: $X \to X$ by the following construction

$$(O\chi)(t) := \int_0^t \left(\frac{1}{\tau^{\beta}\Gamma(1-\beta)}\right) * (\nu_1(\beta,\tau)\chi(\tau) + \nu_0(\beta,\tau)\Phi(\tau,\chi))d\tau.$$

We have the following existence result:

Theorem 4.4. *Consider the hybrid conformable Eq. (34). If*

$$|\Phi(t,\chi(t)) - \Phi(t,\eta(t))| < \ell|\chi(t) - \eta(t)|$$

for some positive constant $\ell < \frac{\Gamma(1-\beta)-(1-\beta)\mathfrak{T}^{\beta}}{\beta\mathfrak{T}^{1-\beta}}$, $\mathfrak{T} \in (0,\infty)$. *Then O has a unique fixed point in X.*

Proof. As in Theorem 3.2, we consider ν_0 and ν_1 to be as follows:

$$\nu_1(\beta,t) = (1-\beta)t^{\beta}, \quad \nu_0(\beta,t) = \beta t^{1-\beta}, \quad t \in (0,\mathfrak{T}), \mathfrak{T} < \infty.$$

$$\begin{aligned} &\mathfrak{S}(O\chi_1(t), O\chi_2(t), O\chi_3(t)) = \max\{|O\chi_\imath(t) - O\chi_\jmath(t)| : \imath,\jmath = 1,2,3, \imath \neq \jmath\} \\ &\le \max\{|\nu_1(\beta,t)\chi_\imath(t) + \nu_0(\beta,t)\Phi(\chi_\imath(\chi_\imath(t)) \\ &\quad -\big(\nu_1(\beta,t)\chi_\jmath(t) + \nu_0(\beta,t)\Phi(\chi_\jmath(\chi_\jmath(t))\big)|\frac{\mathfrak{T}}{\beta^2\Gamma(1-\beta)} : \imath,\jmath = 1,2,3, \imath \neq \jmath\} \\ &\le \max\{\Big(\nu_1(\beta,t)|\chi_\imath - \chi_\jmath| + \nu_0(\beta,t)\ell|\chi_\imath - \chi_\jmath|\Big)\frac{\mathfrak{T}}{\beta^2\Gamma(1-\beta)} : \imath,\jmath = 1,2,3, \imath \neq \jmath\} \\ &\le \max\{\Big((1-\beta)\mathfrak{T}^{\beta}|\chi_\imath - \chi_\jmath| + \beta\mathfrak{T}^{1-\beta}\ell|\chi_\imath - \chi_\jmath|\Big)\frac{\mathfrak{T}}{\beta^2\Gamma(1-\beta)} : \imath,\jmath = 1,2,3, \imath \neq \jmath\} \\ &= \max\{[(1-\beta)\mathfrak{T}^{\beta} + \beta\mathfrak{T}^{1-\beta}\ell]\left(\frac{\mathfrak{T}}{\beta^2\Gamma(1-\beta)}\right)|\chi_\imath - \chi_\jmath| : \imath,\jmath = 1,2,3, \imath \neq \jmath\} \\ &:= r\mathfrak{S}(\chi_1,\chi_2,\chi_3). \end{aligned}$$

Since $[(1-\beta)\mathfrak{T}^{\beta} + \beta\mathfrak{T}^{1-\beta}\ell]\Big(\frac{\mathfrak{T}}{\beta^2\Gamma(1-\beta)}\Big) < 1 \Rightarrow \frac{[(1-\beta)\mathfrak{T}^{\beta} + \beta\mathfrak{T}^{1-\beta}\ell]}{\Gamma(1-\beta)} < \frac{\beta^2}{\mathfrak{T}} \le 1$. Hence, O is bounded in the unit ball B_r of radius $0 < r < 1$. In the similar manner of Theorem 3.2, we conclude that the operator O has a unique fixed point according to Lemma 3.1. □

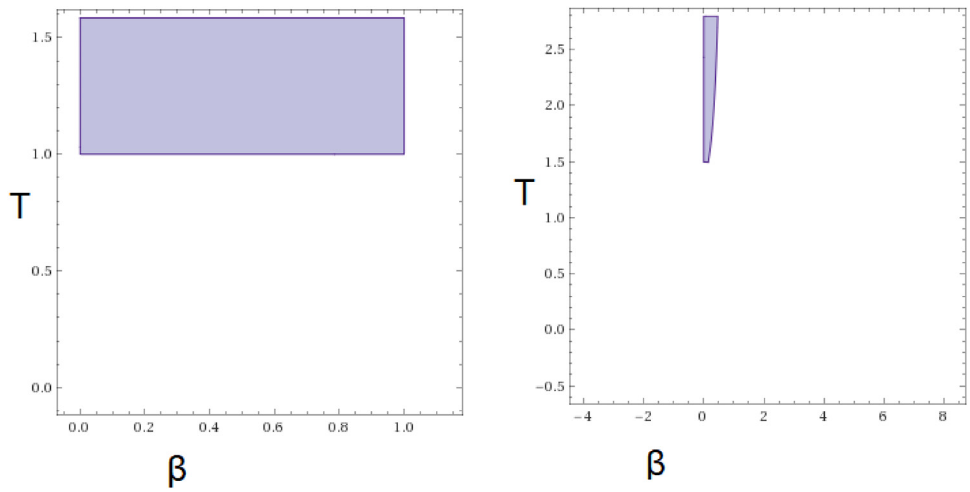

Fig. 12. The upper bound of the inequalities $\ell < \frac{1-(1-\beta)\mathfrak{T}^{\beta}}{\beta\mathfrak{T}^{1-\beta}}$ and $\ell < \frac{\Gamma(1-\beta)-(1-\beta)\mathfrak{T}^{\beta}}{\beta\mathfrak{T}^{1-\beta}}$ for the conformable and the hybrid conformable calculus respectively.

Remark 4.4.1. As a comparison, the assumptions of Theorems 3.2 and 4.4 are different in terms of the upper bound of ℓ. Fig. 12 shows the relation between β and $\mathfrak{T}$ for each inequality. We conclude that the solution exists when $\mathfrak{T} \in [1, \infty)$ for Eq. (13) and $\mathfrak{T} \in [1.5, \infty)$ for (34).

Remark 4.4.2.

- Dynamic Systems and Control are the main technical area within the biological studies. The Dynamic Systems and Controls area efforts on approaches for scheming and controlling natural systems such as ABS. Basically application parts contain novel schemes, bio-cell in the cellular dynamics and control of power. By including, integral control delivers a different (controllable) equilibrium point; the stability and reach-ability of this point are investigated by the uniqueness of the fixed point. Our consequences demonstrated that that integral control in the suggested ABS saves the normal state of the controlled variable for both linear and nonlinear systems.
- The paths are not diverging, but they are bound through performance (for the ABS, the attractiveness of the solution is indicating an asymptotically stable equilibrium point). The conditions we have used in Section 4 can be realized as an addition, auxiliary, of the steady state situation. The process in Section 4 can be utilized in systems that display periodic oscillations

5. Conclusion

Artificial biological system (ABS) is formulated as a conformable dynamical system using two recent types of conformable calculus. A comparison is illustrated between the two suggested dynamic systems. The important study in this investigation is to provide a stable solution. We introduced the sufficient conditions for the unique solution and under the optimal integral controller, we proved its stability. The main tool in this investigation is the iterative fixed point theorem of self-mapping (this because, the ABS is self training). Moreover, the harmony and oscillatory behaviors are presented based on conformable second order differential equation which is the generalized Yates equation. Through the results, we confirm that the new generalizations of ABS indicated more flexibility and convergence and accuracy. For the future works, one can apply different types of fractional calculus, conformable calculus and deformed calculus to generalize ABS.

Declaration of Competing Interest

The authors declare no conflict of interest.

CRediT authorship contribution statement

Rabha W. Ibrahim: Formal analysis, Methodology. **Dania Altulea:** Conceptualization, Writing - review & editing.

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