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Of Asymmetric Central Bank Preferences**

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# Optimal Monetary Policy in a Model of Asymmetric Central Bank Preferences

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## Abstract

This paper considers optimal monetary in the context of the central bank adopting a asymmetric objective function. We exploit a procedure, due to Varian and Zellner, to derive policies under commitment and discretion. Our results show that under asymmetric preferences, many of the extant results on the time consistency problem no longer hold. A striking feature of the optimal policy solutions is that a committed policymaker is not unambiguously preferred to his discretionary counterpart. Moreover, the form of the optimal discretionary solution indicates that the usual mechanisms to eliminate the inflation bias are inappropriate.

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# 1 Introduction

A well-known feature of the debate on rules versus discretion, and the subsequent literature following Kydland and Prescott(1997) and Barro and Gordon(1983) is the so called "time-consistency" problem of discretionary monetary policy. In a recent paper, Broadbent and Barro(1997) draw a useful distinction between the positive theory of monetary policy as underpinning a description of actual policy making, and research which focuses on prescriptive mechanisms to eliminate the well-known inflationary bias under discretion, as in Rogo $\alpha$ (1985), Walsh (1995), Persson and Tabellini (1994) and Svensson(1997).

A recent strand in the literature, drawing on actual policy perspectives, has been to highlight issues of how central banks operate in practice, and the implications for the design of policy rules of convexities in objectives and trade-offs. Examples include Fischer(1994), Mishkin and Posen(1997), Clark, Laxton and Rose(1995) and Bean(1997). In striking contrast to the conventional inflationary bias concern under central bank discretion, Fischer highlights the issue of whether deflationary bias is a more likely outcome. To quote,

"An important reason to expose central bankers to elected officials is that, just as the latter may have an inflationary bias, the former may easily develop a deflationary bias. Shielded as they are from public opinion, contained within an anti-inflationary temple, central bankers can all too easily deny that cyclical unemployment can be reduced by easing monetary policy."

Fischer's concern for central bank deflationary bias finds support in Mishkin and Posen(1997). They cite evidence to support the claim that central bank policy has favored inflation outcomes that erred on the side of being too low in the case of Canada, and that the Bank of England's initial inflation target 2.5% or less was consistent with the probable asymmetry of the output inflation trade-off. Willem Buiter, in commenting on the European Central Bank's recently announced objectives notes:

" The ECB has recently decided to define it (the inflation target) as a year-on-year increase in the harmonised index of consumer prices of 2 per cent or less. This is an asymmetric inflation target: inflation should not be above 2 per cent, but is allowed to be below that level"  
- The Sunday Telegraph, October 18 1998.

Amongst financial journalists too, there is a perception that the authorities preferences are not quadratic. For example, D. Smith (Sunday Times, 9 November 1997) writes:

“There is a bias towards over caution in policy built into the new arrangements, at least for a while. If George [Governor of the Bank of England] has to write to Brown [Chancellor of Exchequer] in two years’ time and apologize for the fact that inflation is 1%, and therefore outside his effective target range [2.5%], he would do so a happy man. If he had to do so with inflation at 5%, he would probably slip his resignation letter into the same envelope.”

The focus of this paper is the analysis of optimal policy when the central bank has asymmetric preferences. The desirability of doing so is hardly in dispute. To date however, modelling procedures have, perforce, assumed quadratic or linear preferences for reasons of well-known tractability. We model preferences by exploiting a procedure due originally to Varian(1974) and Zellner(1992) in the context of Bayesian analysis. The Linex form, which we discuss later, has the convenient properties that readily allows for asymmetric loss and also exhibits the quadratic form as a special case. Hence, the procedures enable us to not only implement the essential nonlinearities in central bank preferences but to also derive the conventional results as a special case of the model.

Our findings bear on both strands of Broadbent and Barro’s distinction between plausible description of actual behavior and the issue of mechanism design. We show, within the usual commitment/discretion framework how Fischer’s conjecture on deflation bias arises under commitment. Moreover, it is possible that under commitment, the expected deviation of inflation from its target is larger than is the case when the authorities pursue discretionary policy, thus reversing the usual inequality that obtains under symmetric loss.

Likewise, our analysis of mechanisms proposed to eliminate the inflation bias reveals similar sensitivity under asymmetric preferences. Thus, Persson and Tabellini (1990) show that by targeting output to its natural rate the authorities can eliminate the inflation bias. We show, however, that this result that does not hold under asymmetric loss. Implementing the Walsh type contract, depending as it does on the parameters of the loss function implies, under asymmetric loss, that the authorities’ compensation can be

positively, rather than negatively, related to inflation!. Our results moreover, and echoing the findings of Svensson(1997), illustrate the difficulties of practically implementing a Walsh-type contract. However, the alternative proposals on inflation targeting, as proposed in Svensson, face similar difficulties. In general, the optimal design of a mechanism involves the combination of both, a Walsh-type contract and an inflation target, though the appropriate information required to implement such a framework would suggest an impracticability.

Underlying Fischer's concern as to the dangers of deflation bias is the issue of potential conflicts of preferences as between the central bank and the public. Our framework allows for a welfare analysis of alternative policies under central bank asymmetric preferences in conjunction with the usual social welfare loss as represented by a quadratic loss function. We show that the welfare loss can be greater under commitment than discretionary policy, a feature that is reinforced by the extent to which the authorities mimic the conservative bank features of Rogoff(1985).

Our analysis has limited convex behavior to that of central bank preferences. Bean(1997), Clark et al(1995) and Huang, Nobay and Peel(1998) address the issue of optimal policy in the presence of convexities in the Phillips Curve, and show how policy displays a bias towards conservatism. Whilst it is possible, in principle, to implement our Linex procedure to allow for convexity in the Lucas supply curve, we have, for technical reasons, not been able to incorporate both nonlinearities within the model, although preliminary analysis is suggestive of qualitatively similar results to those reported here.

In summary, the results presented in this paper underline the fact that even limited realism beyond the conventional approach to modelling the authorities preferences delivers results that are substantively at variance with the conventional literature on optimal monetary policy. In particular, the literature does not offer a robust underpinning to important practical issues of central bank independence and governance.

The rest of the paper is structured as follows. In the next section we set out the properties of the Linex function, outline the model structure and solve for the optimal monetary policies, under both commitment and discretion, in the model embodying asymmetric preferences. The concluding section draws on the results and discusses them in the context of the conventional literature.

## 2 The Model

The Linex loss function, as discussed in Christoffersen and Diebold(1994) is due originally to Varian(1974) and Zellner(1992) in the context of Bayesian econometric analysis. It is given by

$$L = \frac{e^{\theta x} - \theta x - 1}{\theta^2} \quad (1)$$

where  $\theta$  is a constant and  $x$  is a variable. For  $\theta > 0$ , the function is approximately exponential for  $x > 0$  and linear for  $x < 0$ . The properties are reversed for  $\theta < 0$ . As  $\theta$  approaches zero  $L$  approximates  $\frac{x^2}{2}$  the quadratic function. The diagram below illustrates the quadratic form (dotted lines) and the Linex function when  $\theta = -0.75$

It illustrates the essential asymmetries of not achieving objectives. Consider  $x$  below to be the deviations from a full employment target - then, recessions are viewed as more costly than booms, relative to the familiar quadratic loss schedule.



THE LINEX REPRESENTATION

In addition, if  $x$  is given by the process:  $x = \bar{x} + \epsilon$ ; where  $\bar{x}$  is the conditional mean of the process and  $\epsilon$ ; an error term is conditionally normal and with conditional variance  $\sigma^2$ , the expected value of (1),  $EL$ , is given by:

$$EL = \frac{e^{\theta(\bar{x} + \frac{\sigma^2}{2})} - \theta\bar{x} - 1}{\theta^2} \quad (2)$$

As shown in Christensen and Diebold (1994), the optimal forecast of  $x$ , which minimizes (2), is no longer given by  $\bar{x}$  as with quadratic loss but by:  $\bar{x} + \frac{\sigma}{2} \bar{x}^2$ . In other words, there is an optimal degree of bias given by  $\frac{\sigma}{2} \bar{x}^2$ .

## 2.1 The Central Bank's Preference Specification and the Lucas Supply Schedule

Following our discussion above, we specify the authorities preferences as

$$L = \frac{e^{a(\pi - \pi^*)} - a(\pi - \pi^*) - 1}{a^2} + \bar{A} \frac{e^{b(y - y^*)} - b(y - y^*) - 1}{b^2} \quad (3)$$

where  $\pi$  is the inflation rate,  $\pi^*$  is the inflation target;  $y$  is output,  $y^*$  is the target output level, and  $\bar{A}$  is a positive constant. As  $\bar{A}$  represents the relative weights placed by the authorities on inflation and output deviations, we could consider its value as representing the "degree of conservatism" in the sense of Rogoff (1985) - lower values implying a more conservative banker. The asymmetric characterization of preferences, that is, viewing inflation outcomes above targets and output outcomes below targets as being more costly than the alternatives, implies that  $a > 0$  and  $b < 0$ .

We assume, as in the standard literature, that the supply schedule is given by:

$$y = y_n + \mu(\pi - \pi^e) + u \quad (4)$$

where  $\mu$  is positive constant coefficient, the superscript  $e$  denotes the rational expectation taken at the beginning of each period, and  $u$  is a random shock. For simplicity  $y_n$  is normalized to zero. As in Svensson (1997),  $\pi$  is assumed to be the control variable under the central bank's control.

## 2.2 The Optimal Commitment Policy

As is well understood, the monetary authority's minimization problem, in the case of commitment, involves the additional constraint that the ex ante expected inflation,  $\pi^e$ , must be equal to its committed inflation rate,  $E[\pi]$ . To derive the commitment solution we obtain two first-order conditions with respect to  $\pi$  and  $\pi^e$ , respectively:

$$\frac{e^{a(\pi - \pi^*)} - 1}{a} + \bar{A} \mu \frac{e^{b(y - y^*)} - 1}{b} - \Lambda = 0 \quad (5)$$

$$E \left[ -\hat{\Lambda} \mu \frac{e^{b(y-y^*)} - 1}{b} + \Lambda \right] = 0 \quad (6)$$

where  $\Lambda$  is the Lagrange multiplier of  $\pi^e = E[\pi]$ . To eliminate  $\Lambda$ , we can add (5) and (6), which gives:

$$e^{a(\pi - \pi^*)} + \frac{a\hat{\Lambda}\mu}{b} [e^{b(y-y^*)} - E e^{b(y-y^*)}] = 1 \quad (7)$$

Equation (7) is the optimal policy rule under commitment. Taking the expectation of equation (7) and assuming that  $\pi$  is conditionally normal we obtain:

$$e^{a(E\pi - \pi^* + \frac{a\sigma_\pi^2}{2})} = 1 \quad (8)$$

where  $\frac{3}{4}\sigma_\pi^2$  denotes the variance of  $\pi$ : Thus, we finally obtain,

$$E\pi = \pi^* - \frac{a\frac{3}{4}\sigma_\pi^2}{2} \quad (9)$$

Notice that the above result obtains for any specification of output preferences,  $f(y-y^*)$ , given that the commitment solution contains the term  $E\{f(y-y^*) - Ef(y-y^*)\}$ , which is zero, given rational expectations. The assumption that  $\pi$  is conditionally normal imposes restrictions on the error  $u$ ; which will as a consequence be nonnormally distributed. Since assuming that  $u$  is conditionally normal does not allow a closed form solution, we have, for ease of exposition assumed conditional normality of  $\pi$ ; although results employing Taylor expansions in the case where  $\pi$  is assumed nonnormal reveal that the results are qualitatively similar.

It will be immediately obvious that the solution for expected inflation demonstrates an important effect of asymmetric preference on monetary policy. In the existing literature, because the authorities' utility function is quadratic and thus symmetric with respect to missing their target, they can, under commitment, always hit the inflation target, i.e.,  $E\pi = \pi^*$ . In our model, the central bank may be viewed as risk averse in inflation (if  $a > 0$ ). Consequently, a "deflation premium",  $-\frac{1}{2}a\frac{3}{4}\sigma_\pi^2$ ; is a component of expected inflation. Thus, under commitment and asymmetric preferences, Fischer's conjecture of deflationary bias arises as an optimal solution. We further discuss this issue when we consider some welfare implications later.



### 3 The Optimal Discretionary Policy

Here, the optimal rule is derived by minimizing the constrained loss function of the monetary authority only with respect to the actual ex post inflation  $\pi$ . From this, we obtain the first-order condition with respect to  $\pi$ :

$$e^{a(\pi - \pi^*)} + \frac{a\lambda}{b} [e^{b(y - y^*)} - 1] = 1 \quad (10)$$

Equation (10) is the optimal policy rule under discretion. Taking the expectation of equation (10), when  $a = 0$ , and  $y$  is assumed to be conditionally normal we obtain:

$$E\pi + \frac{\lambda}{b} \left[ e^{b(-y^* + \frac{b\sigma_y^2}{2})} - 1 \right] = 0 \quad (11)$$

where  $\sigma_y^2$  denotes the variance of  $y$ :

Taking the expectation of (10) when  $b = 0$ , and  $\pi$  is assumed to be conditionally normal we obtain

$$e^{a(E\pi - \pi^* + a\frac{\sigma_\pi^2}{2})} - a\lambda\mu^* = 1 \quad (12)$$

and

$$E\pi = \pi^* - \frac{1}{2}a\sigma_\pi^2 + \frac{1}{a} \ln \{1 + a\lambda\mu^*\} \quad (13)$$

In marked contrast to the quadratic preferences solution in Persson and Tabellini(1990), we see, from eqn. 13 above that reducing the output target to the natural output level, so that  $y^* = 0$ , does not remove the inflation bias. Indeed, were it the case that the Persson-Tabellini result held, the claims by policy-making economists such as King(1996) and Blinder(1997) that central bankers do not target output above the potential or natural, would imply that in reality the extant literature on inflation bias has focused on a non- issue. Be that as it may, it is hard to motivate a reason for society or the central banker penalizing deviations from target output symmetrically. Moreover; from a central bank perspective, recessions precipitate public questioning of the rationale for their independence. In a seminal paper, Cukierman and Meltzer(1986), modelled the output objective in a linear manner to reflect the asymmetry between booms and recessions. However, their linear specification has the questionable implication that the central bank would be willing to accept any increase in output variance for a marginal decrease in

inflation. The quadratic specification remedies this feature but at the price of assuming symmetric costs.

In general, asymmetric preferences over output deviations are sufficient to generate an inflationary bias. As shown by eqn.9 above, the equilibrium

inflation level under commitment will be lower than is the case under symmetric preferences by the deflation bias term  $-\frac{1}{2}a\gamma_y^2$ : Note however, from eqn, 13 that the standard inflationary bias result under discretion, namely  $\hat{\mu}y^*$ , does not hold unambiguously - equilibrium inflation can in principle be nearer (or equal ) to  $\gamma_y^*$  since the outcome will depend upon the terms  $-\frac{1}{2}a\gamma_y^2 + \frac{1}{a} \ln \{1 + a\hat{\mu}y^*\}$ :

In our framework, incorporating a Walsh linear inflation contract is straightforward . In the discretionary case, and where  $t$  is the constant tax parameter, it is given by the first order condition that

$$e^{a(E\gamma_y - \frac{1}{2}a^2 + a\frac{\sigma_\pi^2}{2})} - a\hat{\mu}y^* + ta = 1 \quad (14)$$

It is clear from (14) that when preferences are asymmetric, to remove the inflation bias through using a Walsh-type contract would require an inflation contract that depends on parameters of the loss function and could require the authorities' compensation to be positively, rather than negatively, related to inflation. This is another reason why, as argued in Svensson(1997), that inflation targets may be much easier to implement than linear inflation contracts in the real world. Note however from (13) that the inflation target which removes the inflation bias also depends on the risk parameter. Consequently when preferences are asymmetric, the design of either inflation targeting or Walsh contracts requires knowledge of the central bankers preferences. As illustrated in Havrilesky (1991), this is not likely to be the case. Hence, the design and implementation of such schemes in practise is likely to be an impracticable task

We proceed to solve for the variances of inflation and output in the discretionary case. We obtain that

$$\gamma_y^2 = \frac{(e^{a^2\frac{\sigma_\pi^2}{2}} - 1)(a\hat{\mu}y^* + 1 - at)^2}{a^2\hat{\mu}^2} \quad (15)$$

$$\sigma_u^2 = \sigma_{\pi}^2 \mu^2 + 2\bar{A}^{-1}(a\bar{A}\mu y^* + 1 - a\tau)e^{-\frac{a^2\sigma_{\pi}^2}{2}}\sigma_{\pi}^2(1 + 0.5a^2\sigma_{\pi}^2 + 0.125a^4(\sigma_{\pi}^2)^2) + \sigma_y^2 \quad (16)$$

Note that whilst eqn.15 above is an exact solution, eqn. 16 is derived by taking expectations of moments upto order 6 on the assumption of conditional normality, given the cross-product terms in the solution.

Inspection of the nonlinear equations (15) and (16) reveal a number of interesting features. Firstly, note that the variances of inflation and output are not independent of the target level of output,  $y^*$  as is the case with quadratic preferences. A higher target level of output lowers the variance of inflation and increases the variance of output. It follows that for a standard social welfare function, which has quadratic arguments in inflation and output deviations, that a zero target level of output is not in general welfare maximizing! Numerical calculations demonstrate that, ceteris paribus, the welfare maximising target level of output is positively related to the variance of the supply shock when  $a$  is positive. To illustrate, we undertake the following experiment. Consider the following quadratic social welfare function

$$w(x) = 0.5(\sigma_{\pi}^2 + \sigma_{\pi}^2 + \sigma_y^2 + y^{*2}) \quad (17)$$

and values of the parameters  $\bar{A} = 1$ ;  $\sigma_{\pi} = 1$ ;  $\mu = 1$ ;  $a = 0.1$ ;  $\sigma_u^2 = 15$ ;  $y^* = 0$

We solve the above on the assumption that  $y^* = 0$ ; the baseline result in the conventional literature, to derive the following values.

$$\begin{aligned} w(x) &= 3.785 \\ \sigma_{\pi}^2 &= 3.7324 \\ E\pi &= -0.18662 \\ \sigma_y^2 &= 3.8029 \end{aligned}$$

and re-evaluate the model with one marginal change, namely that  $y^* = 0.1$ , which yields values :

$$\begin{aligned} w(x) &= 3.7763 \\ \sigma_{\pi}^2 &= 3.6953 \\ E\pi &= -0.00853 \\ \sigma_y^2 &= 3.8401 \end{aligned}$$

As these results illustrate, welfare is higher in the case where we allow a small increase in  $y^*$ . Hence, from a welfare perspective, a nonzero  $y^*$  can

be optimal within a framework in which the authorities pursue asymmetric preferences. Since the condition  $y^* = 0$  corresponds to commitment, the result implies that a committed central banker with asymmetric preferences can be worse from a welfare perspective than his discretionary counterpart. Further, numerical calculations confirm that the welfare maximizing target level of output is positively related to the variance of the supply shock when  $\alpha$  is positive. We can see, intuitively, that as the variance of supply shocks increase, the variance of inflation rises and expected inflation falls. If these terms are given relatively high weight in the social welfare function then raising the output target will, at the margin, reduce these terms by more than the increase in the variance of output and deadweight loss, and as a consequence increase welfare.

Note too that unlike the standard discretionary solution with quadratic preferences and a linear supply schedule, the variance of inflation and output depend on  $\alpha$ ; the parameter of the linear inflation contract. As a consequence there is, in principle, a rationale for implementation of both an inflation target and a linear inflation contract. The inflation target could be employed to achieve zero expected inflation and the parameter of the inflation contract chosen to minimize the social welfare function. It will be a complex function of the parameters in the model including naturally the variance of the supply shock. We also note from (15)

contracts which eliminates the inflationary bias depends on these parameters. We also showed that unlike with quadratic loss there is an independent role for both an inflation target and a linear Walsh contract. We also showed that when the central bank has asymmetric preferences over output deviations reducing the central bank's target level of output to the natural rate does not eliminate the inflationary bias and that the variances of inflation and output are not independent of the target level of output. Indeed, welfare can increase with increases in the target level of output.

Departures from linearity, even when they occur in one feature of the model have been shown to reverse many of the features of what is now the standard time consistency paradigm. To the extent that asymmetric preferences represent more realism than the linear quadratic formulation, these results serve to underline the need for incorporating other convex features into modelling optimal monetary policy, as in Bean(1997)

## 5 References

- Barro, Robert and David Gordon. 1983, "Rules, Discretion and Reputation in a Model of Monetary Policy," *Journal of Monetary Economics*, 12: 101-122.
- Bean, Charles. 1997 " The Convex Phillips Curve and Macroeconomic Policymaking under Uncertainty, mimeo L.S.E.
- Blinder, A (1997)"What Central Bankers Could Learn from Academics- and ViceVersa," *Journal of Economic Perspectives*,11,pp.3-19
- Broadbent,Ben and Robert Barro.1997, "Central bank preferences and macroeconomic equilibrium", *Journal of Monetary Economics*, 39, pp 17-43
- Christoerson., P.F. and Diebold, F., 1994, "Optimal Prediction under Asymmetric Loss," *National Bureau of Economic Research Working Paper* 167.
- Cukierman,A.and A.H.Meltzer,(1986)"A Theory of Ambiguity, credibility, and Inflation under Discretion and Asymmetric Information".*Econometrica* 54,pp1099-1128.
- Fischer,S.(1994)"Modern Central Banking" in Forrest Capie,Charles Goodhart,Stanley Fischer and Norbert Schnadt, *The Nature Of Central Banking*, Cambridge University Press,Cambridge,U.K.
- Havrilesky,T.M(1991)"Screening FOMC Members for their Biases and Dependability", *Economics And Politics*,3,139-149.
- King,M(1996)"How Should Central Banks Reduce Inflation?-Conceptual Issues", in *Achieving Price Stability* ,Federal Reserve Bank of Kansas City Symposium Series,,pp53-91
- Kydland, Finn and Edward C. Prescott, 1977, "Rules rather than Discretion: the Inconsistence of Optimal Plans," *Journal of Political Economy*, 85, 473-90.
- Mishkin ,F.And Posen,A.(1997)"Inflation Targeting :Lessons from four Countries", *Federal Reserve Bank Of New York Economic Policy Review* 3.

Persson, T. and Tabellini, G. (1990) *Macroeconomic Policy, Credibility and Politics*, Harwood, London.

Persson, T. and Tabellini, G. (1994) "Designing Institutions for Monetary Stability", *Carnegie-Rochester Conference Series on Public Policy*. 39, 53-84.

Rogoff, Kenneth. 1985, "The Optimal Degree of Commitment to an Intermediate Monetary Target," *Quarterly Journal of Economics*, 100: 1169-1190.

Svensson, Lars. 1997, "Optimal Inflation Targets, 'Conservative' Central Banks, and Linear Inflation Contracts," *American Economic Review*, 87, 98-114..

Varian, H. 1974, "A Bayesian Approach to Real Estate Assessment," in S.E. Feinberg and A. Zellner (eds), *Studies in Bayesian Economics in Honor of L.J. Savage*, Amsterdam: North Holland, 195-208.

Walsh, Carl E. 1995, "Optimal Contracts for Central Bankers," *American Economic Review*, 85: 150-167.

Zellner, A. 1992, "Bayesian Estimation and Prediction Using Asymmetric Loss Functions," *Journal of the American Statistical Association*, 81: 446-451.