Robust Leaderless Consensus of Euler-Lagrange Systems with Interconnection Delays

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Abstract— **In this work, a distributed control method to achieve the leaderless consensus of heterogeneous Euler-Lagrange (EL) systems with bounded time-varying communication delays while simultaneously rejecting periodic external disturbances is reported. The robust controller has a simple-to-implement structure of proportional-integral-derivative scheme that employs the internal model approach to reject the disturbance. We consider that the network of EL-systems is interconnected through an undirected weighted graph that is static and we assume that the information exchange between any connected nodes is subjected to bounded variable time-delays. The efficacy of the proposed method is shown in a numerical simulation using a network of ten robotic manipulators.**

Index Terms— **Disturbance Rejection, Euler-Lagrange Systems, Consensus, Time-Delays, Internal Model**

I. Introduction

Consensus for multiple systems means that the state variables of all agents converge to a common agreement value [1]. Several solutions to this problem and for linear agents are well-studied under many different scenarios [2], [3]. However, these solutions become more complex if one considers the agents' nonlinear dynamics [4], [5], communication delays [6] and input disturbances [7], [8], [9]. There are two possible consensus problems: the *leaderless*, where all agents are required to agree at a common non–specified value, and the *leader–follower*, where the agents have to converge to a common pre-specified point [10].

A variety of physical (mechanical and electrical) systems can be represented by nonlinear system equations through the well-known Euler–Lagrange (EL) equations of motion [11]. The first results on consensus (synchronization) for agents with EL-dynamics have been reported in [12] and in [6] for nonidentical EL-systems with interconnecting delays. Since then, a plethora of different controllers have been proposed to ensure consensus, from simple Proportional plus damping (P+d) schemes [4], [13] to more elaborate adaptive [14], [6], [15] and sliding-mode controllers [16].

In this work we focus on ensuring the leaderless consensus of EL-systems while rejecting non-vanishing time-varying external disturbances. Closely related to this work are the solutions provided in [7] and in [8], where a robust controller is proposed, via the internal model approach, for the consensus of EL-systems but without accounting for delays in the interconnections. However, in real-life scenarios, time-delays naturally appear when communicating the state variables among agents [17]. Moreover, recently, the sliding-mode technique has also been employed to deal with parameter uncertainty and with external perturbations [18], [19], [20]. However, these solutions can be prone to chattering [21], [22].

The controller that we propose here is smooth and it is composed of two terms, a Proportional plus damping (P+d) scheme, which drives the agents towards a consensus position, and an extra integral term, which is used to reject the external disturbances. The integral term design hinges upon the internal model approach [23], [24] and the resulting scheme has the structure of a simple-to-implement PID controller. We assume that the EL-systems are interconnected through an undirected weighted graph that is static and that the information exchange between any connected nodes is subjected to bounded variable time-delays. For this scenario we provide a sufficient condition for the proportional and the derivative gains of the controller to ensure that the network finds an agreement position and that the velocities converge to zero, globally. Up to the authors' knowledge, this work provides the first solution to the leaderless consensus problem for EL-agents when variable time-delays appear in the interconnections and with disturbance rejection capabilities with a uniformly continuously differentiable controller, which does not exhibit a chattering phenomenon. Therefore, this work extends the previous P+d scheme reported in [13] to the case when external disturbances arise. Simulations, with ten robot manipulators are shown to provide evidence of the performance of the controller.

II. Background

This paper deals with the leaderless consensus of a network of n degrees-of-freedom EL-systems. The network has N heterogeneous EL-systems and each ith-system (also referred to as *agent*) has the following dynamics

$$\mathbf{M}_i(\mathbf{q}_i)\ddot{\mathbf{q}}_i + \mathbf{C}_i(\mathbf{q}_i,\dot{\mathbf{q}}_i)\dot{\mathbf{q}}_i + \frac{\partial}{\partial \mathbf{q}_i}U_i(\mathbf{q}_i) = \boldsymbol{\tau}_i + \mathbf{d}_i, \quad (1)$$

where $\mathbf{q}_i \in \mathbb{R}^n$ is the generalized position coordinates, the matrix $\mathbf{M}_i : \mathbb{R}^n \to \mathbb{R}^{n\times n}$ is the generalized inertia matrix, $U_i : \mathbb{R}^n \to \mathbb{R}$ is the potential energy function, $\boldsymbol{\tau}_i \in \mathbb{R}^n$ is the control input and $\mathbf{d}_i \in \mathbb{R}^n$ is an external disturbance generated by an exosystem (defined later below). The matrix $\mathbf{C}_i(\mathbf{q}_i,\dot{\mathbf{q}}_i) := \dot{\mathbf{M}}_i(\mathbf{q}_i)\dot{\mathbf{q}}_i - \frac{1}{2}\frac{\partial}{\partial \mathbf{q}_i}\dot{\mathbf{q}}_i^\top \mathbf{M}_i(\mathbf{q}_i)\dot{\mathbf{q}}_i$ is defined via

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the Christoffel symbols of the first kind. The dynamics of each agent satisfies the following properties [25], [26].

P1. The inertia matrix is symmetric and positive definite, for all $\mathbf{q}_i \in \mathbb{R}^n$.
P2. The matrix $\dot{\mathbf{M}}_i(\mathbf{q}_i) - 2\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)$ is skew-symmetric.
P3. The Coriolis matrix is bounded, for all $\mathbf{q}_i \in \mathbb{R}^n$, as $|\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)\dot{\mathbf{q}}_i| \leq k_{ci}|\dot{\mathbf{q}}_i|^2$ for $k_{ci} > 0$.

We further assume that the external disturbance satisfies the following,

A1. The disturbance $\mathbf{d}_i(t) \in \mathbb{R}^n$ is generated by an exosystem of the form

$$\dot{\mathbf{w}}_i = \mathbf{S}_i \mathbf{w}_i, \qquad \mathbf{d}_i = \mathbf{C}_i \mathbf{w}_i, \tag{2}$$

where $\mathbf{w}_i \in \mathbb{R}^{p_i}$, $\mathbf{S}_i \in \mathbb{R}^{p_i \times p_i}$ and $\mathbf{C}_i \in \mathbb{R}^{n \times p_i}$. The exosystem is assumed to be neutrally stable, i.e., all the eigenvalues of $\mathbf{S}_i$ are different and they lie on the imaginary axis. $\triangleleft$

The interconnection of the EL-agents is modeled through a graph given by the standard Laplacian matrix $\mathbf{L} := [\ell_{ij}] \in \mathbb{R}^{N \times N}$, whose elements are defined as

$$\ell_{ii} = \sum_{j \in \mathcal{N}_i} a_{ij}, \quad \ell_{ij} = -a_{ij}, \tag{3}$$

where $a_{ij} > 0$ if $j \in \mathcal{N}_i$ and $a_{ij} = 0$, otherwise. The set $\mathcal{N}_i$ contains all the agents transmitting information to the i-th agent. Regarding the agents' interconnection, we assume further that

A2. The graph is *undirected, static* and *connected.* $\triangleleft$
A3. The information exchange, from the j-th agent to the i-th agent, is subjected to a variable time-delay $T_{ji}(t)$ with a known upper bound ${}^*T_{ji}$. Hence $0 \leq T_{ji}(t) \leq {}^*T_{ji} < \infty$. Moreover, the time-delays are differentiable and their derivatives are bounded. $\triangleleft$

Remark 1: By construction, $\mathbf{L}$ has zero row sum, i.e., $\mathbf{L}\mathbf{1}_N = \mathbf{0}$. Moreover, Assumption A2 ensures that $\text{rank}(\mathbf{L}) = N - 1$, that $\mathbf{L}$ has a single zero-eigenvalue and that the rest of the spectrum has positive real parts [2]. Further, it also holds that $\mathbf{L} = \mathbf{L}^\top$. $\triangleleft$

The problem that we aim to solve in this work is

LC (**Leaderless Consensus** problem): For a network of N heterogeneous n-DOF EL-systems (1) satisfying A1–A3, design a distributed controller such that all positions, globally and asymptotically converges to a consensus point, i.e.,

$$\lim_{t \to \infty} \mathbf{q}_i(t) = \mathbf{q}_c; \qquad \lim_{t \to \infty} \dot{\mathbf{q}}_i(t) = \mathbf{0},$$

for some $\mathbf{q}_c \in \mathbb{R}^n$.

III. Robust Leaderless Consensus of EL-Agents

The robust controller that we propose here is composed of an integral term, which is designed using the internal model principle, and a P+d scheme. This is precisely the object study of this section.

A. Internal Model Dynamics

In order to counteract the input disturbance effects, an internal model-based estimation term is employed in the controller. This term employs the following realization

$$\dot{\mathbf{x}}_i = \mathbf{A}_i \mathbf{x}_i - \mathbf{B}_i \mathbf{u}_i, \tag{4}$$

where, matrices $\mathbf{A}_i \in \mathbb{R}^{l_i \times l_i}$ and $\mathbf{B}_i \in \mathbb{R}^{l_i \times n}$ are such that $\mathbf{A}_i + \mathbf{A}_i^\top = \mathbf{0}$ and the pair $(\mathbf{A}_i, \mathbf{B}_i)$ is observable. We should underscore that the eigenvalues of matrix $\mathbf{A}_i$ are the same as those of $\mathbf{S}_i$. In this case, as in [7], there exists a transformation matrix $\mathbf{T}_i \in \mathbb{R}^{l_i \times p_i}$, such that

$$\mathbf{T}_i \mathbf{S}_i = \mathbf{A}_i \mathbf{T}_i, \quad \mathbf{B}_i^\top \mathbf{T}_i + \mathbf{C}_i = \mathbf{0}. \tag{5}$$

$\mathbf{u}_i \in \mathbb{R}^n$ is the input to the internal model dynamics and it will be defined latter.

Defining the estimation error coordinates

$$\tilde{\mathbf{x}}_i := \mathbf{x}_i - \mathbf{T}_i \mathbf{w}_i,$$

yields the error dynamics

$$\begin{aligned} \dot{\tilde{\mathbf{x}}}_i &= \mathbf{A}_i \mathbf{x}_i - \mathbf{B}_i \mathbf{u}_i - \mathbf{T}_i \mathbf{S}_i \mathbf{w}_i, \\ &= \mathbf{A}_i(\mathbf{x}_i - \mathbf{T}_i \mathbf{w}_i) - \mathbf{B}_i \mathbf{u}_i \\ &= \mathbf{A}_i \tilde{\mathbf{x}}_i - \mathbf{B}_i \mathbf{u}_i, \end{aligned} \tag{6}$$

where, to obtain the first equality, we have employed (2) and (4), respectively. For the second equation, we use (5).

Defining, additionally,

$$\tilde{\mathbf{d}}_i := \mathbf{B}_i^\top \mathbf{x}_i + \mathbf{d}_i,$$

and using (2) and (5), we have that

$$\begin{aligned} \tilde{\mathbf{d}}_i &= \mathbf{B}_i^\top \mathbf{x}_i + \mathbf{C}_i \mathbf{w}_i = \mathbf{B}_i^\top (\mathbf{x}_i - \mathbf{T}_i \mathbf{w}_i) \\ &= \mathbf{B}_i^\top \tilde{\mathbf{x}}_i. \end{aligned} \tag{7}$$

Consider now the Lyapunov candidate function

$$H_i(\tilde{\mathbf{x}}_i) = \frac{1}{2}|\tilde{\mathbf{x}}_i|^2,$$

then $\dot{H}_i$ evaluated along (6) yields

$$\dot{H}_i = \tilde{\mathbf{x}}_i^\top (\mathbf{A}_i \tilde{\mathbf{x}}_i - \mathbf{B}_i \mathbf{u}_i) = -\tilde{\mathbf{x}}_i^\top \mathbf{B}_i \mathbf{u}_i,$$

where we have used the fact that $\mathbf{A}_i = -\mathbf{A}_i^\top$. Hence, this means that (6) is passive from the input $\mathbf{B}_i \mathbf{u}_i$ to the output $\tilde{\mathbf{x}}_i$ and this property is employed in the controller design.

B. Robust P+d Scheme

Our proposed robust P+d with gravity cancellation controller is given by

$$\boldsymbol{\tau}_i = \mathbf{B}_i^\top \mathbf{x}_i - p_i \sum_{j \in \mathcal{N}_i} a_{ij} \left(\mathbf{q}_i - \mathbf{q}_j(t - T_{ji}(t))\right) - d_i \dot{\mathbf{q}}_i + \frac{\partial}{\partial \mathbf{q}_i} U_i, \tag{8}$$

where $p_i > 0$ and $d_i > 0$ are the proportional and the damping injection gains, respectively. These controllers are the robust extension of the scheme reported in [13].

The closed-loop system of (1) with (8) yields

$$\begin{aligned}\ddot{\mathbf{q}}_i =& -\mathbf{M}_i^{-1}(\mathbf{q}_i)\left[\mathbf{C}_i(\mathbf{q}_i,\dot{\mathbf{q}}_i)\dot{\mathbf{q}}_i + d_i\dot{\mathbf{q}}_i - \mathbf{B}_i^\top \mathbf{x}_i - \mathbf{d}_i\right]\\ &- p_i\mathbf{M}_i^{-1}(\mathbf{q}_i)\sum_{j\in\mathcal{N}_i} a_{ij}\left(\mathbf{q}_i - \mathbf{q}_j(t-T_{ji}(t))\right),\end{aligned}$$

and hence, using (7) it holds that

$$\begin{aligned}\ddot{\mathbf{q}}_i =& -\mathbf{M}_i^{-1}(\mathbf{q}_i)\left[\mathbf{C}_i(\mathbf{q}_i,\dot{\mathbf{q}}_i)\dot{\mathbf{q}}_i + d_i\dot{\mathbf{q}}_i - \mathbf{B}_i^\top \tilde{\mathbf{x}}_i\right]\\ &- p_i\mathbf{M}_i^{-1}(\mathbf{q}_i)\sum_{j\in\mathcal{N}_i} a_{ij}\left(\mathbf{q}_i - \mathbf{q}_j(t-T_{ji}(t))\right).\end{aligned} \tag{9}$$

Note that, if we take the kinetic energy of each agent to be

$$K_i(\dot{\mathbf{q}}_i,\mathbf{q}_i) = \frac{1}{2}\dot{\mathbf{q}}_i^\top \mathbf{M}_i(\mathbf{q}_i)\dot{\mathbf{q}}_i,$$

then, simple calculations show that

$$\begin{aligned}\dot{K}_i =& -d_i|\dot{\mathbf{q}}_i|^2 - p_i\dot{\mathbf{q}}_i^\top \sum_{j\in\mathcal{N}_i} a_{ij}\left(\mathbf{q}_i - \mathbf{q}_j(t-T_{ji}(t))\right)\\ &+ \dot{\mathbf{q}}_i^\top \mathbf{B}_i^\top \tilde{\mathbf{x}}_i.\end{aligned}$$

Therefore, we can see that

$$\dot{H}_i + \dot{K}_i = -d_i|\dot{\mathbf{q}}_i|^2 - p_i\dot{\mathbf{q}}_i^\top \sum_{j\in\mathcal{N}_i} a_{ij}\left(\mathbf{q}_i - \mathbf{q}_j(t-T_{ji}(t))\right),$$

provided that the input of the internal model scheme $\mathbf{u}_i$ is given by $\dot{\mathbf{q}}_i$.

We can now state our main result as follows.

Proposition 1: For any given proportional gains p_i and any time-delay bounds ${}^*T_{ji}$, $i = 1,\ldots,N$, if the damping injection gains d_i satisfies

$$d_i > \frac{1}{2}p_i\sum_{j\in\mathcal{N}_i} a_{ij}\left(\alpha_i + \frac{{}^*T_{ij}^2}{\alpha_j}\right), \tag{10}$$

for all $i = 1,\ldots,N$ and for some $\alpha_i > 0$ then the controller (4), with $\mathbf{u}_i = \dot{\mathbf{q}}_i$, and (8) solves the **LC** problem globally. $\square$

Proof: For all i, we consider a Lyapunov candidate function V_i given by

$$V_i = \frac{1}{p_i}\left(K_i + H_i\right) + \frac{1}{4}\sum_{j\in\mathcal{N}_i} a_{ij}|\mathbf{q}_i - \mathbf{q}_j|^2.$$

By evaluating $\dot{V}_i$ along the trajectories of closed-loop system (6) and (9), we obtain

$$\begin{aligned}\dot{V}_i =& -\frac{d_i}{p_i}|\dot{\mathbf{q}}_i|^2 - \sum_{j\in\mathcal{N}_i} a_{ij}\dot{\mathbf{q}}_i^\top\left(\mathbf{q}_i - \mathbf{q}_j(t-T_{ji}(t))\right)\\ &+ \frac{1}{2}\sum_{j\in\mathcal{N}_i} a_{ij}(\dot{\mathbf{q}}_i - \dot{\mathbf{q}}_j)^\top(\mathbf{q}_i - \mathbf{q}_j).\end{aligned}$$

Since the Laplacian matrix is symmetric [4], it holds that

$$\frac{1}{2}\sum_{i=1}^N\sum_{j\in\mathcal{N}_i} a_{ij}(\dot{\mathbf{q}}_i - \dot{\mathbf{q}}_j)^\top(\mathbf{q}_i - \mathbf{q}_j) = \sum_{i=1}^N\sum_{j\in\mathcal{N}_i} a_{ij}\dot{\mathbf{q}}_i^\top(\mathbf{q}_i - \mathbf{q}_j).$$

Hence, by defining $\mathcal{V} := \sum_{i=1}^N V_i$, it follows that

$$\begin{aligned}\dot{\mathcal{V}} =& -\sum_{i=1}^N\left[\frac{d_i}{p_i}|\dot{\mathbf{q}}_i|^2 + \sum_{j\in\mathcal{N}_i} a_{ij}\dot{\mathbf{q}}_i^\top\left(\mathbf{q}_j - \mathbf{q}_j(t-T_{ji}(t))\right)\right],\\ =& -\sum_{i=1}^N\left[\frac{d_i}{p_i}|\dot{\mathbf{q}}_i|^2 + \sum_{j\in\mathcal{N}_i} a_{ij}\dot{\mathbf{q}}_i^\top\int_{t-T_{ji}(t)}^t \dot{\mathbf{q}}_j(\sigma)\mathrm{d}\sigma\right],\end{aligned}$$

where, for the second equality, we have employed the fact that

$$\int_{t-T_{ji}(t)}^t \dot{\mathbf{q}}_j(\sigma)\mathrm{d}\sigma = \mathbf{q}_j - \mathbf{q}_j(t-T_{ji}(t)).$$

Now, using Young's and Cauchy-Schwartz' inequalities we obtain that, for any $\alpha_i > 0$,

$$\begin{aligned}-\dot{\mathbf{q}}_i^\top\int_{t-T_{ji}(t)}^t \dot{\mathbf{q}}_j(\sigma)\mathrm{d}\sigma \leq& \frac{\alpha_i}{2}|\dot{\mathbf{q}}_i|^2 + \frac{1}{2\alpha_i}\left|\int_{t-T_{ji}(t)}^t \dot{\mathbf{q}}_j(\sigma)\mathrm{d}\sigma\right|^2\\ \leq& \frac{\alpha_i}{2}|\dot{\mathbf{q}}_i|^2 + \frac{{}^*T_{ji}}{2\alpha_i}\int_{t-{}^*T_{ji}}^t |\dot{\mathbf{q}}_j(\sigma)|^2\mathrm{d}\sigma.\end{aligned}$$

Using these inequalities we can bound $\dot{\mathcal{V}}$ as

$$\begin{aligned}\dot{\mathcal{V}} \leq& -\sum_{i=1}^N\left[\frac{d_i}{p_i}|\dot{\mathbf{q}}_i|^2 - \sum_{j\in\mathcal{N}_i} a_{ij}\frac{\alpha_i}{2}|\dot{\mathbf{q}}_i|^2\right]\\ &+ \sum_{i=1}^N\sum_{j\in\mathcal{N}_i} a_{ij}\frac{{}^*T_{ji}}{2\alpha_i}\int_{t-{}^*T_{ji}}^t |\dot{\mathbf{q}}_j(\sigma)|^2\mathrm{d}\sigma.\end{aligned}$$

Let us now consider

$$W_i := \sum_{j\in\mathcal{N}_i} a_{ij}\frac{{}^*T_{ji}}{2\alpha_i}\int_{-{}^*T_{ji}}^0\int_{t+\sigma}^t |\dot{\mathbf{q}}_j(\theta)|^2 d\theta\mathrm{d}\sigma, \tag{11}$$

and note that $\dot{W}_i$ is given by

$$\dot{W}_i = \sum_{j\in\mathcal{N}_i} a_{ij}\frac{{}^*T_{ji}}{2\alpha_i}\left[{}^*T_{ji}|\dot{\mathbf{q}}_j|^2 - \int_{t-{}^*T_{ji}}^t |\dot{\mathbf{q}}_j(\sigma)|^2\mathrm{d}\sigma\right].$$

By defining

$$\mathcal{E} := \sum_{i=1}^N\left[V_i(\mathbf{q}_i,\dot{\mathbf{q}}_i) + W_i(\dot{\mathbf{q}}_{jt})\right],$$

we obtain

$$\dot{\mathcal{E}} \leq -\sum_{i=1}^N\left[\left(\frac{d_i}{p_i} - \frac{\alpha_i\ell_{ii}}{2}\right)|\dot{\mathbf{q}}_i|^2 - \sum_{j\in\mathcal{N}_i} a_{ij}\frac{{}^*T_{ji}^2}{2\alpha_i}|\dot{\mathbf{q}}_j|^2\right],$$

where ℓ_{ii} is defined in (3).

Now, as in [13], for compactness of presentation, we introduce $\mathbf{Q} := \left[|\dot{\mathbf{q}}_1|^2 \cdots |\dot{\mathbf{q}}_N|^2\right]^\top$ and

$$\mathbf{\Psi} = \begin{bmatrix} \frac{d_1}{p_1} - \frac{\alpha_1\ell_{11}}{2} & \cdots & -\frac{{}^*T_{N1}^2 a_{1N}}{2\alpha_1}\\ \vdots & \ddots & \vdots\\ -\frac{{}^*T_{1N}^2 a_{N1}}{2\alpha_N} & \cdots & \frac{d_N}{p_N} - \frac{\alpha_N\ell_{NN}}{2}\end{bmatrix},$$

so that we can write

$$\dot{\mathcal{E}} \leq -\mathbf{1}_N^\top\mathbf{\Psi}\mathbf{Q} = -\sum_{i=1}^N \lambda_i|\dot{\mathbf{q}}_i|^2,$$

where

$$\lambda_i = \frac{d_i}{p_i} - \frac{1}{2}\sum_{j\in\mathcal{N}_i} a_{ij}\left(\alpha_i + \frac{{}^*T_{ij}^2}{\alpha_j}\right).$$

Setting d_i such that (10) holds, ensures that λ_i are strictly positive numbers. It implies that

$$\dot{\mathcal{E}} \leq -\sum_{i=1}^{N} \lambda_i |\dot{\mathbf{q}}_i|^2 \leq 0.$$

Since $\mathcal{E}$ is positive definite then $\dot{\mathbf{q}}_i \in \mathcal{L}_2$. Moreover, $\dot{\mathbf{q}}_i, \tilde{\mathbf{x}}_i, |\mathbf{q}_i - \mathbf{q}_j| \in \mathcal{L}_\infty$ for all $i \in \bar{N}$ and $j \in \mathcal{N}_i$.

Now, the error $\mathbf{q}_i - \mathbf{q}_j(t - T_{ji}(t))$ can be written as

$$\mathbf{q}_i - \mathbf{q}_j(t - T_{ji}(t)) = \mathbf{q}_i - \mathbf{q}_j + \int_{t-T_{ji}(t)}^{t} \dot{\mathbf{q}}_j(\sigma)\mathrm{d}\sigma, \quad (12)$$

and it can be proved that $|\mathbf{q}_i - \mathbf{q}_j(t-T_{ji}(t))| \in \mathcal{L}_\infty$ provided that $|\mathbf{q}_i - \mathbf{q}_j| \in \mathcal{L}_\infty$ and that $\dot{\mathbf{q}}_i \in \mathcal{L}_2$.

Since all signals in the right-hand-side of the closed-loop system equations in (9) are bounded, then $\ddot{\mathbf{q}}_i \in \mathcal{L}_\infty$. Hence, Barbalat's lemma allows us to conclude that $\lim\limits_{t\to\infty} \dot{\mathbf{q}}_i(t) = 0$. This, in turn, implies that

$$\lim_{t\to\infty} \int_0^t \ddot{\mathbf{q}}_i(\sigma)\mathrm{d}\sigma = -\dot{\mathbf{q}}_i(0).$$

Furthermore, under Assumption A3, the time-derivative of the right-hand-side of the closed-loop system equations (9) is also bounded. Thus, $\frac{\mathrm{d}}{\mathrm{d}t}\ddot{\mathbf{q}}_i \in \mathcal{L}_\infty$. In these conditions, Barbalat's lemma also ensures that $\lim\limits_{t\to\infty} \ddot{\mathbf{q}}_i(t) = 0$.

Since velocities asymptotically converge to zero, then from (12) and (9), the following limits hold

$$\lim_{t\to\infty} p_i \sum_{j\in\mathcal{N}_i} a_{ij}\left(\mathbf{q}_i(t) - \mathbf{q}_j(t)\right) = \lim_{t\to\infty} \mathbf{B}_i^\top \tilde{\mathbf{x}}_i(t).$$

Moreover, the convergence of velocities and accelerations to zero along with the fact that accelerations are uniformly continuous, ensure that $\lim\limits_{t\to\infty} \frac{\mathrm{d}}{\mathrm{d}t}\ddot{\mathbf{q}}_i(t) = 0$. Hence, after differentiating the accelerations in (9), we can establish that $\lim\limits_{t\to\infty} \mathbf{B}_i^\top \dot{\tilde{\mathbf{x}}}_i(t) = \mathbf{0}$. By substituting this limit to the second equation of (9), it holds that

$$\lim_{t\to\infty} \mathbf{B}_i^\top \dot{\tilde{\mathbf{x}}}_i(t) = \lim_{t\to\infty} \mathbf{B}_i^\top \mathbf{A}_i \tilde{\mathbf{x}}_i(t) = \mathbf{0}. \quad (13)$$

Inductively, it follows that

$$\lim_{t\to\infty} \mathbf{B}_i^\top \mathbf{A}_i^2 \tilde{\mathbf{x}}_i(t) = \cdots = \lim_{t\to\infty} \mathbf{B}_i^\top \mathbf{A}_i^{l_i} \tilde{\mathbf{x}}_i(t) = \mathbf{0}. \quad (14)$$

Hence, by invoking the Cayley-Hamilton theorem, there exists a set of real numbers $\{c_k\}_{k=1}^{l_i}$ such that

$$\mathbf{A}_i^{l_i} + c_1 \mathbf{A}_i^{l_i-1} + \cdots + c_{l_i-1}\mathbf{A}_i + c_{l_i}\mathbf{I} = \mathbf{0}. \quad (15)$$

Using (15), we can establish that

$$\mathbf{B}_i^\top \tilde{\mathbf{x}}_i = -\frac{1}{c_{l_i}} \mathbf{B}_i^\top \left(\mathbf{A}_i^{l_i} + c_1 \mathbf{A}_i^{l_i-1} + \cdots + c_{l_i-1}\mathbf{A}_i\right)\tilde{\mathbf{x}}_i = \mathbf{0},$$

where to obtain the second equality we have employed (13) and (14). Since $\lim\limits_{t\to\infty} \mathbf{B}_i^\top \tilde{\mathbf{x}}_i(t) = \mathbf{0}$, it holds that

$$\lim_{t\to\infty} \sum_{j\in\mathcal{N}_i} a_{ij}\left(\mathbf{q}_i(t) - \mathbf{q}_j(t)\right) = \mathbf{0}. \quad (16)$$

Finally, by concatenating the N positions as $\mathbf{q} := [\mathbf{q}_1^\top, ..., \mathbf{q}_N^\top]^\top$, we can arrive at

$$\lim_{t\to\infty} (\mathbf{L} \otimes \mathbf{I}_n)\mathbf{q}(t) = \mathbf{0},$$

which by the properties of the Laplacian matrix, together with (16), implies that there exists $\mathbf{q}_c \in \mathbb{R}^n$, such that

$$\lim_{t\to\infty} \mathbf{q}(t) = \mathbf{1}_N \otimes \mathbf{q}_c.$$

■

IV. Simulation result

A. Simulation setup

Using a network with ten 2-DOF revolute joint manipulators, this section presents some simulations that illustrate the solution to the control problem reported in this paper. For simplicity, the interconnection variable time-delay for all agents is the same, and it emulates an ordinary UDP/IP Internet delay with a normal Gaussian distribution with mean, variance, and seed equal to 0.45, 0.005, and 0.35, respectively [27]. The Laplacian matrix of the system is given by

$$L = \begin{bmatrix} 1.4 & 0 & -0.3 & 0 & 0 & 0 & 0 & -0.4 & 0 & -0.7 \\ 0 & 0.9 & 0 & -0.8 & 0 & 0 & 0 & 0 & 0 & -0.1 \\ -0.3 & 0 & 0.3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.8 & 0 & 1 & 0 & 0 & 0 & 0 & -0.2 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 0 & -0.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.6 & 0 & -0.6 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.5 & 0 & 1.4 & 0 & 0 & -0.9 \\ -0.4 & 0 & 0 & 0 & 0 & -0.6 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -0.2 & 0 & 0 & 0 & 0 & 0.2 & 0 \\ -0.7 & -0.1 & 0 & 0 & 0 & 0 & -0.9 & 0 & 0 & 1.7 \end{bmatrix}.$$

Each manipulator dynamics follows the EL equations (1), whose inertia and Coriolis matrices are given by

$$\mathbf{M}_i(\mathbf{q}_i) = \begin{bmatrix} \alpha_i + 2\beta_i c_{i2} & \delta_i + \beta_i c_{i2} \\ \delta_i + \beta_i c_{i2} & \delta_i \end{bmatrix},$$

and

$$\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i) = \delta_i \begin{bmatrix} -s_{i2}\dot{q}_{i2} & -s_{i2}(\dot{q}_{i1} + \dot{q}_{i2}) \\ s_{i2}\dot{q}_{i1} & 0 \end{bmatrix},$$

respectively. In these expressions, c_{ik}, s_{ik} are a short notation of $\cos(q_{ik})$ and $\sin(q_{ik})$; q_{ik} is the angular position of link k of manipulator i, with $k \in 1, 2$; $\alpha_i = l_{i2}^2 m_{i2} + l_{i1}^2(m_{i1} + m_{i2})$, $\beta_i = l_{i1} l_{i2} m_{i2}$, and $\delta_i = l_{i2}^2 m_{i2}$, where l_{ik} and m_{ik} are the length and mass of each link, respectively. The exosystem's matrices that model the disturbance $\mathbf{d}_i(t)$ in (2) are

$$\mathbf{S}_i = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}; \qquad \mathbf{C}_i = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

The network is composed of three different groups of manipulators, where members of each group has the same parameters. The physical parameters are $m_1 = 4$ kg, $m_2 = 2$ kg, and $l_1 = 0.2 = l_2 = 0.4$ m, for Agents 1, 2, and 3; $m_1 = 3$ kg, $m_2 = 2.5$ kg, and $l_1 = 0.6$ m, $l_2 = 0.5$ m, for Agents 4, 5, and 6; $m_1 = 3.5$ kg, $m_2 = 2.5$ kg, and $l_1 = 0.3$ m, $l_2 = 0.35$ m, for Agents 7, 8, 9, and 10

The proportional gains p_i for the controllers (8) are all set to 10 Nm. By letting $\alpha_i = 1$ and by using ${}^*T_{ji} = 0.7$ s and $p_i = 10$ Nm, condition (10) is given by $d_i \geq 8.5 l_{ii}$, where l_{ii} corresponds to the ith-diagonal element of the Laplacian matrix. Setting the damping gains as $d_1 = 12$, $d_4 = 9$, and $d_2 = 7,7$, $d_3 = 2.6$, $d_4 = 9.5$, $d_5 = 4.3$, $d_6 = 5,2$, $d_7 = d_9 = 7$, $d_8 = 9$, and $d_10 = 14$, ensure that condition (10) hold for all i.

The internal model-based estimation term in (4) is given by

$$\dot{\mathbf{x}}_i = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \mathbf{x}_i - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \dot{\mathbf{q}}_i,$$

for agents 1,2,3,4 and 5; and

$$\dot{\mathbf{x}}_i = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \mathbf{x}_i - \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \dot{\mathbf{q}}_i,$$

for agents 6,7,8,9 and 10.

The initial positions for each agent are set as $\mathbf{q}_1(0) = [11, 8]^\top$, $\mathbf{q}_2(0) = [10, 7]^\top$, $\mathbf{q}_3(0) = [9, 6]^\top$, $\mathbf{q}_4(0) = [8, 5]^\top$, $\mathbf{q}_5(0) = [7, 4]^\top$, $\mathbf{q}_6(0) = [6, 3]^\top$, $\mathbf{q}_7(0) = [5, 2]^\top$, $\mathbf{q}_8(0) = [4, 1]^\top$, $\mathbf{q}_9(0) = [3, 0]^\top$, and $\mathbf{q}_{10}(0) = [2, -1]^\top$. The initial velocities are the same as the initial positions.

B. Simulation results

Fig. 1 presents the simulation results for the positions $\mathbf{q}_i$. Note that the EL-systems asymptotically converge to the consensus point $\mathbf{q}_c = [6.144, 1.072]^\top$. Fig. 2 depicts the velocity behavior, from which one can observe that the generalized velocities asymptotically converge to zero.

As a comparison, we now apply the control method presented in [13] with the same conditions. Fig. 3 presents the simulation results for the positions $\mathbf{q}_i$. Clearly, we can observe that without the disturbance rejection term, consensus cannot be achieved.

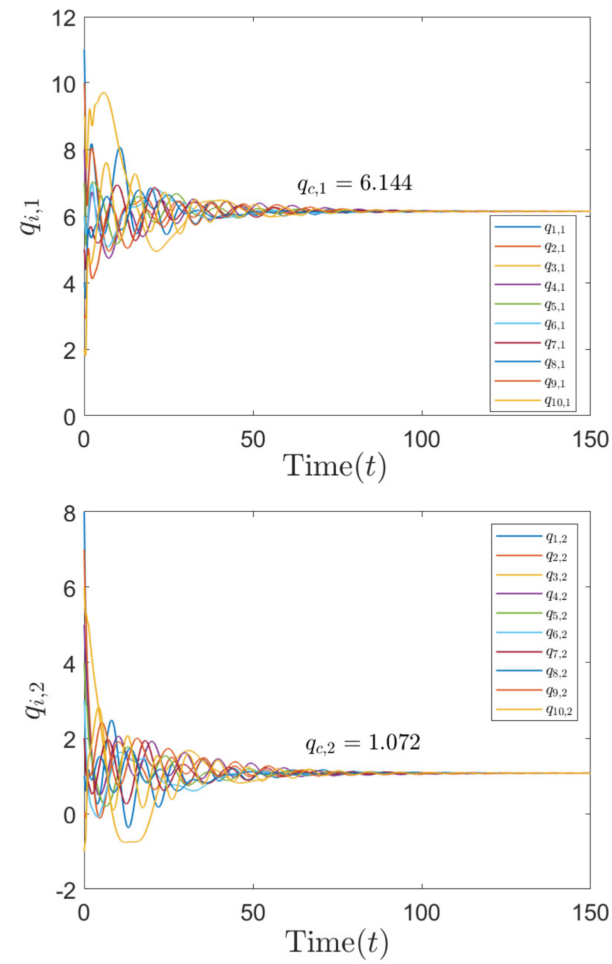

Fig. 1. Position trajectories (left plot: the first joints; right plot: the second joints) of the ten robots initialized at $\left[\begin{smallmatrix}11\\8\end{smallmatrix}\right]$, $\left[\begin{smallmatrix}10\\7\end{smallmatrix}\right]$, $\left[\begin{smallmatrix}9\\6\end{smallmatrix}\right]$, $\left[\begin{smallmatrix}8\\5\end{smallmatrix}\right]$, $\left[\begin{smallmatrix}7\\4\end{smallmatrix}\right]$, $\left[\begin{smallmatrix}6\\3\end{smallmatrix}\right]$, $\left[\begin{smallmatrix}5\\2\end{smallmatrix}\right]$, $\left[\begin{smallmatrix}4\\1\end{smallmatrix}\right]$, $\left[\begin{smallmatrix}3\\0\end{smallmatrix}\right]$, and $\left[\begin{smallmatrix}2\\-1\end{smallmatrix}\right]$, respectively.

V. Conclusions

Using the internal model approach [23], [24], we have modified the P+d controllers [13] to globally solve the leaderless consensus problem in a network of heterogeneous EL-systems perturbed by external disturbances and with variable time-delays. Simulation results validate the robustness of the proposed P+d controllers. The proposed controller requires velocities to be measurable and this may not be practical in some applications. As a future research avenue, following the results in [28] and in [29], we intend to relax such requirement.

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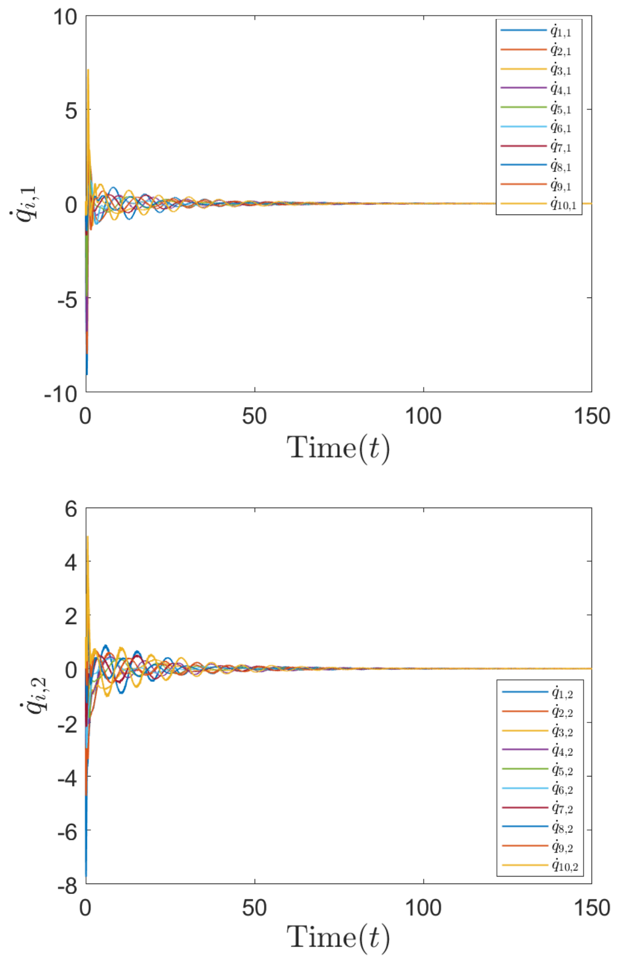

Fig. 2. Velocity trajectories (left plot: the first joints; right plot: the second joints) of the ten robots, initialized at $\left[\begin{smallmatrix}11\\8\end{smallmatrix}\right]$, $\left[\begin{smallmatrix}10\\7\end{smallmatrix}\right]$, $\left[\begin{smallmatrix}9\\6\end{smallmatrix}\right]$, $\left[\begin{smallmatrix}8\\5\end{smallmatrix}\right]$, $\left[\begin{smallmatrix}7\\4\end{smallmatrix}\right]$, $\left[\begin{smallmatrix}6\\3\end{smallmatrix}\right]$, $\left[\begin{smallmatrix}5\\2\end{smallmatrix}\right]$, $\left[\begin{smallmatrix}4\\1\end{smallmatrix}\right]$, $\left[\begin{smallmatrix}3\\0\end{smallmatrix}\right]$, and $\left[\begin{smallmatrix}2\\-1\end{smallmatrix}\right]$, respectively.

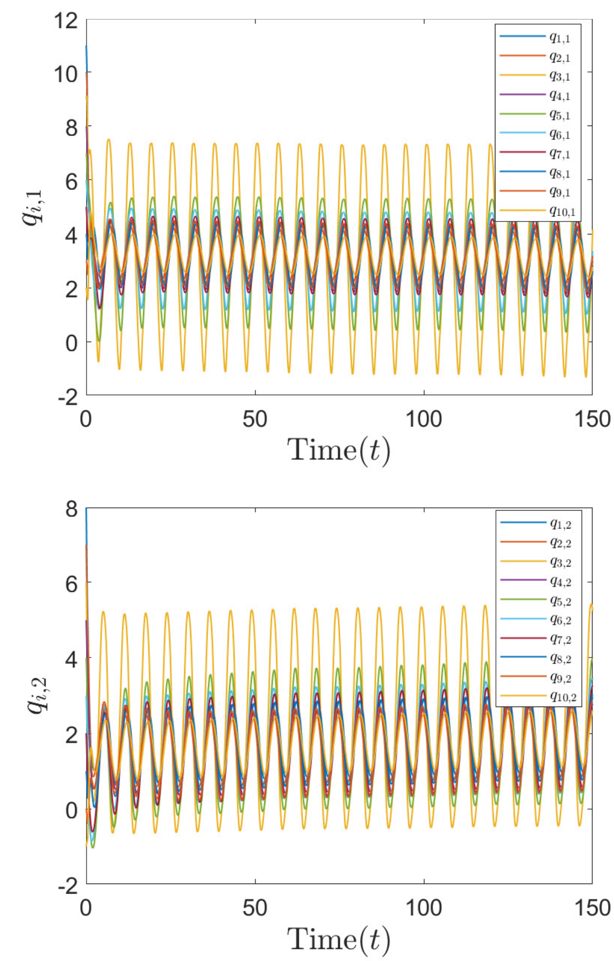

Fig. 3. Position trajectories (left plot: the first joints; right plot: the second joints) of the ten robots initialized at $\left[\begin{smallmatrix}11\\8\end{smallmatrix}\right]$, $\left[\begin{smallmatrix}10\\7\end{smallmatrix}\right]$, $\left[\begin{smallmatrix}9\\6\end{smallmatrix}\right]$, $\left[\begin{smallmatrix}8\\5\end{smallmatrix}\right]$, $\left[\begin{smallmatrix}7\\4\end{smallmatrix}\right]$, $\left[\begin{smallmatrix}6\\3\end{smallmatrix}\right]$, $\left[\begin{smallmatrix}5\\2\end{smallmatrix}\right]$, $\left[\begin{smallmatrix}4\\1\end{smallmatrix}\right]$, $\left[\begin{smallmatrix}3\\0\end{smallmatrix}\right]$, and $\left[\begin{smallmatrix}2\\-1\end{smallmatrix}\right]$, respectively, using controller in [13].

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