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The Non-Isolated Resolving Number of Some Corona Graphs

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Abstract. An ordered set $W = \{w_1, w_2, \dots, w_k\} \subseteq V(G)$ and a vertex v in a connected graph G, the representation of v with respect to W is the ordered k-tuple $r(v|W) = \big(d(v,w_1), d(v,w_2), \dots, d(v,w_k)\big)$, where $d(x,y)$ represents the distance between the vertices x and y in G. The set W is called a resolving set for G if every vertex of G has distinct representations. A resolving set with the minimum number of vertices is called a basis for G and its cardinality is called the metric dimension of G, denoted by $\dim(G)$. A resolving set W is called a non-isolated resolving set if the induced subgraph $\langle W \rangle$ has no isolated vertices. The minimum cardinality of a non-isolated resolving set of G is called the non-isolated resolving number of G, denoted by $nr(G)$. The corona product between a graph G and a graph H, denoted by $G \odot H$, is a graph obtained from one copy of G and $|V(G)|$ copies $H_1, H_2, \dots, H_n$ of H such that all vertices in H_i are adjacent to the i-th vertex of G. We study the non-isolated resolving sets of some corona graphs. We determine $nr(G \odot H)$ where G is any connected graph and H is a complete graph, a cycle, or a path.

1. Introduction

All graphs in this paper are finite, simple, and connected. Let $G = (V, E)$ be a graph. The *distance* $d(u,v)$ between two vertices u and v in a connected graph G is the length of a shortest $u - v$ path in G. For $W = \{w_1, w_2, \dots, w_k\} \subseteq V(G)$ and a vertex v in a connected graph G, the *representation* of v with respect to G is the ordered k-tuple $r(v|W) = \big(d(v,w_1), d(v,w_2), \dots, d(v,w_k)\big)$. If every distinct vertices $x, y \in V(G)$ satisfy $r(x|W) \neq r(y|W)$, then W is called a resolving set. A resolving set with the minimum cardinality is called a *basis* of G. Its cardinality is called the *metric dimension* of G, denoted by $\dim(G)$.

The metric dimension problem was first studied by Harary and Melter [8] and independently by Slater [15]. Slater considered the minimum resolving set of a graph as the location of the placement of a minimum number of sonar/loran detecting devices in a network. Thus, the position of every vertex in the network can be uniquely describe by its distances to the devices in the set.

The metric dimension problem is a difficult problem. Garey and Johnson [7] have shown that determining the metric dimension of any graph is an NP-Problem. However, some results for certain classes of graph has been obtained, which can be seen in [2,5,9,10,11,12,13,16].

Now in this paper, let us consider the other version of the resolving set problem, which is called a non-isolated resolving number. In this version, if an induced subgraph of G by a resolving set W does not contain an isolated vertex, then W is called a *non-isolated resolving set*. The non-isolated resolving set with minimum cardinality is called an *nr*-set. The non-isolated resolving number of G, denoted by $nr(G)$, is the cardinality of nr-set of G.

This non-isolated resolving problem was introduced by Chitra and Arumugam [6]. They have proven that $nr(G) \leq 2\dim(G)$. They have characterized all connected graphs of order $n \geq 3$ with $nr(G) = n-1$. They have also determined an exact value of the non-isolated resolving number of some graphs included paths, complete graphs, friendship graphs, complete bipartite graphs, and the Cartesian product of graphs.

In another paper, Yunika *et al.* [17] determined the non-isolated resolving number of some exponential graphs. Meanwhile, Avadayappan *et al.* [1] determined the non-isolated resolving number of double broom graphs, the join of a complete graph and a path.

In this paper, we consider the corona product between two connected graphs G and H. The *corona* graph $G\odot H$ is a graph obtained from one copy of G and $V(G)$ copies $H_1, H_2, \dots, H_{|V(G)|}$ of H where every vertex of H_i is adjacent to the i-th vertex of G We recall that a graph $G+H$ is a graph with $V(G+H) = V(G) \cup V(H)$ and $E(G+H) = E(G) \cup E(H) \cup \{xy|x \in V(G), y \in V(H)\}$. Note that $G\odot H$ contains an induced subgraph which is isomorphic to $K_1 + H$. Some of our results provide a connection between $nr(G\odot H)$ and $\dim(K_1+H)$. In order to prove that, we use the following usefull lemma, which has been proved in [14].

Lemma 1 [14] Let Q be a connected graph. Then there exists a basis S of $G+K_1$ such that $S \subseteq V(G)$.

2. Main Results

In this section, we study the non-isolated resolving set of $G\odot H$ where G is a connected graphs and H is either a complete graph, a path, or a cycle as stated in Theorem 1,2,3, respectively.

Theorem 1 Let m and n be two positive integers. Let G be a connected graph G of order $n \geq 1$. Then

$$nr(G\odot K_m) = \begin{cases} nm, & \text{if m = 2,} \\ n(m-1), & \text{if } m \geq 3. \end{cases}$$

Proof.

Let $V(G) = \{a_1, a_2, \dots, a_n\}$ and $H = (G\odot K_m)$, where $V(H) = V(G) \cup \{v_i^l v_j^l | 1 \leq l \leq n, 1 \leq i < j \leq m\}$ and $E(H) = E(G) \cup \{v_i^l v_j^l | 1 \leq l \leq n, 1 \leq i < j \leq m\} \cup \{a_l v_j^l | 1 \leq l \leq n, 1 \leq i < j \leq m\}$. We distinguish two cases.

Case 1.1 : $m = 2$

For $l \in \{1,2,\dots,n\}$, we define the set $W_l = \{v_1^l, v_2^l\}$. Let $W = \bigcup_{l=1}^{n} W_l$. Note that $|W| = 2n$. Since $v_1^l v_2^l \in E(H)$, it is clear that W does not contain an isolated vertex.

Now, we will show that W is a resolving set of H. Let x and y be two distinct vertices in $V(H) - W$. Then x and y are in G. Let $x = a_p$ $x = a_p$ and $y = a_q$ for some $p, q \in \{1,2,\dots,n\}$ with $p \neq q$. Since $d(x, v_1^p) = 1 \neq 2 \leq d(y, v_1^p), r(x|W_p) \neq r(y|W_p)$. It implies that $r(x|W) \neq r(y|W)$.

By contradiction, suppose that $nr(G\odot K_m) \leq 2n-1$.

Let W be an nr-set of $G\odot K_m$ with $|W| \leq 2n-1$. For $l \in \{1,2,\dots,n\}$, let $W_l = \{v_i^l \in W \middle| i \in \{1,2,\dots,m\}\}$. Since $|W| \leq 2n-1$, there exists $l \in \{1,2,\dots,n\}$ such that $|W_l| \leq 1$. If $|W_l| = 0$, then for $w \in W$ $d(v_1^l, w) = d(v_2^l, w)$, which implies $r(v_1^l|W) = r(v_2^l|W)$, a contradiction. If $|W_l| = 1$, then W contains an isolated vertex, a contradiction.

Case 1.2 : $m \geq 3$

For $l \in \{1,2,\dots,n\}$, we define the set $W_l = \{v_1^l, v_2^l, \dots, v_{m-1}^l\}$. Let $W = \bigcup_{l=1}^{n} W_l$. Note that $|W| = n(m-1)$. Since $v_i^l v_j^l \in E(H)$ for $1 \leq i < j \leq m-1$, it is clear that W does not contain an isolated vertex.

Now, we show that W is a resolving set of H. Let x and y be two distinct vertices in $V(H) - W$. We distinguish three subcases.

Subcases 1.2.1 :
Let $x = v_m^p$ and $y = v_m^q$ for $p, q \in \{1,2, \dots, n\}$ with $p \neq q$. Since $d(x, v_1^p) = 1 \neq 3 \leq d(y, v_1^p)$, we have $r(x|W_p) \neq r(y|W_p)$.
Subcases 1.2.2 :
Let $x = a_p$ and $y = a_q$ for $p, q \in \{1,2, \dots, n\}$ with $p \neq q$. Since $d(x, v_1^p) = 1 \neq 2 \leq d(y, v_1^p)$, we have $r(x|W_p) \neq r(y|W_p)$.
Subcases 1.2.3:
Let $x = v_m^p$ and $y = a_q$ for $p, q \in \{1,2, \dots, n\}$. Since $d(x, v_1^p) = 1 \neq 2 \leq d(y, v_1^p)$, we obtain $r(x|W_p) \neq r(y|W_p)$.
All subcases above imply that $r(x|W) \neq r(y|W)$.
Now, we will prove that $nr(G \odot K_m) \geq n(m-1)$. Let W be an nr-set of $G \odot K_m$. For every $l \in \{1,2, \dots, n\}$, let $W_l = \{v_i^l \in W | i \in \{1,2, \dots, m\}\}$. We have claim that $|W_l| \geq (m-1)$. Otherwise, we have two vertices v_s^l and v_t^l for some $s, t \in \{1,2, \dots, m\}$ such that $v_s^l \notin W_l$ and $v_t^l \notin W_l$. Since $d(v_s^l, x) = d(v_t^l, x)\ \forall x \epsilon W$, we obtain $r(v_s^l|W) = r(v_t^l|W)$, a contradiction. Hence, $|W| \geq n(m-1)$.
Next, we consider the corona graph $H = G \odot Q_n$, where G is any connected graph and Q_n is a path or a cycle with order n. Let S^l be a set of two or more vertices of Q_n^l. Let $v_i^l, v_j^l \in S^l$ be two distinct vertices of Q_n^l. Let $P(v_i^l, v_j^l)$ be a path in Q_n^l from v_i^l to v_j^l. We define a *gap* between v_i^l and v_j^l as $V\left(P(v_i^l, v_j^l)\right) - \{v_i^l, v_j^l\}$ where every vertex in a gap is not element of S^l. The vertices v_i^l and v_j^l we called as end points of gap between v_i^l and v_j^l. Two different gaps are called *neighboring gaps* if they have common end point. In case Q_n^l is a cycle, if $|S^l| = r$, then S^l has r gaps. In case Q_n^l is a path, if $|S^l| = r - 1$, then S^l has r gaps. Note that, for both cases, some of gaps maybe empty. This definition was first introduced by Buczkowski *et al.* [3] to prove the metric dimension of the wheel graph. In addition M. Bača *et al.* [2] using this gap technique to prove metric dimensions of complete bipartite graph minus its Hamiltonian cycle.
Now, let $V(H_l) = V(\{a_l\} + Q_n^l)$, where $a_l \in V(G), Q_n^l \subseteq Q_n$ and W_l be basis of H_l. We observe the following three facts.

(i) *Every gap of W_l contains at most three vertices.* Otherwise, there is a gap containing four vertices $v_j^l, v_{j+1}^l, v_{j+2}^l, v_{j+3}^l$ of Q_n^l, where $1 \leq j \leq n, 1 \leq l \leq m$. However, $r(v_{j+1}^l|W_l) = r(v_{j+2}^l|W_l) = (2,2, \dots, 2)$, a contradiction.
(ii) *At most one gap of W_l contains three vertices.* Otherwise, there exist distinct two gaps $\{v_j^l, v_{j+1}^l, v_{j+2}^l\}$ and $\{v_k^l, v_{k+1}^l, v_{k+2}^l\}$. However, $r(v_{j+1}^l|W_l) = r(v_{k+2}^l|W_l) = (2,2, \dots, 2)$, a contradiction.
(iii) *If a gap of W_l contains at least two vertices, then any neighboring gaps contain at most one vertex.* Otherwise, there exist five consecutive vertices $v_j^l, v_{j+1}^l, v_{j+2}^l, v_{j+3}^l, v_{j+4}^l$ of Q_n^l, such that v_{j+2}^l is the only vertex of W_l. However $r(v_{j+1}^l|W_l) = r(v_{j+3}^l|W_l)$, a contradiction.

Suppose now that W_l is any set of vertices (a basis or not) of Q_n^l that satisfies (i)-(iii), and let v be any vertex of $V(H_l) - W_l$. There are four possibilities.

(1) *v belongs to a gap of size 1 of* W_l. Let v_i^l and v_j^l be the neighboring vertices of W_l that determine this gap. Then v is adjacent to v_i^l and v_j^l and has distance 2 from all other vertices of W_l. Since $n \geq 7$, no other vertices of H_l has this property and so $r(v|W_l) \neq r(x|W_l)$ for $v \neq x$.
(2) *v belongs to a gap of size 2 of* W_l. Then we may assume that $v_j^l, v_{j+1}^l = v, v_{j+2}^l, v_{j+3}^l$ are vertices of Q_n^l, where $v_{j+1}^l, v_{j+3}^l \in W_l$ and $v_{j+2}^l \notin W_l$. Then v is adjacent to v_j^l and has distance 2 from

all other vertices of W_l. By property (iii), only v has this property and so $r(v|W_l) \neq r(x|W_l)$ for $v \neq x$.

(3) *v belongs to a gap of size 3 of* W_l. Then there exist vertices $v_j^l, v_{j+1}^l, v_{j+2}^l, v_{j+3}^l, v_{j+4}^l$ of Q_n^l, only v_{j+1}^l and v_{j+4}^l which of belong to W_l. Assume first that $v = v_{j+1}^l$. Then v is adjacent to v_j^l and has distance 2 from all other vertices of W_l. By property (iii), v is the only vertex of H_l with this property and so $r(v|W_l) \neq r(x|W_l)$ for $v \neq x$. Next, we assume that $v = v_{j+2}^l$. Then $r(v|W_l) = (2,2,2, \dots,2)$. By properties (i) and (ii), no other vertex of H_l has this representation.

(4) $u = a_l$. Then $r(u|W_l) = (1,1, \dots,1)$ and is u the only vertex of H_l with this representation.

Consequently, any set W having properties (i)-(iii) is a resolving set of H.

The following lemma will be used to prove the upperbound of the Theorem 2 and Theorem 3.

Lemma 2 For $n \geq 7$, let Q_n be a path or a cycle. Then very basis S of $K_1 + Q_n$ contains an isolated vertex.

Proof.

Suppose there is a basis S of $K_1 + Q_n$ that does not contain an isolated vertex.

Case 2.1: $|S|$ is even.

Let $|S| = 2q$

Subcases 2.1.1 : Q_n is a cycle.

For some integer $q \geq 1$. By (iii) at most q gaps contain more than one vertex and, by (i) and (ii), all contain at most two vertices except possibly one containing three vertices. So, the number of vertices belonging to the gaps of S is at most $2q + 1$. Since S does not contain an isolated vertex, we have q empty gaps. Hence $n - 2q \leq 2q + 1$, which implies that $|S| = 2q \geq \left\lceil\frac{n-1}{2}\right\rceil$. In [3], Buczkowski *et al.* has been proven that $\dim(C_n + K_1) = \left\lfloor\frac{2n+2}{5}\right\rfloor$. Since $\left\lceil\frac{n-1}{2}\right\rceil = \left\lceil\frac{5n-5}{10}\right\rceil > \left\lfloor\frac{4n+4}{10}\right\rfloor = \left\lfloor\frac{2n+2}{5}\right\rfloor$, we have a contradiction.

Subcases 2.1.2 : Q_n is a path.

For some integer $q \geq 1$. By (iii) at most q gaps contain more than one vertex and, by (i) and (ii), all contain at most two vertices except possibly one containing three vertices. So, the number of vertices belonging to the gaps of S is at most $2q - 1$. Since S does not contain an isolated vertex, we have q empty gaps. Hence $n - 2q \leq 2q - 1$, which implies that $|S| = 2q \geq \left\lceil\frac{n+1}{2}\right\rceil$. In [4], Cáceres *et al.* has been proven that $\dim(P_n + K_1) = \left\lfloor\frac{2n+2}{5}\right\rfloor$. Since $\left\lceil\frac{n+1}{2}\right\rceil = \left\lceil\frac{5n+5}{10}\right\rceil > \left\lfloor\frac{4n+4}{10}\right\rfloor = \left\lfloor\frac{2n+2}{5}\right\rfloor$, we have a contradiction.

Case 2.2 : $|S|$ is odd.

Let $|S| = 2q + 1$.

Subcases 2.2.1: Q_n is a cycle.

For some integer $q \geq 1$. By (iii) at most q gaps contain more than one vertex and, by (i) and (ii), all contain at most two vertices except possibly one containing three vertices. So, the number of vertices belonging to the gaps of S is at most $2q + 1$. Since S does not contain an isolated vertex, we have $q + 1$ empty gaps. Hence $n - 2q - 1 \leq 2q + 1$, which implies that $|S| = 2q \geq \left\lceil\frac{n-2}{2}\right\rceil$. In [3], Buczkowski *et al.* has been proven that $\dim(C_n + K_1) = \left\lfloor\frac{2n+2}{5}\right\rfloor$. Since $\left\lceil\frac{n-2}{2}\right\rceil = \left\lceil\frac{5n-10}{10}\right\rceil > \left\lfloor\frac{4n+4}{10}\right\rfloor = \left\lfloor\frac{2n+2}{5}\right\rfloor$, we have a contradiction.

Subcases 2.2.2: Q_n is a path.

For some integer $q \geq 1$. By (iii) at most q gaps contain more than one vertex and, by (i) and (ii), all contain at most two vertices except possibly one containing three vertices. So, the number of vertices belonging to the gaps of S is at most $2q - 1$. Since S does not contain an isolated vertex, we have $q + 1$ empty gaps. Hence $n - 2q - 1 \leq 2q - 1$, which implies that $|S| = 2q + 1 \geq \left\lceil\frac{n+2}{2}\right\rceil$. In [4], Cáceres *el* *al.* has been proven that $\dim(P_n + K_1) = \left\lfloor \frac{2n+2}{5} \right\rfloor$. Since $\left\lceil \frac{n+2}{2} \right\rceil = \left\lceil \frac{5n+10}{10} \right\rceil > \left\lfloor \frac{4n+4}{10} \right\rfloor = \left\lfloor \frac{2n+2}{5} \right\rfloor$, we have a contradiction.

Theorem 2 For $n \geq 7$, let G be a connected graph of order $m \geq 1$ then,

$$nr(G \odot C_n) = [\dim(K_1 + C_n) + 1]m$$

Proof.

Let $V(G) = \{a_1, a_2, \dots, a_m\}$, $V(C_n) = \{v_1, v_2, \dots, v_n\}$. For $n \geq 7$, let $H = (G \odot C_n)$, $V(H) = V(G) \cup \{v_i^l | 1 \leq i \leq n, 1 \leq l \leq m\}$ and $E(H) = E(G) \cup \{v_i^l v_{i+1}^l, v_1^l v_n^l | 1 \leq i < j \leq n-1\} \cup \{a_l v_j^l, 1 \leq l \leq m\}$. Let $V(K_1 + C_n) = \{v_0\} \cup V\{C_n\}$ and $E(K_1 + C_n) = E(C_n) \cup \{v_0 v_i | 1 \leq i \leq n\}$. Let B be a basis of $K_1 + C_n$. In [14] it is proven that there exists a basis B of $K_1 + Q$ for a connected graph Q, such that all vertices of B are from Q.

For $l \in \{1,2,\dots,m\}$, we define the set $W_l = \{v_i^l | v_i \in B\} \cup \{a_l\}$. Note that, $|W_l| = \dim(K_1 + C_n) + 1$. Let $W = \bigcup_{i=1}^{m} W_l$. Since $v_i^l a_l \in E(H)$, then W does not contain an isolated vertex.

Now, we will show that W is a resolving set of H. Let x and y be two different vertices in $V(H) - W$.

i. Let $x = v_a^l$ and $y = v_b^l$, with $a, b \in \{1,2,\dots,n\}$, $a \neq b$. Since x and y are the vertices in a copy of $K_1 + C_n$ and B is a basis of $K_1 + C_n$, then x and y resolve by W_l. Therefore, $r(x|W) \neq r(y|W)$.

ii. Let $x = v_a^l$ and $y = v_b^p$, with $a, b \in \{1,2,\dots,n\}$, $l, p \in \{1,2,\dots,m\}$ $l \neq p$. Since $d(x, a_l) = 1 \neq 2 \leq d(y, a_l)$ then $r(x|W) \neq r(y|W)$.

By contradiction, suppose that $nr(G \odot C_n) \leq [\dim(K_1 + C_n) + 1]m - 1$.

Let W be an nr-set of H with $|W| \leq [\dim(K_1 + C_n) + 1]m - 1$. For $l \in \{1,2,\dots,m\}$, let $W_l = \left\{v_i^l, a_l \in W \middle| i \in \{1,2,\dots,m\}\right\}$. Then there exists $l \in \{1,2,\dots,m\}$ such that $|W_l| \leq \dim(K_1 + C_n)$. Since an induced subgraph of H by $\{a_l, v_i^l | 1 \leq i \leq n\}$ is isomorphic to $K_1 + C_n$, say H_l, then $|W_l|$ must be $\dim(K_1 + C_n)$. So, it is clear that every two different vertices in H_l, has different representation with respect to W_l. However, by Lemma 2, every basis S of $K_1 + C_n$ contains an isolated vertex. Therefore, we have a contradiction.

Theorem 3 For $n \geq 7$, let G be a connected graph of order $m \geq 1$, then

$$nr(G \odot P_n) = [\dim(K_1 + P_n) + 1]m.$$

Proof.

Let $V(G) = \{a_1, a_2, \dots, a_m\}$, $V(P_n) = \{v_1, v_2, \dots, v_n\}$. For $n \geq 7$, let $H = (G \odot P_n)$, $V(H) = V(G) \cup \{v_i^l | 1 \leq i \leq n, 1 \leq l \leq m\}$ and $E(H) = E(G) \cup \{v_i^l v_{i+1}^l | 1 \leq i < j \leq n-1\} \cup \{a_l v_j^l, 1 \leq l \leq m\}$. Let $V(K_1 + P_n) = \{v_0\} \cup V\{P_n\}$ and $E(K_1 + P_n) = E(P_n) \cup \{v_0 v_i | 1 \leq i \leq n\}$. Let B be a basis of $K_1 + P_n$. In [14] it is proven that there exists a basis B of $K_1 + Q$ for a connected graph Q, such that all vertices of B are from Q.

For $l \in \{1,2,\dots,m\}$, we define the set $W_l = \{v_i^l | v_i \in B\} \cup \{a_l\}$. Note that, $|W_l| = \dim(K_1 + P_n) + 1$. Let $W = \bigcup_{i=1}^{m} W_l$. Since $v_i^l a_l \in E(H)$, then W does not contain an isolated vertex.

Now, we will show that W is a resolving set of H. Let x and y be two different vertices in $V(H) - W$.

i. Let $x = v_a^l$ and $y = v_b^l$, with $a, b \in \{1,2,\dots,n\}$, $a \neq b$. Since x and y are the vertices in a copy of $K_1 + P_n$ and B is a basis of $K_1 + P_n$, then x and y resolve by W_l. Therefore, $r(x|W) \neq r(y|W)$.

ii. Let $x = v_a^l$ and $y = v_b^p$, with $a, b \in \{1,2,\dots,n\}$, $l, p \in \{1,2,\dots,m\}$ $l \neq p$. Since $d(x, a_l) = 1 \neq 2 \leq d(y, a_l)$ then $r(x|W) \neq r(y|W)$.

By contradiction, suppose that $nr(G \odot P_n) \leq [\dim(K_1 + P_n) + 1]m - 1$.

Let W be an nr-set of H with $|W| \leq [\dim(K_1 + P_n) + 1]m - 1$. For $l \in \{1,2,\dots,m\}$, let $W_l = \left\{v_i^l, a_l \in W \middle| i \in \{1,2,\dots,m\}\right\}$. Then there exists $l \in \{1,2,\dots,m\}$ such that $|W_l| \leq \dim(K_1 + P_n)$. Since an induced subgraph of H by $\{a_l, v_i^l | 1 \leq i \leq n\}$ is isomorphic to $K_1 + P_n$, say H_l, then $|W_l|$ must be $\dim(K_1 + P_n)$. So, it is clear that every two different vertices in H_l, has different representation with respect to W_l. However, by Lemma 2, every basis S of $K_1 + P_n$ contains an isolated vertex. Therefore, we have a contradiction.

3. Conclusion

In this paper, we have studied non-isolated resolving set of the corona product $G \odot H$ where G is any connected graphs and H is complete graph, a cycle or a paths. We obtain an exact value of non-isolated resolving number of them.

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GENERAL COMMENTS

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