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Article

Non-Isolated Resolving Sets of Corona Graphs with Some Regular Graphs

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Abstract: Let G be a connected, simple, and finite graph. For an ordered set $W = \{w_1, w_2, \dots, w_k\} \subseteq V(G)$ and a vertex v of G , the representation of v with respect to W is the k -vector $r(v|W) = (d_G(v, w_1), \dots, d_G(v, w_k))$. The set W is called a resolving set of G , if every two vertices of G has a different representation. A resolving set containing a minimum number of vertices is called a basis of H . The number of elements in a basis of G is called the metric dimension of G and denoted by $dim(G)$. In this paper, we considered a resolving set W of G where the induced subgraph of G by W does not contain an isolated vertex. Such a resolving set is called a non-isolated resolving set. A non-isolated resolving set of G with minimum cardinality is called an nr -set of G . The cardinality of an nr -set of G is called the non-isolated resolving number of G , denoted by $nr(G)$. Let H be a graph. The corona product graph of G with H , denoted by $G \odot H$, is a graph obtained by taking one copy of G and $|V(G)|$ copies of H , namely $H^1, H^2, \dots, H^{|V(G)|}$, such that the i -th vertex of G is adjacent to every vertex of H^i . If the degree of every vertex of H is k , then H is called a k -regular graph. In this paper, we determined $nr(G \odot H)$ where G is an arbitrary connected graph of order n at least two and H is a k -regular graph of order t with $k \in \{t-2, t-3\}$.

Keywords: corona product graph; k -regular graph; metric dimension; non-isolated resolving number; non-isolated resolving set

MSC: 05C12; 05C76



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1. Introduction

Let G be a simple, finite, and connected graph with the vertex set $V(G)$ and the edge set $E(G)$. For u and v in $V(G)$, the distance between two different vertices u and v is the length of the shortest $u-v$ path in G , denoted by $d_G(u, v)$. Let $W = \{w_1, w_2, \dots, w_k\}$ be an ordered set of vertices of G . The representation of $v \in V(G)$ with respect to W is the k -vector $r(v|W) = (d_G(v, w_1), \dots, d_G(v, w_k))$. If every two vertices of G has a different representation, then the set W is called a resolving set of G . A resolving set containing a minimum number of vertices is called a basis of G . The number of elements in a basis of G is called the metric dimension of G and denoted by $dim(G)$.

The metric dimension concept of a graph was first introduced by Slater [1] and Harary and Melter [2]. Since then, many results in the metric dimension have been obtained. Some of them can be seen in [3–17]. A resolving set is needed to manage robotic navigation [18], to identify chemical compounds [19], and to optimize the placement of threat detection sensors [20].

Practically, we need a threat detection sensor that also has the function to check whether its neighbor sensor is malfunctioning or not. Hence, we need an optimization of the placement of threat detection sensors such that the location of each sensor must not be isolated. In 2015, Chitra and Arumugam [21] introduced the concept of a non-isolated resolving set. Let W be a subset of $V(H)$. A resolving set W is called a *non-isolated resolving set*, if the induced subgraph of H by W has no isolated vertex. A non-isolated resolving set of H with minimum cardinality is called an *nr-set* of H . The cardinality of an *nr-set* of H is called the *non-isolated resolving number* of H , denoted by $nr(H)$. Since a non-isolated resolving set is also a resolving set of a connected graph H of order n , we have:

$$1 \leq \dim(H) \leq nr(H) \leq n - 1. \tag{1}$$

Chitra and Arumugam [21] determined the non-isolated resolving number of some classes of graphs such as complete bipartite graphs, complete graphs, paths, and friendship graphs. The non-isolated resolving number of the graph depends on the structure of the graph. We can obtain a structure of a graph by using an operation between two graphs. Determining a relation, in terms of a non-isolated resolving number, between the original graph and the resulting graph under a graph operation is also interesting to be considered. Chitra and Arumugam also determined the non-isolated resolving number of the Cartesian product between a path and a path, a cycle, and a complete graph. In addition, they provided an upper bound for the non-isolated resolving number of the Cartesian product of a connected graph with a complete graph. In 2018, Hasibuan et al. [22] continued the investigation of the non-isolated resolving set of the Cartesian product of some graphs. They showed the non-isolated number of the Cartesian product between a path and some simple graphs, namely a complete graph, a cycle, a bipartite, and a friendship graph.

Now, we consider corona product graphs. Let G be a connected graph of order $n \geq 2$ and H be a graph of order $t \geq 2$. The *corona product graph* $G \odot H$ is a graph obtained by taking one copy of G and $|V(G)|$ copies of H , namely $H^1, H^2, \dots, H^{|V(G)|}$, such that the i -th vertex of G is adjacent to every vertex of H^i . If the degree of every vertex of H is k , then H is said to be *k-regular*. Abidin et al. [23] studied the non-isolated resolving set of $G \odot H$ where H is a complete graph. Note that a simple and $(t - 1)$ -regular graph is isomorphic to a complete graph of order t .

In this paper, we determined $nr(G \odot H)$ where G is any connected graph of order at least two and H is a k -regular graph of order t for $k = t - 2$ or $k = t - 3$. From now on, we only consider finite and simple graphs. We also define $[a, b] = \{n \in \mathbb{Z} | a \leq n \leq b\}$.

2. Non-Isolated Resolving Set of $G \odot H_t$ Where H_t Is a $(t - 2)$ -Regular Graph

Let H_t be a $(t - 2)$ -regular graph of order t . A subset M of $E(H_t)$ is called a *matching* in H_t if no two of its elements are adjacent in H_t . If M is a matching in H_t with the property that every vertex of H_t is incident with an edge of M , then M is called a *perfect matching* in H_t [24]. If H_t is a $(t - 2)$ -regular graph of order $t \geq 2$, it is easy to check that t is an even positive integer and H_t is isomorphic to a complete graph of order t minus a perfect matching in H_t . In this section, we determine a non-isolated resolving number of the corona product of a connected graph with a $(t - 2)$ -regular graph of order t .

Theorem 1. Let n be a positive integer and t be an even integer at least two. If G is a connected graph of order n and H_t is a $(t - 2)$ -regular graph of order t , then:

$$nr(G \odot H_t) = \begin{cases} 2n, & \text{if } t = 2; \\ \frac{nt}{2}, & \text{if } t \geq 4. \end{cases}$$

Proof. Let $V(G) = \{u_1, u_2, u_3, u_4, \dots, u_n\}$ and $t = 2m$ for some positive integer m . By the definition of the corona product, for every $l \in [1, n]$, let $V(H_l^t) = \{v_1^l, v_2^l, \dots, v_{2m}^l\}$ be the vertex set of H_l^t such that $E(H_l^t) = \{v_i^l v_j^l | i \text{ and } j \text{ in } [1, 2m] \text{ with } i < j \text{ and } j \neq m + i\}$. Therefore, we can say that $V(G \odot H_t) = V(G) \cup \bigcup_{l=1}^n V(H_l^t)$ and $E(G \odot H_t) = E(G) \cup$

$\cup_{l=1}^n E(H_l^i) \cup \{v_j^l u_l | l \in [1, n], j \in [1, 2m]\}$. We distinguish two cases.

Case 1. $m = 1$:

First, we show that $nr(G \odot H_t) \leq 2n$. We define $W_l = \{u_l, v_1^l\}$ for every $l \in [1, n]$. Let $W = \cup_{l=1}^n W_l$. Since $u_l v_1^l \in E(G \odot H_t)$, there is no isolated vertex in W . Let x and y be two different vertices in $V(G \odot H_t) - W = \{v_2^l | l \in [1, n]\}$. Let $x = v_2^p$ and $y = v_2^q$ for some p and q in $[1, n]$ with $p \neq q$. Since $d_{G \odot H_t}(x, v_1^p) = 2 < d_{G \odot H_t}(y, v_1^p)$, we obtain $r(x|W_p) \neq r(y|W_p)$. Therefore, $r(x|W) \neq r(y|W)$.

Next, we show that $nr(G \odot H_t) \geq 2n$. Suppose that W' is an nr -set of $G \odot H_t$ with $|W'| < 2n$. For every $a \in [1, n]$, let $W'_a = W' \cap \{V(H_a^i) \cup \{u_a\}\}$. Since $|W'| < 2n$, there exists $b \in [1, n]$ such that $|W'_b| \leq 1$. Since W'_b does not contain an isolated vertex, there are two different vertices v_1^b and v_2^b in $V(H_b^i) - W'$ satisfying $r(v_1^b|W') = r(v_2^b|W')$. Therefore, we obtain a contradiction.

Case 2. $m \geq 2$:

We show that $nr(G \odot H_t) \leq nm$. For every $l \in [1, n]$, we define $W_l = \{v_1^l, v_2^l, \dots, v_m^l\}$. Since an induced subgraph of $G \odot H_t$ by W_l is isomorphic to a complete graph on order m , it is easy to see that there is no isolated vertex in W_l . Let $W = \cup_{l=1}^n W_l$. Let x and y be any two different vertices in $V(H_t) - W$. We consider four subcases.

Subcase 2.1. x and y in $V(H_t^p)$ for some $p \in [1, n]$:

Let $x = v_{m+a}^p$ and $y = v_{m+b}^p$ for some a and b in $[1, m]$ with $a \neq b$. Since $d_{G \odot H_t}(x, v_b^p) = 1 < d_{G \odot H_t}(y, v_b^p)$, we have $r(x|W_p) \neq r(y|W_p)$. Therefore, $r(x|W) \neq r(y|W)$.

Subcase 2.2. $x \in V(H_t^p)$ for some $p \in [1, n]$ and $y \in V(G)$:

Let $x = v_{m+a}^p$ for some $a \in [1, m]$ and $y = u_q$ for some $q \in [1, n]$. If $p = q$, then $d_{G \odot H_t}(x, v_a^p) = 2 > 1 = d_{G \odot H_t}(y, v_a^p)$. This means that $r(x|W_p) \neq r(y|W_p)$. If $p \neq q$, then there exists $v_c^p \in W_p - \{v_a^p\}$ such that $d_{G \odot H_t}(x, v_c^p) = 1 < d_{G \odot H_t}(y, v_c^p)$. This means that $r(x|W_p) \neq r(y|W_p)$. Therefore, $r(x|W) \neq r(y|W)$.

Subcase 2.3. $x \in V(H_t^p)$ and $y \in V(H_t^q)$ for some p and q in $[1, n]$ with $p \neq q$:

Let $x = v_{m+a}^p$ and $y = v_{m+b}^q$ for some a and b in $[1, m]$. Since $d_{G \odot H_t}(x, v_a^p) = 2 < d_{G \odot H_t}(y, v_a^p)$, we obtain $r(x|W_p) \neq r(y|W_p)$. Therefore, $r(x|W) \neq r(y|W)$.

Subcase 2.4. x and y in $V(G)$ with $x \neq y$:

Let $x = u_p$ and $y = u_q$ for some p and q in $[1, n]$ with $p \neq q$. Since $d_{G \odot H_t}(x, v_1^p) = 1 < d_{G \odot H_t}(y, v_1^p)$, we obtain $r(x|W_p) \neq r(y|W_p)$. Therefore, $r(x|W) \neq r(y|W)$.

Next, we show that $nr(G \odot H_t) \geq nm$. Suppose that W' is an nr -set of $G \odot H_t$ with $|W'| < nm$. Then, there exists $p \in [1, n]$ such that $|W' \cap V(H_t^p)| < m$. There exist two different vertices v_a^p and v_{a+m}^p for some $a \in [1, m]$ that are not in W' . Note that for every $c \in [1, 2m] - \{a, a+m\}$, we obtain $d_{G \odot H_t}(v_a^p, v_c^p) = 1 = d_{G \odot H_t}(v_{a+m}^p, v_c^p)$, and for every $z \in V(G \odot H_t) - V(H_t^p)$, we have $d_{G \odot H_t}(v_a^p, z) = d_{G \odot H_t}(v_{a+m}^p, z)$. Therefore, we have $r(v_a^p|W') = r(v_{a+m}^p|W')$. This is a contradiction. \square

An illustration of Theorem 1 can be seen in Figure 1.

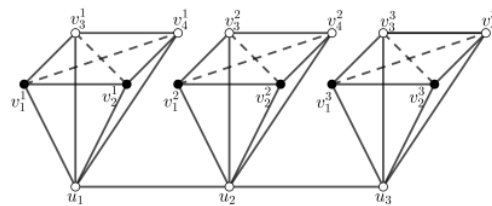


Figure 1. $P_3 \odot K_4$ with its nr -set $W = \{v_1^1, v_2^1, v_1^2, v_2^2, v_1^3, v_2^3\}$.

3. Non-Isolated Resolving Set of $G \odot H_t$ Where H_t Is a $(t - 3)$ -Regular Graph

In this section, we investigate $nr(G \odot H_t)$ where H_t is a $(t - 3)$ -regular graph for any $t \geq 3$. It is easy to check that H_t is connected if and only if $t \geq 5$. Note that for a connected graph G , a graph $G \odot H_t$ is still connected for $t \in \{3, 4\}$. In case $t \in [3, 4]$, we show that $nr(G \odot H_t)$ only depends on the order of G , which can be seen in Section 3.3. In case $t \geq 5$, we prove that for most cases of $nr(G \odot H_t)$, it depends on $nr(H_t)$.

Now, assume $t \geq 5$. In Lemma 1, we show that H_t is isomorphic to the join product of B_{m_1}, B_{m_2}, \dots , and B_{m_q} for some $q \in [1, \lfloor \frac{t}{3} \rfloor]$, where $B_{m_i} = K_{m_i} - E(C_{m_i})$ with K_{m_i} a complete graph of order m_i and C_{m_i} a cycle graph of order m_i . We recall the join product of G_1 and G_2 , denoted by $G_1 + G_2$, which is a graph with the vertex set $V(G_1 + G_2) = V(G_1) \cup V(G_2)$ and the edge set $E(G_1 + G_2) = E(G_1) \cup E(G_2) \cup \{uv \mid u \in V(G_1), v \in V(G_2)\}$. We also prove that for some cases H_t , $nr(H_t)$ depends on $nr(B_k)$ for every $k \in [1, q]$.

Lemma 1. For an integer $t \geq 5$, let H_t be a $(t - 3)$ -regular graph. Then, there exists $q \in [1, \lfloor \frac{t}{3} \rfloor]$ such that $H_t = B_{m_1} + B_{m_2} + \dots + B_{m_q}$, where for every $i \in [1, q]$, $B_{m_i} = K_{m_i} - E(C_{m_i})$ with m_i is an integer at least three and $m_1 + \dots + m_q = t$.

Proof. Let H_t be a $(t - 3)$ -regular graph. Let $3 \leq m_i \leq t$ and B_{m_i} be a complete graph of order m_i minus a Hamiltonian cycle. It is shown that $H_t = B_{m_1} + B_{m_2} + \dots + B_{m_q}$ by using the following algorithm.

Input a $(t - 3)$ -regular graph H_t with t vertices:

1. Let $i := 1$;
2. Let $G_i = H_t$;
3. Choose two non-adjacent vertices in G_i , say v_1^i and v_2^i and say another unlabeled vertex that is not adjacent to v_2^i as v_3^i ;
4. Let $j := 3$;
5. Let $v \neq v_{j-1}^i$ be a vertex that is not adjacent to v_j^i ;
6. If $v_1^i v_j^i \in E(G_i)$, then $v_{j+1}^i = v$, and define $j := j + 1$, then repeat Step 5. Otherwise, go to Step 7;
7. Let B_{m_i} be an induced subgraph of G_i by $\{v_1^i, v_2^i, \dots, v_j^i\}$;
8. Define $G_{i+1} = G_i - B_{m_i}$;
9. If $V(G_{i+1}) \neq \emptyset$, then define $i := i + 1$, and repeat Step 3. Otherwise, finish.

Let q be the last number i obtained from the above algorithm. We obtain $H_t = B_{m_1} + B_{m_2} + \dots + B_{m_q}$. \square

Let us consider a graph $G \odot H_t$, where G is a connected graph of order n and H_t is a $(t - 3)$ -regular graph. For $i \in [1, n]$, for every x and y in $V(H_t^i)$ and $z \notin V(G \odot H_t) - V(H_t^i)$, we have $d_{G \odot H_t}(x, z) = d_{G \odot H_t}(y, z)$, which implies any two vertices in H_t^i are only resolved in $V(H_t^i)$. Therefore, it is necessary to determine $nr(H_t^i)$ for every $i \in [1, n]$. Note that H_t^i is a $(t - 3)$ -regular graph.

Let $t \geq 5$ and $B_m \subseteq H_t$ such that $B_m = K_m - E(C_m)$ for some $m \in [3, t]$. It is easy to check that B_m is connected if and only if $m \geq 5$. If H_t is only the join of some B_3 or B_4 , H_t is still connected. This case of H_t is investigated in Section 3.2. Now, we assume $m \geq 5$. Note that for every two vertices $v \in V(B_m)$ and $u \in V(H_t) - V(B_m)$, we obtain $vu \in E(H_t)$. Therefore, every two vertices x and y in $V(B_m)$ is not resolved by u . Therefore, an nr -set of H_t must contain some vertices in B_m , which resolves B_m .

Lemma 2. Let H_t be a $(t - 3)$ -regular graph of order $t \geq 5$. Let W be an nr -set of H_t . Let $B_m \subseteq H_t$ where $B_m = K_m - E(C_m)$ for some $m \in [5, t]$:

- (i) If $m = t$, then W is an nr -set of B_t ;
- (ii) If $m < t$, then $|W \cap V(B_m)| \geq \dim(B_m)$.

Proof. (i) Since $m = t$, we have $B_t = H_t$. Hence, W is also an nr -set of B_t ;

- (ii) Since H_t is a $(t - 3)$ -regular graph, by Lemma 1, there is $q \in [1, \lfloor \frac{t}{3} \rfloor]$ such that $H_t = B_{m_1} + B_{m_2} + \dots + B_{m_q}$ where $B_{m_i} = K_{m_i} - E(C_{m_i})$ and $m_1 + \dots + m_q = t$. Let W be an nr -set of H_t and $W_i = W \cap V(B_{m_i})$ for every $i \in [1, q]$. Next, it is proven by contradiction. Suppose there is $j \in [1, q]$ such that $|W_j| < \dim(B_{m_j})$. As a consequence, there are two vertices u and v in $V(B_{m_j}) - W_j$ such that $r(u|W_j) = r(v|W_j)$. Since every $w \in W - W_j$ is adjacent to all vertices in B_{m_j} , we have $r(u|W) = r(v|W)$. We obtain a contradiction. □

In order to determine $nr(H_t)$, according to Lemma 2, we need to determine $\dim(K_m - E(C_m))$ for some $m \in [5, t]$, which can be seen in Section 3.1. Note that $K_m - E(C_m)$ is a complete graph of order m minus a Hamiltonian cycle. In Section 3.2, we provide the non-isolated resolving number of H_t . Some results in Section 3.1 can be used to determine $nr(H_t)$. All results in Sections 3.1 and 3.2 were used to determine $nr(G \odot H_t)$, which can be seen in Section 3.3.

3.1. Resolvability of a Complete Graph Minus a Hamiltonian Cycle

By Lemma 2, we need to determine a basis or an nr -set of B_{m_i} . It is easy to check that B_m is connected when $m \geq 5$. We use a gap between two vertices to determine a (non-isolated) resolving set of B_m . Let Q be a subset of $V(B_m)$. Let u and v be two different vertices in Q . A gap of Q between u and v is defined as $V(P(u, v)) - \{u, v\}$ where $P(u, v)$ is a $u - v$ path in C_m , all inner vertices of which are not in Q . Note that every pair of vertices u and v in Q has at most two different gaps. The vertices u and v are called the end point of a gap of Q between u and v . If two different gaps of Q have a common end point, then those two gaps are called neighboring gaps. Therefore, if $|Q| = r$, then Q has r gaps, and some of the gaps may be empty. This technique was introduced in [25]. For illustration, let $Q = \{v_1, v_5, v_7, v_{10}, v_{11}, v_{14}\}$ as described in Figure 2. We have some gaps, namely $\{v_2, v_3, v_4\}, \{v_6\}, \{v_8, v_9\}, \{\}, \{v_{12}, v_{13}\}$, and $\{v_{15}\}$.

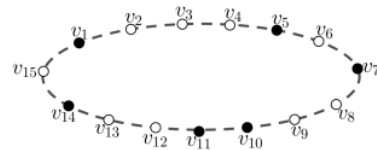


Figure 2. $B_{10} = K_{10} - E(C_{10})$ with its $Q = \{v_1, v_5, v_7, v_{10}, v_{11}, v_{14}\}$.

Lemma 3. Let $m \geq 5$ and $B_m = K_m - E(C_m)$. Then, W is a resolving set of B_m if and only if W satisfies all the following properties:

- (i) Every gap of W contains at most three vertices;
- (ii) At most one gap of W contains three vertices;
- (iii) If a gap of W contains at least two vertices, then its neighboring gaps contain at most one vertex.

Proof. Let $V(B_m) = \{v_1, v_2, \dots, v_m\}$ such that $E(C_m) = \{v_1v_2, v_2v_3, \dots, v_{m-1}v_m, v_mv_1\}$ and $v_1 \in W$.

(\rightarrow) Let W be a resolving set of B_m :

- (i) Suppose there is a gap of W containing four vertices $v_k, v_{k+1}, v_{k+2}, v_{k+3}$ for some $k \in [2, m - 3]$. We obtain $r(v_{k+1}|W) = r(v_{k+2}|W) = (1, 1, \dots, 1)$. We obtain a contradiction. Hence, every gap of W contains at most three vertices;
- (ii) Suppose there are two different gaps containing three vertices v_k, v_{k+1}, v_{k+2} and v_l, v_{l+1}, v_{l+2} for some k and l in $[2, m - 2]$ with $k \neq l$. We obtain $r(v_{k+1}|W) = r(v_{l+1}|W) = (1, 1, \dots, 1)$. This is a contradiction. Therefore, the number of gaps of W containing three vertices is at most one;

(iii) Suppose there are five vertices $v_k, v_{k+1}, v_{k+2}, v_{k+3}, v_{k+4}$ for some $k \in [2, m - 4]$ such that v_{k+2} is the only vertex of W . Since $d_{B_m}(v_{k+1}, v_{k+2}) = 2 = d_{B_m}(v_{k+3}, v_{k+2})$ and for every $z \in W - \{v_{k+2}\}$, $d_{B_m}(v_{k+1}, z) = 1 = d_{B_m}(v_{k+3}, z)$, we obtain $r(v_{k+1}|W) = r(v_{k+3}|W)$. This is a contradiction. We conclude that if A is a gap of W containing at least two vertices, then the neighboring gaps of A contain at most one vertex.

(\leftarrow) Let W satisfy the three above properties. We show that W is a resolving set of B_m . Let v be any vertex in $V(B_m) - W$. We divide the proof into three cases as follows:

1. v belongs to a gap of size one of W :
Let v_i and v_{i+2} be the end points of a gap containing v . Then, the vertex v has a distance of two to both v_i and v_{i+2} . Since $m \geq 5$, for every $u \in V(B_m) - \{v, v_i, v_{i+2}\}$, we have $d_{B_m}(u, v_i) = 1$ or $d_{B_m}(u, v_{i+2}) = 1$. This implies $r(v|W) \neq r(u|W)$;
2. v belongs to a gap of size two of W :
Let v_i and v_{i+3} be the end points of a gap containing v . Without loss of generality, let $v = v_{i+1}$, then $d_{B_m}(v, v_i) = 2$ and $d_{B_m}(v, v_{i+3}) = 1$. If there is $u \in V(B_m) - W$ with $u \neq v$ and $d_{B_m}(u, v_i) = 2$ and $d_{B_m}(u, v_{i+3}) = 1$, then there is $w \in W - \{v_i\}$ such that $d_{B_m}(w, u) = 2$ and $d_{B_m}(w, v) = 1$. Therefore, for every $x \in V(B_m) - \{v\}$, we obtain $r(v|W) \neq r(x|W)$;
3. v belongs to a gap of size three of W :
Let v_i and v_{i+4} be the end points of a gap containing v . Let $v = v_{i+2}$, then $r(v|W) = (1, 1, 1, \dots, 1)$. By Properties (i) and (ii) of Lemma 3, no other vertex of B_m has this representation. Now, let $v = v_{i+1}$ or $v = v_{i+3}$. Without loss of generality, let $v = v_{i+1}$, then $d_{B_m}(v, v_i) = 2$ and $d_{B_m}(v, v_{i+4}) = 1$. If there is $u \in \{V(B_m) - W\}$ with $u \neq v$, $d_{B_m}(u, v_i) = 2$ and $d_{B_m}(u, v_{i+4}) = 1$, then there is $w \in W - \{v_i\}$ such that $d_{B_m}(w, u) = 2$ and $d_{B_m}(w, v) = 1$. Therefore, for every $x \in V(B_m) - \{v\}$, we obtain $r(v|W) \neq r(x|W)$.

We conclude that every vertex in $V(B_m)$ has a distinct representation with respect to W . Therefore, W is a resolving set of B_m . \square

Now, we are ready to determine $\dim(K_m - E(C_m))$ and $nr(K_m - E(C_m))$ where $m \geq 5$.

Theorem 2. Let $m \geq 5$ and $B_m = K_m - E(C_m)$. Then:

$$nr(B_m) = \dim(B_m) = \lfloor \frac{2m+2}{5} \rfloor.$$

Proof. First, we show that $\dim(B_m) \geq \lfloor \frac{2m+2}{5} \rfloor$. Let W be a resolving set of B_m . Based on Properties (i)–(iii) of Lemma 3, there is at most one gap of W that contains three vertices. Furthermore, at most $\lfloor \frac{|W|}{2} \rfloor - 1$ gaps contain two vertices, and the rest of the gaps contain at most one vertex. Let us consider two cases below:

1. $|W| = 2p$ for some $p \in \mathbb{N}$:
The number of vertices in all gaps of W is at most $3p + 1$. Therefore, $m - 2p \leq 3p + 1$. This implies that $|W| = 2p \geq \lceil \frac{2}{5}m - \frac{2}{5} \rceil \geq \lfloor \frac{2m+2}{5} \rfloor$;
2. $|W| = 2p + 1$ for some $p \in \mathbb{N}$:
The number of vertices in all gaps of W is at most $3p + 2$. Therefore, $m - 2p - 1 \leq 3p + 2$. This means that $|W| = 2p + 1 \geq \lceil \frac{2}{5}m - \frac{6}{5} + 1 \rceil \geq \lfloor \frac{2m+2}{5} \rfloor$.

For an upper bound, let $V(B_m) = \{v_1, \dots, v_m\}$ such that $E(C_m) = \{v_1v_2, \dots, v_{m-1}v_m, v_mv_1\}$. We show that $\dim(B_m) \leq \lfloor \frac{2m+2}{5} \rfloor$. Let us consider five cases below:

1. $m = 0 \pmod{5}$:
Therefore, $m = 5l$ for some $l \in \mathbb{N}$. Thus, $\lfloor \frac{2m+2}{5} \rfloor = 2l$. We define $W = \{v_{5j+1}, v_{5j+4} | j \in [0, l - 1]\}$. Note that W contains $2l$ vertices and satisfies Properties (i)–(iii) of Lemma 3;
2. $m = 1 \pmod{5}$:
Therefore, $m = 5l + 1$ for some $l \in \mathbb{N}$. Thus, $\lfloor \frac{2m+2}{5} \rfloor = 2l$. For $l = 1$, we define $W = \{v_1, v_5\}$. For $l \geq 2$, we define $W = \{v_{5j+1}, v_{5j+4} | j \in [0, l - 2]\} \cup \{v_{5l-4}, v_{5l}\}$. Note that W contains $2l$ vertices and satisfies Properties (i)–(iii) of Lemma 3;

3. $m = 2(\text{mod } 5)$:
Therefore, $m = 5l + 2$ for some $l \in \mathbb{N}$. Thus, $\lfloor \frac{2m+2}{5} \rfloor = 2l + 1$. We define $W = \{v_{5j+1}, v_{5j+4} | j \in [0, l-1]\} \cup \{v_{5l+1}\}$. Note that W contains $2l + 1$ vertices and satisfies Properties (i)–(iii) of Lemma 3;
4. $m = 3(\text{mod } 5)$:
Therefore, $m = 5l + 3$ for some $l \in \mathbb{N}$. Thus, $\lfloor \frac{2m+2}{5} \rfloor = 2l + 1$. For $l = 1$, we define $W = \{v_1, v_5, v_7\}$. For $l \geq 2$, we define $W = \{v_{5j+1}, v_{5j+4} | j \in [0, l-2]\} \cup \{v_{5l-4}, v_{5l}, v_{5l+2}\}$. Note that W contains $2l + 1$ vertices and satisfies Properties (i)–(iii) of Lemma 3;
5. $m = 4(\text{mod } 5)$:
Therefore, $m = 5l + 4$ for some $l \in \mathbb{N}$. Thus, $\lfloor \frac{2m+2}{5} \rfloor = 2l + 2$. We define $W = \{v_{5j+1}, v_{5j+4} | j \in [0, l]\}$. Note that W contains $2l + 2$ vertices and satisfies Properties (i)–(iii) of Lemma 3.
Therefore, W with $|W| = \lfloor \frac{2m+2}{5} \rfloor$ is a resolving set of B_m . We obtain that $\dim(B_m) = \lfloor \frac{2m+2}{5} \rfloor$. Next, it is easy to check that W does not contain an isolated vertex. Consequently, W is also a non-isolated resolving set. Hence, $nr(B_m) \leq \dim(B_m)$. By (1), we have $nr(B_m) \geq \dim(B_m)$. We conclude that $nr(B_m) = \dim(B_m)$. \square

3.2. nr -Set of a $(t - 3)$ -Regular Graph

Let H_t be a $(t - 3)$ -regular graph of order $t \geq 5$. By Lemma 1, a $(t - 3)$ -regular graph is isomorphic to $B_{m_1} + B_{m_2} + \dots + B_{m_q}$ for some $q \in [1, \lfloor \frac{t}{3} \rfloor]$, where $B_{m_i} = K_{m_i} - E(C_{m_i})$ and $m_i \geq 3$ for every $i \in [1, q]$. Let W be an nr -set of H_t . By Lemma 2 (i), if $q = 1$, then W is an nr -set of $K_t - E(C_t)$. Note that an nr -set of $K_t - E(C_t)$ for any $t \geq 5$ was discussed in Section 3.1. If H_t contains B_{m_i} with $m_i \in [5, t - 3]$, by Lemma 2 (ii), then $|W \cap V(B_{m_i})| \geq \dim(B_{m_i})$. A basis of B_{m_i} also was discussed in Section 3.1. Note that for $q \geq 2$, it is possible to have B_3 or B_4 in H_t . However, B_m with $m \in \{3, 4\}$ is a disconnected graph. In Lemma 4, we provide a property of an nr -set of H_t that contains B_3 or B_4 .

Lemma 4. Let H_t be a $(t - 3)$ -regular graph where $t \geq 6$. Let $B_m \subseteq H_t$ where $B_m = K_m - E(C_m)$ for some $m \in \{3, 4\}$. Then, B_m contributes at least two vertices in an nr -set of H_t .

Proof. Suppose there exists an nr -set W' of H_t that contains at most one vertex of B_m . Note that for every $v \in V(B_m)$ and $z \in V(H_t) - V(B_m)$, there are two vertices x and y in $V(B_m)$ such that $d_{B_m}(x, v) = 2 = d_{B_m}(y, v)$ and $d_{B_m}(x, z) = 1 = d_{B_m}(y, z)$. Hence, $r(x|W') = r(y|W')$. We have a contradiction. \square

Let $H_t = B_{m_1} + B_{m_2} + \dots + B_{m_q}$ with $3 \leq m_i \leq m_{i+1}$ for every $i \in [1, q - 1]$. We assume that H_t contains b subgraphs that are isomorphic to B_3 or B_4 . By Lemma 2 (ii) and Lemma 4, we have $nr(H_t) \geq 2b + \dim(B_{m_{b+1}}) + \dim(B_{m_{b+2}}) + \dots + \dim(B_{m_q})$. Let $S \subseteq V(H_t)$ and $S_i = S \cap V(B_{m_i})$ for every $i \in [b + 1, q]$. We define gaps of S in H_t as the union of gaps of S_{b+1}, S_{b+2}, \dots , and S_q . In order to determine $nr(H_t)$, we also need to consider some necessary conditions for the nr -set of H_t , which can be seen in the following lemma.

Lemma 5. Let H_t be a $(t - 3)$ -regular graph of order $t \geq 5$. Let W be a non-isolated resolving set of H_t . Then, W satisfies the two conditions below:

- (i) Every gap of W contains at most three vertices;
- (ii) At most one gap of W contains three vertices.

Proof. Let W be a non-isolated resolving set of H_t . For every $i \in [1, q]$, let $V(B_{m_i}) = \{v_{h,i} | h \in [1, m_i]\}$ such that $E(C_{m_i}) = \{v_{1,i}v_{2,i}, v_{2,i}v_{3,i}, \dots, v_{m_i-1,i}v_{m_i,i}, v_{m_i,i}v_{1,i}\}$ and $v_{1,i} \in W$:

- (i) Suppose there is a gap of W containing four vertices $v_{k,i}, v_{k+1,i}, v_{k+2,i}, v_{k+3,i}$ for some $k \in [2, m_i - 3]$ and $i \in [1, q]$. We obtain $r(v_{k+1,i}|W) = r(v_{k+2,i}|W) = (1, 1, \dots, 1)$. We obtain a contradiction. Hence, every gap of W contains at most three vertices;

- (ii) By considering Lemma 4, suppose there are two different gaps containing three vertices $v_{k,i}, v_{k+1,i}, v_{k+2,i}$ in $V(B_{m_i})$ of H_t and $v_{l,j}, v_{l+1,j}, v_{l+2,j}$ in $V(B_{m_j})$ of H_t for some $k \in [2, m_i - 2]$ and $l \in [2, m_j - 2]$ with $i \neq j$. We obtain $r(v_{k+1,i}|W) = r(v_{l+1,j}|W) = (1, 1, \dots, 1)$. We have a contradiction. Hence, the number of gaps of W containing three vertices is at most one.

□

In order to determine an nr -set of H_t , by Lemma 5, we need to investigate the gap property for a basis of B_{m_i} , which can be seen in the lemma below.

Lemma 6. *Let H_t be a $(t - 3)$ -regular graph of order $t \geq 5$. Let $B_m \subseteq H_t$ such that $B_m = K_m - E(C_m)$ for some $m \in [5, t]$, then every basis of B_m has a gap containing at least three vertices if and only if $m \equiv 1$ or $3 \pmod{5}$.*

Proof. Let $V(B_m) = \{v_1, v_2, \dots, v_m\}$ such that $E(C_m) = \{v_1v_2, v_2v_3, \dots, v_{m-1}v_m, v_mv_1\}$.

(\rightarrow) Suppose $m \not\equiv 1$ or $3 \pmod{5}$. We distinguish three cases as follows:

Case 1. $m \equiv 0 \pmod{5}$:

Let $m = 5l$ for some integer l at least two. Thus, $\lfloor \frac{2m+2}{5} \rfloor = 2l$. We define $W = \{v_{5j+1}, v_{5j+4} | j \in [0, l-1]\}$.

Case 2. $m \equiv 2 \pmod{5}$:

Let $m = 5l + 2$ for some $l \in \mathbb{N}$. Thus, $\lfloor \frac{2m+2}{5} \rfloor = 2l + 1$. We define $W = \{v_{5j+1}, v_{5j+4} | j \in [0, l-1]\} \cup \{v_{5l+1}\}$.

Case 3. $m \equiv 4 \pmod{5}$:

Let $m = 5l + 4$ for some $l \in \mathbb{N}$. Thus, $\lfloor \frac{2m+2}{5} \rfloor = 2l + 2$. We define $W = \{v_{5j+1}, v_{5j+4} | j \in [0, l]\}$.

From all cases above, we have that every gap of W contains at most two vertices. By Lemma 3 and Theorem 2, we conclude that W is a basis of B_m .

(\leftarrow) Suppose that every gap of the basis of B_m contains at most two vertices. We have two cases:

Case 1. $m \equiv 1 \pmod{5}$:

Let $m = 5l + 1$ for some $l \in \mathbb{N}$. By using Theorem 2, we obtain $dim = 2l$. By Properties (iii) of Lemma 3, there are $\frac{dim(B_m)}{2}$ gaps that contain one vertex and at most $\frac{dim(B_m)}{2}$ gaps that contain two vertices. By Theorem 2, we obtain $|V(B_m)| \leq 5l$, a contradiction with $m = 5l + 1$.

Case 2. $m \equiv 3 \pmod{5}$:

Let $m = 5l + 3$ for some $l \in \mathbb{N}$. By using Theorem 2, we obtain $dim = 2l + 1$. By Properties (iii) of Lemma 3, there are $\frac{dim(B_m)+1}{2}$ gaps that contain one vertex and at most $\frac{dim(B_m)-1}{2}$ gaps that contain two vertices. By Theorem 2, we obtain $|V(B_m)| \leq 5l + 2$, a contradiction with $m = 5l + 3$. □

The lemma below is useful to determine the non-isolated number of H_t .

Lemma 7. *Let H_t be a $(t - 3)$ -regular graph of order $t \geq 5$. Let $B_m \subseteq H_t$ such that $B_m = K_m - E(C_m)$ for some $m \in \{3, 4\}$. There is an nr -set W of H_t such that for every $v \in V(B_m)$, $r(v|W) \neq (1, 1, \dots, 1)$.*

Proof. Let $V(B_m) = \{v_1, v_2, \dots, v_m\}$ such that $E(C_m) = \{v_1v_2, \dots, v_{m-1}v_m, v_mv_1\}$. Let $S = \{v_1, v_m\}$. Let S' be an nr -set of $V(H_t) - V(B_m)$. We define $W = S \cup S'$. We distinguish two cases as follows:

1. For $m = 3$, we have $r(v_1|S) = (0, 2)$, $r(v_2|S) = (2, 2)$, and $r(v_3|S) = (2, 0)$;
2. For $m = 4$, we have $r(v_1|S) = (0, 2)$, $r(v_2|S) = (2, 1)$, $r(v_3|S) = (1, 2)$, and $r(v_4|S) = (2, 0)$.

Therefore, for all $v \in V(B_m)$, we obtain $r(v|W) \neq (1, 1, \dots, 1)$. □

In the next theorem, we provide the non-isolated resolving number of a $(t - 3)$ -regular graph where $t \geq 5$.

Theorem 3. Let H_t be a $(t - 3)$ -regular graph of order $t \geq 5$. Let $q \in [1, \lfloor \frac{t}{3} \rfloor]$ such that $H_t = B_{m_1} + B_{m_2} + \dots + B_{m_q}$, where $B_{m_i} = K_{m_i} - E(C_{m_i})$ and $3 \leq m_i \leq m_{i+1}$, for every $i \in [1, q - 1]$. Let b be the number of B_3 and B_4 . Let d be the number of B_g with $g = 1$ or $3 \pmod{5}$ and $g \geq 6$. Then,

$$nr(H_t) = \begin{cases} nr(B_t), & \text{if } q = 1; \\ 2b, & \text{if } b = q \geq 2; \\ 2b + \sum_{i=b+1}^q nr(B_{m_i}), & \text{if } q \geq 2, d \in \{0, 1\}, \text{ and } b < q; \\ 2b + \sum_{i=b+1}^q nr(B_{m_i}) + d - 1, & \text{if } q \geq 2, d \geq 2, \text{ and } b < q. \end{cases}$$

Proof. We divide the proof into four cases.

Case 1. $q = 1$:

According to Lemma 1, we obtain $H_t = B_t$. Therefore, $nr(H_t) = nr(B_t)$.

Case 2. $b = q \geq 2$:

By Lemma 4, we only need to show that $nr(H_t) \leq 2b$. For every $i \in [1, q]$, let $V(B_{m_i}) = \{v_{1,i}, v_{2,i}, \dots, v_{m_i,i}\}$ such that $E(C_{m_i}) = \{v_{1,i}v_{2,i}, \dots, v_{m_i,i}v_{1,i}\}$. We define $W_i = \{v_{1,i}, v_{m_i,i}\}$ of H_t . Let $W = \bigcup_{i=1}^q W_i$. Note that $|W| = 2q$ and W does not contain an isolated vertex. For $m_i = 3$, we have $r(v_{2,i}|W_i) = (2, 2)$. For $m_i = 4$, we have $r(v_{2,i}|W_i) = (2, 1) \neq (1, 2) = r(v_{3,i}|W_i)$. Since $x \in V(B_{m_i})$ is adjacent to $y \in V(B_{m_j})$ with $i \neq j$, we have $r(x|W) \neq r(y|W)$. Therefore, $nr(H_t) = 2q = 2b$.

Case 3. $q \geq 2, d \in \{0, 1\}$, and $b < q$:

Let W be a non-isolated resolving set of H_t . Let $W_i = W \cap B_{m_i}$ for every $i \in [1, q]$. By Lemma 4, we obtain $|W_j| \geq 2$ for every $j \in [1, b]$. Since $b < q$, we have $m_k \geq 5$ for every $k \in [b + 1, q]$. By Lemma 2 (ii) and Theorem 2, we obtain $|W_k| \geq \dim(B_{m_k}) = nr(B_{m_k})$. Therefore, we obtain $nr(H_t) \geq 2b + \sum_{i=b+1}^q nr(B_{m_i})$.

Next, for every $i \in [1, q]$, let $V(B_{m_i}) = \{v_{1,i}, v_{2,i}, \dots, v_{m_i,i}\}$ such that $E(C_{m_i}) = \{v_{1,i}v_{2,i}, \dots, v_{m_i,i}v_{1,i}\}$. We construct a non-isolated resolving set of H_t with $2b + \sum_{i=b+1}^q nr(B_{m_i})$ elements. For every $k \in [1, b]$, let $W_k = \{v_{1,k}, v_{m,k}\}$. For every $j \in [b + 1, q]$, choose a basis W_j of B_{m_j} such that if $m_j = 0, 2$, or $4 \pmod{5}$, then every gap of W_j contains at most two vertices. We define $W = \bigcup_{k=1}^b W_k \cup \bigcup_{j=b+1}^q W_j$.

We show that W is a non-isolated resolving set of H_t . Note that W does not contain an isolated vertex. For $m_k = 3$, we have $r(v_{2,k}|W_k) = (2, 2)$. For $m_k = 4$, we have $r(v_{2,k}|W_k) = (2, 1) \neq (1, 2) = r(v_{3,k}|W_k)$. For every $m_j \geq 5$, since W_j is a basis of B_{m_j} , every two different vertices x and y of B_{m_j} satisfies $r(x|W_j) \neq r(y|W_j)$. Thus, for $i \in [1, q]$, every pair of distinct vertices x and y in $V(B_{m_i})$ satisfies $r(x|W_i) \neq r(y|W_i)$.

Now, for distinct integers r and s in $[1, q]$, let us consider $x \in V(B_{m_r}) - W_r$ and $z \in V(B_{m_s}) - W_s$. If $r \in [1, b]$, or $m_r = 0, 2$, or $4 \pmod{5}$ with $m_r \geq 5$, then there exists a vertex in W_r that is not adjacent to x , which implies $r(x|W_r) \neq (1, 1, \dots, 1) = r(z|W_r)$. If $m_r = 1$ or $3 \pmod{5}$ with $m_r \geq 5$, then there exists a vertex in W_s that is not adjacent to z , which implies $r(z|W_s) \neq (1, 1, \dots, 1) = r(x|W_s)$. Therefore, we have $r(x|W) \neq r(z|W)$. We conclude that W is a non-isolated resolving set of H_t . Since $|W| = 2b + \sum_{i=b+1}^q \dim(B_{m_i})$ and by considering Theorem 2, we obtain that $nr(H_t) \leq 2b + \sum_{i=b+1}^q \dim(B_{m_i}) = 2b + \sum_{i=b+1}^q nr(B_{m_i})$.

Case 4. $q \geq 2, d \geq 2$, and $b < q$:

Let W be a non-isolated resolving set of H_t . Let $W_i = W \cap B_{m_i}$ for every $i \in [1, q]$. By Lemma 4, we obtain $|W_j| \geq 2$ for every $j \in [1, b]$. Since $b < q$, we have $m_k \geq 5$ for every $k \in [b + 1, q]$. By Lemma 2 (ii) and Theorem 2, we obtain $|W_k| \geq \dim(B_{m_k}) = nr(B_{m_k})$. Since there are $d \geq 2$ gaps containing three vertices, by Lemma 5 (ii), we have $nr(H_t) \geq 2b + \sum_{i=b+1}^q nr(B_{m_i}) + d - 1$.

Next, for every $i \in [1, q]$, let $V(B_{m_i}) = \{v_{1,i}, v_{2,i}, \dots, v_{m_i,i}\}$ such that $E(C_{m_i}) = \{v_{1,i}v_{2,i}, \dots, v_{m_i,i}v_{1,i}\}$. We construct a non-isolated resolving set of H_t with $2b + \sum_{i=b+1}^q nr(B_{m_i}) + d - 1$ elements. For every $k \in [1, b]$, let $W_k = \{v_{1,k}, v_{m_k,k}\}$. For every $j \in [b + 1, q]$, choose a basis W_j of B_{m_j} such that if $m_j = 0, 2$, or $4 \pmod{5}$, then every gap of W_j contains at most two vertices. For $j \in [b + 1, q]$, if $m_j = 1$ or $3 \pmod{5}$, then by Lemma 3.7, W_j has a gap containing three vertices. For those W_j , without loss of generality, let $v_{1,j}, v_{2,j}, v_{3,j}$ be a gap of W_j with three vertices. Let $A = \{v_{2,j} | j \in [b + 1, q], m_j = 1 \text{ or } 3 \pmod{5}\}$. Note that $|A| = d$. Let $\alpha = \min\{j \in [b + 1, q] | m_j = 1 \text{ or } 3 \pmod{5}\}$, we define $A' = A - \{v_{2,\alpha}\}$. We define $W = \cup_{k=1}^b W_k \cup \cup_{j=b+1}^q W_j \cup A'$.

We show that W is a non-isolated resolving set of H_t . Note that W does not contain an isolated vertex. For $m_k = 3$, we have $r(v_{2,k} | W_k) = (2, 2)$. For $m_k = 4$, we have $r(v_{2,k} | W_k) = (2, 1) \neq (1, 2) = r(v_{3,k} | W_k)$. For every $m_j \geq 5$ with $j \in [b + 1, q]$, since W_j is a basis of B_{m_j} , every two different vertices x and y of B_{m_j} satisfies $r(x | W_j) \neq r(y | W_j)$. Thus, for $i \in [1, q]$, for two distinct vertices x and y in $V(B_{m_i})$, we obtain $r(x | W_i) \neq r(y | W_i)$.

Now, for distinct integers r and s in $[1, q]$, let us consider $x \in V(B_{m_r}) - W$ and $z \in V(B_{m_s}) - W$. If $r \in [1, b]$, or $m_r = 0, 2$, or $4 \pmod{5}$ with $m_r \geq 5$, then there exists a vertex in W_r that is not adjacent to x . Hence, $r(x | W_r) \neq (1, 1, \dots, 1) = r(z | W_r)$. If $m_s = 0, 2$, or $4 \pmod{5}$ with $m_s \geq 5$, then there exists a vertex in W_s that is not adjacent to z , which implies $r(z | W_s) \neq (1, 1, \dots, 1) = r(x | W_s)$. If $m_r = 1$, or $3 \pmod{5}$ with $m_r \geq 5$ and $m_s = 1$, or $3 \pmod{5}$ with $m_s \geq 5$, without loss of generality, and assuming $r < s$, then there exists a vertex in $W \cap V(B_{m_s})$ that is not adjacent to z , which implies $r(z | W_s \cup \{v_{2,m_s}\}) \neq (1, 1, \dots, 1) = r(x | W_s \cup \{v_{2,m_s}\})$. Therefore, we have $r(x | W) \neq r(z | W)$. We conclude that W is a non-isolated resolving set of H_t . Since $|W| = 2b + \sum_{i=b+1}^q dim(B_{m_i}) + d - 1$ and by considering Theorem 2, we obtain that $nr(H_t) \leq 2b + \sum_{i=b+1}^q dim(B_{m_i}) + d - 1 = 2b + \sum_{i=b+1}^q nr(B_{m_i}) + d - 1$. \square

Some illustrations of Theorem 3 can be seen in some figures of H_{32} below. An illustration of Theorem 3 of Case 1, Case 2, and Case 3 can be seen in Figures 3–5, respectively. In those figures, we have H_{32} as $B_{32}, 4B_3 + 5B_4$, and $B_3 + B_4 + B_{12} + B_{13}$, respectively. The black vertices represent the elements of a chosen nr -set. For Case 4, $H_{32} = 2B_4 + B_{11} + B_{13}$ with its nr -set in Figure 6. For every $j \in [1, 4]$, let W_j be an nr -set of B_{m_j} . There is one gap of W_3 containing three vertices, which are $v_{1,3}, v_{2,3}, v_{3,3}$, and there is one gap of W_4 containing three vertices, which are $v_{1,4}, v_{2,4}, v_{3,4}$. Note that vertices $v_{2,3}$ and $v_{2,4}$ have the same representation. Hence, we should add vertex $v_{2,3}$ to an nr -set of H_{32} .



Figure 3. $H_{32} = B_{32}$ with its nr -set.

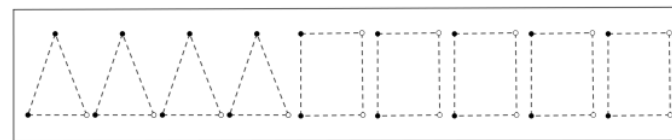


Figure 4. $H_{32} = 4B_3 + 5B_4$ with its nr -set.

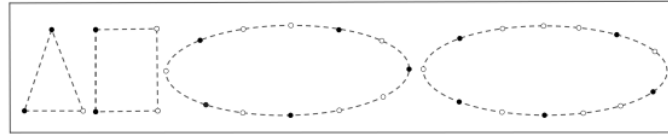


Figure 5. $H_{32} = B_3 + B_4 + B_{12} + B_{13}$ with its nr -set.

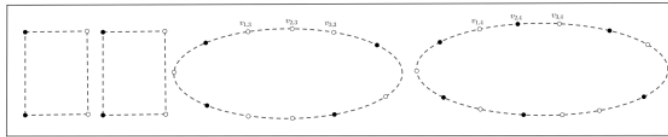


Figure 6. $H_{32} = 2B_4 + B_{11} + B_{13}$ with its nr -set.

3.3. nr -Set of $G \odot H_t$ Where H_t Is a $(t - 3)$ -Regular Graph

Let H_t be a $(t - 3)$ -regular graph of order $t \geq 3$. Note that H_t is isomorphic to a complete graph K_t minus the edge set of some disjoint cycles.

Theorem 4. Let G be a connected graph of order $n \geq 2$ and H_t be a $(t - 3)$ -regular graph of order $t \geq 3$. Then,

$$nr(G \odot H_t) = \begin{cases} 3n, & \text{if } t = 3 \text{ or } t = 4; \\ n \times nr(H_t), & \text{if } t \geq 5. \end{cases}$$

Proof. Let $V(G) = \{u_1, u_2, \dots, u_n\}$. For every $i \in [1, n]$, we define H_t^i as a copy of H_t in $G \odot H_t$ where all of its vertices are adjacent to u_i . Let $V(H_t^i) = \{v_1^i, v_2^i, \dots, v_t^i\}$. We divide a proof into three cases.

Case 1. $t = 3$:

First, we show that $nr(G \odot H_t) \leq 3n$. For every $i \in [1, n]$, we define $W_i = \{v_1^i, v_2^i, u_i\}$. Let $W = \bigcup_{i=1}^n W_i$. Note that $|W| = 3n$. Since $u_i v_1^i$ and $u_i v_2^i \in E(G \odot H_t)$, W does not contain an isolated vertex. Next, we show that W is a resolving set of $G \odot H_t$. Let x and y be two distinct vertices in $V(G \odot H_t) - W$. Let $x = v_3^p$ and $y = v_3^q$ for some p and q in $[1, n]$ with $p \neq q$. Since $d_{G \odot H_t}(x, u_p) = 1 < d_{G \odot H_t}(y, u_p)$, this implies that $r(x|W) \neq r(y|W)$.

Now, we show that $nr(G \odot H_t) \geq 3n$. Suppose that $nr(G \odot H_t) \leq 3n - 1$. Let W' be an nr -set of $G \odot H_t$ and $W'_i = W'_i \cap V(H_t^i)$ for every $i \in [1, n]$. Then, there exists $j \in [1, n]$ such that $1 \leq |W'_j| \leq 2$. We have two subcases:

- i. If $u_j \in W'_j$, then there exist two vertices x and y in $V(H_t^j)$ such that $d_{G \odot H_t}(x, u_j) = 1 = d_{G \odot H_t}(y, u_j)$. This implies that $r(x|W') = r(y|W')$, a contradiction;
- ii. If $u_j \notin W'_j$, then W' contains an isolated vertex, a contradiction.

Case 2. $t = 4$:

Without loss of generality, let $E(H_t) = \{v_1^i v_3^i, v_2^i v_4^i\}$. First, we show that $nr(G \odot H_t) \leq 3n$. For every $i \in [1, n]$, we define $W_i = \{v_1^i, v_2^i, u_i\}$. Let $W = \bigcup_{i=1}^n W_i$. Note that $|W| = 3n$. Since $u_i v_1^i, u_i v_2^i \in E(G \odot H_t)$, W does not contain an isolated vertex. Next, we show that W is a resolving set of $G \odot H_t$. Let x and y be two distinct vertices in $V(G \odot H_t) - W$. We have two subcases:

- i. x and y in H_t^j for some $j \in [1, n]$:
Let $x = v_3^j$ and $y = v_4^j$. Since $d_{G \odot H_t}(x, v_1^j) = 1 < d_{G \odot H_t}(y, v_1^j)$, we obtain $r(x|W) \neq r(y|W)$;
- ii. $x \in H_t^j$ and $y \in H_t^k$ for some j and k in $[1, n]$ with $j \neq k$:
Let $x = v_a^j$ and $y = v_b^k$ for some a and b in $\{3, 4\}$. Since $d_{G \odot H_t}(x, u_j) = 1 < d_{G \odot H_t}(y, u_j)$, we obtain $r(x|W) \neq r(y|W)$.

Now, we show that $nr(G \odot H_t) \geq 3n$. Suppose that $nr(G \odot H_t) \leq 3n - 1$. Let W' be an nr -set of $G \odot H_t$ and $W'_i = W_i \cap V(H_t^i)$ for every $i \in [1, n]$. Then, there exists $j \in [1, n]$ such that $|W'_j| \leq 2$. We have two subcases:

- i. If $u_j \in W'_j$, then there exist two vertices x and y in $V(H_t^j)$ such that $d_{G \odot H_t}(x, u_j) = 1 = d_{G \odot H_t}(y, u_j)$. This implies that $r(x|W') = r(y|W')$, a contradiction;
- ii. For $u_j \notin W'_j$, if W' contains an isolated vertex, then we have a contradiction. Otherwise, then there exist two vertices x and y in $V(H_t^j)$ such that $d_{G \odot H_t}(x, z) = 2 = d_{G \odot H_t}(y, z)$ for every $z \in W'_j$. This implies that $r(x|W') = r(y|W')$, a contradiction.

Case 3. $t \geq 5$:

We prove that $nr(G \odot H_t) \leq n(nr(H_t))$. For every $i \in [1, n]$, let W_i be an nr -set of H_t^i . We define $W = \bigcup_{i=1}^n W_i$. Note that there is no isolated vertex in W . Next, we show that every two distinct vertices in $G \odot H_t$ has a distinct representation with respect to W . Let x and y be different vertices in $V(G \odot H_t) - W$. We divide this into four subcases:

- i. x and y in $V(H_t^j)$ for some $j \in [1, n]$:
Since W_j is an nr -set of H_t^j , we have $r(x|W_j) \neq r(y|W_j)$. This means that $r(x|W) \neq r(y|W)$;
- ii. $x \in V(H_t^j)$ and $y \in V(G)$ for some $j \in [1, n]$:
If $y = u_j$, then for some $k \in [1, n]$ with $k \neq j$ and for any $z \in W_k$, we have $d_{G \odot H_t}(x, z) = d_{G \odot H_t}(y, z) + 1$. If $y = u_k$, then for any $z \in W_k$, we have $d_{G \odot H_t}(x, z) = d_{G \odot H_t}(x, y) + d_{G \odot H_t}(y, z)$. Therefore, $r(x|W_k) \neq r(y|W_k)$, and this means that $r(x|W) \neq r(y|W)$;
- iii. $x \in V(H_t^j)$ and $y \in V(H_t^k)$ for some j and k in $[1, n]$ with $j \neq k$:
For any $z \in W_k$, we have $d_{G \odot H_t}(x, z) \geq 3$, but $d_{G \odot H_t}(y, z) \leq 2$. Therefore, $r(x|W_j) \neq r(y|W_j)$, so that $r(x|W) \neq r(y|W)$;
- iv. x and y in $V(G)$:
Let $x = u_j$ and $y = u_k$ for some j and k in $[1, n]$ with $j \neq k$. For any $z \in W_j$, we have $d_{G \odot H_t}(x, z) = 1 < d_{G \odot H_t}(y, z)$. Therefore $r(x|W_j) \neq r(y|W_j)$. This means that $r(x|W) \neq r(y|W)$.

Next, we show that $nr(G \odot H_t) \geq n(nr(H_t))$. Suppose that $nr(G \odot H_t) \leq n(nr(H_t)) - 1$. Let W' be an nr -set of $G \odot H_t$. For every $i \in [1, n]$, let $W'_i = W' \cap V(H_t^i)$. Then, there exists $j \in [1, n]$ such that $|W'_j| \leq nr(H_t) - 1$. Hence, there exist two vertices x and y in $V(H_t^j)$ such that $d_{G \odot H_t}(x, w) = d_{G \odot H_t}(y, w)$ for every $w \in W'_j$. Since for every $v \in V(G \odot H_t) - V(H_t^j)$, we have $d_{G \odot H_t}(x, v) = d_{G \odot H_t}(y, v)$. Therefore, $r(x|W') = r(y|W')$. This is a contradiction. \square

4. Conclusions

In this paper, we studied the non-isolated resolving number of the corona product $G \odot H$ where G is an arbitrary connected graph of order n at least two and H is a k -regular graph of order t with $k \in \{t-2, t-3\}$. We obtained that the non-isolated resolving number of $G \odot H$ is a function of $nr(H)$. We made the hypothesis that $nr(G \odot H)$ for any k -regular graph H is also a function of $nr(H)$.

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Sent: Thu, 27 Jan 2022 09:42:11 +0700 (WIB)

Subject: [Mathematics] Manuscript ID: mathematics-1592863 - Article Processing Charge Confirmation

Dear Mrs. Abidin,

Thank you very much for submitting your manuscript to Mathematics:

Journal name: Mathematics

Manuscript ID: mathematics-1592863

Type of manuscript: Article

Title: Non-Isolated Resolving Sets of Corona Graphs with Some Regular Graphs

Authors: Wahyuni Abidin *, A.N.M Salman, Suhadi Widodo Saputro

Received: 26 January 2022

E-mails: wahyuniabidin@s.itb.ac.id, msalman@math.itb.ac.id, suhadi@math.itb.ac.id

We confirm that, if accepted for publication, the following Article Processing Charges (APC), 1800 CHF, will apply to your article:

Journal APC: 1800 CHF

Total APC: 1800 CHF

Please also confirm who will handle the Invoice:

Name: Wahyuni Abidin

Address: Mrs. Wahyuni Abidin

Combinatorial Mathematics Research Group, Faculty of Mathematics and Natural Sciences, Institut Teknologi Bandung

Institut Teknologi Bandung

12345 Bandung

Indonesia

E-Mail: wahyuniabidin@s.itb.ac.id

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Thank you in advance for your cooperation. I look forward to hearing from you.

Kind regards,
Emma He
Section Managing Editor
E-Mail: emma.he@mdpi.com

--

Ms. Emma He
MDPI Branch Office, Beijing
Suite 305, Zhongjia Mansion, Building No.13,
Taiyangyuan Community, Dazhongsi East Road,
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E-Mail: mathematics@mdpi.com
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From: Mata Wang

January 29, 2022 7:54 PM

To: Wahyuni Abidin

Cc: Mathematics Editorial Office A.N.M Salman Suhadi Widodo Saputro

Dear Mrs. Abidin,

Thank you for submitting your manuscript to /Mathematics/. Here are three things to confirm if your paper can be accepted for publication in our journal in the future.

1. We need you to confirm that, if accepted for publication, the following Article Processing Charges (APC), 1800 CHF, will apply to your article:

Journal APC: 1800 CHF
Total APC: 1800 CHF

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Your confirmation would help us process your manuscript efficiently with no delay. Thank you for your cooperation in advance.

Best regards,

Mata Wang
Assistant Editor
E-Mail: mata.wang@mdpi.com
Mathematics (IF 2.258 ; <https://www.mdpi.com/journal/mathematics>)

[Mathematics] Manuscript ID: mathematics-1592863 - Need Your Confir 3 messag



From: wahyuniabidin@students.itb.ac.id

January 31, 2022 9:34 AM

To: Mata Wang



Dear Mata Wang
Assistant Editor

Thank you very much for your information. We confirm if our paper is accepted for publication in your journal.

1. We will pay the article processing charge if our manuscript is accepted.
2. We confirm that all authors do not support open reviews. However, all authors support open access publishing.

Best regards,
Wahyuni Abidin

----- Original Message -----

From: mata wang <mata.wang@mdpi.com>

To: Wahyuni Abidin <wahyuniabidin@s.itb.ac.id>

Cc: mathematics@mdpi.com, A.N.M Salman <msalman@math.itb.ac.id>, Suhadi Wido Saputro <suhadi@math.itb.ac.id>

Sent: Sat, 29 Jan 2022 18:54:49 +0700 (WIB)

Subject: [Mathematics] Manuscript ID: mathematics-1592863 - Need Your Confirmation

Dear Mrs. Abidin,

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Journal APC: 1800 CHF

Total APC: 1800 CHF

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In other words, if you selected "Open Review", readers can see your published paper with the review reports and your responses. If you did not select "Open Review", readers can only see your published paper.

Your confirmation would help us process your manuscript efficiently with no delay. Thank you for your cooperation in advance.

Best regards,

Mata Wang
Assistant Editor

E-Mail: mata.wang@mdpi.com

Mathematics (IF 2.258 ; <https://www.mdpi.com/journal/mathematics>)



From: Mata Wang on behalf of Mathematics Editorial Office February 23, 2022 2:10 PM

To: Wahyuni Abidin

Cc: A.N.M Salman, Suhadi Wido Saputro, Mathematics Editorial Office

Reply To: Mata Wang

Dear Mrs. Abidin,

We sent a revision request for the following manuscript on 18 February 2022.

Manuscript ID: mathematics-1592863

Type of manuscript: Article

Title: Non-Isolated Resolving Sets of Corona Graphs with Some Regular Graphs

Authors: Wahyuni Abidin *, A.N.M Salman, Suhadi Wido Saputro

Received: 26 January 2022

E-mails: wahyuniabidin@s.itb.ac.id, msalman@math.itb.ac.id,
suhadi@math.itb.ac.id

May we kindly ask you to update us on the progress of your revisions? If you have finished your revisions, please upload the revised version together with your responses to the reviewers as soon as possible.

You can find your manuscript and review reports at this link:

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Thank you in advance for your kind cooperation and we look forward to hearing from you soon.

Kind regards,
Mata Wang
Assistant Editor
E-Mail: mata.wang@mdpi.com

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From: wahyuniabidin@students.itb.ac.id

February 24, 2022 5:58 PM

To: Mata Wang

Thank you for your attention.

We will post revisions on suggestions from reviewers on **Friday 25 February 2022**.

Kind regards,
W. Abidin

----- Original Message -----

From: Mathematics Editorial Office <mathematics@mdpi.com>

To: Wahyuni Abidin <wahyuniabidin@s.itb.ac.id>

Cc: A.N.M Salman <msalman@math.itb.ac.id>, Suhadi Wido Saputro <suhadi@math.itb.ac.id>, Mathematics Editorial Office <mathematics@mdpi.com>

Sent: **Wed, 23 Feb 2022 13:10:59 +0700** (WIB)

Subject: [Mathematics] Manuscript ID: mathematics-1592863 - Revision Reminder

Dear Mrs. Abidin,

We sent a revision request for the following manuscript on **18 February 2022**.

Manuscript ID: mathematics-1592863 

Type of manuscript: Article

Title: Non-Isolated Resolving Sets of Corona Graphs with Some Regular Graphs

Authors: Wahyuni Abidin *, A.N.M Salman, Suhadi Wido Saputro

Received: **26 January 2022**

E-mails: wahyuniabidin@s.itb.ac.id, msalman@math.itb.ac.id, suhadi@math.itb.ac.id

May we kindly ask you to update us on the progress of your revisions? If you have finished your revisions, please upload the revised version together with your responses to the reviewers as soon as possible.

You can find your manuscript and review reports at this link:

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Thank you in advance for your kind cooperation and we look forward to hearing from you soon.

Kind regards,
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From: susy@mdpi.com on behalf of Mathematics Editorial Office February 25, 2022 10:22 PM

To: Wahyuni Abidin

Cc: A.N.M Salman, Suhadi Widodo Saputro

Reply To: Mata Wang



Dear Mrs. Abidin,

Thank you very much for resubmitting the modified version of the following manuscript:

Manuscript ID: mathematics-1592863

Type of manuscript: Article

Title: Non-Isolated Resolving Sets of Corona Graphs with Some Regular Graphs

Authors: Wahyuni Abidin *, A.N.M Salman, Suhadi Widodo Saputro

Received: 26 January 2022

E-mails: wahyuniabidin@s.itb.ac.id, msalman@math.itb.ac.id, suhadi@math.itb.ac.id

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A member of the editorial office will be in touch with you soon regarding progress of the manuscript.



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From: Mata Wang on behalf of Mathematics Editorial Office February 27, 2022 2:20 PM

To: Wahyuni Abidin

Cc: A.N.M Salman, Suhadi Wido Saputro, Mathematics Editorial Office

Reply To: Mata Wang



Dear Mrs. Abidin,

Thank you very much for providing the revised version of your paper:

Manuscript ID: mathematics-1592863

Type of manuscript: Article

Title: Non-Isolated Resolving Sets of Corona Graphs with Some Regular Graphs

Authors: Wahyuni Abidin *, A.N.M Salman, Suhadi Wido Saputro

Received: 26 January 2022

E-mails: wahyuniabidin@s.itb.ac.id, msalman@math.itb.ac.id,
suhadi@math.itb.ac.id

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We will continue processing your paper and will keep you informed about the status of your submission.

Kind regards,

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From: Mathematics Editorial Office

March 10, 2022 2:52 PM

To: Wahyuni Abidin

Cc: Mata Wang

Reply To: info@sciprofiles.com Mata Wang



Dear Mrs. Abidin,

Congratulations on your paper being accepted for publication in Mathematics.

To enhance your visibility, as well as that of co-authors, we would like to invite you to set up profiles on SciProfiles (<https://sciprofiles.com>), which will allow us to add a link for each author to their permanent homepage in the final version of the paper.

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Many thanks in advance for your collaboration. If you have any questions, please feel free to contact info@sciprofiles.com.

Kind regards,

Dr. Shu-Kun Lin, Founder and President
<https://sciprofiles.com/profile/2>



From: Mata Wang on behalf of Mathematics Editorial Office

March 10, 2022 2:52 PM

To: Wahyuni Abidin

Cc: A.N.M Salman, Suhadi Wido Saputro, Mathematics Editorial Office, Mata Wang

Reply To: Mata Wang

Dear Mrs. Abidin,

Congratulations on the acceptance of your manuscript, and thank you for your interest in submitting your work to Mathematics:

Manuscript ID: mathematics-1592863

Type of manuscript: Article

Title: Non-Isolated Resolving Sets of Corona Graphs with Some Regular Graphs

Authors: Wahyuni Abidin *, A.N.M Salman, Suhadi Wido Saputro

Received: 26 January 2022

E-mails: wahyuniabidin@s.itb.ac.id, msalman@math.itb.ac.id, suhadi@math.itb.ac.id

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Kind regards,
Francisco Chiclana
Editor-in-Chief



From: [Mata Wang](#) on behalf of [Mathematics Editorial Office](#)

March 10, 2022 2:52 PM

To: [Wahyuni Abidin](#) [A.N.M Salman](#) [Suhadi Widodo Saputro](#)

Cc: [Mathematics Editorial Office](#) [Mata Wang](#)

Reply To: [Mata Wang](#)

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When you submitted, you chose No Funding in the system. Your manuscript has now been accepted. Please carefully check and ensure that the funding information is correct in any places where it appears in your manuscript.

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Type of manuscript: Article

Title: Non-Isolated Resolving Sets of Corona Graphs with Some Regular Graphs

Authors: Wahyuni Abidin *, A.N.M Salman, Suhadi Widodo Saputro

Received: 26 January 2022

E-mails: wahyuniabidin@s.itb.ac.id, msalman@math.itb.ac.id,
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From: Mata Wang on behalf of Mathematics Editorial Office

March 11, 2022 12:47 PM

To: Wahyuni Abidin

Cc: A.N.M Salman, Suhadi Widodo Saputro, Mathematics Editorial Office, Mata Wang

Reply To: Mata Wang

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Manuscript ID: mathematics-1592863

Type of manuscript: Article

Title: Non-Isolated Resolving Sets of Corona Graphs with Some Regular Graphs

Authors: Wahyuni Abidin *, A.N.M Salman, Suhadi Widodo Saputro

Received: 26 January 2022

E-mails: wahyuniabidin@s.itb.ac.id, msalman@math.itb.ac.id,

suhadi@math.itb.ac.id

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Kind regards,
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Assistant Editor
E-Mail: mata.wang@mdpi.com

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From: wahyuniabidin@students.itb.ac.id

March 14, 2022 8:40 AM

To: Mata Wang



Dear Ms. Mata Wang,

There are some changes as follows:

1. The second author's name was changed to Anm Salman.
2. The first author's affiliation has a change in the manuscript, namely "Doctoral Program of Mathematics, Faculty of Mathematics and Natural Sciences, Institut Teknologi Bandung,, Jl. Ganesha No.10, Bandung 40132, Indonesia".
3. The first author also added the affiliation of the original institute. However, it is put in a footnote.
Please update information on the system.

Thanks you.

Kind Regards,

Wahyuni Abidin

. . .



From: susy@mdpi.com on behalf of Wahyuni Abidin

March 14, 2022 8:41 AM

To: Mata Wang

Cc: Mathematics Editorial Office wahyuniabidin@s.itb.ac.id

Dear Editor,

Proofreading has been completed for the following manuscript:

Manuscript ID: mathematics-1592863

Type of manuscript: Article

Title: Non-Isolated Resolving Sets of Corona Graphs with Some Regular Graphs

Authors: Wahyuni Abidin *, A.N.M Salman, Suhadi Widodo Saputro

Received: 26 January 2022

E-mails: wahyuniabidin@s.itb.ac.id, msalman@math.itb.ac.id,
suhadi@math.itb.ac.id

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Mathematics Editorial Office

Postfach, CH-4020 Basel, Switzerland

Office: St. Alban-Anlage 66, CH-4052 Basel

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From: susy@mdpi.com on behalf of [Mathematics Editorial Office](#) March 14, 2022 9:45 AM

To: [Wahyuni Abidin](#)

Cc: [A.N.M Salman](#) [Suhadi Widodo Saputro](#)

Reply To: [Mata Wang](#)

Dear Mrs. Abidin,

Thank you very much for resubmitting the modified version of the following manuscript:

Manuscript ID: [mathematics-1592863](#)

Type of manuscript: Article

Title: Non-Isolated Resolving Sets of Corona Graphs with Some Regular Graphs

Authors: Wahyuni Abidin *, A.N.M Salman, Suhadi Widodo Saputro

Received: 26 January 2022

E-mails: wahyuniabidin@s.itb.ac.id, msalman@math.itb.ac.id,
suhadi@math.itb.ac.id

https://susy.mdpi.com/user/manuscripts/review_info/d9ff5010fe9a302682187c11030f9bcd

A member of the editorial office will be in touch with you soon regarding progress of the manuscript.

Kind regards,

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Office: St. Alban-Anlage 66, CH-4052 Basel

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From: Mata Wang

March 14, 2022 2:46 PM

To: Wahyuni Abidin

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Best regards,

Ms. Mata Wang
Assistant Editor
E-Mail: mata.wang@mdpi.com
Skype: live:.cid.6d48320430262cc2



From: billing@mdpi.com

March 15, 2022 4:47 PM

To: wahyuniabidin@s.itb.ac.id

Cc: [A.N.M Salman](#) [Suhadi Wido Saputro](#) [Mata Wang](#) [MDPI Billing](#)
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Combinatorial Mathematics Research Group, Faculty of Mathematics and Natural Sciences, Institut Teknologi Bandung
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From: pupovac@mdpi.com on behalf of Mathematics Editorial Office March 17, 2022 5:59 PM

To: wahyuniabidin@sitb.ac.id msalman@math.itb.ac.id suhadi@math.itb.ac.id

Cc: billing@mdpi.com website@mdpi.com Mathematics Editorial Office
pupovac@mdpi.com Mata Wang

Reply To: Mata Wang

Dear Authors,

We are pleased to inform you that your article "Non-Isolated Resolving Sets of Corona Graphs with Some Regular Graphs" has been published in Mathematics and is available online:

Abstract: <https://www.mdpi.com/2227-7390/10/6/962>

PDF Version: <https://www.mdpi.com/2227-7390/10/6/962/pdf>

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Comments and
Suggestions for Authors

A set W is called a resolving set of G , if every two vertices of G has different representation. A resolving set containing a minimum number of vertices is called a basis of H . The number of elements in a basis of G is called the metric dimension of G and is denoted by $\dim(G)$. This paper deals with a resolving set W of G where the induced subgraph of G by W does not contain an isolated vertex. A non-isolated resolving set of G with minimum cardinality is called an nr -set of G . The cardinality of a nr -set of G is called the non-isolated resolving number of G , and is denoted by $nr(G)$. In this paper, $nr(G \times H)$ is calculated, where $G \times H$ is corona product of of the graphs G and H , and is obtained by taking one copy of G and $|V(G)|$ copies of H , G is an arbitrary connected graph of order n at least 2 and H is a k -regular graph of order t with $k = t - 2, t - 3$.

The paper is easy to follow, clearly presented, English is good. However, the I am not convinced about the use of this work in the process of knowledge. Why do we need to know the results of this paper? The paper should have a Conclusions section where the meaning of this work should be highlighted, and some future perspectives should be given.

Submission Date 26 January 2022

Date of this review 04 Feb 2022 22:50:24



Dear Reviewer 1 of Mathematics Journal,

We would like to thank you for the opportunity to resubmit a revised version of our manuscript entitled non-isolated resolving sets of corona graphs with some regular graphs. We appreciate the reviewers for your precious time in reviewing our paper and providing valuable comments. It was your valuable and insightful comments that led to possible improvements in the current version.

The manuscript has been revised carefully to address reviewers' comments. Below response to the reviewers' comments and concerns. We hope the revised manuscript meet your standard.

Reviewer 1

Reviewer Comments	Author's Respond	Modification
1. The paper is easy to follow, clearly presented, English is good. However, the I am not convinced about the use of this work in the process of knowledge. Why do we need to know the results of this paper? The paper should have a Conclusions section where the meaning of this work should be highlighted, and some future perspectives should be given.	Thank you for the suggestion. As suggested by reviewers, we have added a conclusion section and future perspectives.	[Page 12, line 485-490]

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Submission Date 26 January 2022

Date of this review 06 Feb 2022 01:09:27



Review report
“Non-isolated resolving sets of corona graphs with some
regular graphs”

by
W. Abidin, A.N.M. Salman and S.W. Saputro

Comments

This manuscript establishes the non-resolving number of a graph which is the corona product between a connected graph G and a k -regular graph H of order t where $k \in \{t - 2, t - 3\}$. The results are significant and interesting. The proofs are correct and the presentation is systematic with well-written English. I recommend to “Accept” after minor revision. Please find the comments in the followings.

- Page 1, Line 12: “a k -regular graph” should read “a k -regular graph”. Please change “-” (minus) to “-” (hyphen). There are several places that should be changed too.
- Page 2, Lines 58-59: “If G is a connected and simple graph” should read “If G is a connected graph”. That is, remove “and simple” and assume throughout in the part of “Introduction” that all the graphs will be simple. Please remove “simple and” for H_t too.
- Pages 3-4: Personally, I think Lemma 1 is obvious. It is fine to state it without the proof in case the authors would like to shorten the length of the paper.
- Page 4, Line 135: There is no definition of “ H_t^i ” or I miss something?
- Page 4, Lines 149 and 151: Remove “For $m = t$ ” and “For $m < t$ ”.
- Page 6, Line 259: “an $(t - 3)$ -regular” should read “a $(t - 3)$ -regular”.
- Page 7, Line 266: “whose contains” should read “who contain”.
- Page 7, Lines 270-272: Rewrite the sentence from “Since for every...” to “... $d_{B_m}(y, z)$ ”.
- Page 10, Line 409: Would it be better to change “case 3.1” to be “Case 1” the same as in the proof? If so, please change the other cases too.
- Page 12, Line 485: “pandant” should read “pendant”.

Dear Reviewer 2 of Mathematics Journal,

We would like to thank you for the opportunity to resubmit a revised version of our manuscript entitled non-isolated resolving sets of corona graphs with some regular graphs. We appreciate the reviewers for your precious time in reviewing our paper and providing valuable comments. It was your valuable and insightful comments that led to possible improvements in the current version.

The manuscript has been revised carefully to address reviewers' comments. Below point-by-point response to the reviewers' comments and concerns. We hope the revised manuscript meet your standard.

Reviewer Comments	Author's Respond	Modification
1. Page 1, Line 12: "a k -regular graph" should read "a k -regular graph". Please change "-" (minus) to "-" (hyphen). There are several places that should be changed too.	Thank you the correction. As suggested by reviewer, we have changed "-" (minus) to "-" (hyphen). It is on lines 12, 20, 49, 58, 124, 263, 302.	<p>We have revised</p> <ul style="list-style-type: none"> - "k-regular" [line 12] into "k-regular" [line 12] - "$u - v$" [line 20] into "$u-v$" [line 20] - "$(t - 1)$-regular" [line 45] into "$(t - 1)$-regular" [line 49] - "$(t - 2)$ -regular" [line 53] into "$(t - 2)$-regular" [line 58] - "$(t - 3)$-regular" [line 120] into "$(t - 3)$-regular" [line 124] - "$(t - 3)$-regular" [line 259] into "$(t - 3)$-regular" [line 263] - "$(t - 3)$-regular" [line 299] into "$(t - 3)$-regular" [line 302]
2. Page 2, Lines 58-59: "If G is a connected and simple graph" should read "If G is a connected graph". That is, remove "and simple" and assume throughout in the part of "Introduction" that all the graphs will be simple. Please remove "simple and" for H_t too.	As suggested by reviewer, we have replaced <ul style="list-style-type: none"> - "If G is a connected and simple graph" with "If G is a connected graph. - "H_t is a simple and $(t - 2)$-regular graph" with "H_t is a $(t - 2)$-regular graph" 	<ul style="list-style-type: none"> - The revision of page 2 (line 58) is in page 2 (line 62). - The revision of page 2 (line 59) is in page 2 (line 63).
3. Pages 3-4: Personally, I think Lemma 1 is obvious. It is fine to state it without the proof in case the authors would like to shorten the length of the paper.	Thank you for the advice. However, we continue to present the proof of Lemma 1 to provide an explanation to readers who do not immediately see that it is true.	

4. Page 4, Line 135: There is no definition of " H_t^i " or I miss something?	Thank you for the suggestion. We have given an explanation of " H_t^i " which is contained in the definition of corona product graph. [page 2, line 45-47]	
5. Page 4, Lines 149 and 151: Remove "For $m = t$ " and "For $m < t$ ".	Thank you for the suggestion. We have removed the sentence "For $m = t$ " and "For $m < t$ ". [Page 4, line 153 and 154]	
6. Page 6, Line 259: "an $(t - 3)$ -regular" should read "a $(t - 3)$ -regular"	Thank you for the suggested. We have replaced "an $(t - 3)$ -regular" with "a $(t - 3)$ -regular".	The revision of page 6 (line 259) is in page 7 (line 263).
7. Page 7, Lines 270-272: Rewrite the sentence from "Since for every..." to "... $d_{B_m}(y, z)$ ".	Thank you for your comment. We have changed the sentence to "Note that, for every $v \in V(B_m)$ and $z \in V(H_t) - V(B_m)$, there are two vertices x and y in $V(B_m)$ such that $d_{B_m}(x, v) = 2 = d_{B_m}(y, v)$ and $d_{B_m}(x, z) = 1 = d_{B_m}(y, z)$."	<p>- We have revised</p> <p>[original text] Since for every $v \in V(B_m)$ and $z \in V(H_t) - V(B_m)$, there are two vertices x and y in $V(B_m)$ such that $d_{B_m}(x, v) = 2 = d_{B_m}(y, v)$ and $d_{B_m}(x, z) = 1 = d_{B_m}(y, z)$. [Page 7, line 270-272]</p> <p>to</p> <p>[revised text] Note that, for every $v \in V(B_m)$ and $z \in V(H_t) - V(B_m)$, there are two vertices x and y in $V(B_m)$ such that $d_{B_m}(x, v) = 2 = d_{B_m}(y, v)$ and $d_{B_m}(x, z) = 1 = d_{B_m}(y, z)$. [Page 7, line 275-276]</p>
8. Page 10, Line 409: Would it be better to change "case 3.1" to be "Case 1" the same as in the proof? If so, please change the other cases too.	Thank you for pointing this out. We have replaced "case 3.1, case 3.2, and case 3.3" with "Case 1, Case 2, and Case 3". [Page 10, line 412]	
9. Page 12, Line 485: "pandant" should read "pendant".	Thank you for the correction. We have replaced "pandant" with "pendant". [Page 12, line 494]	

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Submission Date 26 January 2022

Date of this review 17 Feb 2022 04:28:56



Referee report for
Non-Isolated Resolving Sets of Corona Graphs with Some Regular Graphs
W. Abidin, A.N.M. Salman and S.W. Saputro
for MDPI Mathematics

The non-isolated resolving number of graph G is the minimum cardinality of a non-isolated resolving set. A resolving set is a set of vertices $W \subset V(G)$ such that the path distances $(d_G(v, w_1), \dots, d_G(v, w_k))$ ($W = \{w_1, \dots, w_k\}$) determine v uniquely for any $v \in V(G)$. Here d_G is the path distance in G (which is assumed connected). W is non-isolated if the induced subgraph of W in G does not contain an isolated vertex. The non-isolated resolving number $nr(G)$ of G is the minimum cardinality of a non-isolated resolving set. In this paper, $nr(G \odot H_t)$ is calculated for the corona product of a connected graph G and H_t is either $(t - 2)$ or $(t - 3)$ -regular graph of order t . The corona product $G \odot H$ is the graph obtained by placing a copy H^i of H over each vertex v_i of G and letting v_i be adjacent to each vertex of H^i .

In the case that H_t is a simple $(t - 2)$ -regular graph of order t and G is a simple connected graph of order n , $nr(G \odot H_t)$ is equal to $2n$ if $t = 2$, and to $nt/2$ if $t \geq 4$. The case in which H_t is $(t - 3)$ -regular is more complicated. It is proved that $nr(G \odot H_t)$ is equal to $3n$ if $t \in \{3, 4\}$ and is equal to $nr(H_t)n$ if $t \geq 5$ where H_t is $(t - 3)$ -regular and G has order n . The numbers $nr(H_t)$ are computed separately (Theorem 3) by expressing $H_t = B_{m_1} + \dots + B_{m_q}$ where $m_1 \geq 3$, the m_i are nondecreasing, and $B_m = K_m - E(C_m)$. Here K_m is the complete graph of order m and C_m is a (Hamiltonian) cycle of order m . Here $+$ is the join: $G + H$ is the graph with $V(G + H) = V(G) \cup V(H)$ and whose edges are the union of those of G , of H , and of the complete bipartite graph between $V(G)$ and $V(H)$. The authors are encouraged to define the corresponding graph differences $G - H$ precisely, since expressions of this form are used throughout.

The work is of moderate interest. The concept of a resolving set is important, but that of a non-isolated resolving set is newer and less familiar. Although reference [13]—where the concept of $nr(G)$ is introduced—is cited, and the concept has been investigated further, it should at least be stated why $nr(G)$ is an important concept. Specifically, it might be mentioned if there are examples of graphs in which $nr(G)$ matches other familiar definitions of dimension when the resolving number does not. Besides this, it should be mentioned explicitly why Corona products are important in the context of metric dimension.

The paper is laid out in a way that the arguments are reasonably clear to follow. The arguments in Section 3 are a little more technical but seem okay.

As mentioned, the difference of two graphs needs to be defined more precisely.

In line 69 it should be clarified why the subscript two in "it must be $x = v_2^p$ and $y = v_2^q$."

In line 110, "joint product" should be "join product," see also line 140.

In line 170, it could be helpful to have an illustration of the notion of a gap as described "...whose all inner vertices are not in Q ."

Dear Reviewer 3 of Mathematics Journal,

We would like to thank you for the opportunity to resubmit a revised version of our manuscript entitled non-isolated resolving sets of corona graphs with some regular graphs. We appreciate the reviewers for your precious time in reviewing our paper and providing valuable comments. It was your valuable and insightful comments that led to possible improvements in the current version.

The manuscript has been revised carefully to address reviewers' comments. Below point-by-point response to the reviewers' comments and concerns. We hope the revised manuscript meet your standard.

Reviewer Comments	Author's Respond	Modification
1. Why $nr(G)$ is an important concept. Specifically, it might be mentioned if there are examples of graphs in which $nr(G)$ matches other familiar definitions of dimension when the resolving number does not.	Thank you for the suggestion. In the introduction to the last paragraph, we have added a sentence about the importance of the concept of $nr(G)$.	[The sentences that have been added are as follows] Practically, we need a threat detection sensor which also has a function to check whether its neighbour sensor is malfunctioning or not. Hence, we need an optimization of placement of threat detection sensors such that the location of each sensor must be not isolated. [Page 1, last paragraph on line 1-4]
2. Why corona products are important in the context of metric dimension.	Thank you for the suggestion. On page 2 lines 33-37, we give the reasons for the importance of the corona product.	[The sentences that have been added are as follows] The non-isolated resolving number of the graph is depended on the structure of the graph. We can obtain a structure of a graph by using an operation between two graphs. Determining a relation, in terms of non-isolated resolving number, between the original graph and the resulting graph under a graph operation is also interesting to be considered. [page 2, line 33-37]
3. In line 69 it should be clarified why the subscript two in "it must be $x = v_2^p$ and $x = v_2^q$ "	Thank you for the suggestion. Since $\{v_2^l \mid l \in [1, n]\} \notin W$. Therefore, we have changed the sentence to "Let x and y be two different vertices in $V(G \odot H_t) - W = \{v_2^l \mid l \in [1, n]\}$. Let $x = v_2^p$ and $y = v_2^q$ for some p and q in $[1, n]$ with $p \neq q$."	- We have revised [original text] Let x and y be two different vertices in $V(G \odot H_t) - W$. Then it must be $x = v_2^p$ and $y = v_2^q$ for some p and q in $[1, n]$ with $p \neq q$. [page 2, line 68-69]

		<p>to [revised text] Let x and y be two different vertices in $V(G \odot H_t) - W = \{v_2^l l \in [1, n]\}$. Let $x = v_2^p$ and $y = v_2^q$ for some p and q in $[1, n]$ with $p \neq q$. [page 2, line 72-74]</p>
<p>4. In line 110, "joint product" should be "join product," see also line 140.</p>	<p>Thank you for pointing this out. We have replaced "joint product" with "join product" and "joint" with "join".</p>	<ul style="list-style-type: none"> - We have revised "joint product" [line 110] into "join product". [line 114] - We have revised "joint" [line 140] into "join". [line 144]
<p>5. In line 170, it could be helpful to have an illustration of the notion of a gap as described "...whose all inner vertices are not in Q."</p>	<p>As a reviewer's suggestion, we've added an illustration of the notation of a gap. [Page 5, line 177-179]</p>	



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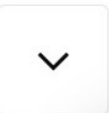
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Article type	Article
Title	Non-Isolated Resolving Sets of Corona Graphs with Some Regular Graphs
Journal	<i>Mathematics</i>
Volume	10
Issue	6

Abstract Let G be a connected, simple, and finite graph. For an ordered set and a vertex v of G , the representation of v with respect to W is the k -vector . The set W is called a resolving set of G , if every two vertices of G has a different representation. A resolving set containing a minimum number of vertices is called a basis of H . The number of elements in a basis of G is called the metric dimension of G and denoted by . In this paper, we considered a resolving set W of G where the induced subgraph of G by W does not contain an isolated vertex. Such a resolving set is called a non-isolated resolving set. A non-isolated resolving set of G with minimum cardinality is called an n -set of G . The cardinality of an n -set of G is called the non-isolated resolving number of G , denoted by . Let H be a graph. The corona product graph of G with H , denoted by , is a graph obtained by taking one copy of G and copies of H , namely , such that the i -th vertex of G is adjacent to every vertex of . If the degree of every vertex of H is k , then H is called a k -regular graph. In this paper, we determined where G is an arbitrary connected graph of order n at least two and H is a k -regular graph of



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Figure Count	6
Table Count	0
Reference Count	25

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Editor Decision

Decision	Accept in current form
Decision Date	9 March 2022



Author Information

Submitting Author Wahyuni Abidin

Corresponding Author Wahyuni Abidin

