Centro de Investigação Operacional

# Profitable Mixed Capacitated Arc Routing and Related Problems

Enrique Benavent, Ángel Corberán, Luís Gouveia,
Maria Cândida Mourão and Leonor Santiago Pinto

CIO – Working Paper 4/2013

# Profitable Mixed Capacitated Arc Routing and Related Problems

Enrique Benavent[①]; Ángel Corberán[①]; Luís Gouveia[②④]; Maria Cândida Mourão[③④]; Leonor Santiago Pinto[③⑤]

[①] Dep. Estadística e Investigación Operativa, Univ. Valencia, Spain

[②] Universidade de Lisboa, Faculdade de Ciências, DEIO, Lisboa, Portugal

[③] Universidade Técnica de Lisboa, Instituto Superior de Economia e Gestão, Lisboa, Portugal

[④] Centro de Investigação Operacional, Portugal

[⑤] CEMAPRE – Centro de Matemática Aplicada à Previsão e Decisão Económica, Portugal

## Abstract

Mixed Capacitated Arc Routing Problems (MCARP) aim to identify a set of vehicle trips that, starting and ending at a depot node, serve a given number of links, regarding the vehicles capacity, and minimizing a cost function. If both profits and costs on arcs are considered, the Profitable Mixed Capacitated Arc Routing Problem (PMCARP) may be defined. We present compact flow based models for the PMCARP, where two types of services are tackled, mandatory and optional. Adaptations of the models to fit into some other related problems are also proposed. The models are evaluated, according to their bounds quality as well as the CPU times, over large sets of test instances. New instances have been created from benchmark ones in order to solve variants that have been introduced here for the first time. Results show the new models performance within CPLEX and compare, whenever available, the proposed models against other resolution methods.

**Keywords:** Routing; Arc Routing Problems; Profits; Flow-based Models.

## 1. Introduction

We consider arc routing problems where a profit is associated with the service of the arcs in a given subset. In these problems we are faced with two conflicting objectives: maximizing the total collected profit and minimizing the travelling cost. This conflict can be addressed in different ways: i) by bicriteria optimization, ii) by combining both goals in the same objective function, in the so called *profitable problems*, iii) or by considering one as the objective and the other as a constraint. In the latter case, problems are known as *orienteering* or as *prize-collecting* problems, depending on if they consist of maximizing the collected profit or minimizing the travelled cost, respectively. More in detail, a problem is called *profitable* when it consists of finding the route that maximizes the difference between the total collected profit and the traveling cost. In the *orienteering* (or team orienteering, if a fleet of identical vehicles is considered) problem the objective is to maximize the collected profit with a constraint that the maximum cost (or time or length) of the route does not exceed a given limit. Finally, in the *prize-collecting* problem we look for a minimum cost route collecting at least a given amount of profit. This characterization follows the one proposed by (Feillet, Dejax, & Gendreau, 2005a) for the node routing case.

Profitable, orienteering and some other related capacitated arc routing problems defined on mixed graphs are considered in this paper, and, as far as we know, some of them are introduced here for the first time. In these problems, a mixed graph is given with three different types of links: mandatory, optional and deadheading. Mandatory and optional links are also called demand links or tasks and have an associated profit. All links have a deadheading cost associated with their traversal. In general, the objective is to find a set of tours that maximize a profit function, servicing all the mandatory tasks and maybe some of the optional ones, and respecting some side constraints. The profit of a demand link is available only once, and it is obtained when the service is performed.

Specifically, in this article we present and evaluate computationally compact flow-based formulations for several mixed capacitated arc routing problems with profits. Single-commodity flow models provide a general framework for modelling many routing problems. The pioneering work of Gavish and Graves (Gavish & Graves, 1978) provided this kind of models for several routing problems. The reader is also referred to Toth and Vigo (Toth & Vigo, 2002) for other examples. There are several reasons that explain the wide use of these models. They are easy to understand, easy to implement and allow additional constraints to be handled easily. However, many of the routing problems modelled so far by the single-commodity flow models are node routing problems and not much has been done with such models for arc routing. This is the main purpose of this work, to provide and evaluate single commodity flow models for several arc routing problems with profits.

The Mixed Capacitated Arc Routing Problems (MCARP or MP for short) is an NP-hard problem since it generalizes the undirected CARP (Golden & Wong, 1981), which is known to be NP-hard. Therefore, all the versions of the MCARP studied in this paper are also NP-hard.

In this paper, we consider: i) the Profitable Mixed Capacitated Arc Routing Problem (PMP), which tries to find a single vehicle tour maximizing the difference between the collected net profit and the total deadheading cost; ii) the penalised version of the PMP, which consists of maximizing an objective function that includes the total net profit, the total deadheading cost and the penalties paid for the not serviced tasks; iii) the Orienteering Mixed Capacitated Arc Routing Problem (OMP) where the objective is to find a tour maximizing the total collected (gross) profit and no deadheading cost is considered, although the tour length cannot be greater than a time limit $L$; iv) the simpler case of the OMP where no demands are taken into account and which is called the Uncapacitated Orienteering Problem (UOMP). Finally, multiple vehicle versions of the same problems are also studied.

The model presentation starts with the characterization of a set of feasible solutions that is the basis of all problems. The different problems are then defined, where, and as said above, both one vehicle and multiple vehicle cases are considered. For the multiple vehicle problems, an aggregate model is also presented and studied. Although non valid, the aggregate models generally produce good upper bounds, as it is confirmed by the computational experience. Benchmark instances are used for the variants already known from the literature and new ones have been adapted for the problems here proposed for the first time. Results show the new models performance within CPLEX and compare, whenever available, the proposed models against other resolution methods.

## 2. Literature review

The first arc routing problem dealing with profits maximization is the Maximum Benefit Chinese Postman Problem (MBCPP) introduced by (Malandraki & Daskin, 1993), who studied its directed version. In the MBCPP, a profit (also called benefit) is collected each time a demand arc is traversed, although the profit decreases as the number of traversals increases. As far as we know, no other paper on arc routing problems with profits has been published until the mid of the first decade of 2000, when, then, a number of new articles on the subject has appeared.

Among the profitable problems, the Profitable Rural Postman Problem (PRPP, also called Prize-collecting Arc Routing Problem and Privatized Rural Postman Problem) was the focus of the work by (Aráoz, Fernández, & Zoltan, 2006) and (Araóz, Fernández, & Meza, 2009b). In this problem, only the edges in a given subset *R* have an associated profit and it is assumed that this profit can be collected only once, independently of the number of times the edge is traversed. This problem can be considered a special case of a MBCPP in which only a positive benefit is associated with the edges in *R*, while all the other edges have null benefit. The capacitated version of the PRPP was studied by (Irnich, 2010). A related problem, the Clustered Prize-collecting Arc Routing Problem (CPARP) was studied in (Aráoz, Fernández, & Franquesa, 2009) and (Corberán, Fernández, Franquesa, & Sanchis, 2011) for undirected and windy graphs, respectively. In the CPARP the connected components defined by the edges with profits (demand edges) are considered, and it is required that if an edge is serviced, then all the demand edges of its component are also serviced. That is, for each component either all or none of its demand edges have to be serviced. Besides the above mentioned paper by (Malandraki & Daskin, 1993), (Pearn & Wang, 2003), (Pearn & Chiu, 2005) and (Corberán, Plana, Rodriguez-Chía, & Sanchis, 2011b) also discuss and study the MBCPP. (Feillet, Dejax, & Gendreau, 2005b) considered a more general problem, the Profitable Arc Tour Problem (PATP). In this case, the objective is to find a set of cycles in the graph that maximizes the difference between the collected profit and the travel costs; there are limits on the number of times that profit is available on each arc and the cycles cannot exceed a given length. (Deitch & Ladany, 2000) defined an orienteering problem where the objective is to design the route for a touristic bus that maximizes the "attractiveness" of the sites visited and the scenic routes traversed. The team orienteering version of the undirected Capacitated Arc Routing Problem is handled in (Archetti, Feillet, Hertz, & Speranza, 2010) and the uncapacitated version on a directed graph in (Archetti, Corberán, Plana, Sanchis, & Speranza, 2013). Table 1 summarizes the main characteristics of all these problems.

Several applications are mentioned in the literature for routing problems with profits. For instance, the orienteering problem (Vansteenwegen, Souffriau, & Oudheusden, 2011), appears in sport games where a set of checkpoints is given, each one with an associated score, and competitors try to maximize the collected score that is obtained by visiting a subset of checkpoints within a given time frame. (Hochbaum & Olinick, 2001) cite the problem of maximizing the reliability of cycles in telecommunication survival networks. The problem faced by private service companies that try to maximize operational profits (instead of minimizing the costs as in the public sector) by collecting a subset of demand edges also fits the class of profitable problems, as well as the previously mentioned bus touring problem (Deitch & Ladany, 2000). Finally, let us mention that the work of (Feillet, Dejax, & Gendreau, 2005b) focus a tactical freight transportation-planning problem arising in the car industry. In this context, a set of trips need to be planned for transporting freight between plants. Trips can either be round trips or direct trips. A round trip has the same origin and destination and is restricted to a given length. A direct trip is not constrained but is more expensive, even if, for example, a direct trip $i$ - $j$ is cheaper than a round trip $i$ - $j$ - $i$ leaving the truck empty on its way back. Authors point out that this problem can be expressed as a PATP, where the set of freight transportation demands would be the set of arcs with profits, the number of times these transportation operations have to be planned would correspond to the number of times profits can be collected, and the differences in cost between round trips and direct trips would define the profits.

<**Insert Table 1:** Main characteristics of the arc routing problems with profits in the Literature.>

## 3. Mixed arc routing problems with profits

The problems under study are defined on a mixed graph $\Gamma = (N, A' \cup E)$. Edges in $E$, characterize narrow two way streets that may be served by only one traversal (zigzag services). Arcs, in $A'$, represent either one way or large two way streets that must be served in both directions, in which case the street is modelled with two reverse arcs. An homogenous vehicle fleet is based at a *depot* node, $0 \in N$.

Two types of links in $A' \cup E$ are distinguished: *demand links* or *tasks,* and *deadheading links*. Tasks are either *mandatory* ($A_M \subseteq A', E_M \subseteq E -$ links that must be served by a vehicle) or *optional* ($A_O \subseteq A', E_O \subseteq E -$ links that may be served but are not compulsory). Node set $N$ represents the depot, the street crossings or the dead-end streets. $N$ also includes a depot copy, the artificial node $0' \in N$, joined to the original depot 0 by two deadheading reverse arcs of zero cost. Then we may assume that any vehicle will use exactly one arc leaving $0'$ and one arc entering it, and, if needed, vehicle tours may use node 0 as an intermediate node.

A *vehicle tour* is a closed walk starting and ending at the depot copy. Each link contained in the tour can be just traversed (this is called deadheading) or, in the case of a demand link, it can also be served. The total demand of the links served by the vehicle cannot exceed its capacity. Each task has an associated profit that is collected at most once, whenever it is served. Each time a link is deadheaded, task or not, a cost is taken into account. The net profit associated with the service of a demand link is defined as the difference between its (gross) profit and its traversing cost.

In what follows we present the notation used in order to define and model the arc routing problems with profits considered here.

- $G = (N, A)$ is a directed graph, derived from $\Gamma$, by replacing each edge in $E$ by two arcs with opposite directions, i.e., $A = A' \cup \{(i,j),(j,i): (i,j) \in E\}$.
- $B = |A_M| + |A_O| + |E_M| + |E_O|$.
- $R \subseteq A$ is the set of arcs in $G$ associated with the tasks, and its cardinality is $|R| = B + |E_M| + |E_O|$.
- For each task $(i,j) \in R$: $p_{ij}$ is its net profit, $q_{ij}$ its demand, and $t_{ij}^s$ is the time needed to serve it.
- $c_{ij}$ and $t_{ij}^d$ are the deadheading cost and time of traversing arc $(i,j) \in A$, respectively.
- $K$ is the maximum number of vehicles, thus the maximum number of tours allowed.
- $W$ is the capacity of each vehicle and $L$ is the tour time limit.

The problems we are studying here basically consist of finding a set of no more than $K$ vehicle trips, satisfying the vehicles capacity, starting and ending at the depot, servicing all the mandatory tasks, and some of the optional ones. In some of the models additional constraints, as limiting the total time of the trips, will be considered. The goal is to maximize the collected profit, although in some models other terms are also included in the objective function.

Single vehicle models are studied in Section 3.1, while multiple vehicles ones are discussed in Section 3.2.

### *3.1. Single vehicle mixed capacitated arc routing problems with profits*

We start by characterizing the feasible region for single vehicle mixed arc routing problems with a compact model using flow variables. Note that a feasible solution is a single tour, satisfying the capacity constraint, which serves all the mandatory tasks and some of the optional ones. As in (Gouveia, Mourão, & Pinto, 2010) we define the following variables:

- $x_{ij} = \begin{cases} 1 & \text{if } (i,j) \in R \text{ is served} \\ 0 & \text{otherwise} \end{cases}$
- $y_{ij}$ is the number of times that arc $(i,j) \in A$ is deadheaded.
- $f_{ij}$ is the flow traversing arc $(i,j) \in A \setminus \{(0,0')\}$. It is related to the remaining demand in the tour, or in a subtour of it.

The set of feasible solutions is characterized by the following set of inequalities (see (Gouveia, Mourão, & Pinto, 2010)):

$$\sum_{j:(i,j)\in A} y_{ij} + \sum_{j:(i,j)\in R} x_{ij} = \sum_{j:(j,i)\in A} y_{ji} + \sum_{j:(j,i)\in R} x_{ji} \qquad i \in N \tag{1}$$

$$x_{ij} = 1 \qquad \forall (i,j) \in A_M \tag{2}$$

$$x_{ij} \le 1 \qquad \forall (i,j) \in A_O \tag{3}$$

$$x_{ij} + x_{ji} = 1 \qquad \forall (i,j) \in E_M \tag{4}$$

$$x_{ij} + x_{ji} \le 1 \qquad \forall (i,j) \in E_O \tag{5}$$

$$y_{0'0} = 1 \tag{6}$$

$$\sum_{j:(j,i)\in A} f_{ji} - \sum_{j:(i,j)\in A} f_{ij} = \sum_{j:(j,i)\in R} q_{ji}\, x_{ji} \qquad i \in N\backslash\{0'\} \tag{7}$$

$$f_{0'0} = \sum_{(i,j)\in R} q_{ij}\, x_{ij} \tag{8}$$

$$f_{ij} \le W\left(y_{ij} + x_{ij}\right) \qquad \forall (i,j) \in A \setminus \{(0,0')\} \tag{9}$$

$$x_{ij} \in \{0,1\} \qquad \forall (i,j) \in R \tag{10}$$

$$f_{ij} \ge 0 \qquad \forall (i,j) \in A \setminus \{(0,0')\} \tag{11}$$

$$y_{ij} \ge 0, \text{ integer} \qquad \forall (i,j) \in A \tag{12}$$

Conditions (1) imply the continuity of the tour at each node; the service of each mandatory arc and edge is guaranteed by (2) and (4), respectively; while optional services are allowed by (3) and (5); (6) fixes node 0' as the starting point of the trip, and jointly with (8) and (9) guarantees the vehicle capacity; (7) and (8) are flow conservation constraints that together with the linking constraints (9) force the connectivity of the vehicle tour. Note that (7) are typical generalized flow conservation constraints on each node $i$, ensuring that if arc $(j, i)$ is served, then $q_{ji}$ units of flow are absorbed by node $i$. Conditions (9) imply that a flow variable is positive only if the corresponding arc is traversed by the vehicle trip, being then essential to impose connectivity, as stated above.

This set contains only $(2|A| - 1 + |R|)$ variables and the number of constraints is $2|N| + |A| + B$. In addition, note that if (2), (4) and (6) are properly used the model could even be simplified. Actually, these figures reduce, respectively, to $2|A| - 2 + |A_0| + 2|E_0| + |E_M|$ and $2|N| + |A| + |A_0| + |E_0| - 1$. However, and to emphasize the relationship with the following formulations, these conditions are preserved.

We next present compact formulations for single vehicle mixed capacitated arc routing problems with profits. Basically all of them share the set of feasible solutions defined by (1)–(12). They differ accordingly to their objective functions and/or some additional constraints. In fact, the so-called profitable problems are characterized by maximizing the difference between the net profit and the deadheading cost, while in the so-called orienteering problems the maximization of the gross profit is subject to a time limit constraint. Three main problems are discussed next: the profitable, the penalised profitable, and the orienteering mixed capacitated arc routing problems. As will be noted in Section 4, their associated polynomial models provide quite reasonable computational results.

***Profitable mixed capacitated arc routing problem***

We consider first the profitable mixed capacitated arc routing problem (PMP), which tries to find a single vehicle tour maximizing the difference between the total net profit and the total deadheading cost. A valid formulation for the PMP, denoted by F1, consists of constraints (1)–(12) and the following objective function:

$$\max\left(\sum_{(i,j)\in R} p_{ij}\, x_{ij} - \sum_{(i,j)\in A} c_{ij}\, y_{ij}\right) \tag{13}$$

Note that the objective function includes a fixed term associated with the total net profit of the mandatory tasks. Since the model has a polynomial number of variables and constraints, it can be directly used within an ILP package like CPLEX.

***Penalised profitable mixed capacitated arc routing problem***

Consider now that, associated with each task $(i,j) \in R$, there is a penalty $\beta_{ij}$ that is paid if the task is not served. The penalised profitable mixed problem (PPMP) consists of maximizing an objective function that includes the total net profit, the total deadheading cost and the penalties paid for the not serviced tasks. Therefore, the formulation of the PPMP, denoted by F1P, consists of the objective function

$$\max\left(\sum_{(i,j)\in R} p_{ij}\, x_{ij} - \sum_{(i,j)\in A} c_{ij}\, y_{ij} - \sum_{(i,j)\in R} \beta_{ij}\left(1 - x_{ij}\right)\right) \tag{14}$$

and constraints (1) to (12).

***Orienteering mixed capacitated arc routing problem***

In the orienteering mixed capacitated arc routing problem (OMP) the objective is to find a tour maximizing the total collected (gross) profit and no deadheading costs are considered. However, the tour, in addition to the vehicle capacity constraint, has to satisfy a time limit $L$.

Thus, to model the problem (OMP), we consider the objective function:

$$\max \sum_{(i,j)\in R} p'_{ij}\, x_{ij} \tag{15}$$

where $p'_{ij}$ denotes the gross profit associated with servicing task $(i,j) \in R$. The following set of constraints, guaranteeing that the time limit is not exceeded, has to be added:

$$\sum_{(i,j)\in R} t^s_{ij}\, x_{ij} + \sum_{(i,j)\in A} t^d_{ij}\, y_{ij} \le L \qquad (16).$$

This new model, with objective function (15) and constraints (1) to (12) and (16), is denoted by F1O.

Note that, as for the capacity constraints, the time limit constraints (16) may be written by means of flow inequalities needing new flow variables $g_{ij}$ associated with each arc $(i,j) \in A \setminus \{(0,0')\}$:

$$\sum_{j:(j,i)\in A} g_{ji} - \sum_{j:(i,j)\in A} g_{ij} = \sum_{j:(j,i)\in R} t^s_{ji}\, x_{ji} + \sum_{j:(j,i)\in A} t^d_{ji}\, y_{ji} \qquad i \in N\setminus\{0'\} \tag{17}$$

$$g_{0'0} = \sum_{(i,j)\in R} t^s_{ij}\, x_{ij} + \sum_{(i,j)\in A} t^d_{ij}\, y_{ij} \tag{18}$$

$$g_{ij} \leq L\left(x_{ij} + y_{ij}\right) \qquad \forall (i,j) \in A \setminus \{(0,0')\} \qquad (19)$$

$$g_{ij} \geq 0 \qquad \forall (i,j) \in A \setminus \{(0,0')\} \qquad (20).$$

In the simpler case in which no demands are considered, we have the uncapacitated orienteering problem (UOMP), which can be formulated by using the objective function (15) and constraints (1)−(6), (10), (12) and (17)−(20). This formulation is denoted by F1OU.

***Strengthened models***

In an attempt of improving the linear relaxation bounds, and hopefully to speed up the integer solver, some valid inequalities can be added to the previous models. The valid inequalities we have used are described next and are taken from (Gouveia, Mourão, & Pinto, 2010).

$$f_{ij} \geq q_{ij}\, x_{ij} \qquad \forall (i,j) \in R \qquad (21)$$

$$f_{ij} \geq y_{ij} - 1 \qquad \forall (i,j) \in A \setminus (R \cup \{(0,0')\}) \qquad (22).$$

The first set of constraints specifies that the value of the flow on an arc served by the vehicle should be at least equal to its demand, while the second set relates the flow in deadheading arcs with the number of times they are deadheaded. Similar constraints can be added for the flow variables $g_{ij}$.

Each one of the presented models, when strengthened with constraints (21) and (22), is denoted by SF# (being F# the designation of the original model), yielding respectively models SF1, SF1P, and SF1O. Concerning model SF1OU, constraints (21) and (22) must be replaced by

$$g_{ij} \geq t_{ij}^{s}\, x_{ij} + t_{ij}^{d}\, y_{ij}, \qquad \forall (i,j) \in R \qquad (23),$$

$$g_{ij} \geq t_{ij}^{d}\, y_{ij}, \qquad \forall (i,j) \in A \setminus (R \cup \{(0,0')\}) \qquad (24)$$

Table 2 summarizes the models proposed before for single vehicle mixed capacitated arc routing problems with profits. Next we extend these compact models to the multiple vehicles cases.

***3.2. Multiple vehicles mixed capacitated arc routing problems with profits***

The previously described models are here extended to the case where $K$ vehicles are considered. A feasible solution for a multiple vehicles mixed capacitated arc routing problem with profits (K-MP) is then a set of $K$ tours, satisfying the capacity constraints, servicing all the mandatory tasks and some of the optional ones. Like in the single vehicle case, this section begins with the characterization of the feasible solutions by means of a compact model that uses flow variables. Then, the different profit problems for the multiple vehicles case are presented. Basically all these problems have a similar set of feasible solutions and differ in their objective functions and/or in some additional constraints.

Again, the use of flow variables enables the description of feasible vehicle tours with a polynomial number of variables and constraints (see (Gouveia, Mourão, & Pinto, 2010)). For $k = 1, \dots, K$, we define:

- $x_{ij}^k = \begin{cases} 1 & \text{if } (i,j) \in R \text{ is served by vehicle } k \\ 0 & \text{otherwise} \end{cases} \quad \forall (i,j) \in R;$
- $y_{ij}^k$ is the number of times that arc $(i,j) \in A$ is deadheaded during trip $k$;
- $f_{ij}^k$ is the flow traversing arc $(i,j) \in A \setminus \{(0,0')\}$ related with the remaining demand in tour $k$ or in a subtour of it.

The set of feasible solutions is defined by:

$$\sum_{j:(i,j)\in A} y_{ij}^k + \sum_{j:(i,j)\in R} x_{ij}^k = \sum_{j:(j,i)\in A} y_{ji}^k + \sum_{j:(j,i)\in R} x_{ji}^k \qquad i \in N, \quad k = 1, \dots, K \tag{25}$$

$$\sum_{k=1}^{K} x_{ij}^k = 1 \qquad \forall (i,j) \in A_M \tag{26}$$

$$\sum_{k=1}^{K} x_{ij}^k \le 1 \qquad \forall (i,j) \in A_O \tag{27}$$

$$\sum_{k=1}^{K} \left(x_{ij}^k + x_{ji}^k\right) = 1 \qquad \forall (i,j) \in E_M \tag{28}$$

$$\sum_{k=1}^{K} \left(x_{ij}^k + x_{ji}^k\right) \le 1 \qquad \forall (i,j) \in E_O \tag{29}$$

$$y_{0'0}^k \le 1 \qquad k = 1, \dots, K \tag{30}$$

$$\sum_{j:(j,i)\in A} f_{ji}^k - \sum_{j:(i,j)\in A} f_{ij}^k = \sum_{j:(j,i)\in R} q_{ji}\, x_{ji}^k \qquad i \in N \setminus \{0'\}; \quad k = 1, \dots, K \tag{31}$$

$$f_{0'0}^k = \sum_{(i,j)\in R} q_{ij}\, x_{ij}^k \qquad k = 1, \dots, K \tag{32}$$

$$f_{ij}^k \le W\left(y_{ij}^k + x_{ij}^k\right) \qquad \forall (i,j) \in A \setminus \{(0,0')\}; \quad k = 1, \dots, K \tag{33}$$

$$x_{ij}^k \in \{0, 1\} \qquad \forall (i,j) \in A \setminus \{(0,0')\}; \quad k = 1, \dots, K \tag{34}$$

$$f_{ij}^k \geq 0 \qquad \forall (i,j) \in A \setminus \{(0,0')\}; \quad k = 1, \dots, K \tag{35}$$

$$y_{ij}^k \geq 0 \quad \text{and integer} \qquad \forall (i,j) \in A; \quad k = 1, \dots, K \tag{36}.$$

Conditions (25) guarantee the continuity of the tours at each node; the service of each mandatory arc and edge is guaranteed by (26) and (28), respectively; while optional services are allowed by (27) and (29); (30) fixes node $0'$ as the starting point of each tour and, jointly with (32) and (33), ensures that vehicles capacity is satisfied; (31) and (32) are the flow conservation constraints, which together with the linking constraints (33) guarantee the connectivity of the trips. Note that (31) are typical generalized flow conservation constraints on each node $i$, meaning that if arc $(j,i)$ is served by vehicle $k$, then $q_{ji}$ units of flow are absorbed by node $i$. Conditions (33) imply that a flow variable is positive only if the corresponding arc is traversed by the tour, being then essential to ensure connectivity, as stated above.

This set of feasible solutions is characterized with only $K(2|A| - 1 + |R|)$ variables and $K(2|N| + |A|) + B$ constraints. Next we present compact formulations for several multiple vehicles mixed capacitated arc routing problems with profits.

***Profitable multiple vehicles mixed capacitated arc routing problem***

In the profitable multiple vehicles mixed capacitated arc routing problem, denoted by K-PMP, the objective function represents the total net profit collected minus the deadheading cost. Thus, a valid formulation for the K-PMP, referred as FK, incorporates constraints (25)−(36) and the following objective:

$$\max \sum_{k=1}^{K} \left( \sum_{(i,j)\in R} p_{ij}\, x_{ij}^k - \sum_{(i,j)\in A} c_{ij}\, y_{ij}^k \right) \tag{37}$$

Note that inequalities (30) imply that if a vehicle is used then it leaves the artificial depot only once. This would allow the addition to the model of fixed costs associated with the use of the vehicles.

***Penalised profitable multiple vehicles mixed capacitated arc routing problem***

In this problem, for each task $(i,j) \in R$, a penalty $\beta_{ij}$ is paid if arc $(i,j)$ is not served. The penalised profitable multiple vehicles mixed arc routing problem (K-PPMP) consists of maximizing an objective function including the total net profit, the total deadheading cost and the penalties paid for the arcs not served.

$$\max \sum_{k=1}^{K} \left( \sum_{(i,j)\in R} p_{ij}\, x_{ij}^{k} - \sum_{(i,j)\in A} c_{ij}\, y_{ij}^{k} \right) - \sum_{(i,j)\in R} \beta_{ij} \left( 1 - \sum_{k=1}^{K} x_{ij}^{k} \right) \qquad (38)$$

The formulation of the K-PPMP, denoted by FKP, consists of (38) and constraints (25) to (36).

***Team orienteering mixed capacitated arc routing problem***

The team orienteering mixed capacitated arc routing problem (K-OMP) generalizes the orienteering problem to the case of multiple vehicles. The aim is to find a set of tours maximizing the total collected (gross) profit, satisfying the capacity constraints and a time limit $L$ for each vehicle tour. A similar problem, defined on an undirected graph and without mandatory tasks, is studied in (Archetti, Feillet, Hertz, & Speranza, 2010).

The objective function of the K-OMP is:

$$\max \sum_{k=1}^{K} \left( \sum_{(i,j)\in R} p'_{ij}\, x_{ij}^{k} \right) \qquad (39)$$

where $p'_{ij}$ denotes the gross profit of servicing task $(i,j) \in R$. The constraints, guaranteeing that the time limit per trip is not exceeded, are:

$$\sum_{(i,j)\in R} t_{ij}^{s}\, x_{ij}^{k} + \sum_{(i,j)\in A} t_{ij}^{d}\, y_{ij}^{k} \le L \qquad k = 1, \dots, K \qquad (40).$$

This new model, which maximizes (39) subject to (25) to (36), and (40), is denoted by FKO.

As in the single vehicle case, the time limit constraints (40) can be replaced by the following new flow inequalities that have to be added for each $k = 1, \dots, K$:

$$\sum_{j:(j,i)\in A} g_{ji}^{k} - \sum_{j:(i,j)\in A} g_{ij}^{k} = \sum_{j:(j,i)\in R} t_{ji}^{s}\, x_{ji}^{k} + \sum_{j:(j,i)\in A} t_{ji}^{d}\, y_{ji}^{k} \qquad i \in N \setminus \{0'\} \qquad (41)$$

$$g_{0'0}^{k} = \sum_{(i,j)\in R} t_{ij}^{s}\, x_{ij}^{k} + \sum_{(i,j)\in A} t_{ij}^{d}\, y_{ij}^{k} \qquad (42)$$

$$g_{ij}^{k} \le L\left(x_{ij}^{k} + y_{ij}^{k}\right) \qquad \forall (i,j) \in A \setminus \{(0,0')\} \qquad (43)$$

$$g_{ij}^{k} \ge 0 \qquad \forall (i,j) \in A \setminus \{(0,0')\} \qquad (44).$$

If no demands are considered, a simpler uncapacitated model, denoted by FKOU, can be defined. It consists of the objective function (39), and constraints (25) to (30), (34) to (36), and (41) to (44).

***Strengthened multiple vehicles models***

The above formulations can be strengthened by the addition of some valid inequalities proposed in (Gouveia, Mourão, & Pinto, 2010). Let us denote by $Q_M$ the total mandatory demand, i.e. $Q_M = \sum_{(i,j)\in A_M \cup E_M} q_{ij}$. Then the following inequality, which states the minimum number of vehicles that have to be used in order to serve all the mandatory demand, is valid.

$$\sum_{k=1}^{K} y_{0'0}^{k} \geq \left\lceil \frac{Q_M}{W} \right\rceil \tag{45}$$

The following constraints, similar to the ones described for the single vehicle case, are also valid.

$$f_{ij}^{k} \geq q_{ij}\, x_{ij}^{k} \qquad \forall (i,j) \in R; \qquad k = 1, \dots, K \tag{46}$$

$$f_{ij}^{k} \geq y_{ij}^{k} - 1 \qquad \forall (i,j) \in A \setminus (R \cup \{(0,0')\}) \qquad k = 1, \dots, K \qquad (47).$$

Note that the multiple vehicles formulations are highly symmetric since any permutation of the vehicle routes defines different solutions that are in fact identical in practice. The existence of this kind of alternative integer solutions, only differing in their vehicle indexes, may lead to huge computing times. The next set of constraints breaks some of these symmetries, and is used for the instances with no mandatory tasks:

$$y_{0'0}^{k} \geq y_{0'0}^{k+1} \qquad k = 1, \dots, K-1 \qquad (48).$$

In a solution with $P < K$ tours, these inequalities remove all the equivalent solutions with vehicle indexes greater than $P$.

In the presence of mandatory tasks, and in order to avoid at least partially this symmetry, we introduce the following set of constraints that have also been used in (Benavent, Corberán, Plana, & Sanchis, 2009). Let $(l_1, \dots, l_S)$ be any ordering of the mandatory links, where $S = \text{Min}\{K; |A_M| + |E_M|\}$. The idea is to force the numbering of vehicles to follow the numbering of the smallest index link they service, which can be done as follows:

$$
\begin{aligned}
& z_{l_1}^1 = 1 \\
& z_{l_i}^k \le \sum_{j=1}^{i-1} z_{l_j}^{k-1} \quad k = 3, \dots, S; \quad i \ge 2 \\
& z_{l_i}^k = 0 \quad k = i+1, \dots, S; \quad i = 1, \dots, S-1,
\end{aligned}
\tag{49}
$$

where $z_{l_i}^k = x_{l_i}^k$ if $l_i \in A_M$, and $z_{l_i}^k = x_{jt}^k + x_{tj}^k$ if $l_i = (j,t) \in E_M$.

Vehicle 1 will serve link $l_1$. The second set of constraints state that if a mandatory link $l_i$ is served by vehicle $k$ then at least one "previous" link $l_j,\ j = 1, \dots, i-1$, has to be served by the vehicle $k-1$. The last set of constraints prevents links $l_i,\ i = 1, \dots, S-1$ from being served by vehicles with indices greater than $i$.

We have noticed that the value of the above inequalities depends to a great extent on the ordering chosen for the mandatory links. A good choice is as follows: the first mandatory link is the farthest one from the depot; the second link is the farthest one both from the depot and from the first link; and so on.

Model FK# strengthened with constraints (45) to (49) is denoted by SFK#, thus resulting in SFK, SFKP and SFKO models. As in the single vehicle case, constraints (46) and (47) may be adapted and written with Flow variables $g_{ij}^k$ to get the strengthened uncapacitated model SFKOU.

***Aggregate relaxations of multiple vehicles models***

The number of variables and constraints of multiple vehicles formulations, although polynomial, is too large. This makes difficult to solve them optimally. The aggregated versions of these formulations provide valid relaxations that can be used to get upper bounds in short computing times. These aggregate formulations are next presented and discussed. We define $x_{ij} = \sum_{k=1}^{K} x_{ij}^k\ \forall (i,j) \in R$; $y_{ij} = \sum_{k=1}^{K} y_{ij}^k$ and $f_{ij} = \sum_{k=1}^{K} f_{ij}^k\ \ \forall (i,j) \in A$. Note that $x_{ij}$ takes value one, if and only if, task $(i,j)$ is served by any vehicle, while $y_{ij}$ is the total number of times that arc $(i,j)$ is deadheaded.

The aggregate formulation of FK#, denoted Agg(FK#), is obtained by adding each family of constraints (30) to (36) for all vehicles and using the above defined aggregate variables. The models thus obtained are analogous to the ones presented for the single vehicle cases. Note that the aggregate models are non-valid formulations for the original problems. Nevertheless, these models are useful to obtain valid and good upper bounds, as it is confirmed by the computational results.

It can be shown (see (Gouveia, Mourão, & Pinto, 2010)) that the linear programming relaxation bounds of the aggregate and original models are equal. In fact, from the variables definition in Agg(FK) it is easy to transform a feasible solution of the linear relaxation of FK, LFK, into a feasible solution of the linear relaxation of Agg(FK) with the same objective value. And vice-versa, given a feasible solution of the linear relaxation of Agg(FK), $\bar{x}_{ij}, \bar{y}_{ij}, \bar{f}_{ij}$, a feasible solution of LFK with the same objective value may also be defined by: $x_{ij}^k = \frac{\bar{x}_{ij}}{K}, y_{ij}^k = \frac{\bar{y}_{ij}}{K},\ f_{ij}^k = \frac{\bar{f}_{ij}}{K}$.

The aggregate models may be strengthened with the aggregate versions of the inequalities (45) to (47). Strengthened aggregate models are represented by Agg(SF#). Multiple vehicles models, including the corresponding relaxations, are summarized in Table 2, lines 9−18. It can also be proved that the optimal values of the linear relaxations of the models and their strengthened aggregate versions are equal.

## 4. Computational results

The proposed models are analysed over some benchmark instances from the literature and others that have been generated for those problems that were not previously studied. The computational results were obtained using CPLEX 12.1 in a computer with 2 AMD Opteron 6172 processors (24 cores) at 2.1GHz and with 64 GB RAM. A time limit of one hour was established.

### *4.1. Data instances*

Single vehicle models are tested on instances generated from the sets of instances alba, madri and alda proposed by (Corberán, Mejía, & Sanchis, 2005) for the Mixed General Routing Problem (MGRP). The MGRP consists of finding a minimum cost tour traversing a given subset of required edges and arcs ($E_R \cup A_R$) and visiting a given subset of required vertices ($V_R$). Multiple vehicle models are tested on instances based on the ones used in (Belenguer, Benavent, Lacomme, & Prins, 2006) and (Gouveia, Mourão, & Pinto, 2010) for the mixed CARP, and on the ones proposed in (Archetti, Feillet, Hertz, & Speranza, 2010) and (Archetti, Corberán, Plana, Sanchis, & Speranza, 2013) for the team orienteering arc routing problem.

**Table 2: Problems and instances.**

| Line | Problem | Special characteristics | Objective function | Constraints | Model | Instances |
|---|---|---|---|---|---|---|
| 1 | PMP | $K = 1$ | (13) | (1)-(12) | F1 | palba; pmadri; palda |
| 2 | | | | (1)-(12), (21), (22) | SF1 | |
| 3 | PPMP | $K = 1$ | (14) | (1)-(12) | F1P | ppalba; ppmadri; ppalda |
| 4 | | | | (1)-(12), (21), (22) | SF1P | |
| 5 | OMP | $K = 1$ | (15) | (1)-(12), (16) | F1O | oalba; omadri; oalda |
| 6 | | | | (1)-(12), (16), (21), (22) | SF1O | |
| 7 | UOMP | $K = 1$ Uncapacitated | (15) | (1)-(6), (10), (12), (17)-(20) | F1OU | uoalba; uomadri; uoalda |
| 8 | | | | (1)-(6), (10), (12), (17)-(20), (23),(24) | SF1OU | |
| 9 | K-PMP | – | (37) | (25)-(36) | FK | pmval; plpr |
| 10 | | | | (25)-(36), (45)-(49) | SFK | |
| 11 | K-PMP relaxation | – | Agg(37) | Agg(25)-Agg(36) | Agg(FK) | pmval; plpr |
| 12 | | | | Agg(25)-Agg(36), Agg(45)-Agg(47) | Agg(SFK) | |
| 13 | K-PPMP | – | (38) | (25)-(36), (45)-(49) | SFKP | ppmval; pplpr |
| 14 | K-PPMP relaxation | – | Agg(38) | Agg(25)-Agg(36), Agg(45)-Agg(47) | Agg(SFKP) | |
| 15 | K-OMP | – | (39) | (25)-(36), (40), (45)-(49) | SFKO | tval; toval |
| 16 | K-UOMP | Uncapacitated | | (25)-(30), (34)-(36), (41)-(49) * | SFKOU | thertz |
| 17 | K-OMP relaxation | – | Agg(39) | Agg(25)-Agg(36), Agg(40), Agg(45)-Agg(47) | Agg(SFKO) | tval; toval |
| 18 | K-UOMP relaxation | Uncapacitated | | Agg(25)-Agg(30), Agg(34)-Agg(36), Agg(41)-Agg(47) * | Agg(SFKOU) | thertz |

* (46)-(47) should be rewritten using flow variables $g_{ij}$.

The above instances are modified, when necessary, to generate instances for all the problems under study. The additional data required is generated as follows.

For each required link $(i, j)$ in the original data sets:

**(i)** **mandatory/optional links**: Let $r$ be a random number between 0 and 1. Given a value for $m \in \{0{,}25; 0{,}5; 0{,}75\}$, each required link is made mandatory if $r < m$, and optional otherwise. Note that by varying $m$, each initial instance is transformed into three instances.

**(ii)** **net profit:** $p_{ij} = \lfloor \bar{c} + u_{ij} + 0{,}5 \rfloor$, where $\bar{c}$ is the average cost of the links, and $u_{ij}$ is a random uniform number generated in the interval $(0{,}8c_{ij}; 1{,}5c_{ij})$.

**(iii)** **gross profit:** $p'_{ij} = p_{ij} + c_{ij}$.

**(iv)** **demand**: $q_{ij}$ is a random uniform integer number in $(0{,}75c_{ij}; 1{,}5c_{ij})$.

**(v)** **penalty:** $\beta_{ij}$ is a random uniform integer number in $\left(0{,}1p_{ij}; 0{,}5p_{ij}\right)$.

**(vi)** **deadheading and service times** (in minutes): we compute first a speed $v_{ij}$ as a random number in $(20; 50)$, and then

**deadheading time**: $t_{ij}^d = \left\lceil \frac{c_{ij}}{v_{ij}} 60 \right\rceil$;

**service time**: $t_{ij}^s = \gamma_{ij} t_{ij}^d$, where:

$$\gamma_{ij} = \begin{cases} 2 & \text{if} \quad q_{ij} \leq 0{,}9Q_{AM} \\ 3 & \text{if} \quad 0{,}9Q_{AM} < q_{ij} < 1{,}1Q_{AM} \\ 4 & \text{if} \quad q_{ij} \geq 1{,}1Q_{AM} \end{cases}$$

and $Q_{AM} = \frac{Q_M}{|A_M + E_M|}$ .

Moreover, the vehicle capacity, in the single vehicle models ($K = 1$), and the time limit are generated as follows:

**(vii)** **time limit per trip**: $L$ was set to 95% of the time spent by a feasible solution computed with F1.

**(viii)** **vehicle capacity**: $W = round(Q_M + \alpha\, Q_O)$ with $Q_O$ representing the total optional demand and $\alpha \in \{0{,}50;\ 0{,}80\}$.

### *4.2. Results for single vehicle problems*

Single vehicle models are tested on instances based on the benchmark MGRP ones of (Corberán, Mejía, & Sanchis, 2005). The authors generated three sets of MGRP instances, alba, alda and madri, from the street networks of three Spanish towns (Albaida, Aldaya and Madrigueras). The original data has the following characteristics:

- alba: $|\mathrm{N}| = 116$; $|\mathrm{E} \cup \mathrm{A}| = 174$; $7 \leq |\mathrm{E_R}| \leq 148$; $3 \leq |\mathrm{A_R}| \leq 74$.
- madri: $|\mathrm{N}| = 196$; $|\mathrm{E} \cup \mathrm{A}| = 316$; $13 \leq |\mathrm{E_R}| \leq 250$; $2 \leq |\mathrm{A_R}| \leq 158$.
- alda: $|N| = 214$; $|E| = 224$; $|A| = 127$; $0 \leq |E_R| \leq 209$; $0 \leq |A_R| \leq 118$.

Since in the MGRP there are required nodes that have to be necessarily visited and this is not the case in our models, each $v \in V_R$ not incident with a required link is here replaced by the arc task $(v', v'')$, each arc with end node $v$ is replaced by an arc entering at $v'$, and arcs leaving $v$ are replaced by arcs leaving $v''$, while each edge $(v, w)$ incident with $v$ is substituted by two arcs $(v'', w)$ and $(w, v')$. The cost of arc

$(v', v'')$, $c_{v'v''}$, is a random number generated in the interval $(0{,}8\bar{c}; 1{,}2\bar{c})$, where $\bar{c}$ represents the average cost of all the links in the original graph.

Table 3 shows the main characteristics of the generated sets of instances. Their names can be found in the first column and include the percentage of optional links among the required ones in the original instances. For example, *palba*25 denotes the set of instances gathered from set *alba* by generating profits and randomly selecting 25% of the required links in the original instance as optional. The number of instances and the average values of the characteristics in each set of instances are shown in the other columns. Moreover, for each instance, two different vehicle capacities are considered by varying $\alpha \in \{0{,}5;\ 0{,}8\}$ in (viii). The name of the instance sets will also include this characteristic, thus *palba*25_W50 and *palba*25_W80, for example, denote the set of instances in *palba*25 where vehicle capacity is obtained using $\alpha = 0{,}5$ and $\alpha = 0{,}8$, respectively, in (viii).

**Table 3: Instance characteristics (average values)**

| Name | # of instances | $\|V\|$ | $\|A\|$ | $\|A_M\|$ | $\|A_O\|$ | $\|E\|$ | $\|E_M\|$ | $\|E_O\|$ |
|---|---|---|---|---|---|---|---|---|
| palba25 | 25 | 146,8 | 165,7 | 14,4 | 45,6 | 76,8 | 14,2 | 44,2 |
| palba50 | 25 | 146,8 | 165,7 | 30,4 | 29,6 | 76,8 | 28,9 | 29,6 |
| palba75 | 25 | 146,8 | 165,7 | 44,8 | 15,2 | 76,8 | 43,9 | 14,5 |
| pmadri25 | 25 | 243,2 | 292,6 | 28,7 | 79,8 | 131,6 | 24,1 | 72,3 |
| pmadri50 | 25 | 243,2 | 292,6 | 55,8 | 52,8 | 131,6 | 47,6 | 48,7 |
| pmadri75 | 25 | 243,2 | 292,6 | 81,7 | 26,8 | 131,6 | 72,5 | 23,9 |
| palda25 | 31 | 263,7 | 320,5 | 27,3 | 83,0 | 152,1 | 27,8 | 79,2 |
| palda50 | 31 | 263,7 | 320,5 | 54,5 | 55,8 | 152,1 | 54,2 | 52,7 |
| palda75 | 31 | 263,7 | 320,5 | 81,5 | 28,8 | 152,1 | 80,8 | 26,2 |

### *Profitable mixed capacitated arc routing problem*

The computational results obtained for the profitable mixed capacitated arc routing problem with models F1 and SF1 on the above sets of instances are shown in Table 4. Third column presents the average percentage values of the optional demand effectively collected in the corresponding optimal solutions. Average gap values (in percentage) for models F1 and SF1 are displayed in columns 4 and 6, respectively. Gap values are computed from the differences between the best upper bounds found by the branch-and-bound procedure, within a time limit of an hour, and the optimal values, if known, or the profit of the best feasible solutions found. Columns 8 and 9 show the average gap values for the upper bounds obtained with the linear relaxations of the above models, which are denoted as LF1 and LSF1, respectively. Columns headed by #OS indicate the number of instances solved to optimality with each integer model. Finally, last three rows present the minimum, average and maximum computing times for each model in all the instances.

As may be seen in Table 4, model F1 is able to solve to optimality most of the instances (450 out of 486) in small computing times, which proves the effectiveness of this model in solving medium size instances. On the other hand, SF1 requires slightly larger computing times and solves optimally 437 instances, probably due to the fact that a less number of branch-and-bound nodes can be explored. Concerning linear relaxations LF1 and LSF1, it can be seen that they are very low time consuming, and that both provide similar upper bounds that are not good in general.

**Table 4: Results for the profitable mixed capacitated arc routing problem (PMP)**

| | | | F1 | | SF1 | | LF1 | LSF1 |
|---|---|---|---|---|---|---|---|---|
| **Name** | **# of instances** | $\%Q_O$ | **gap (%)** | **#OS** | **gap (%)** | **#OS** | **gap (%)** | **gap (%)** |
| palba25_W80 | 25 | 79,7 | 0,0 | 24 | 0,0 | 24 | 26,2 | 26,2 |
| palba50_W80 | 25 | 79,3 | 0,0 | 25 | 0,0 | 25 | 28,0 | 28,0 |
| palba75_W80 | 25 | 77,7 | 0,0 | 25 | 0,0 | 25 | 31,7 | 31,7 |
| pmadri25_W80 | 25 | 79,9 | 0,2 | 21 | 0,3 | 22 | 15,0 | 15,0 |
| pmadri50_W80 | 25 | 79,7 | 0,1 | 21 | 0,3 | 22 | 15,4 | 15,4 |
| pmadri75_W80 | 25 | 79,0 | 0,1 | 21 | 0,2 | 21 | 16,4 | 16,4 |
| palda25_W80 | 31 | 79,9 | 0,0 | 27 | 0,2 | 24 | 10,4 | 10,4 |
| palda50_W80 | 31 | 79,9 | 0,1 | 27 | 0,1 | 25 | 10,9 | 10,9 |
| palda75_W80 | 31 | 79,6 | 0,1 | 29 | 0,1 | 28 | 10,9 | 10,8 |
| palba25_W50 | 25 | 49,9 | 0,0 | 23 | 0,0 | 23 | 38,5 | 38,4 |
| palba50_W50 | 25 | 49,9 | 0,0 | 25 | 0,0 | 25 | 51,3 | 51,2 |
| palba75_W50 | 25 | 49,7 | 0,0 | 25 | 0,0 | 25 | 39,4 | 39,3 |
| pmadri25_W50 | 25 | 50,0 | 0,2 | 23 | 0,4 | 22 | 20,1 | 20,1 |
| pmadri50_W50 | 25 | 49,9 | 0,1 | 23 | 0,2 | 21 | 19,1 | 19,1 |
| pmadri75_W50 | 25 | 49,9 | 0,1 | 23 | 0,1 | 22 | 18,8 | 18,8 |
| palda25_W50 | 31 | 50,0 | 0,0 | 28 | 0,2 | 25 | 13,9 | 13,9 |
| palda50_W50 | 31 | 50,0 | 0,1 | 30 | 0,2 | 29 | 13,9 | 13,9 |
| palda75_W50 | 31 | 49,9 | 0,1 | 30 | 0,2 | 29 | 12,5 | 12,5 |
| **Total/average** | **486** | | **0,1** | **450** | **0,1** | **437** | **21,8** | **21,3** |

| | | F1 | SF1 | LF1 | LSF1 |
|---|---|---|---|---|---|
| **Cpu time (s)** | **Min** | 0,7 | 0,2 | 0,0 | 0,01 |
| | **Av** | 436,7 | 553,7 | 0,0 | 0,01 |
| | **max** | 3600 | 3600 | 0,1 | 0,01 |

### *Penalised profitable mixed capacitated arc routing problem*

In order to test models F1P and SF1P for the penalised profitable problem (PPMP), arc penalties, computed as shown in (v), have been added to the preceding instances to obtain the sets of instances denoted by ppalba, ppmadri and ppalda. The results are shown in Table 5, which has the same structure as Table 4.

**Table 5: Results for the penalised profitable mixed capacitated arc routing problem (PPMP)**

| Name | # of instances | $\%Q_O$ | F1P gap (%) | F1P #OS | SF1P gap (%) | SF1P #OS | LF1P gap (%) | LSF1P gap (%) |
|---|---|---|---|---|---|---|---|---|
| ppalba25_W80 | 25 | 79,8 | 0,0 | 25 | 0,0 | 25 | 36,5 | 36,5 |
| ppalba50_W80 | 25 | 79,6 | 0,0 | 25 | 0,0 | 25 | 34,5 | 34,4 |
| ppalba75_W80 | 25 | 78,8 | 0,0 | 25 | 0,0 | 24 | 35,5 | 35,4 |
| ppmadri25_W80 | 25 | 80,0 | 0,1 | 19 | 0,3 | 20 | 19,9 | 19,9 |
| ppmadri50_W80 | 25 | 79,8 | 0,2 | 22 | 0,3 | 21 | 18,4 | 18,4 |
| ppmadri75_W80 | 25 | 79,4 | 0,1 | 23 | 0,2 | 23 | 18,1 | 18,1 |
| ppalda25_W80 | 31 | 80,0 | 0,0 | 24 | 0,2 | 22 | 13,8 | 13,8 |
| ppalda50_W80 | 31 | 79,9 | 0,1 | 25 | 0,2 | 23 | 13,2 | 13,1 |
| ppalda75_W80 | 31 | 79,7 | 0,1 | 30 | 0,1 | 30 | 11,9 | 11,9 |
| ppalba25_W50 | 25 | 49,9 | 0,0 | 24 | 0,0 | 24 | 166,8 | 166,5 |
| ppalba50_W50 | 25 | 49,9 | 0,0 | 25 | 0,0 | 25 | 58,4 | 58,3 |
| ppalba75_W50 | 25 | 49,7 | 0,0 | 25 | 0,0 | 25 | 52,4 | 52,4 |
| ppmadri25_W50 | 25 | 50,0 | 0,2 | 23 | 0,5 | 22 | 39,8 | 39,8 |
| ppmadri50_W50 | 25 | 50,0 | 0,3 | 22 | 0,3 | 21 | 27,4 | 27,4 |
| ppmadri75_W50 | 25 | 49,9 | 0,1 | 24 | 0,1 | 23 | 22,4 | 22,4 |
| ppalda25_W50 | 31 | 50,0 | 0,0 | 28 | 0,5 | 26 | 25,6 | 25,6 |
| ppalda50_W50 | 31 | 50,0 | 0,0 | 30 | 0,2 | 28 | 19,3 | 19,3 |
| ppalda75_W50 | 31 | 49,9 | 0,2 | 30 | 0,2 | 29 | 14,6 | 14,6 |
| Total/average | 486 | | 0,1 | 449 | 0,2 | 436 | 34,9 | 35,8 |

| | | F1P | SF1P | LF1P | LSF1P |
|---|---|---|---|---|---|
| Cpu time (s) | Min | 1,1 | 0,1 | 0,0 | 0,01 |
| | Av | 469,0 | 586,7 | 0,0 | 0,01 |
| | max | 3600 | 3600 | 0,1 | 0,01 |

As may be seen in Table 5, model F1P solves to optimality all but 37 instances, while SF1P cannot solve 50 of them. The penalties that have to be paid if no service is made on the optional tasks seem to make the problem harder to solve, with worse linear relaxation bounds.

### *Orienteering mixed capacitated arc routing problem*

The sets of instances oalba, omadri and oalda, used to test the models proposed for the orienteering mixed capacitated arc routing problem, have been generated from the sets palba, pmadri and palda, respectively, as follows. First, the net profit of each task is substituted by the gross profit computed as in (iii). Then, service and deadheading times for the arcs are computed as in (vi), and (vii) is used to calculate the time limit $L$. The total number of instances in Table 6 and Table 7 reduces to 477 due to infeasibilities caused by the time limit constraint.

**Table 6: Results for the orienteering mixed capacitated arc routing problem (OMP)**

| Name | # of instances | $\%Q_O$ | F1O gap (%) | #OS | gap (%) | SF1O #OS | LF1O gap (%) | LSF1 gap (%) |
|---|---|---|---|---|---|---|---|---|
| oalba25_W80 | 25 | 76,4 | 0,0 | 24 | 0,0 | 24 | 4,9 | 4,9 |
| oalba50_W80 | 25 | 72,5 | 0,0 | 25 | 0,0 | 25 | 4,8 | 4,8 |
| oalba75_W80 | 25 | 58,0 | 0,0 | 25 | 0,0 | 25 | 5,3 | 5,3 |
| omadri25_W80 | 25 | 77,1 | 0,1 | 21 | 0,1 | 21 | 3,4 | 3,4 |
| omadri50_W80 | 25 | 73,5 | 0,1 | 21 | 0,2 | 20 | 3,5 | 3,5 |
| omadri75_W80 | 24 | 61,7 | 0,2 | 20 | 0,3 | 20 | 3,9 | 3,9 |
| oalda25_W80 | 31 | 78,4 | 0,0 | 28 | 0,1 | 22 | 2,8 | 2,8 |
| oalda50_W80 | 31 | 75,0 | 0,2 | 26 | 0,2 | 24 | 3,3 | 3,3 |
| oalda75_W80 | 31 | 65,1 | 0,2 | 28 | 0,2 | 28 | 3,8 | 3,8 |

| Name | # of instances | $\%Q_O$ | F1O gap (%) | F1O #OS | SF1O gap (%) | SF1O #OS | LF1O gap (%) | LSF1 gap (%) |
|---|---|---|---|---|---|---|---|---|
| oalba25_W50 | 25 | 48,5 | 0,0 | 23 | 0,0 | 23 | 6,4 | 6,4 |
| oalba50_W50 | 25 | 43,6 | 0,0 | 25 | 0,0 | 24 | 0,0 | 6,6 |
| oalba75_W50 | 21 | 28,2 | 0,0 | 21 | 0,0 | 21 | 7,3 | 7,3 |
| omadri25_W50 | 25 | 48,4 | 0,2 | 20 | 0,3 | 20 | 5,1 | 5,1 |
| omadri50_W50 | 25 | 44,7 | 0,3 | 20 | 0,4 | 20 | 5,3 | 5,3 |
| omadri75_W50 | 22 | 28,2 | 0,0 | 22 | 0,0 | 22 | 6,4 | 6,4 |
| oalda25_W50 | 31 | 49,1 | 0,0 | 23 | 0,4 | 17 | 4,1 | 4,1 |
| oalda50_W50 | 31 | 45,8 | 0,2 | 25 | 0,7 | 24 | 5,0 | 5,0 |
| oalda75_W50 | 30 | 31,5 | 0,2 | 28 | 0,1 | 29 | 6,2 | 6,2 |
| Total/average | 477 | | 0,1 | 425 | 0,2 | 409 | 4,5 | 4,9 |

| | | F1 | SF1 | LF1 | LSF1 |
|---|---|---|---|---|---|
| Cpu time (s) | Min | 1,1 | 0,6 | 0,01 | 0,02 |
| | Av | 572,7 | 715,5 | 0,07 | 0,02 |
| | max | 3600 | 3600 | 0,18 | 0,01 |

The results included in Table 6 show that most of the instances are solved to optimality with both models, F1O (425 of 477) and SF1O (409 of 477). The bounds provided by the linear relaxations are tighter than the previous ones.

To evaluate the uncapacitated orienteering mixed arc routing models, F1OU and SF1OU, we consider the instance sets soalba, somadri and soalda that are obtained from oalba, omadri and oalda just by ignoring the arc demands and the vehicle capacity. Although the vehicle capacity is not considered, the names of the instances have the suffix “80” or “50” to identify the different values of the time limit L. In fact, instances with “80” suffix, reflect that L has been set to 95% of the time used by a feasible solution in F1 for instances “W_80”, while in “50” the L value is computed similarly but for “W_50”.

**Table 7: Results for the uncapacitated orienteering mixed CARP (UOMP)**

| Name | # of instances | $\%Q_O$ | F1OU gap (%) | F1OU #OS | SF1OU gap (%) | SF1OU #OS | LF1OU gap (%) | LSF1OU gap (%) |
|---|---|---|---|---|---|---|---|---|
| uoalba25_80 | 25 | 76,6 | 0,0 | 24 | 0,0 | 24 | 6,6 | 6,6 |
| uoalba50_80 | 25 | 72,5 | 0,0 | 25 | 0,0 | 25 | 6,2 | 6,2 |
| uoalba75_80 | 25 | 58,0 | 0,0 | 25 | 0,0 | 25 | 6,6 | 6,6 |
| uomadri25_80 | 25 | 77,2 | 0,1 | 20 | 0,1 | 19 | 4,3 | 4,3 |
| uomadri50_80 | 25 | 73,5 | 0,1 | 20 | 0,2 | 21 | 4,2 | 4,2 |
| uomadri75_80 | 25 | 61,0 | 0,4 | 20 | 0,5 | 21 | 4,7 | 4,7 |
| uoalda25_80 | 31 | 78,5 | 0,0 | 27 | 0,3 | 21 | 3,9 | 3,9 |
| uoalda50_80 | 31 | 75,1 | 0,2 | 26 | 0,4 | 24 | 3,9 | 3,9 |
| uoalda75_80 | 31 | 64,8 | 0,3 | 29 | 0,4 | 26 | 4,2 | 4,2 |
| uoalba25_50 | 25 | 49,6 | 0,0 | 24 | 0,0 | 24 | 12,5 | 12,5 |
| uoalba50_50 | 25 | 43,7 | 0,0 | 25 | 0,0 | 25 | 19,3 | 11,5 |
| uoalba75_50 | 21 | 28,2 | 0,0 | 21 | 0,0 | 21 | 10,4 | 10,4 |
| uomadri25_50 | 25 | 49,5 | 0,3 | 21 | 0,4 | 21 | 8,2 | 8,2 |
| uomadri50_50 | 25 | 45,0 | 0,3 | 21 | 0,8 | 20 | 7,4 | 7,4 |
| uomadri75_50 | 22 | 28,2 | 0,0 | 22 | 0,1 | 20 | 7,6 | 7,6 |
| uoalda25_50 | 31 | 50,3 | 0,0 | 25 | 0,9 | 20 | 7,2 | 7,2 |
| uoalda50_50 | 31 | 45,9 | 0,2 | 28 | 0,4 | 25 | 7,2 | 7,2 |
| uoalda75_50 | 29 | 31,4 | 0,3 | 26 | 0,2 | 26 | 7,5 | 7,5 |
| Total/average | 477 | | 0,1 | 429 | 0,3 | 408 | 7,3 | 6,9 |

| | | F1OU | SF1OU | LF1OU | LSF1OU |
|---|---|---|---|---|---|
| Cpu time (s) | Min | 0,4 | 0,4 | 0,0 | 0,02 |
| | av | 608,8 | 755,5 | 0,1 | 0,06 |
| | max | 3600 | 3600 | 0,2 | 0,17 |

Again, results in Table 7 show that most of the instances are solved to optimality with both models, F1O (429 of 477) and SF1O (408 of 477). The bounds provided by the linear relaxations are also tighter than the ones in Table 4 and Table 5.

### 4.3. Results for multiple vehicle problems

The performance of the multiple vehicle profitable and penalised models is analysed using instances pmval and pmlpr, generated from the mval and mlpr instances used in (Belenguer, Benavent, Lacomme, & Prins, 2006) and (Gouveia, Mourão, & Pinto, 2010) for the mixed CARP. Team orienteering models are studied on the tval, toval and thertz instances generated by (Archetti, Feillet, Hertz, & Speranza, 2010) and (Archetti, Corberán, Plana, Sanchis, & Speranza, 2013) from the val instances of (Benavent, Campos, Corberán, & Mota, 1992) and the Hertz instances (Hertz, Laporte, & Nanchen-Hugo, 1999). Additional data for the above instances have been generated as described in Section 4.1.

The following tables showing the results obtained with the multiple vehicle models also include those obtained with the aggregate models, which are not valid but provide good upper bounds in short computing times.

#### *Profitable mixed capacitated arc routing problem with multiple vehicles*

The main characteristics of the instance sets are shown in Table 8. As in Table 3, their names can be found in the first column and include the percentage of optional links among the required ones in the original instances. The number of instances and the average values of their characteristics for each set of instances are shown in the other columns. Again, for each instance, two different vehicle capacities are considered by varying $\alpha \in \{0{,}5;\ 0{,}8\}$ in (vii).

**Table 8: Instance characteristics (average values)**

| Name | # | $\lvert V\rvert$ | $\lvert A\rvert$ | $\lvert A_M\rvert$ | $\lvert A_O\rvert$ | $\lvert E\rvert$ | $\lvert E_M\rvert$ | $\lvert E_O\rvert$ | $K$ |
|---|---|---|---|---|---|---|---|---|---|
| pmval25 | 34 | 36 | 63 | 15 | 48 | 25 | 6 | 19 | 7 |
| pmval50 | 34 | 36 | 63 | 31 | 32 | 25 | 12 | 13 | 7 |
| pmval75 | 34 | 36 | 63 | 47 | 16 | 25 | 19 | 6 | 7 |
| pmlpr25 | 15 | 168 | 335 | 68 | 88 | 88 | 21 | 67 | 11 |
| pmlpr50 | 15 | 168 | 335 | 135 | 129 | 88 | 44 | 44 | 11 |
| pmlpr75 | 15 | 168 | 335 | 198 | 66 | 88 | 65 | 23 | 11 |

Table 9 shows the computational results obtained for this problem with models FK, Agg(FK) and SFK on the above sets of instances with a time limit of one hour. The name of each set includes the percentage of mandatory demand and the percentage value used to compute the vehicles capacity. Note that the profitable mixed capacitated arc routing problem with multiple vehicles is a hard problem and none of the models we have tried was able to find even a feasible solution in many of the instances. In fact, second column of Table 9 gives the number of instances for which a feasible solution was found by models FK and SFK. Average gap values (in percentage) for models FK, Agg(FK), SFK and LFK (linear relaxation of FK) are computed as $\frac{(\bullet)-\mathrm{LB}}{\mathrm{LB}} \times 100$, where $(\bullet)$ represents the upper bound obtained with the corresponding model and LB is the best lower bound obtained from FK and SFK. Columns headed by #OS, for models FK and SFK, show the number of instances solved to optimality with these models. Since model Agg(FK) cannot produce feasible solutions, column #OV gives the number of times the optimal value was reached with this model. Note that the average gaps shown in the table have been computed taking into account only the instances for which a feasible solution is known.

**Table 9: Results for the multiple profitable mixed capacitated arc routing problem (K-PMP)**

| | | FK | | Agg(FK) | | SFK | | LFK |
|---|---|---|---|---|---|---|---|---|
| **Name** | **#FS** | **gap(%)** | **#OS** | **gap(%)** | **#OV** | **gap(%)** | **#OS** | **gap(%)** |
| pmval25_W50 | 34 | 3,37 | 1 | 2,85 | 0 | 2,51 | 3 | 4,35 |
| pmval50_W50 | 34 | 5,11 | 2 | 4,51 | 1 | 4,27 | 5 | 6,02 |
| pmval75_W50 | 33 | 3,67 | 4 | 3,22 | 2 | 3,25 | 6 | 4,59 |
| pmval25_W80 | 33 | 3,15 | 2 | 2,62 | 1 | 2,58 | 2 | 3,90 |
| pmval50_W80 | 33 | 3,51 | 5 | 3,04 | 4 | 5,59 | 6 | 4,26 |
| pmval75_W80 | 33 | 3,11 | 1 | 2,52 | 3 | 3,72 | 4 | 3,87 |
| pmlpr25_W50 | 6 | 0,28 | 3 | 0,24 | 4 | 0,24 | 5 | 1,23 |
| pmlpr50_W50 | 7 | 1,24 | 5 | 1,12 | 0 | 2,35 | 5 | 2,12 |
| pmlpr75_W50 | 6 | 0,19 | 5 | 0,02 | 5 | 0,19 | 5 | 1,02 |
| pmlpr25_W80 | 7 | 4,19 | 5 | 4,19 | 5 | 4,24 | 5 | 4,69 |
| pmlpr50_W80 | 7 | 0,85 | 4 | 0,84 | 5 | 0,85 | 5 | 1,86 |
| pmlpr75_W80 | 6 | 0,12 | 4 | 0,02 | 5 | 0,02 | 5 | 0,92 |
| **Sum** | 239 | | 41 | | 35 | | 56 | |

| | | FK | Agg(FK) | SFK | LFK |
|---|---|---|---|---|---|
| **Cpu time (s)** | min | 3,20 | 2,92 | 2,11 | 0,20 |
| | **Av** | **3084,9** | **35,25** | **2952,4** | **0,61** |
| | **max** | **3600,0** | **287,02** | **3600,0** | **0,98** |

From Table 9 we may infer that strengthened and non-strengthened models performed very similarly. The results in the table show that SFK is better than FK in the number of optimal solutions found, although they both produce similar gaps. The results obtained with Agg(FK) show that this relaxation achieve small gap values in short computing times. In fact, a good number of optimal values were found for the pmlpr instances. Finally, the linear relaxation LFK provides quite good results in negligible times.

### *Penalised profitable multiple vehicles mixed capacitated arc routing problem*

Instances used to test the models for the penalised case have been generated from instance sets pmval and pmlpr by adding the penalties computed as shown in (v) (see Section 4.1).

Table 10 depicts the results in the same order as Table 9. Results are very similar to the previous ones, but the introduction of the penalties seems to make the problem harder. In fact, gap values tend to be worse and the number of optimally solved instances decreases, although the number of optimal values reached by the aggregated model increases from 35 to 37.

**Table 10: Results for the penalised profitable multiple vehicles mixed capacitated arc routing problem (K-PPMP).**

| | | FK | | Agg(FK) | | SFK | | LFK |
|---|---|---|---|---|---|---|---|---|
| Name | #FS | gap(%) | #OS | gap(%) | #OV | gap(%) | #OS | gap(%) |
| ppmval25_W50 | 34 | 6,05 | 2 | 5,37 | 1 | 4,86 | 4 | 7,53 |
| ppmval50_W50 | 34 | 8,41 | 1 | 7,68 | 1 | 7,48 | 4 | 9,63 |
| ppmval75_W50 | 32 | 3,67 | 4 | 3,10 | 2 | 2,98 | 7 | 4,64 |
| ppmval25_W80 | 32 | 3,42 | 3 | 3,55 | 2 | 2,90 | 3 | 4,33 |
| ppmval50_W80 | 33 | 6,65 | 4 | 6,20 | 4 | 11,81 | 6 | 7,51 |
| ppmval75_W80 | 33 | 5,08 | 3 | 5,20 | 1 | 4,70 | 3 | 6,11 |
| pplpr25_W50 | 7 | 1,37 | 5 | 1,34 | 5 | 9,66 | 5 | 2,09 |
| pplpr50_W50 | 7 | 1,75 | 3 | 1,69 | 3 | 1,75 | 3 | 2,45 |
| pplpr75_W50 | 7 | 2,06 | 3 | 1,94 | 5 | 0,20 | 5 | 3,04 |
| pplpr25_W80 | 7 | 10,99 | 4 | 12,63 | 4 | 10,99 | 4 | 11,31 |
| pplpr50_W80 | 7 | 6,59 | 2 | 6,29 | 4 | 0,49 | 2 | 7,10 |
| pplpr75_W80 | 6 | 0,14 | 2 | 0,04 | 5 | 1,73 | 5 | 0,86 |
| Sum | 239 | | 36 | | 37 | | 51 | |

| | | FK | Agg(FK) | SFK | LFK |
|---|---|---|---|---|---|
| Cpu time (s) | min | 2,56 | 2,73 | 2,16 | 0,46 |
| | Av | 3121,5 | 34,97 | 2999,3 | 0,76 |
| | max | 3600,0 | 229,32 | 3600,0 | 1,29 |

### *Team orienteering mixed capacitated arc routing problem*

Models for the K-OMP are tested with the benchmark instances of (Archetti, Feillet, Hertz, & Speranza, 2010) that were generated from the 34 val ones of (Benavent, Campos, Corberán, & Mota, 1992). These are grouped in two classes:

- tval, containing 102 instances with vehicle capacity equal to 30 and a time limit of 40, and
- toval, containing 102 instances for which the vehicle capacity $W$ ranges from 20 to 250, and the time limit $L$ ranges from 27 to 133.

Furthermore, instances in each class have been grouped accordingly to their size: Group I - 24 nodes; Group II - 30 to 34 nodes; Group III 40 or 41 nodes; and Group IV - 50 nodes. There are no mandatory links in these data sets and the number of vehicles considered is 2, 3 or 4.

Since the results for the initial and strengthened models are very similar, only those obtained for the strengthened ones are shown in Table 11. Average gap values are computed as $\frac{(\bullet)-\mathrm{LB}}{\mathrm{LB}} \times 100$, where LB is the lower bound value obtained with the strengthened model (SFKO) and ($\bullet$) is the upper bound value under analysis, i.e., for the strengthened model (SFKO), for the aggregated model (Agg(SFKO)) and for the linear relaxation (LSFKO). Columns headed by #OS, for model SFKO, and by #OV, for model Agg(SFKO), show the number of instances solved to optimality and the number of times the optimal value was reached with these models, respectively. Last two columns compare the results obtained with model SFKO and those obtained by (Archetti, Feillet, Hertz, & Speranza, 2010). In this last paper a branch-and-price algorithm and several metaheuristics for the K-OMP are presented. Column headed by dif(%) shows the average value of $\frac{\mathrm{LBA}-\mathrm{LB}}{\mathrm{LB}} \times 100$, where LBA is the cost of the best solution found by any of the algorithms proposed by (Archetti, Feillet, Hertz, & Speranza, 2010), while as before LB stands for the cost of the best solution found with model SFKO. Last column shows the number of optimal solutions found by the branch-and-price algorithm of (Archetti, Feillet, Hertz, & Speranza, 2010) in one hour of CPU time.

**Table 11: - Results for the team orienteering mixed capacitated arc routing problem (K-OMP).**

| | | SFKO | | Agg(SFKO) | | LSFKO | Archetti et al. | |
|---|---|---|---|---|---|---|---|---|
| **Name** | **# of instances** | **gap(%)** | **#OS** | **gap(%)** | **#OV** | **gap(%)** | **dif(%)** | **#OS** |
| **tval I** | **27** | **5,94** | **25** | **11,68** | **12** | **17,14** | **0,00** | **18** |
| **tval II** | **30** | **5,09** | **19** | **5,88** | **3** | **10,66** | **0,30** | **30** |
| **tval III** | **21** | **3,97** | **9** | **4,05** | **0** | **7,55** | **0,40** | **21** |
| **tval IV** | **24** | **5,06** | **6** | **4,55** | **1** | **9,49** | **0,20** | **13** |
| **Sum** | **102** | | **59** | | **16** | | | **82** |
| **toval I** | **27** | **1,26** | **21** | **1,70** | **13** | **3,63** | **0,05** | **14** |
| **toval II** | **30** | **1,87** | **12** | **1,90** | **5** | **4,79** | **-0,09** | **1** |
| **toval III** | **21** | **1,98** | **7** | **1,95** | **2** | **6,08** | **-0,20** | **3** |
| **toval IV** | **24** | **2,82** | **9** | **2,14** | **4** | **5,60** | **-0,49** | **0** |
| **Sum** | **102** | | **49** | | **24** | | | **18** |

| | | SFKO | Agg(SFKO) | LSFKO |
|---|---|---|---|---|
| **Cpu time (s)** | **Min** | **0,1** | **0,1** | **0,03** |
| | **Av** | **1861,1** | **1478,0** | **1,2** |
| | **max** | **3600,0** | **3600,0** | **10,7** |

Model SFKO is able to optimally solve 59 out of the 102 tval instances and 49 out of 102 toval instances. In comparison, Agg(SFKO) is worse than SFKO since, although in some cases it gives a better gap, the number of optimal values reached is much lower than the number of optimal solutions obtained by SFKO. Linear relaxation LSFKO is solved very quickly but produces large gaps. The comparison with the work of (Archetti, Feillet, Hertz, & Speranza, 2010) shows that SFKO is able to provide feasible solutions of similar quality, despite the fact that the results given by (Archetti, Feillet, Hertz, & Speranza, 2010) are obtained using not only an exact method but also several metaheuristics. It can be seen that the (Archetti, Feillet, Hertz, & Speranza, 2010) results are slightly better for tval instances, while SFKO is slightly better for toval ones.

The strengthened model SFKOU for the uncapacitated team orienteering mixed arc routing problem (K-UOMP) is tested on instances proposed by (Archetti, et al., 2012) for the team orienteering arc routing problem, that were generated from those described in (Hertz, et al., 1999). These instances are defined on directed graphs, i.e. there are no edge tasks (nor mandatory neither optional). The number of nodes varies from 17 to 55, the number of arcs is between 138 and 429, the number of arc tasks ranges from 8% to 24%, and the number of vehicles is between 2 and 4. These adapted instances are renamed as thertz$\mu\sigma$, where $\mu \in \{d, g, r\}$ indicates different graph types, and $\sigma \in \{00, 25, 50\}$ represents the percentage of mandatory tasks among the total number.

Results of the uncapacitated models are shown in Table 12. Third column indicates the number of instances for which a feasible solution was found by model SFKOU. As before, average gap values are computed as $\frac{(\bullet)-\mathrm{LB}}{\mathrm{LB}} \times 100$, where LB is the lower bound value obtained with the strengthened model (SFKOU) and (•) is the upper bound value under analysis. Columns headed by #OS, for model SFKOU, and by #OV, for model Agg(SFKOU), show the number of instances solved to optimality and the number of times the optimal value was reached with these models, respectively. Last two columns show the results obtained with the branch-and-cut algorithm proposed by (Archetti, Corberán, Plana, Sanchis, & Speranza, 2013). As before, column headed by dif(%) is the average value of $\frac{\mathrm{LBA}-\mathrm{LB}}{\mathrm{LB}} \times 100$, where LBA is the cost of the best feasible solution found by the branch-and-cut of (Archetti, Corberán, Plana, Sanchis, & Speranza, 2013).

Note that model SFKOU seems to work well, both in the number of optimal solutions found and in the average gaps, and it performs similarly to the capacitated model. From this table we may also conclude that the branch-and-cut by (Archetti, Corberán, Plana, Sanchis, & Speranza, 2013) performs better for all the sets of instances tested, although it must be noted that this method was specially devised to deal with completely directed instances.

**Table 12: Results for the uncapacitated team orienteering mixed arc routing problem (K-UOMP)**

| | | | SFKOU | | Agg(SFKOU) | | LSFKOU | Archetti&al | |
|---|---|---|---|---|---|---|---|---|---|
| **Instance** | **# of instances** | **# FS** | **gap(%)** | **#OS** | **gap(%)** | **#OV** | **gap(%)** | **dif(%)** | **#OS** |
| thertzd00 | 81 | 81 | 6,61 | 22 | 12,09 | 6 | 16,27 | 1,34 | 65 |
| thertzd25 | 81 | 51 | 2,91 | 25 | 5,61 | 5 | 10,40 | 0,37 | 50 |
| thertzd50 | 81 | 35 | 3,38 | 24 | 10,25 | 10 | 16,51 | 0,16 | 34 |
| thertzg00 | 81 | 81 | 5,28 | 38 | 6,73 | 22 | 11,08 | 0,37 | 61 |
| thertzg25 | 81 | 65 | 3,13 | 36 | 4,36 | 20 | 8,42 | 0,32 | 59 |
| thertzg50 | 81 | 56 | 1,88 | 36 | 4,10 | 28 | 8,26 | 0,17 | 52 |
| thertzr00 | 60 | 58 | 3,12 | 41 | 20,84 | 7 | 28,12 | 0,07 | 58 |
| thertzr25 | 60 | 56 | 4,32 | 44 | 19,93 | 11 | 27,34 | 0,48 | 56 |
| thertzr50 | 60 | 57 | 3,53 | 46 | 14,49 | 21 | 26,54 | 0,60 | 57 |
| **Sum** | 666 | 540 | | 312 | | 130 | | | 492 |

| | SFKOU | Agg(SFKOU) | LSFKOU | Archetti&al |
|---|---|---|---|---|
| **Cpu time (s)** | 0,02 | 0,01 | 0,00 | 0,09 |
| | 1779,3 | 99,1 | 0,3 | 394,9 |
| | 3600,0 | 3600,0 | 4,3 | 3600,0 |

## 5. Final Remarks

Single-commodity flow models provide a general framework for modelling many routing problems. However, many of the variants modelled by these flow models are node routing problems and not much has been done with such models for arc routing problems. In this paper, we have provided and evaluated single-commodity flow models for several arc routing problems with profits, including the single and multiple vehicle cases. For all the studied problems, which include some that have been introduced here for the first time, exact and relaxed models are presented.

The performance of these models is analysed over a large set of benchmark instances derived from some well-known instances in the literature. Whenever possible, results obtained by the proposed models when solved with CPLEX are compared with previous published ones.

From the computational results we may conclude that the behaviour of the proposed models for the single vehicle variants is quite good. Strengthened as well as initial models succeed in finding the optimum of the problems studied, namely: profitable (PMP), penalised (PPMP), and both orienteering versions (OMP and

UOMP). Linear programming relaxations produce, in general, better bounds for the orienteering cases. The corresponding average CPU times are usually a few seconds.

Gaps for multiple vehicles profitable (K-PMP) models are still reasonably good, whilst the penalised (K-PPMP) problem seems to be harder to solve, as can be seen by the increase on the gap values. Aggregated relaxations have the same behaviour, finding better upper bounds in the first case. Note that, in general, these models provide quite quickly good bounds as can be seen by the small CPU time values. Lower bounds for the team orienteering capacitated models are very similar to the bounds reported in (Archetti, Feillet, Hertz, & Speranza, 2010), while, for the uncapacitated case, our models provide worse results than those by (Archetti, Corberán, Plana, Sanchis, & Speranza, 2013).

We stress that the proposed models consist of a base model complemented by a few additional constraints that allow formulating different arc routing problems with profits. Note also that, as models are built for mixed graphs, they can be applied over other types of graphs. Therefore, an advantage of this approach is that new arc routing problems may be easily modelled, simply by adding new constraints to the base model. To sum up, from a base model, aggregated and valid models providing good bounds in short computing times have been derived for several arc routing problems with profits. This approach may thus be considered a new and useful tool to deal with arc routing problems.

**Acknowledgements:** E. Benavent and Á. Corberán wish to thank the Ministerio de Ciencia e Innovación (project MTM2009-14039-C06-02) and the Ministerio of Economía y Competitividad (project MTM2012-36163-C06-02) of Spain for their support. L. Gouveia, M.C. Mourão and L.S. Pinto wish to thank the Fundação para a Ciência e Tecnologia (projects PEsT-OE/EGE/UI0491/2011; PEsT-OE/MAT/UI0152; PTDC/EGE-GES/121406/2010) for their support.

## References

Aráoz, J., Fernández, E., & Franquesa, C. (2009). The Clustered Prize-collecting Arc Routing Problem. *Transportation Science, 43*(3), 287-308.

Araóz, J., Fernández, E., & Meza, O. (2009b). Solving the Prize-collecting Rural Postman Problem. *European Journal of Operations Research, 196*, 886-896.

Aráoz, J., Fernández, E., & Zoltan, C. (2006). Privatized Rural Postman Problems. *Computers and Operations Research, 33*( ), 3432-3449.

Archetti, C., Corberán, Á., Plana, I., Sanchis, J., & Speranza, M. (2013). The Team Orienteering Arc Routing Problem. *Submitted*.

Archetti, C., Feillet, D., Hertz, A., & Speranza, M. (2010). The undirected capacitated arc routing problem with profits. *Computers and Operations Research, 37*, 1860-1869.

Belenguer, J., Benavent, E., Lacomme, P., & Prins, C. (2006). Lower and upper bound for the mixed capacitated arc routing problem. *Computers and Operations Research, 33*, 3363-3383.

Benavent, E., Campos, V., Corberán, A., & Mota, E. (1992). Tha Capacitated Chinese Postman Problem: Lower Bounds. *Networks, 22*(7), 669-690.

Benavent, E., Corberán, A., Plana, I., & Sanchis, J. (2009). Min-Max K-vehicles Windy Rural Postman Problem. *Networks, 54*, 216-226.

Corberán, A., Fernández, E., Franquesa, C., & Sanchis, J. (2011). The Windy Clustered Prize-Collecting Arc Routing Problem. *Transportation Science, 45*, 317-334.

Corberán, A., Mejía, G., & Sanchis, J. (2005). New Results on the Mixed General Routing Problem. (INFORMS, Ed.) *Operations Research, 53*(2), 363-376. doi:10.1287/opre.1040.0168

Corberán, Á., Plana, I., Rodriguez-Chía, A., & Sanchis, J. (2011b). A Branch-and-Cut for the Maximum Benefit Chinese Postman Problem. *Mathematical Programming*. doi:10.1007/s10107-011-0507-6

Deitch, R., & Ladany, S. (2000). The one-period bus touring problem: Solved by an effective heuristic for the orienteering tour problem and improvement algorithm. *European Journal of Operational Research, 127*, 69-77.

Feillet, D., Dejax, P., & Gendreau, M. (2005a). Traveling Salesman Problems with Profits. *Transportation Science, 39*(2), 188-205.

Feillet, D., Dejax, P., & Gendreau, M. (2005b). The Profitable Arc Tour Problem: Solution with a Branch-and-Price Algorithm. *Transportation Science, 39*(4), 539-552.

Gavish, B., & Graves, S. (1978). *The travelling salesman problem and related problems.* Working Paper OR-078-78: Operations Research Center, MIT, Cambridge, MA.

Golden, B. L., & Wong, R. T. (1981). Capacitated arc routing problems. *Networks, 11*, 305-315.

Gouveia, L., Mourão, M. C., & Pinto, L. S. (2010). Lower bounds for the mixed capacitated arc routing problem. *Computers and Operations Research, 37*, 692-699.

Hertz, A., Laporte, G., & Nanchen-Hugo, P. (1999). Improvement procedures for the undirected rural postman problem. *INFORMS Journal of Cumputing, 11*, 53-62.

Hochbaum, D., & Olinick, E. (2001). The bounded Cycle-cover problem. *INFORMS Journal of computing, 13 (2)*, 104-119.

Irnich, S. (2010). The Profitable Capacitated Rural Postman Problem. *TRISTAN 2010* (pp. 408-412). Tromso, Norway: TRISTAN 2010.

Malandraki, C., & Daskin, M. (1993). The Maximum Benefit Chinese Postman Problem and the Maximum Benefit Traveling Salesman Problem. *European Journal of Operational Research, 65*(2), 218-324.

Pearn, W., & Chiu, W. (2005). Approximate solutions for the maximum benefit Chinese postman problem. *International Journal of Systems Science, 36*(13), 815-822.

Pearn, W., & Wang, K. (2003). On the maximum benefit Chinese postman problem. *OMEGA - The International Journal of Management Science, 31*, 269-273. doi:10.1016/S0305-0483(03)00051-3

Toth, P., & Vigo, D. (2002). *The Vehicle Routing Problem.* Philadelphia: SIAM.

Vansteenwegen, P., Souffriau, W., & Oudheusden, D. (2011). The orienteering problem: A survey. *European Journal of Operational Research, 209*(1), pp. 1-10.

**Table 1: Main characteristics of the arc routing problems with profits in the Literature.**

| Classification | Named as | Graph♣ | Depot? | Objective to maximize | Particularities | | | | References |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | Profit collected | Vehicles | | | |
| | | | | | | # | C; U♦ | | |
| Profitable | Maximum benefit CPP | **D** | yes | net profit | Several times | 1 | U | • Profit paid as a decreasing function with the increasing number of traversals;<br>• Includes routing cost. | (Malandraki & Daskin, 1993)<br>(Pearn & Chiu, 2005) |
| | Maximum benefit CPP | **U** | yes | | | 1 | U | | (Pearn & Wang, 2003)<br>(Corberán, Plana, Rodriguez-Chía, & Sanchis, 2011b) |
| | Profitable arc tour problem | **D** complete graph | no | net profit | | K | U | • Limits on the number of times each profit is available;<br>• Maximum length per cycle;<br>• Includes routing cost. | (Feillet, Dejax, & Gendreau, 2005b) |
| | Prize-collecting RPP | **U** | yes | net profit | once | 1 | U | • Includes routing cost; | (Aráoz, Fernández, & Zoltan, 2006)<br>(Araóz, Fernández, & Meza, 2009b) |
| | Profitable capacitated RPP | **U** | yes | net profit | | 1 | U | • Time limit;<br>• Includes routing cost. | (Irnich, 2010) |
| | Clustered prize-collecting ARP | **U** | yes | net profit | | 1 | U | • Includes routing cost;<br>• Edges are serviced in clusters (for each cluster, either all or none of its edges are serviced) | (Aráoz, et al., 2009a) |
| | Windy clustered prize-collecting ARP | **W** | yes | net profit | | 1 | U | • Edges are serviced in clusters (for each cluster, either all or none of its edges are serviced);<br>• Includes routing cost. | (Corberán, Fernández, Franquesa, & Sanchis, 2011) |
| Orienteering/ Team Orienteering | Bus touring problem | **U** | yes | attractiveness | | 1 | U | • Time limit. | (Deitch & Ladany, 2000) |
| | Capacitated ARP with profits | **U** | yes | profit | | K | C | • Time limit per vehicle;<br>• Objective does not include routing cost. | (Archetti, Feillet, Hertz, & Speranza, 2010) |
| | Team orienteering ARP | **D** | yes | profit | | K | U | • Time limit per vehicle;<br>• Objective does not include routing cost. | (Archetti, et al., 2012)<br>(Archetti, et al., 2013) |

♣**Graph: D**: directed; **U**: undirected; **W**: windy. #: vehicles number ($K > 1$). ♦**Vehicles: C**:capacitated; **U**:uncapacitated