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Subsample stability, change detection and dynamics of oil and metal markets: A recursive approach

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ABSTRACT

The analysis of historical price data for patterns and using such patterns for predictions and policy recommendations has become ubiquitous in the existing economics literature. These predictions and recommendations are premised on the stability of the statistical properties and inter-variable dynamics for which a single regime or few number of regimes can capture. This, however, is a strong assumption with serious repercussions if violated. In this study, the appropriateness of the stability assumption is questioned using various recursive regressions to test stability, consistency of stationarity and stability in inter-variable dynamics between crude oil, gold, silver, and platinum prices. Using monthly data sourced from the World Bank Commodity Price Data (Pink Sheet) from January 1, 9960 to March 2022, our empirical analysis found level prices of oil, gold, and platinum to be consistently non-stationary with rare exceptions. The level price of silver however is found to be inconsistent with multiple regime switches while the logged series of all variables yielded non-stationarity. The default is stationarity for all the variables when price series are logged differenced and/or differenced for oil, silver, and platinum. Differenced gold prices resulted in inconsistent stationarity with multiple regime changes. Even if rare, the stationarity of all the variables is dependent on time and sample size due to the inconsistence in the stationarity verdict. On the bi-variate relationship in the long run, only level silver prices are found to be cointegrated with oil while logged silver prices are inconsistently cointegrated with logged oil prices. Also, in the short-run, only log of oil prices is found to Granger cause log of silver prices. It is thus recommended that researchers and policy makers be tempered in extrapolating statistical findings in general and the price and inter-price dynamics of oil, gold, silver and platinum into the future.

1. Introduction

Economic modelling has become the mainstay in policy circles with policy makers increasingly relying on quantitative economic models to analyse, simulate and track defined policies (Ronald, 1995). Quantitative economic models generally contend a logical and/or quantitative relationship between a set of variables in the short run and/or in the long run. The quantitative relationship in particular is not known a priori and is estimated based on some historical data. These historical data are themselves assumed to have some appealing statistical properties. One of such appealing statistical properties in the case of time series and to a lesser extent panel data is stationarity. The stationarity of historical time-series data implies a time invariant mean, variance, and autocorrelation of the series. The concept of stationarity is so fundamental in time series and panel data parameter estimations that data that fails to meet this property are most often transformed somehow to become stationary. Since the econometrician often has access to finite data, there is no guarantee that data that is stationary in the past will be stationary going forward. This is often taken for granted however with estimated historical stationarity assumed to persist with correlation and causal relation(s) established on the assumption of such stationarity used in predictive models. Indeed, evidence of unit root, the fundamental method used in testing stationarity, is claimed by Nelson et al. (2001) to be overstated with standard tests having low power compared to trend stationary alternatives with structural breaks in trend level or growth rates. Structural break is said to have occurred if at least a single parameter changes at some date in the sample period which is often assumed against reason to be instantaneous (Hansen, 2001).

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When stationarity is established using standard unit root tests like Dickey and Fuller (1981) and/or Phillips and Perron (1988), the norm is to insinuate causality using methods like that proposed by Granger (1969). Noting the sensitivity of causality to the sample period used, Psaradakis et al. (2005) proposed a Markov-switching Granger causality test to cater for the possibility of a change in the parameters. But the switching from one regime to another is again treated as an instantaneous event which is unlikely. Even when stationarity is not established, other statistical methods like Johansen cointegration and the Autoregressive Distributed Lag Models (ARDL) bounds test that contend a long-term 'stable' relationship between the variables exists are inferred. Again, some researchers like Hall et al. (1997) have tried to ameliorate the possibility of unstable cointegrating relationship by accommodating for the possibility of different parametric regimes in the estimation of the cointegrating relationships. But again, such studies assume instantaneous effect of regime switches which is unlikely.

Policies or measures taken based on the results of parametric estimates of logical or quantitative relationships using the standard concepts and measures of stationarity, Granger causality, and cointegration; Johansen or ARDL are likely to be misleading with unintended consequences if there is inconstancy in parameters (Shahbaz et al., 2022). Thus, instability or change in the future trajectory of variable and/or relationship of variables. To tackle this problem of inconstant parameters, Shahbaz et al. (2022) devised a Rolling-window Autoregressive Distributed Lag (RADL) model to capture evolving parameters akin to Markov-switching regimes but without the presumption of instantaneous one-time shift in the parameters. This is done by maintaining a specific sample size that rolls forward re-estimating the parameters upon every roll. Balcilar et al. (2019) also used rolling and recursive rolling approaches to determine the time varying links between oil and gold while Thoma (1994) utilises a recursive regression within a VAR framework to estimate the time varying causality between money and income and subsample instability.

The methodology introduced by Shahbaz et al. (2022) and Balcilar et al. (2019) fail to utilize the full information set and do not comprehensively capture the future trajectory of the cointegrating and causal relationship among variables. All these studies also assume off a change in the stationarity behaviour of the individual variables only catering for the inconstancy of cointegrating and/or causal relationship. This study expands the recursive regression of Thoma (1994) by formulating a recursive ADF, a recursive ARDL bounds test, a recursive Johansen cointegration and a recursive Granger causality test. These concepts are applied to monthly price data sourced from the World Bank Commodity Prices (Pink-Sheets) of oil, gold, silver, and platinum prices from 1960 January to 2022 March to determine if there is stability in the stationarity and inter-price relationship between crude oil and metal(s) prices across time. The utilised World Bank Commodity price series is that of Dubai Fateh Fateh 32 API for oil, the London afternoon fixing average daily rates of 99.5% quality for gold, 99.9% refined London afternoon fixing for Silver and 99.9% refined London afternoon fixing for platinum.

This study ameliorates the shortcomings of Shahbaz et al. (2022) and Balcilar et al. (2019) by exhaustively examining the relationship between oil and metals (gold, silver, and platinum) prices taking into consideration not only the time varying equilibrium relations and/or causality but by also looking at the time varying stochastic properties (stationarity) of the prices of oil, gold, silver, and platinum. Our empirical analysis found level prices of oil, gold, and platinum to be consistently non-stationary with rare exceptions. Silver prices however is found to be inconsistent with multiple regime switches whiles the logged series of all variables yielded non-stationarity. The default is stationarity for all the variables when price series are logged differenced and/or differenced for oil, silver and platinum. Differenced gold prices resulted in inconsistent stationarity with multiple regime changes. Even if rare, the stationarity of all the variables is dependent on time and sample size due to the inconsistency in the stationarity verdict. The inconsistent stationarity results of oil, gold, silver, and platinum prices as found in this study implies that researchers must be circumspect and tempered in their conclusions as any conclusions drawn from any sample study could be a coincidence of sample period and sample size and thus non-generalisable. On the bi-variate relationship in the long run, only silver prices are found to be cointegrated with oil while logged silver prices are inconsistently cointegrated with logged oil prices. Also, in short-run, only log of oil prices is found to Granger cause the log of silver prices. This implies that, whereas cointegration can generally be considered to exist between silver and oil prices, logged silver prices and logged oil prices are cointegrated conditional on the sample period and sample size. This again means that generalisations should be tempered as should be inference of causality other than from oil prices to silvers prices.

The remaining part of paper is organised as follows: Section-II briefly reviews the literature on the stationarity properties of crude oil, gold, silver, and platinum prices and the relationship between the variables. Section-III presents a brief description of the data. Section-IV elaborates the recursive ADF, ARDL, Johansen and Granger causality tests. Section-V details and discusses the empirical results and conclusions are drawn in Section-VI.

2. Literature review

Crude oil is the most actively traded commodity in the commodity markets with its role no longer seen as just an input to production but as a barometer of economic activity and a financial asset (Venditti and Veronese, 2020). This has made the price of oil and consequently its returns to be followed keenly by policy makers with the hope of deciphering its future trend and implications for real and nominal variables like output and inflation (Venditti and Veronese, 2020). This is often done using historical price data. The stochastic properties of such historical price data have important implications for designing and analysing policies, assessing the validity of theories (Landajo et al., 2021) and accurately forecasting (Maslyuk and Smyth, 2008) the future trend of markets. Several studies have examined the stochastic properties of oil and have arrived at different and even contradictory conclusion. Pindyck (1999) for example found oil prices before 1996 to be stationary but failed to reject the null of unit root after 1996. Maslyuk and Smyth (2008) using weekly spot, 3 months and 6 months' price data spanning 1991 to 2004 could not reject the null of unit root for the West Texas Intermediate (WTI) and UK Brent oil using both the Augmented Dickey-Fuller (ADF) test and Phillip-Perron test. Even when the LM test with breaks was used, the null of unit root could not be rejected. Also, Zhang and Wei (2010) concluded non-stationarity of the prices of crude oil using daily data from 4^{th} January 2000 to 31st, March 2008. In contrast, Sari et al. (2010) using daily data from 1/04/1999 to 10/19/2007 found crude oil price to be trend and intercept stationary. Also, crude oil spot prices were found to be intercept stationary at levels using the DF-GLS test of Elliott et al. (1992) by Balcilar et al. (2015) using a 5-day daily spot prices data from January 1987 to February 2012. Similarly, the stochastic properties of gold, by far the most established portfolio risk management tool (Gaspareniene et al., 2018), has been explored by several studies (Gaspareniene et al., 2018; Hassani et al., 2015; Ismail et al., 2009; Rathnayaka and Seneviratna, 2019; Tripathy, 2017). Using monthly data from January 1972 to December 2013, ADF and Bai and Perron (2003) test of structural breaks, Hassani et al. (2015) concluded that, gold prices were non-stationary with structural breaks on 1980(4), 1984(4), 2001(8), 2007(9), and 2007(12). Similarly, Rathnayaka and Seneviratna (2019) used daily gold prices from October 2017 to December 2017 and concluded that gold prices were non-stationary as did by Zhang and Wei (2010) using daily data from January 4, 2000, to March 31, 2008. Also, Godil et al. (2020) used an ADF and the Zivot and Andrews (2002) (ZA) test to test the stationarity of oil and gold prices and concluded that both were first differenced stationary. However, Varela (1999) found gold, silver, and copper future prices to be stationary.

Again, studies examining the stationarity of silver, a precious metal with a long history and extensive markets (Vigne et al., 2017), have not been conclusive. For example, Adewuyi et al. (2020) used 6 conventional tests of stationarity (ADF, PP, DF-GLS, KPSS, ERSPO, and Ng-Perron) to determine the stationarity of silver and several precious and industrial metals using data from 1960 to 2017. They concluded that monthly and quarterly silver prices are stationary at levels contrary to the findings of Yang et al. (2012) who used Pfaffenzeller et al. (2007) annual dataset from 1900 to 2007 in a panel framework and concluded silver prices to be non-stationary. Addison and Ghoshray (2020) explore the stationarity properties of several industrial metals using unit root tests robust to non-stationary volatility and found the price of silver to be stationary. Platinum unlike gold and silver is largely an industrial metal with industrial demand dictating supply and private investment demand barely accounting for about 10% of annual demand (Hillier et al., 2006). Nonetheless, interest in the stochastic, investing and hedging properties of this seemingly obscure metal is heightening. In Adewuyi et al. (2020), monthly and quarterly platinum prices from 1960 to 2017were found to be stationary as did Sari et al. (2010) using daily closing spot prices from January 4, 1999, to October 19, 2007. Rubbaniy et al. (2011) used data from 1985 to 2010 to conclude that platinum prices are non-stationary with hedging properties.

From the foregoing, it is safe to conclude that the stationarity properties of the price of oil and precious metals considered such as gold, silver, and platinum are all empirically indeterminate with contradictory results by different researchers. These contradictions in stationarity results could be methodological and/or sampling (Balcilar et al., 2019). Be that as it may, stochastic properties of metal and oil prices are studied as a means to determining and/or forecasting own price future trajectory and/or the relationship between the metal (oil) prices and other commodities and variables, real and/or nominal using a wide array of methods and sampling periods. Generally, studies on the oil-metals nexus can be broadly categorised into two groups based on their object of study and the methodology employed. The first group of studies are those that seek to determine the long-run equilibrium relationship between oil and metals using cointegration measures like Johansen (1991, 1995), Johansen and Juselius (1990) and Pesaran et al. (2001)'s bounds testing approaches.

The second group focus on the causal relationship between oil and metals using Granger causality type of estimations. For example, Sari et al. (2010) used Johansen (1991, 1995), Johansen and Juselius (1990) and Pesaran et al. (2001) to study the long-run relationship for spot prices of oil and metals (gold, silver, platinum, and palladium) and exchange rate movement using daily data from January 4, 1999 to October 19, 2007. Their empirical evidence by Johansen (1991, 1995) reported no level relationship among the spot prices of oil and precious metals considered. Johansen and Juselius (1990) methodology gave contradictory conclusions with the maximum eigenvalue test contending no level relationship whiles the trace test concluded the existence of level relationship between the variables. Similarly, the results of Pesaran et al. (2001) bounds test also concluded no cointegration among the variables. Thus, they found the absence of level relationship between oil prices and metals prices. On similar lines, Chang et al. (2013) could not establish a long run relationship between oil and gold price using Johansen trace and Maximum eigen-value statistics. In contrast, however, Balcilar et al. (2015) employed Johansen (1991, 1995) and Johansen and Juselius (1990) cointegration methods, and Stock and Watson (1988) multivariate cointegration tests to examine the equilibrium relationship between crude oil, gold, silver, platinum and palladium prices. Their empirical analysis reported the presence of a strong equilibrium relationship among all 5 variables. Similarly, Zhang and Wei (2010) found that crude oil and gold prices in particular to be strongly cointegrated using Engle and Granger (1987) test of cointegration.

On correlation and causality, Balcilar et al. (2019) conducted an exhaustive review on the time-varying correlational and causal relationship between gold and oil prices and reported that reviewed studies purport a strong correlation (Kim and Dilts, 2011), no correlation (Sari et al., 2010; Soytas et al., 2009), both are correlated with their long-term drivers (Bampinas and Panagiotidis, 2015), bidirectional causality (Bildirici and Turkmen, 2015), unidirectional causality running from oil to gold (Zhang and Wei, 2010). Zhang and Wei (2010) in particular found a strong positive correlation between crude oil and gold price, with a unidirectional causality running from crude oil to gold prices. Hammoudeh and Yuan (2008) researched on the impact of oil on mean and volatility of gold, silver and copper using GARCH class of models and daily data from January 2, 1990 to May 1, 2006. It is concluded that oil prices have a calming effect on the volatility of gold and silver but not copper. Sari et al. (2010) found a strong bidirectional causality between oil and silver whiles variance in gold, platinum and palladium were largely due to own price innovation. Turhan et al. (2014) applied a DCC-MIDAS approach using daily prices of crude oil, gold, S&P 500 and the US dollar index from May 17, 1983 to August 27, 2013 and found that gold and crude spot prices are found to have an increasing positive correlation. Baffes (2007) used annual data of 35 commodities from 1960 to 2005 by employing an OLS regression to determine the pass-through of crude oil prices to the other major commodities i.e. gold, silver, lead, nickel, iron ore, aluminium and tin. It is concluded that a high pass-through of oil price shocks to precious metals exists. Soytas et al. (2009) studied the short-run and long-run information transmission from world oil prices to Turkish domestic gold and silver spot prices using daily data from May 2, 2003 to March 1, 2007. They found no significant impact of world oil prices on Turkish domestic precious metal spot prices.

Indeed, the plethora of empirical works using the traditional cointegration and/or causality approaches do not seem to resolve the question of the relationship, if any, between oil prices and metal prices (Shahbaz et al., 2022). This has resulted in a number of works employing several other estimation methods including nonlinear cointegration and nonlinear causality approaches (Bildirici and Türkmen, 2015), regime switching cointegration test (Balcilar et al., 2015) and rolling and recursive cointegration and causality methods (Shahbaz et al., 2022; Balcilar et al., 2019). The rolling-window and recursive cointegration/causality approaches are specifically meant to remedy the inability of regime switching methods to capture multiple regime changes with varying regime duration (Balcilar et al., 2019). Whereas Balcilar et al. (2019) employed the rolling and recursive causality methods to study oil and gold prices association, Shahbaz et al. (2022) used a rolling window ARDL to examine the long run equilibrium relationship between oil and metals (gold, silver, platinum and steal) prices. These studies are however not exhaustive with Balcilar et al. (2019) focusing on only the relationship between oil and gold prices to the exclusion of all other metals, and on short run causality to the exclusion of long-run co-movements. Shahbaz et al. (2022) on the other hand focuses on only a single measure of long-run equilibrium relationship, the rolling-window ARDL method without regards to causality and time varying stochastic properties of the data itself. The rolling-window estimation by design is also limited in that it does not use the full information set (Shahbaz et al., 2022).

3. The data

This study uses monthly oil prices data from January 1960 (1960M01) to March 2022 (2022M03) sourced from World Bank database (https://thedocs.worldbank.org/CMO-Pink-Sheet). The monthly prices of gold, silver and platinum for the same period is sourced from World Bank database (https://thedocs.worldbank.org/CMO-Pink-Sheet). The utilised monthly price series for oil, gold, silver and platinum is that of Dubai Fateh 32 API, the London afternoon fixing average daily rates of 99.5% quality, 99.9% refined London afternoon fixing and 99.9% refined London afternoon fixing respectively.

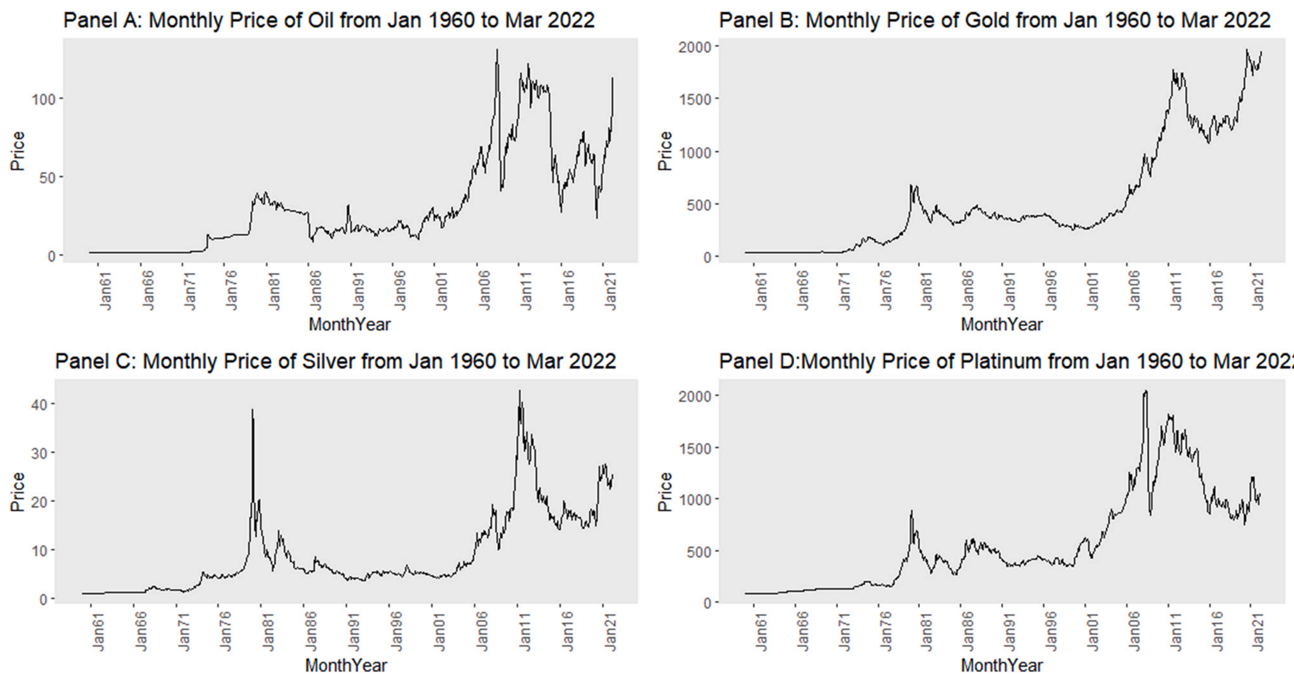

Fig. 1. Price of oil and metals.

Table 1
Descriptive statistics.

	OIL	GOLD	SILVER	PLATINUM
MEAN	30.44	515.71	8.74	568.78
MEDIAN	18.30	362.53	5.36	416.89
MAXIMUM	131.22	1968.63	42.70	2052.45
MINIMUM	1.21	34.94	0.91	78.50
STD DEVIATION	30.23	504.22	7.99	461.10
SKEWNESS	1.27	1.22	1.52	1.06
KURTOSIS	3.81	3.34	5.04	3.30

Fig. 1 presents the price trends of crude oil, gold, silver and platinum. Panel-A depicts the price of crude oil. From barely a dollar per barrel in January 1960, the price of oil increased dramatically in 1973 in response to the oil embargo placed by OPEC in October 1973. By January 1974, the price of oil was trading at $13 from $2.08 the previous year. This was followed by the second oil crisis in 1979 in the wake of the Iranian revolution.

Since then, the price of oil increased steadily hitting an all-time high of $131.22 in July 2008 before the global economic recession led to a crash in oil price to $44.97 before rebounding in January 2009. Over the course of the study, the average price of oil has been $30.44 with a standard deviation of $30.22, positive skewness of 1.27 and leptokurtic kurtosis of 3.8. Global gold prices have also undergone dramatic changes over the course of the study as depicted in Panel-B. From a monthly price of $35.27 in 1960, the price of gold was stable until the collapse of the Bretton woods system in 1971. Since then, the price of gold has been trending positively hitting a high of $1,968.63 in August 2020. On average, gold has been traded for $515.71 since 1960 with a standard deviation of $504. The price of silver has also experienced some oscillations rising to $39 in January 1980 before crashing to $6 in June 1982 and then to an all-time high of $42.69 in April 2011 as depicted in Panel-C of Fig. 1. Over the period, the price of silver averaged $8.74 with a standard deviation of $7.99 as depicted in Table 1. With a minimum of $78.5 in May 1963 and a maximum of $2,052.45 in May 2008, platinum is one of the most valued industrial and to some extent precious metals. The price trend of Platinum is depicted in panel-D of Fig. 1. All the variables are leptokurtic and positively skewed as depicted by the descriptive statistics in Table 1.

4. Methodological framework

This study seeks to determine the stochastic properties of oil, gold, silver and platinum prices and the bi-variate relational dynamics between oil prices and gold, silver, and platinum prices. This section delineates the methodologies employed to arrive at the stationarity and relational conclusions of oil prices and the prices of gold, silver and platinum. Specifically, the recursive ADF, the recursive ARDL and the recursive Johansen cointegration tests and the recursive Granger causality tests are explained.

4.1. Recursive ADF test

Stationarity in time series is a prerequisite to most analysis of economic and financial data as nonstationary time series may produce spurious results with a high probability (Ventosa-Santaularia, 2009). Stationarity is said to exist if a time series has a time invariant mean, variance, and autocorrelation. The most frequently used test of stationarity of economic and financial time series is Augmented Dickey-Fuller (ADF) test due to Dickey and Fuller (1981). For a time series Y_t, with $t = 1, 2, \ldots\ldots T$, an autoregressive regression of the form in equation-1 is estimated.

Case 1:

$$\Delta Y_t = \delta Y_{t-1} + \sum_{i=1}^{p} \beta_i \Delta Y_{t-i} + \varepsilon_t \quad (1)$$

where $\Delta Y_t = Y_t - Y_{t-1}$ and the autoregressive lags ΔY_{t-i}, for $i = 1, ..p$ introduced to render equation-1 not auto correlated. Equation-1 can be specified to accommodate intercept (Case-2) as in equation-2.

Case 2:

$$\Delta Y_t = \alpha + \delta Y_{t-1} + \sum_{i=1}^{p} \beta_i \Delta Y_{t-i} + \varepsilon_t \quad (2)$$

where α is the deterministic intercept. Also, a time trend (Case-3) can be included to capture the time dependent variation in the series as in equation-3.

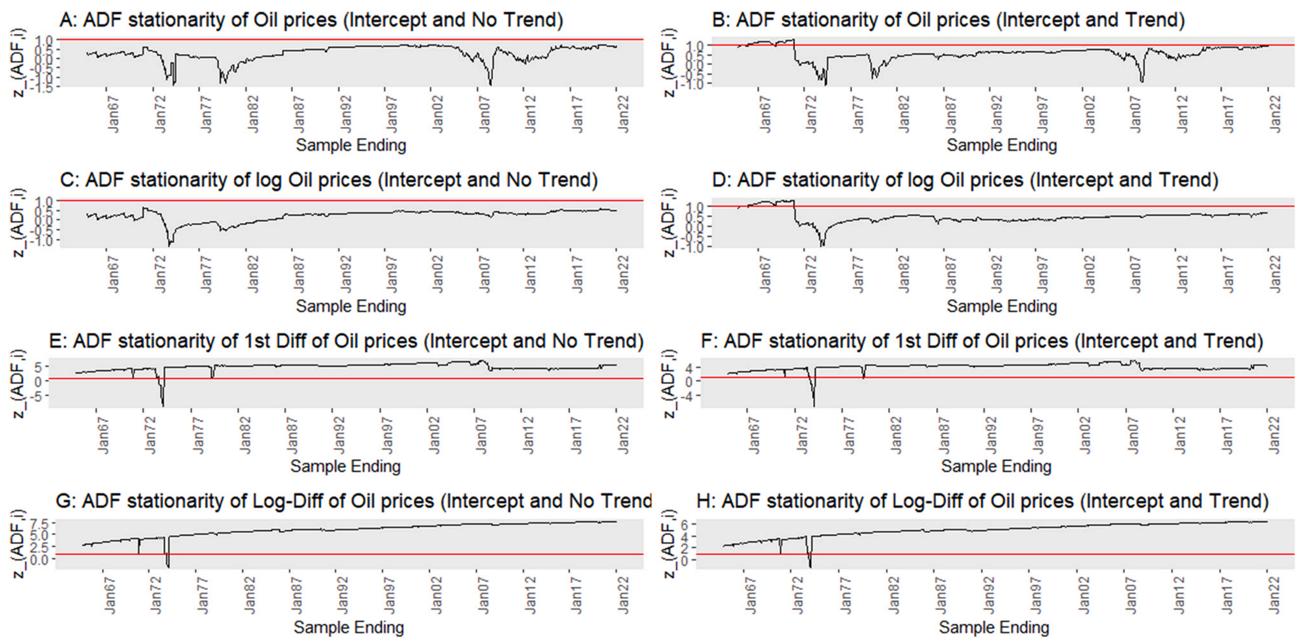

Fig. 2. *Oil Prices Stationarity* $z_{ADF,i}$.

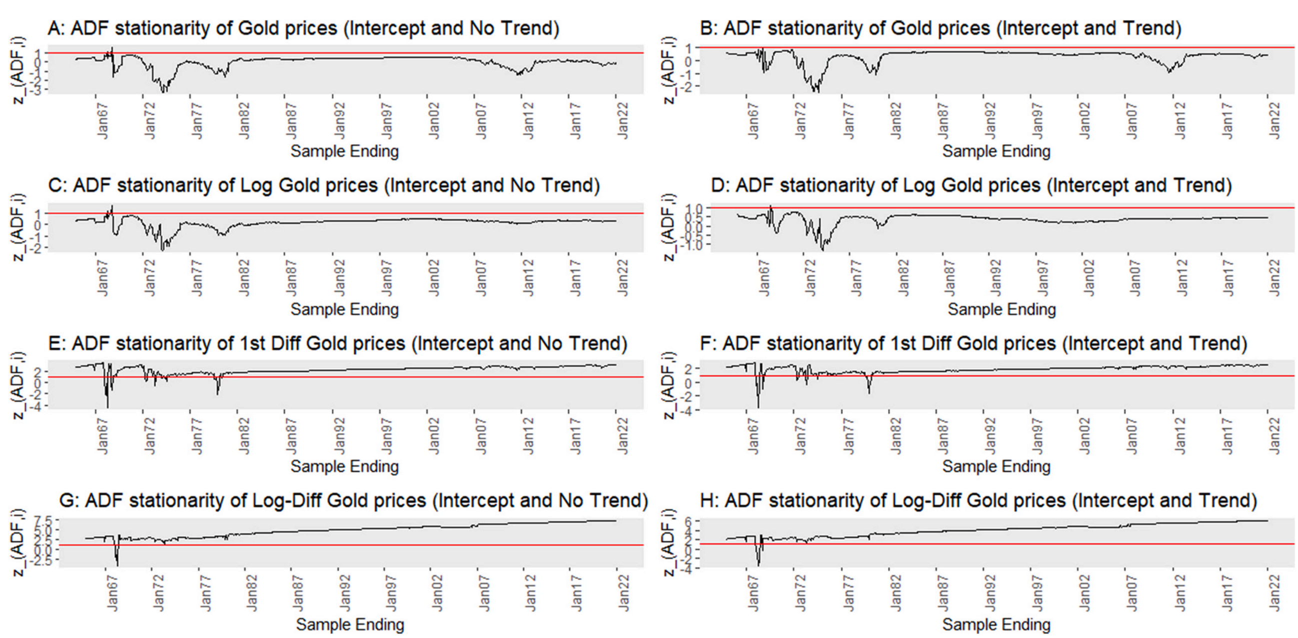

Fig. 3. Gold PricesStationarity $z_{ADF,i}$.

Case 3:

$$\Delta Y_t = \alpha + \varnothing t + \delta Y_{t-1} + \sum_{i=1}^{p} \beta_i \Delta Y_{t-i} + \varepsilon_t \quad (3)$$

where α is again the intercept and $\varnothing$ is the coefficient of the time trend. The test statistic of ADF test is computed as in equation-4 using parameters estimated from either equation-1 or 2 or 3 and compared against the critical values to establish the existence or absence of a unit root.

$$t = \frac{\widehat{\delta}}{SE(\widehat{\delta})} \quad (4)$$

This methodology however uses a snapshot of the series and does not account for the stability in the stochastic properties of the series. As such it is susceptible to the dangers of parameter inconstancy. This implies that a forecast or policy based on the presumption of stationarity as determined by the traditional ADF test could turn out to be wrong going forward. To alleviate this and to capture the stability and whether or not stationarity of the variables are dependent on the sample period, and/or sample size chosen, we choose Y_t such that the sample size used to estimate equation-1 is less than the total data points available. That is, we choose Y_t such that $t = 1, 2, \ldots L,$ where $L < T$. This is then used to estimate equation-1 (2 and 3) and the coefficients obtained are used to estimate equation-4 as in equation-5.

$$t_L = \frac{\widehat{\delta}_L}{SE(\widehat{\delta})_L} \quad (5)$$

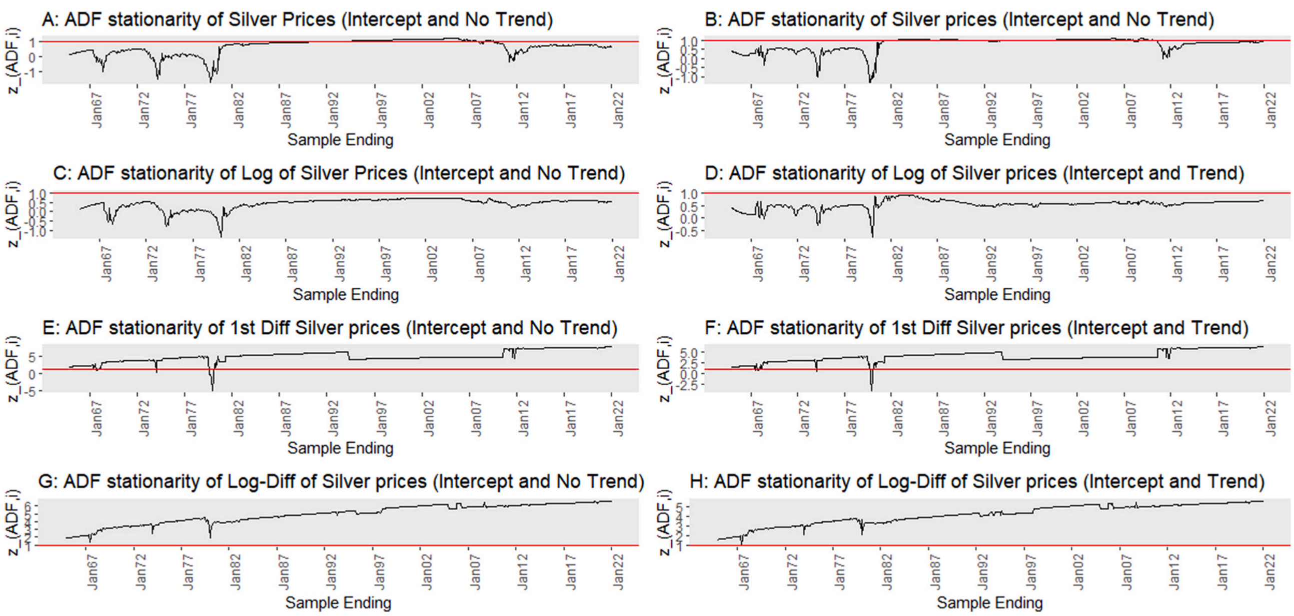

Fig. 4. Silver PricesStationarity $z_{ADF,i}$.

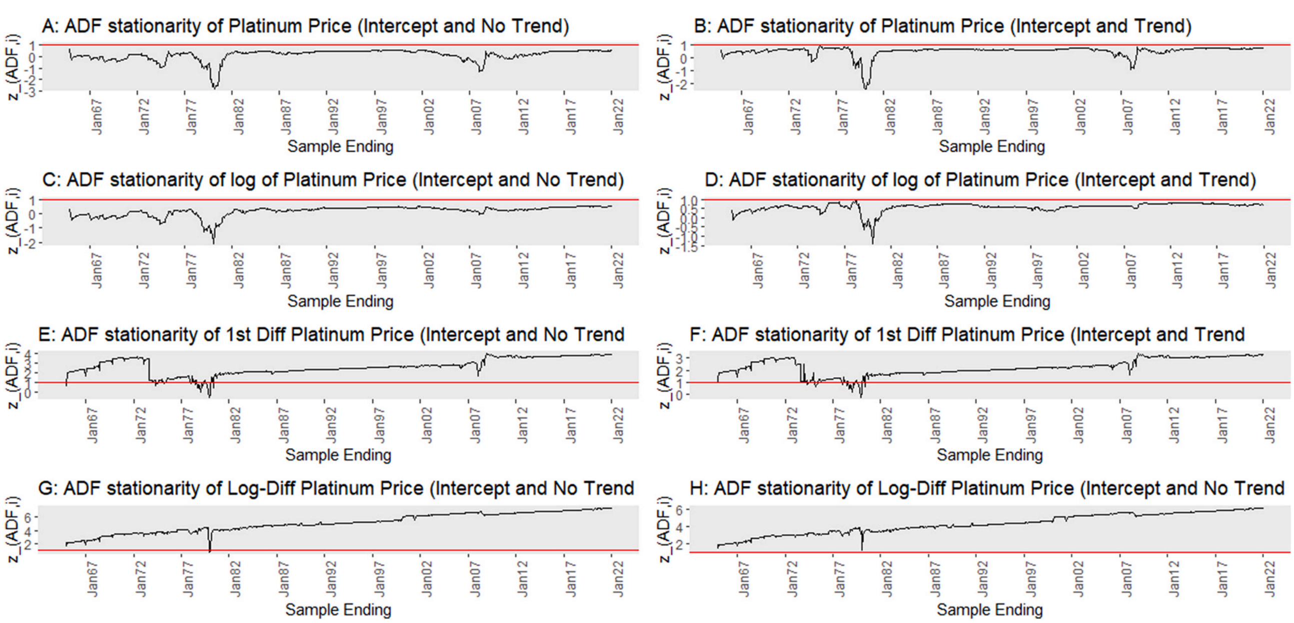

Fig. 5. Platinum PricesStationarity $z_{ADF,i}$.

Subsequently, equation-1 is re-estimated with the sample Y_t for $t = 1, 2, \ldots j$ for $j = L + 1$ and equation-4 is re-estimated with the coefficients obtained from the j samples re-estimated regression equation-1 (or 2 or 3) such that the test statistic becomes as in equation-6 and so on.

$$t_j = \frac{\widehat{\delta}_j}{SE(\widehat{\delta})_j} \tag{6}$$

As such, $T - L + 1$ ADF test statistics are computed. Specifically, L is chosen to be 1964M12 so that the first regression is computed for the sample 1960M01 to 1964M12 which amounts to using a sample size of 60 for the first regression. Subsequently, the appropriate regression and test statistic is re-calculated using 61,62,63 and so on observations until the entire sample of 747 observations is used to estimate the ADF test statistic. This amounts to 688 ADF test statistics.

Noting that for ADF test, the null hypothesis is unit root, and the critical values (CV) are always negative, $t_j < CV_j$ will result in a failure to reject the null of unit root. To determine the sample dependent stability, a sequence of variable $z_{ADF,j} = t_j / CV_j$ is created for $j = 60, 61 \ldots 688$ resulting in a sequence $z_{ADF,60}$, $z_{ADF,61}$, ……$z_{ADF,688}$. For each $z_{ADF,j}$ therefore, the null of unit root is rejected if $z_{ADF,j} > 1$. The $z_{ADF,j}$ for each sample period is plotted with the date that the sample ends on the horizontal axis. For each of the variables, oil, gold, silver and platinum, the $z_{ADF,j}$ of case-2 and 3 are computed using level prices, logged prices, first difference of level prices and first difference of logged prices.

4.2. *Recursive ARDL bounds tests*

The ARDL bounds tests introduced by Pesaran et al. (2001) has become popular in investigating long-run level relationship due to its utility in detecting long-run level relationship among variables

Table 2
Recursive regression ADF unit root analysis.

Variable	Form	Deterministic Term	Conclusion
Oil	Oil price	Intercept	Non-stationary
		Linear trend	Non-stationary with rare exceptions
	Logoil price	Intercept	Non-stationarity
		Linear trend	Non-stationarity with rare exceptions
	1st diff of oil price	Intercept	Stationarity with rare exceptions
		Linear trend	Stationarity with rare exceptions
	1st diff of logoil price	Intercept	Stationarity with rare exceptions
		Linear trend	Stationarity with rare exceptions
Gold	Gold Prices	Intercept	Non-stationary with rare exceptions
		Linear trend	Consistently non-stationary
	Log gold Prices	Intercept	Non-stationarity with rare exceptions
		Linear trend	Non-stationarity with rare exceptions
	1st diff of Gold prices	Intercept	Stationarity with repeated exceptions
		Linear trend	Stationarity with repeated exceptions
	1st diff loggold prices	Intercept	Stationarity with rare exceptions
		Linear trend	Stationarity with rare exceptions
Silver	Silver prices	Intercept	Switching from and to non-stationarity
		Linear trend	Switching from and to non-stationarity
	Logsilver prices	Intercept	Consistent Non-stationarity
		Linear trend	Consistent Non-stationarity
	1st diff of silver prices	Intercept	Stationarity with rare exceptions
		Linear trend	Stationarity with rare exceptions
	1st diff of logsilver prices	Intercept	Consistent Stationarity
		Linear trend	Consistent Stationarity
Platinum	Platinum prices	Intercept	Consistent non-stationarity
		Linear trend	Consistent non-stationarity
	Logplatinum prices	Intercept	Consistent Non-stationarity
		Linear trend	Consistent Non-stationarity
	1st diff of Platinum prices	Intercept	Stationarity with rare exceptions
		Linear trend	Stationarity with rare exceptions
	1st diff of log platinum prices	Intercept	Consistent Stationarity
		Linear trend	Consistent Stationarity

integrated of different orders (Sam et al., 2019). However, it is sample and time dependent and deficient in its ability to capture parametric changes. These shortcomings can be consequential if policy and market actions are taken based on its results. A recursive ARDL ameliorates these shortcomings capturing the stability in the parameters. To determine the stability of the long-run level relationship between crude oil and metals' prices, we employ a bi-variate recursive ARDL bounds test to examine the level relationship between oil and gold, oil and silver and oil and platinum. Just as ADF test, ARDL bounds testing starts by estimating a regression equation and bounds F-statistic is computed. Three forms of the ARDL regressions as in equation-7, 8 and 9 which correspond to case-1 (only no intercept and no trends), case-2 (only unrestricted intercept and no trends) and case-3 (unrestricted intercept and unrestricted trend) are estimated.

Case 1:

$$y_t = \varnothing\left(\pi_0 + \varphi y_{t-1} + \omega X_{t-1}\right) + \sum_{i=1}^{p} \beta_i \Delta y_{t-i} + \sum_{j=1}^{q} \gamma_j \Delta X_{t-j} + \varepsilon_t \quad (7)$$

Case 2:

$$y_t = c_0 + \varnothing\left(\varphi y_{t-1} + \omega X_{t-1}\right) + \sum_{i=1}^{p} \beta_i \Delta y_{t-i} + \sum_{j=1}^{q} \gamma_j \Delta X_{t-j} + \varepsilon_t \quad (8)$$

Case 3:

$$y_t = c_0 + \varnothing\left(\pi_1 t + \varphi y_{t-1} + \omega X_{t-1}\right) + \sum_{i=1}^{p} \beta_i \Delta y_{t-i} + \sum_{j=1}^{q} \gamma_j \Delta X_{t-j} + \varepsilon_t \quad (9)$$

with y_t being a vector of the dependent variable and X_t being a vector of the exogenous variable. c_0 and c_1 are the unrestricted intercept and time trend coefficient-respectively, π_0 and π_1 are the coefficients of the restricted intercepts and time trend respectively. $\varnothing$ captures the long-run equilibrium relations and β_i and γ_j are short-run regression coefficients. The estimated parameters from equations-7 to 8 are used to estimate F-statistic which is compared with the two bounds of asymptotic critical values as simulated by Pesaran et al. (2001) to determine whether or not the variables have a long run level relationship (Shahbaz et al., 2022). If computed F-statistic is greater than the upper bound critical value, there is said to be a long run level relation between the variables whereas the null of no level relation is not rejected if estimated F-statistic is less than lower bound critical value.

In our recursive formulation, we estimate ARDL bounds regression starting with L for $L < T$ and progressively increase the sample size by 1 period until $L = T$ resulting in a sequence of F-statistic, $F_L, F_{L+1}, \ldots\ldots F_T$. Since it is only upper critical bound that is indicative of long-run level relations, the estimated F-statistic is compared with upper critical bound to determine the existence or otherwise of level relationship between the variables which is labelled, $z_{bt,i}$. That is, $z_{bt,i} = \frac{F-Stat_i}{UCV_i}$. As such, a sequence of $z_{bt,L}, z_{bt,L+1}, \ldots\ldots z_{bt,T-L+1}$ is obtained from F-statistic and upper critical bounds. The null of no cointegration is rejected in favour of the alternative if $z_{bt,i} > 1$. The series of $z_{bt,i}$ is again plotted with the last sample month on $x-axis$ to determine the overtime path of cointegration between oil prices and metal prices.

4.3. Recursive Johansen Cointegration test

The other most widely used test of cointegration in the literature is Johansen cointegration test (Pekmezci and Dilek, 2016). This test is a Vector Error Correction Model (VECM) based test which first formulates a VECM of the form in equation-10.

$$\Delta y_{1t} = \Pi y_{t-k} + \Gamma_1 \Delta y_{t-1} + \ldots + \Gamma_{k-1} \Delta y_{t-(k-1)} + \varepsilon_t \quad (10)$$

From equation-10, Δy_t is the first differenced vector of all the endogenous variable(s), Π is the long-run coefficient matrix and Γ_i are short run coefficient matrices of lag i of the differenced variables. Under this formulation, testing for cointegration devolves to testing the rank of Π via its eigenvalues λ. This test statistic is a joint test statistic with the null being that the number of cointegrating vectors is less than or equal to r against a general alternative hypothesis of more than r number of cointegrating equations (Aysan et al., 2022). Again, the traditional Johansen Cointegration test is deficient due to its inability to capture the parameter inconstancy and is sample size and sample period dependent. To remedy this, the recursive formulation as proposed in this study uses the same set up as in equation-10 and progressively increase the sample size by a period until the entire sample is used. This allows for a progressive capture of parametric changes over the entire study period.

To arrive at a decision, the trace test statistic, λ_{trace}, is divided by the

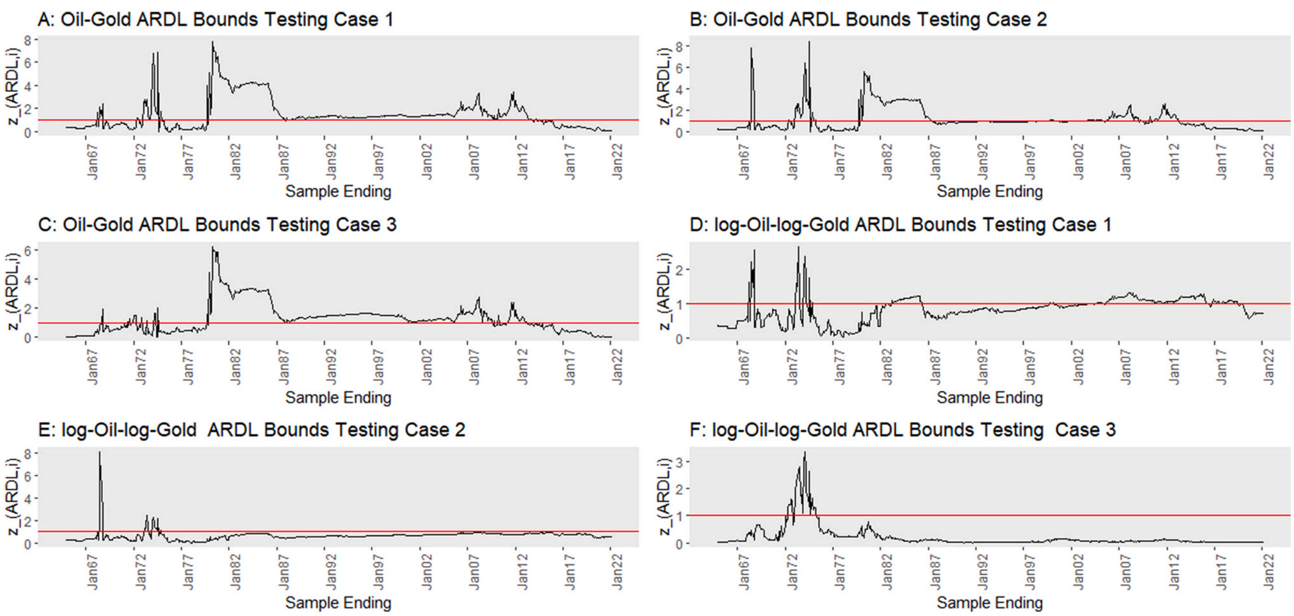

Fig. 6. Oil and gold ARDL bounds test $z_{bt,i}$.

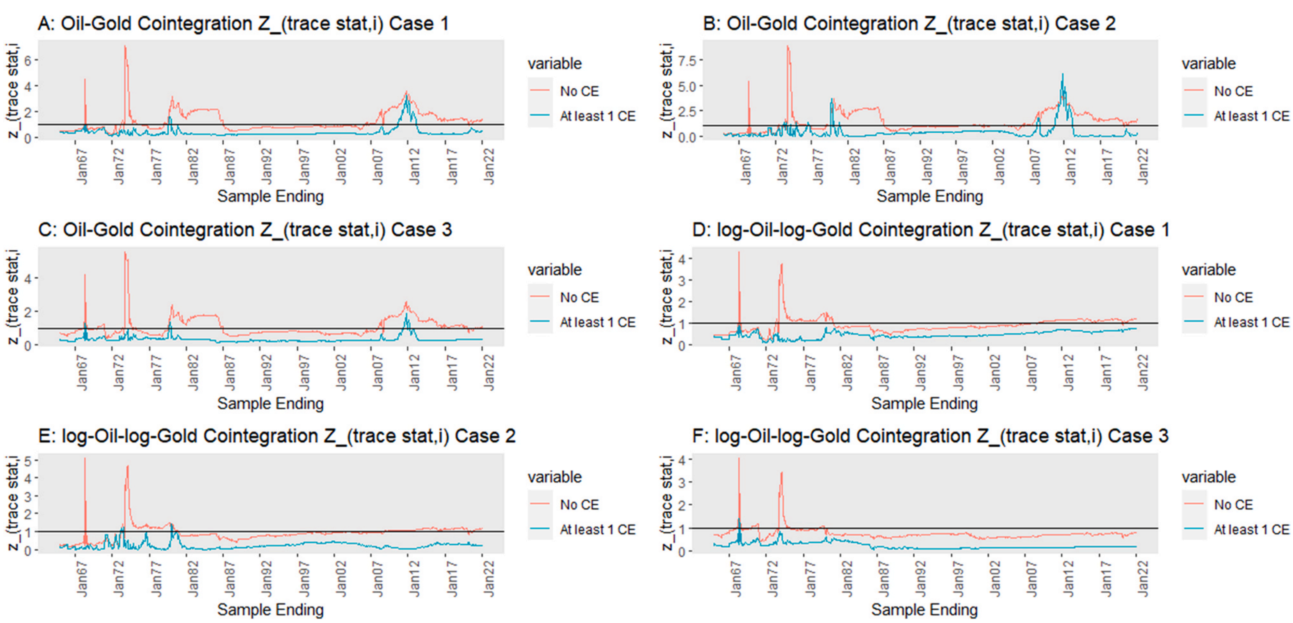

Fig. 7. Oil and gold Johansen trace statistic $z_{\lambda_{(r=-)},i}$.

critical values as before resulting in a sequence of $z_{\lambda_{(r=0)},i} = \frac{\lambda_{trace}}{CV(r=0)}$ and $z_{\lambda_{(r=1)},i} = \frac{\lambda_{trace}}{CV(r=1)}$ for $i = L, L+1, \ldots. T-L+1$. For there to be cointegration between the variables, the bi-variate trace statistic for a maximum rank of zero ($r = 0$) must be rejected whiles failing to reject the maximum rank of 1 ($r = 1$). That is, $z_{\lambda_{(r=0)},i}$ must be greater than 1 and $z_{\lambda_{(r=1)},i}$ must be less than 1. This amounts to rejecting the null of less than or equal to zero (0) cointegrating equation against the alternative of more than zero (0) cointegrating equations for $z_{\lambda_{(r=0)},i}$ whiles failing to reject the null of at least 1 cointegrating equation for $z_{\lambda_{(r=1)},i}$. Again, the $z_{\lambda_{(r=0)},i}$ and $z_{\lambda_{(r=1)},i}$ series are plotted against the sample ending period. For each variable, 3 VECM formulations as in equation-11, 12 and 13 are estimated which are termed case1, 2 and 3 with case-1 having only an intercept in the cointegration equation-11, case-2 includes an intercept in cointegration equation-12 and a deterministic linear trend in the levels data. Case-3 includes a deterministic and linear trend in cointegration equation-13 and a linear trend in the levels data.

Case-1:

$$\Delta y_{1t} = \varphi(\alpha + \beta y_{1t-1} + \delta y_{2t-1}) + \sum_{i=1}^{q} \gamma_{11,i} \Delta y_{1t-i} + \sum_{i=1}^{q} \gamma_{12,i} \Delta y_{2t-i} + \varepsilon_t \tag{11}$$

Case-2:

$$\Delta y_{1t} = \varphi(\alpha + \beta y_{1t-1} + \delta y_{2t-1}) + \theta t + \sum_{i=1}^{q} \gamma_{11,i} \Delta y_{1t-i} + \sum_{i=1}^{q} \gamma_{12,i} \Delta y_{2t-i} + \varepsilon_t \tag{12}$$

Case-3:

$$\Delta y_{1t} = \varphi(\alpha + \vartheta t + \beta y_{1t-1} + \delta y_{2t-1}) + \theta t + \sum_{i=1}^{q} \gamma_{11,i} \Delta y_{1t-i} + \sum_{i=1}^{q} \gamma_{12,i} \Delta y_{2t-i} + \varepsilon_t \tag{13}$$

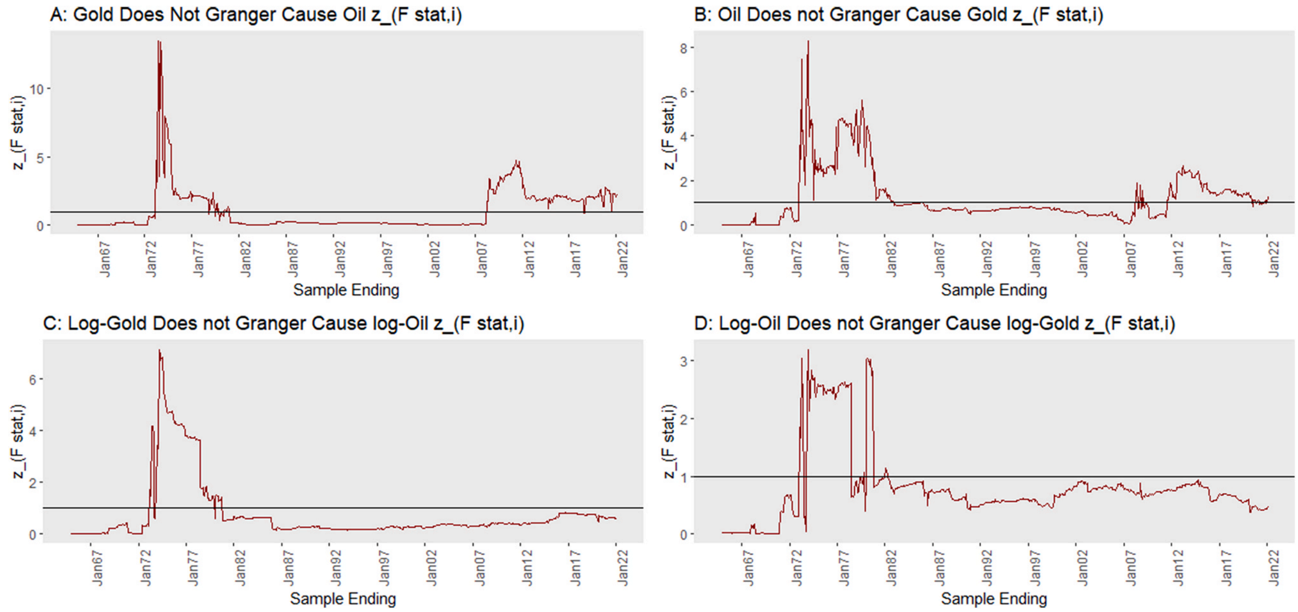

Fig. 8. Oil and gold Granger causality $z_{y_{1t}\rightarrow y_{2t},L}$.

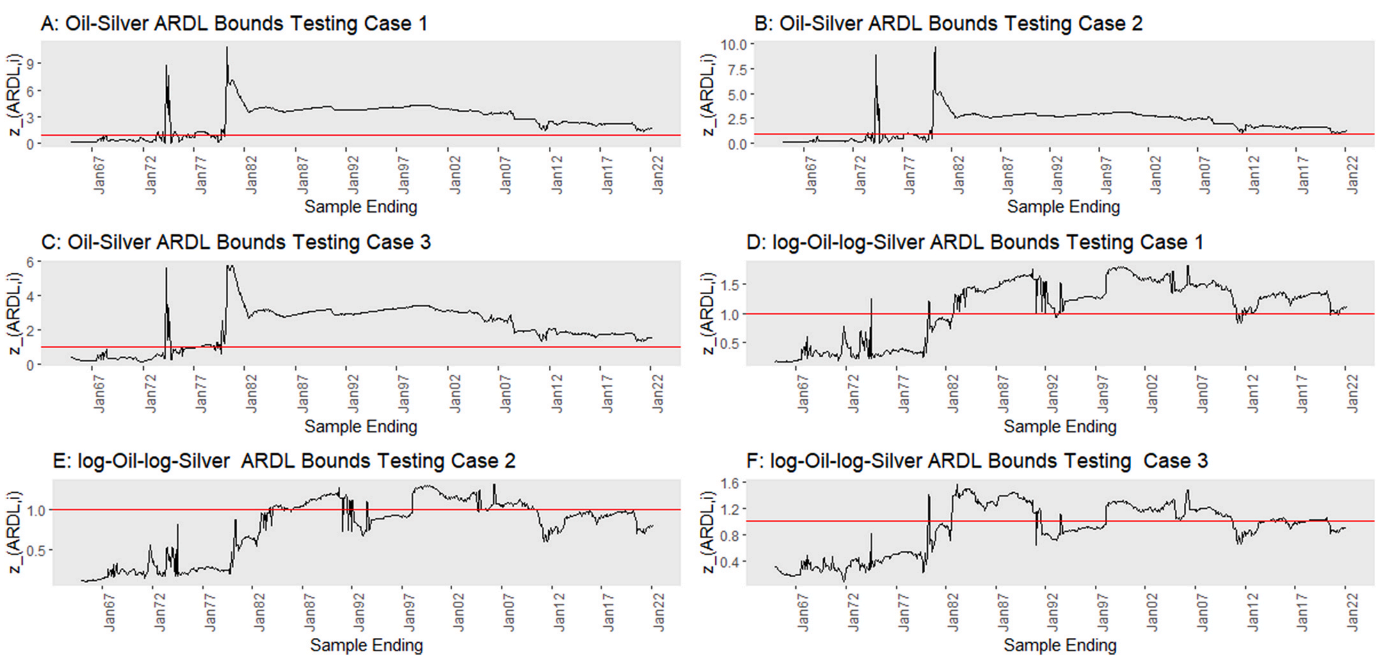

Fig. 9. Oil and silver ARDL bounds tests $z_{bt,i}$.

4.4. Recursive Granger causality test

The ARDL and Johansen cointegration tests focus on the existence or absence of long-run level relationship with no indication about short-run and the direction of "causality". Causality in this sense means the ability to predict the future trajectory of a variable based on the past of another variable (Granger, 1969). Granger causality or the more preferable Granger non-causality is a Vector Autoregressive (VAR) or VECM based short-run predictability measure which relies on F-statistic of the lagged regressors. A bi-variate VECM system in matrix form as in equation-10 can be expanded as in equations-11.

$$\Delta y_{1t} = \alpha_{10} + \alpha_{11} ECT_{t-1} + \sum_{i=1}^{q} \gamma_{11,i} \Delta y_{1t-i} + \sum_{i=1}^{q} \gamma_{12,i} \Delta y_{2t-i} + \varepsilon_{ty_1} \Delta y_{2t}$$

$$= \alpha_{20} + \alpha_{21} ECT_{1t-1} + \sum_{i=1}^{q} \gamma_{21,i} \Delta y_{1t-i} + \sum_{j=1}^{q} \gamma_{22,i} \Delta y_{2t-i} + \varepsilon_{ty_2} \quad (14)$$

where ECT_{it-1} is the error correction term that captures the cointegration of the system and γ s are the short run coefficients. For y_{2t} to Granger causes y_{1t}, at least one $\gamma_{12,i}$ must be significantly different from zero and their joint F-statistic must be significantly different from zero. This same condition applies for $\gamma_{22,i}$ if y_{1t} is to Granger causes y_{2t}. Using the recursive sampling for the regressions, which maps the overtime trajectory of the causal parameters, Granger causality test statistics divided by critical value to obtain $z_{y_{1t}\rightarrow y_{2t},L}$ for the sample size L. $z_{y_{1t}\rightarrow y_{2t},L} > 1$ will

Table 3
Long-run level relationship analysis.

	Test	Form	Case	Conclusion
Oil and Gold	ARDL	Levels	Case 2	Inconsistent
			Case 3	Inconsistent
			Case 4	Inconsistent
		Logged	Case 2	Inconsistent
			Case 3	No cointegration with rare exceptions
			Case 4	No cointegration with rare exceptions
	Johansen	Levels	Case 2	Inconsistent
			Case 3	Inconsistent
			Case 4	Inconsistent
		Logged	Case 2	No cointegration with very rare exceptions
			Case 3	No cointegration with very rare exceptions
			Case 4	No cointegration with very rare exceptions
Oil and Silver	ARDL	Levels	Case 2	Cointegration with rare exceptions
			Case 3	Cointegration with rare exceptions
			Case 4	Cointegration with rare exceptions
		Logged	Case 2	Cointegration with exceptions
			Case 3	Inconsistent
			Case 4	Inconsistent
	Johansen	Levels	Case 2	Cointegration with exceptions
			Case 3	Cointegration with exceptions
			Case 4	Cointegration with exceptions
		Logged	Case 2	No cointegration with rare exceptions
			Case 3	No cointegration with rare exceptions
			Case 4	No cointegration with rare exceptions
Oil and Platinum	ARDL	Levels	Case 2	No cointegration with rare exceptions
			Case 3	No cointegration with rare exceptions
			Case 4	No cointegration with exceptions
		Logged	Case 2	No cointegration with rare exceptions
			Case 3	No cointegration with rare exceptions
			Case 4	No cointegration with rare exceptions
	Johansen	Levels	Case 2	No cointegration with exceptions
			Case 3	No cointegration with exceptions
			Case 4	No cointegration with exceptions
		Logged	Case 2	No cointegration with rare exceptions
			Case 3	No cointegration with rare exceptions
			Case 4	No Cointegration with rare exceptions

Table 4
Granger causality analysis.

Null Hypothesis	Conclusion
Gold does not Granger cause oil	Fail to reject with exceptions
Oil Does not Granger Cause Gold	Fail to reject with exceptions
Log-gold does not Granger cause log-oil	Fail to reject with rare excepetions
Log-oil does not Granger cause log-gold	Fail to reject with rare exceptions
Silver does not Granger cause oil	Fail to reject with exceptions
Oil Does not Granger Cause silver	İnconsistent
Log-silver does not Granger cause log-oil	Fail to reject with rare exceptions
Log-oil does not Granger cause log-silver	Consistently Reject with rare exceptions
platinum does not Granger cause oil	Fail to reject with exceptions
Oil Does not Granger Cause platinum	Fail to reject with exceptions
Log-platinum does not Granger cause log-oil	Fail to reject with rare exceptions
Log-oil does not Granger cause log-platinum	Fail to reject with rare exceptions

amount to the rejection the null of “y_{1t} does not Granger cause y_{2t}” which will be a confirmation of short run causality and the vice versa. A series of $z_{y_{1t}\rightarrow y_{2t},i}$ and $z_{y_{2t}\rightarrow y_{1t},i}$ for $i = L, L+1, \ldots, N-L+1$ are created to investigate the overtime stability and pattern of causality between the variables. These series are plotted against the sample ending period. Plotting the parametric estimates obtained from the different sample sized dataset amounts to capturing changes in the parameters of the bi-variate causality and testing for its consistency which is essential for prediction and policy formulation.

5. Results and discussion

5.1. Recursive ADF tests

5.1.1. Recursive ADF results of oil prices

Fig. 2 presents the recursive ADF test results for the stationarity of oil prices and its transformations. Panel-A and B depict the stationarity of prices of oil in levels with panel-A corresponding to $z_{ADF,i}$ obtained using equation-2 and panel-B corresponding to the $z_{ADF,i}$ statistics obtained using equation-3. As can be seen from Fig. 2 panel-A, oil prices at levels is conclusively non-stationary irrespective of the sample period or sample size when the regression form is as in equation-2. When a linear trend is included as in equation-3 however, a brief period of stationarity for the sample ending 1966M01 to the sample ending 1970M12 is observed. This is a clear evidence that the non-stationarity of oil prices is stable overtime irrespective of whether time trend is included or not. This same pattern is repeated when the level prices are transformed into the log prices with a momentary conclusion of stationarity for 1966M01 to 1970M12 as depicted in panel-C and D. First difference transformation of the level prices however yields a consistent conclusion of stationarity only deviating from this conclusion for the sample period ending 1973M09 to 1974M02. This pattern is repeated irrespective of whether trend is included or not for the first difference and first difference of logged oil prices as in panel-E to H. Overall, we find that the price of oil is thus concluded to be non-stationary even when the price series is log transformed. Prior to 1972 however, rare episodes of stationarity are observed when time trend is included contrary to findings of Balcilar et al. (2015) who found oil prices to be intercept stationary. Sari et al. (2010) assertion is found to have some merit given they contend trend and intercept stationarity which could have coincided with a rare episode of intercept and trend stationarity.

5.1.2. Gold price stationarity

From Fig. 3 (panel-A), gold price is mostly non-stationary with only 2 exceptions for the samples ending 1968M04 and 1968M10 when only intercept is included. In case-3 when a time trend is included, panel-B, the price of gold is consistently non-stationary and stable. In panel-C and D when the price of gold is log transformed, again only samples

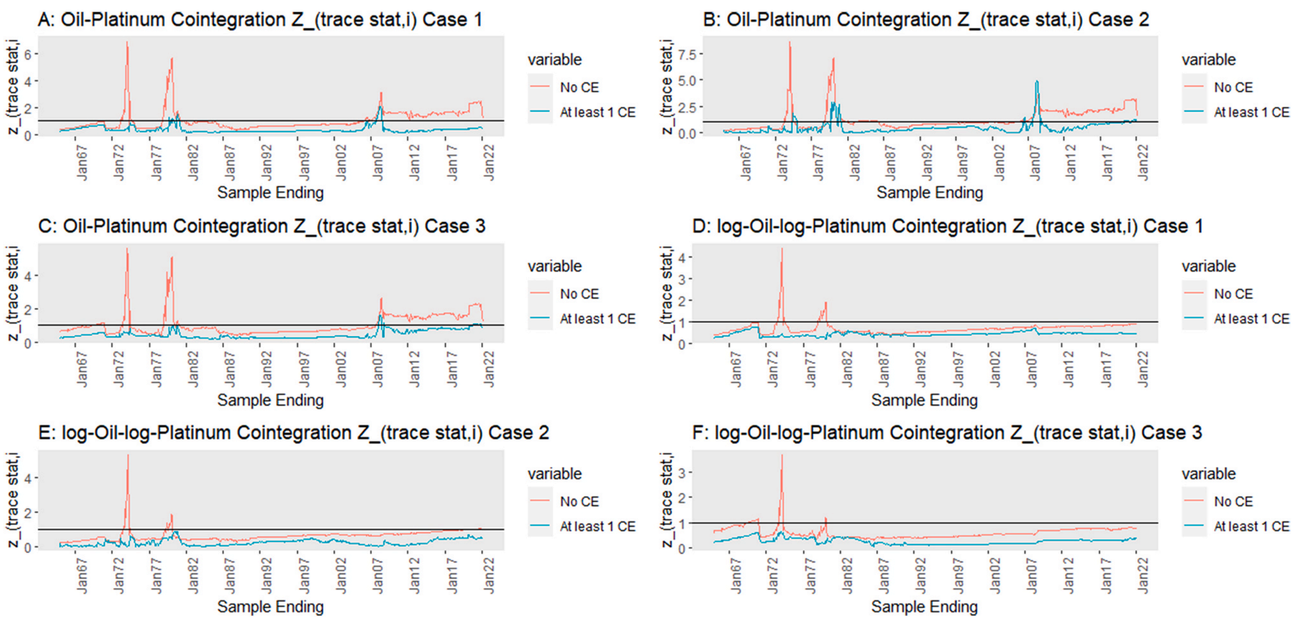

Fig. 10. Oil and silver Johansen trace statistic $z_{\lambda_{(r=-)},i}$.

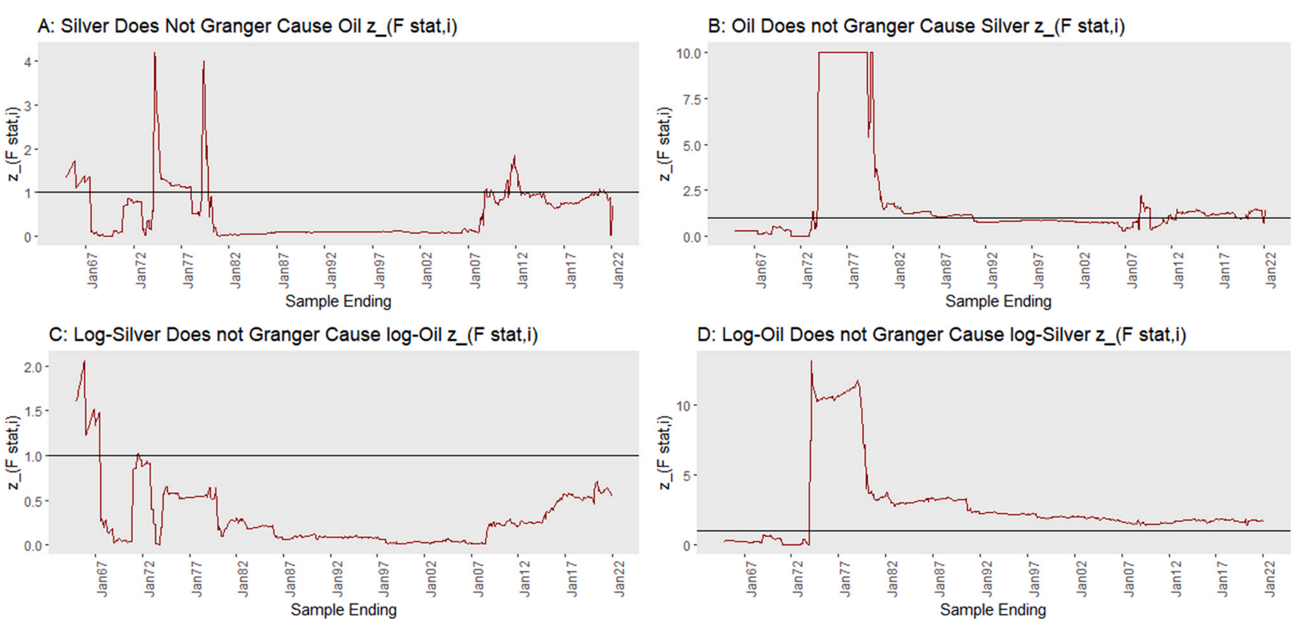

Fig. 11. Oil and silver Granger causality $z_{y_{1t}\to y_{2t},L}$.

ending 1968M04 and 1968M10 for case-2 in panel-C and only for samples ending 1968M07 for case-3 in panel-D. When differenced gold prices are used as in panel-E and F, the default is stationarity with repeated deviations from this conclusion beginning with the samples ending 1968M01 for both case-2 and 3. For samples ending after 1980M05 however, the null of unit root is consistently rejected in favour of the alternative of stationarity in both cases. Panel-G and H which depicts the stationarity of differenced log series generally conclude stationarity with minor exceptions for samples ending 1968M01 to 1968M05 and for samples ending 1968M01 to 1968M09 for case-2 and case-3 respectively. It can therefore be concluded that gold prices are consistently non-stationary and only consistently stationary when differenced log prices are taken. Thus, a random sample size and time of gold prices is likely to result in non-stationarity as found by Hassani et al. (2015) and Rathnayaka and Seneviratna (2019).

5.1.3. *Silver price stationarity*

Silver price stationarity is more ambivalent with the level prices oscillating from stationarity to non-stationary depending on the sample period and/or sample size. For silver, any sample between 1960M01 to 1992M03 yielded a conclusion of non-stationary with or without the inclusion of time trend. For samples ending 1992M03 to 2017M12, the conclusion of unit root is rejected for the level price data as depicted in Panel-A and B of Fig. 4. The log price stationarity as plotted in Panel-C and D however, indicate consistent non-stationarity for samples ending 1964M12 to 2022M03.

First differencing the price data results in consistent stationarity with momentary deviations for samples ending 1967M09 to 1967M11 and for samples ending 1979M08 to 1980M02 for Panel-E. Panel-F also shows a similar pattern with the default of stationarity with momentary deviation. Panel-G and H plot the stationarity conclusion of the differenced log prices consistently and unequivocally rejects the null of unit root in

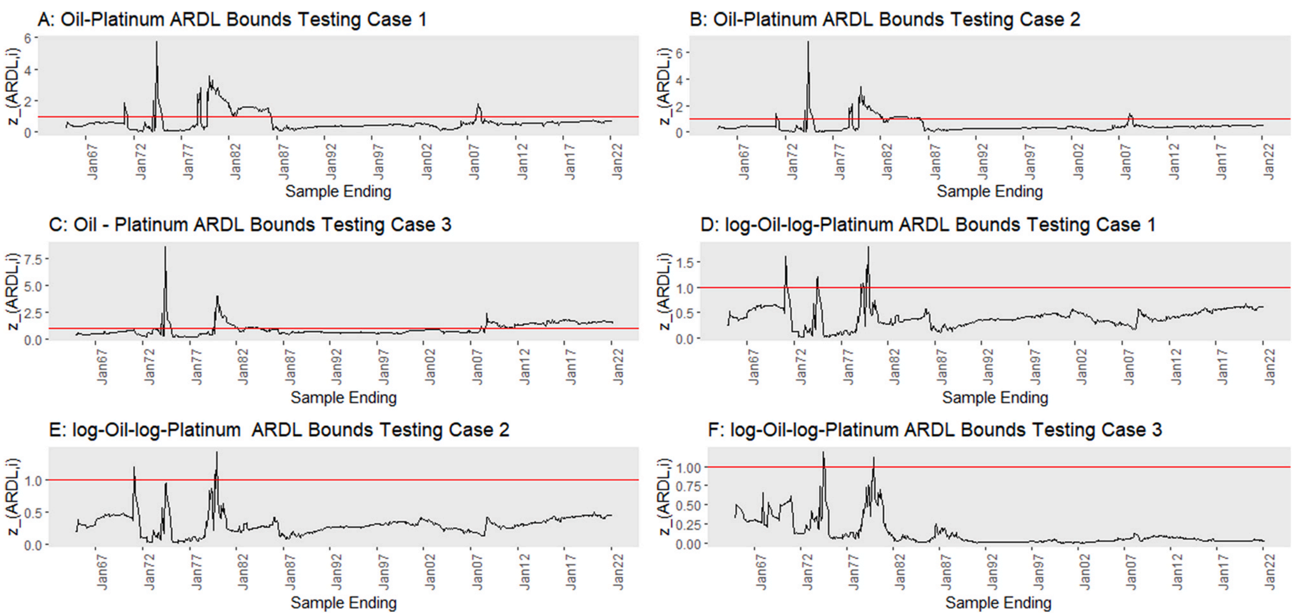

Fig. 12. Oil and platinum ARDL bounds test $z_{bt,i}$.

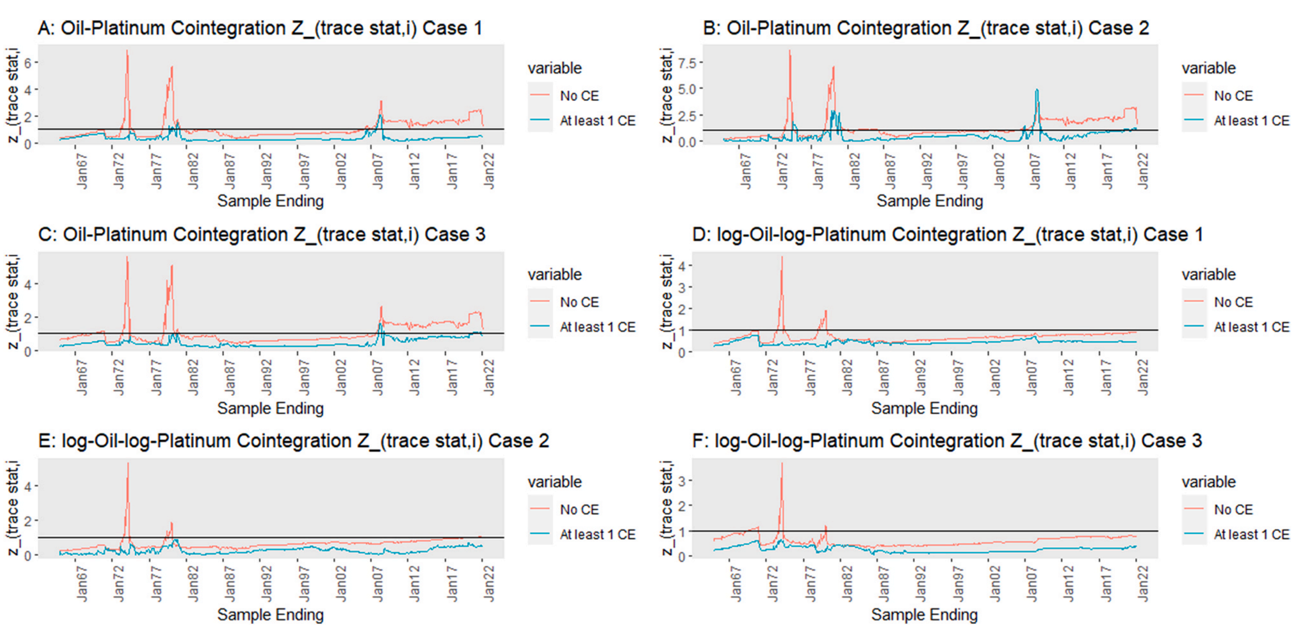

Fig. 13. Oil and platinum Johansen trace statistic $z_{\lambda_{(r=-)},i}$.

favour of stationarity. Thus, the surest transformation that yields stationarity in the price of silver is difference of log prices. This implies the findings of Adewuyi et al. (2020) and Addison and Ghoshray (2020) are rare coincidence of sampling size and time of stationarity.

5.1.4. *Platinum price stationarity*

Platinum is also consistently non-stationary both in logs and in level prices for all sample endings as depicted by Panel-A to Panel-D of Fig. 5 and summarised in Table 2. Overall, differencing the logged prices and to a lesser extent the level price data result in stationarity with rare exceptions to stationarity in the differenced level data as depicted by panel-E and F of Fig. 5. This proves that, the safest transformation of platinum prices to obtained stationarity is the differenced logged prices as in Panel-G and H. This again implies that the findings of Adewuyi et al. (2020) and Sari et al. (2010) who contend stationarity of platinum prices could not be corroborated.

5.2. *The dynamics between oil and gold prices*

The recursive ARDL bounds test and the recursive Johansen cointegration tests as detailed out in Section-IV are estimated to determine the long run relationship and the stability of the long run relationship between oil and gold prices. Also, the short run recursive Granger causalities between oil and gold prices are presented and discussed.

5.2.1. *Oil and gold price dynamics: A recursive ARDL approach*

For the recursive ARDL bounds tests, a $z_{bt,i}$ statistic greater than 1 indicates the rejection of the null of no cointegration among the variables. The long run level relationship between oil and gold is ambivalent with bounds $z_{bt,i}$ statistics lacking consistency. For panel-A to C which depicts case-1, case-2 and case-3 of $z_{bt,i}$ for level prices of oil and gold, the null of no cointegration between oil and gold prices cannot be

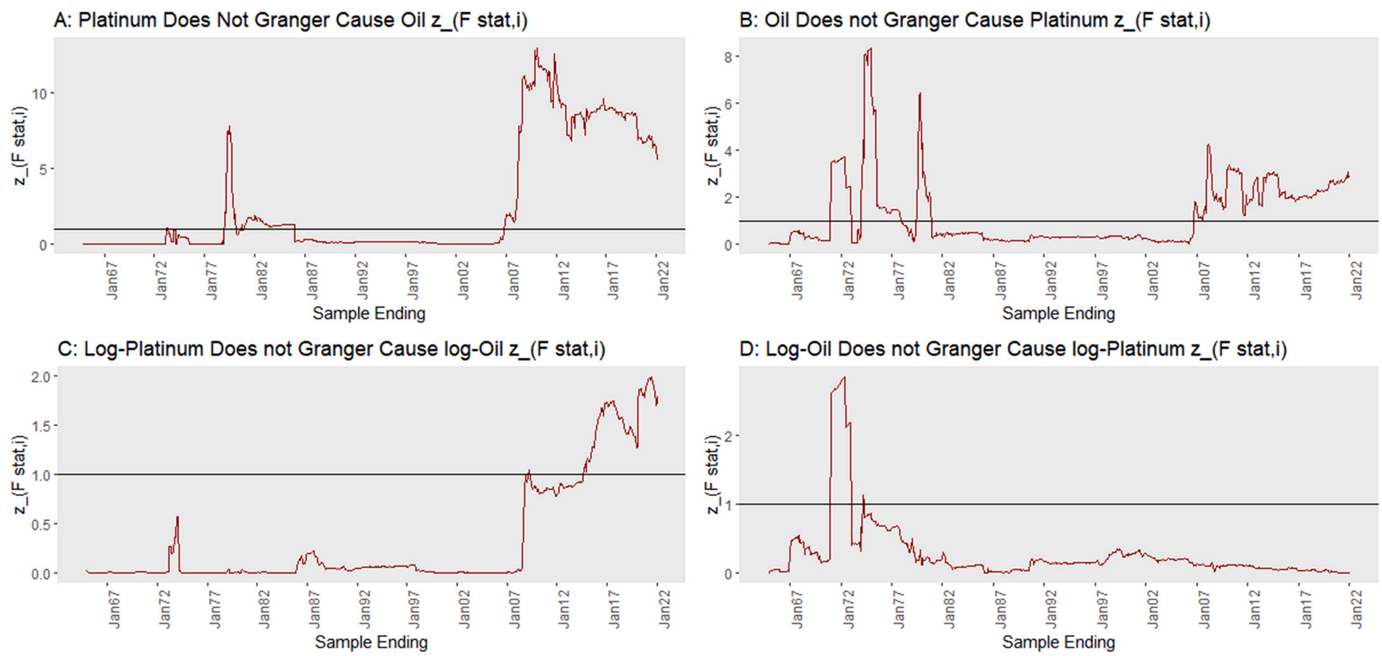

Fig. 14. Oil and platinum Granger causality $z_{y_{1t}\rightarrow y_{2t},L}$.

rejected for samples ending 1964M12 to 1968M03. From 1968M04, the conclusion swings from failing to reject to rejecting the null of no cointegration repeatedly in all 3 cases, a clear indication of non-consistency and instability in the level relationship between the non-transformed oil and gold prices. When the price data is log transformed however, case-1 displays a non-consistent and unstable level relationship as depicted in panel-D of Fig. 6. Case-2 and 3 corresponding to panel-E and F however display a consistent and stable non rejection of the null especially after 1974M12 for case-2 and after 1975M05 for case-3. This implies a verdict that the level prices of oil and gold have a sporadic switch in cointegration regimes and somewhat consistent verdict that there is no long run relationship between oil and gold prices when the price series are log transformed.

5.2.2. Oil and gold price dynamics: A recursive Johansen approach

Fig. 7 displays the cointegration relationship between oil and gold prices per Johansen trace $z_{\lambda_{(r)},i}$. From Fig. 7, all panels show inconsistencies and instabilities with rare episodes of conclusion of cointegration. In panel-A, which is case-1 (only restricted intercept), the relationship is indeterminate with both No Cointegration equation and at least 1 Cointegration equation both rejecting the null for sample endings before 1968M02. From 1968M03 however, the relationship goes into an oscillatory phase switching from cointegration to no cointegration and vice versa between oil and gold prices. A similar pattern is repeated for panel-B, and C corresponding to only unrestricted intercept in the VECM equation (Case-2) and unrestricted intercept and restricted trend in the VECM equation (Case-3). When the prices series are transformed into logs, the sporadic swings are dampened towards consistent rejection of the existence of level relationship between oil and gold prices corroborating the conclusions of ARDL bounds $z_{bt,i}$ plot in Fig. 6. This means that whereas the findings of Chang et al. (2013) is the more likely, that of Balcilar et al. (2015), and Zhang and Wei (2010) are most likely a coincidence in time and sampling size.

5.2.3. Oil and gold price dynamics: A recursive Granger causality approach

The short run relationship and its consistency between oil and gold prices as depicted by the Granger causality $z_{gold_t\rightarrow oil_t,L}$ and $z_{oil_t\rightarrow gold_t,L}$ statistics are presented in Fig. 8. From Fig. 8, the conclusions in Table 4 as regard to gold and oil Granger causality are reached. From Fig. 8 (panel-A), it can be inferred that, the short run causality from gold to oil is time and sample dependent with sample endings before 1973M02 failing to reject the null of "gold not Granger causes oil". The oil to gold causality displays a similar pattern in panel-B with sample endings before 1973M01 all failing to reject the null of no Granger causality. Sample endings beyond 1973M02 for gold to oil and beyond 1973M01 for oil to gold display an oscillatory pattern of causality with periods of causality and periods of non-causality between oil and gold. When the price series are log transformed, a similar pattern is repeated with a much stable relationship towards non-causality after 1980M11 for gold to oil and after 1982M05 for oil to gold. Thus, it is safe to conclude that, there is a transient bi-directional causality between oil and gold prices dependent on time and sample size. This bidirectional causality is lessened when the prices are log transformed with the relationship turning consistently towards non-causality in both directions after 1982M05 as can be seen from panel-C and D of Fig. 8. The conclusions drawn here is similar to that arrived at by Balcilar et al. (2019) who concluded an unstable oil-gold causal relation varying from bidirectional, unidirectional to no causality depending on the sample period.

5.3. The dynamics between oil and silver prices

This section presents and discusses the long run and short run price dynamics between gold and silver prices. Again, the results are obtained using the recursive ARDL bounds test, the recursive Johansen cointegration test and the recursive Granger causality tests as outlined in section 4.

5.3.1. Oil and silver price dynamics: A recursive ARDL approach

The bivariate long-run relationship between oil and silver as measured by the recursive ARDL bounds testing in the various formulation and price transformations are presented in Fig. 9 and summarised in Table 3. In panel-A, the plot of the formulation with "only restricted intercept and no trend" (Case1), all samples ending 1964M12 to 1973M06 fail to reject the null of no cointegration with the exception of the sample ending 1968M06. The samples ending 1973M06 to 1980M02 is inconsistent with several swings from rejection to failing to reject the null of no cointegration. However, from the sample ending 1980M03 however, the null of no level relationship between oil and silver prices has been consistently rejected till the last sample period, 2022M03. A similar pattern is seen in Panel-B and C which represent case-2 and 3 respectively, with a failure to reject the null of no cointegration for all sample endings before 1973M06. Afterwards, there are several swings in conclusion until 1980M02. From then, the null of no cointegration has been consistently rejected using the level price data. However, when the logged prices are used in the estimation as in panel-D to F, all 3 cases imply an inconsistent and sporadic conclusion with swings from rejecting to failing to reject the null.

5.3.2. Oil and silver price dynamics: A recursive Johansen approach

The Johansen cointegration tests $z_{\lambda_{(r=-),i}}$ plots largely corroborate the conclusion of ARDL $z_{bt,i}$ plots. In Fig. 10 (Panel-A to C) which depicts the level relationship of the level prices for all 3 cases, the null of No Cointegration equation could not be rejected consistently with sporadic exceptions for samples before 1979M11. After 1979M11 however, the price of oil and silver is found to be cointegrated with the exceptions of samples ending 2008M04, 2008M05, 2008M06 and 2008M07.

Again when the level prices are log transformed, the relationship breaks down and become inconsistent as can be seen from panel-D to F of Fig. 10. It is therefore safe to say that to a large extent, the level prices of oil and silver are cointegrated irrespective of sample size and sample period with rare exceptions. This conclusion contradicts the findings of Shahbaz et al. (2022).

5.3.3. Oil and silver price dynamics: A recursive Granger causality approach

In the short-run, the causality of the level prices of oil and silver is inconsistent and sporadic as depicted in Fig. 11 panel-A and B. In panel-A, the conclusion is largely a non-rejection of the null of “silver prices not Granger cause oil prices” with rare exceptions. The same conclusion is arrived at with panel-B which depicts inconsistency by failing to reject to rejecting of the null of “oil prices not Granger cause silver prices”. When the price series are log transformed as in panel-C and D, the regime changes dampen with a consistent non rejection of the null of “log of silver prices not Granger causing the log of oil prices” after 1971M08 whiles consistently rejecting the null of “log of oil prices not Granger cause log of silver prices” after 1973M11 as depicted in panel-D. This implies an uni-directional causality from the log prices of oil to log prices of silver finding no support for the strong bidirectional causality documented by Sari et al. (2010).

5.4. The dynamics between oil and platinum prices

This section presents and discusses the long run and short run price dynamics between oil and platinum prices. Again, the results are obtained using the recursive ARDL bounds test, the recursive Johansen Cointegration tests and the recursive Granger causality tests as outlined in section 4.

5.4.1. Oil and platinum price dynamics: A rolling regression ARDL approach

The long-run relationship between oil and platinum prices is inconsistent for case-1 to 3 of ARDL bounds test as is evident from Fig. 12 panel-A to C. Despite the swings and inconsistencies, the evidence leans towards the non-rejection of the null of “no level relationship between level prices of oil and platinum”. The non-rejection of the null is more pronounced when the price series are log transformed as in panel-D to F with rare spikes of deviations from the non-rejection of the null of “no cointegration” between oil and platinum prices. This implies that the oil-platinum price relationship is leaning towards no cointegration per ARDL $z_{bt,i}$.

5.4.2. Oil and platinum price dynamics: A recursive Johansen approa

The conclusions from the recursive Johansen cointegration test plots for oil and platinum prices equally lean towards no cointegration. From panel-A to C of Fig. 13, only sample endings after 2008M08 contend cointegrating relationship between oil and platinum prices. Panel-D to F which depicts the log transformed series is decisively failing to reject the null of no cointegrating relationship between oil and platinum prices. In essence, ARDL $z_{bt,i}$ and Johansen $z_{\lambda_{(r=-),i}}$ plots lean towards the conclusion of no stable and consistent level relationship between oil and platinum prices.

5.4.3. Oil and platinum price dynamics: A recursive Granger causality approach

The short-run Granger causality between oil and platinum is presented in Fig. 14 and summarised in Table 4. Panel-A and B of Fig. 14, which depicts the Granger causality of the level price series go through several regimes of causality and non-causality between oil and platinum. However, when the price series are log transformed, a somewhat consistent non rejection of the null of “platinum not Granger causes oil prices” is reached for sample endings before 2014M09. After 2014M09 however, causality from log platinum prices to log oil prices appears as depicted in panel-C of Fig. 14. Panel-D asserts more strongly a non-causality of log oil prices to log platinum prices with sporadic rejection of the null of log of oil not Granger causing log of platinum for samples ending in 1971M01 to 1972M12. This means to a larger extent, price of oil and its log transformations do not Granger cause the price of platinum and its log transformation. But after 2014M09, there is one-way unidirectional causality from log of platinum prices to log of oil prices.

6. Concluding remarks

Quantitative economic and/or econometric models have become ubiquitous in financial, economic and policy circles. These models are generally based on estimated parameters from historical data assumed to adhere to some strict statistical and stochastic properties. One of such properties is stationarity. Stationarity means a time invariant mean and volatility of a variable which is fundamental to the analysis of time series and to some extent panel data. Once evidence of stationarity is found using historical data, it is assumed to persist and policies and forecast are made based on this assumption. This study sort to determine the validity of the stability and consistency assumption in the stationarity and bivariate relationship between oil and gold, oil and silver and oil and platinum prices. This is accomplished by using monthly price data of oil, gold, silver and platinum from January 1960 to March 2022. A recursive ADF test of stationarity, a recursive ARDL bounds test, a recursive Johansen trace test and a recursive Granger causality test is used to determine the subsample stability and consistency in the stationarity and bivariate relationship between oil and the metals.

From the analysis, oil and gold prices are found to be non-stationary with rare exceptions to this conclusion even when the level price series are log transformed. 1st differencing the logged prices for gold and 1st differencing the logged or level price series for oil however yielded consistent stationarity with rare exceptions. The price series of silver in levels was found to go through multiple regimes and stationarity and non-stationarity whiles the logged silver prices was consistently non-stationary. The differenced level prices of silver resulted in stationarity with rare exceptions whiles the logged differenced series yielded a consistent and stable stationarity. The level and logged price of platinum yielded a consistent and stable non-stationarity conclusion changing to stationarity with exceptions when the level prices is differenced and consistent stationarity when the logged prices is differenced. The long run relationship between oil and gold is inconsistent switching from cointegration to no cointegration with the log transformation resulting in consistent rejection of cointegration with rare exceptions. Level oil and silver prices are found to be cointegrated with rare exceptions whiles the log transformations result in inconsistent rejecting of cointegration. Level platinum prices and their log transformation was found to be consistently not cointegrated with oil prices and log of oil prices with rare exceptions. In short run, none of the metals is found to Granger cause oil prices. However, log of oil price is found to Granger cause log of silver prices with rare exceptions.

The inconsistent stationarity results of oil, gold, silver, and platinum prices as found in this study implies that researchers must be circumspect and tempered in their conclusions as any conclusions drawn from any sample study could be a coincidence of sample period and sample size and thus non-generalisable. On the bi-variate relationship in the long run, only silver prices are found to be cointegrated with oil while logged silver prices are inconsistently cointegrated with logged oil prices. Also, in short-run, only log of oil prices is found to Granger cause the log of silver prices. This implies that, whereas cointegration can generally be considered to exist between silver and oil prices, logged silver prices and logged oil prices are cointegrated conditional on the sample period and sample size. This again means that generalisations should be tempered as should be inference of causality other than from oil prices to silvers prices. It is thus recommended that researchers and policy makers desist as much as possible from extrapolating statistical findings in general and the price and inter-price dynamics of oil, gold, silver, and platinum into the future. Even when extrapolation is required, price series of the non-stationary and unstable variables and inter-variable relations should be modelled explicitly as unstable. The commodities explored here are all natural, and depletable resources. Future researchers may explore the impact of reserves and green alternatives on the price and inter-price stability of these commodities.

CRediT authorship contribution statement

Asad ul Islam Khan: Conceptualization, Formal analysis, Software. **Muhammad Shahbaz:** Conceptualization, Data curation. **Ayuba Napari:** Conceptualization, Visualization, Writing - original draft.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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