Buy on Rumours - Sell on News:

A Manipulative Trading Strategy

By

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Buy on Rumors - Sell on News: Manipulative Trading Strategy

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Abstract

A trader who receives a signal about a future public announcement can exploit this private information twice. First, when he receives his signal, and second, at the time of the public announcement. The second round advantage occurs because the early-informed trader can best infer the extent to which his information is already reflected in the current price. This paper shows that early-informed traders trade very aggressively at the time they receive their signal. They try to manipulate the price in order to enhance their informational advantage at the time of the public announcement. In addition, they speculate by building up a position in period one, which they partially unwind 'on average' in period two. The analysis shows that information leakage makes prices prior to public announcements more informative but reduces informational efficiency in the long run.
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1 Introduction

Investors base their expectations about the future payoffs of an asset on their information. This information affects their trading activity and, thus, the asset price. Information flow is, however, not just a one-way street. Traders who have not received new information are conscious of the fact that the actions of other traders are driven by their information. Thus, uninformed traders can infer part of the other traders' information from the movement of an asset's price.

But even when a trader does receive information he faces a problem if he wants to exploit it in the stock market. To determine his optimal trading moves, he has to figure out how much of this information is new. The current price already reflects this information if prices are (partially) revealing and other traders traded on this or similar information in previous trading rounds. Therefore, the question boils down to whether other market participants have already received related information. The same situation arises in the case of a public announcement. All market participants - with the exception of the trader who acquired this information early - do not know the extent to which this information is already incorporated in the current price.

To address this issue I develop a model where a trader receives an imprecise signal about a forthcoming public announcement. Even if the signal is imprecise, the trader trades on it and moves the price. Only this trader will know the price impact of his trading activity in the trading round prior to the public announcement. Thus he has an additional informational advantage at the time of the public announcement. I show that an early-informed trader can exploit his private information twice. First, when he receives his signal, and second, at the time of the public announcement. This result only holds if other traders draw inferences from the past price even after the public announcement, i.e. if they conduct technical analysis. Other traders face an additional error term in interpreting the past price. Since they do not know the price impact of the early informed traders action, they can not isolate the informational content of the past price from the part resulting from the imprecision of his signal. Paradoxically, it is the imprecision of the early-informed trader's signal which gives him the informational advantage at the time of the public announcement.

In addition to showing that the early-informed trader can exploit his information twice, I show that he trades for speculative reasons. I define 'speculative trading' as trading that is undertaken with the intent to unwind the acquired position after the public announcement. In this context, the early-informed trader can exploit his knowledge of the others' error and 'on average' reverse the position that he built up in the previous trading round.

is enhanced at the time of the public announcement is referred to as 'manipulative trading'. This paper also demonstrates that the trader who receives the information leakage trades in order to manipulate the price in his favor. If the early-informed agent trades very aggressively in the first trading round, the imprecision of his signal has a larger impact on the current price. This imprecision causes the other market participants to make an error while inferring information from the price. The early-informed trader's future capital gains result from correcting the others' misinterpretation. Hence, by trading more aggressively in the first trading round he increases his expected future capital gains in later trading rounds. Even though manipulative trading reduces capital gains prior to the public announcement, the trader more than makes up for this with additional profits afterwards. Manipulative trading behavior contrasts sharply with Kyle (1985) where the insider trades less aggressively today in order to save his informational advantage for future trading rounds. In my setting, he trades more aggressively now in order to enhance his future informational advantage. Therefore, the trading strategy should more appropriately be called "Trade 'Aggressively' on Rumors - Sell on News".

This paper also presents an alternative explanation for aggressive behavior. Recent experimental findings suggest that traders overestimate the importance of their private information (De Bondt and Thaler 1995). This literature attributes this behavior to irrational overconfidence on the part of the traders. My analysis, however, provides a rational explanation for their 'overactivism'.

This paper also highlights the importance of other traders' information in the interpretation of prices and runs counter to the notion of informational efficiency of markets. It illustrates that in some situations, knowledge about what other market participants know can be more important than knowledge about the fundamental value of a stock. This is in the spirit of Keynes' well known beauty contest argument (Keynes 1936). If it is important to know other traders' information in order to interpret the price, then the price cannot be a sufficient statistic for all individual signals. This sheds new light on the strong-form informational efficiency of markets. For the Grossman-Stiglitz Paradox[1] to arise, it is, therefore, not only necessary that all traders are price takers, as illustrated in Jackson (1991), but also that each market participant knows how his information is related to the information of other agents. Rumors are especially detrimental for achieving informationally efficient markets. Even after the truth is announced, rumors still distort the price and should therefore be avoided. This finding lends a rationale for crisis management wherein early public announcements are always recommended.

The remainder of the paper is organized as follows. The related literature is briefly summarized in Section 2. Section 3 outlines the model. It shows that early-informed

[1]The Grossman-Stiglitz Paradox refers to the non-existence of an overall equilibrium with endogenous information acquisition when prices are informationally efficient. If a price is informationally efficient, it reflects all private information, i.e. one can infer a sufficient statistic for all private signals by observing the price. Consequently, no trader has an incentive to gather costly information. However, if nobody collects information, the price cannot be informative and it would be worthwhile to buy a signal.

traders still have an informational advantage at the time of the public announcement and that they trade for speculative as well as manipulative reasons. The impact of information leakage on informational efficiency is illustrated in Section 4. Section 5 extends the analysis to mixed strategies and examines a setting where many informed traders hear a rumor. Conclusion and topics for future research are presented in Section 6.

2 Related Literature

The prior literature focuses primarily on determining the conditions under which traders can manipulate asset prices. Price manipulation can have detrimental implications if it causes the price of an asset to depart even further from its fundamental value, i.e. if it causes the price to become less informationally efficient. The literature distinguishes between trade-based, information-based and action-based stock price manipulation (Allen and Gale 1992).

Classical examples of trade-based manipulation are market 'corners' and short 'squeezes'. Illiquid markets allow some market participants to temporarily exercise some monopoly power and move the price in their favor. However, not too much can be gained because the unwinding of the established position causes the market to move in the opposite direction again. Manipulative trading strategies are profitable if the spot market is less liquid than the futures market. Liquidity allows a trader to go long into futures without affecting price of futures significantly. The trader can then buy the underlying stocks after having established his futures position. If the spot market is illiquid, the price rises and he can short squeeze other traders. Other traders who are short in futures have to buy the underlying stock in order to deliver. Kumar and Seppi (1992) illustrate price manipulation if futures are settled by cash rather than by physical delivery. The intuition is that 'cash settlement' acts as an infinitely liquid market in which pre-existing futures positions are closed out relative to the less liquid spot market. In Allen and Gorton (1992) trade-based manipulation is possible since buy orders are more likely to be from informed traders than sell orders. Therefore, the market is less liquid for upswings than for downturns. Allen and Gale (1992) present a model about trade-based manipulation with higher order uncertainty where all traders are price takers except for one large trader, who is either an informed trader or an uninformed manipulator. Similarly, in Chakraborty (1997) there is also a potentially well informed insider. In addition, there are less informed followers and uninformed liquidity traders. Chakraborty's model illustrates manipulation by the potentially informed insider within a Glosten and Milgrom (1985) setting. Fishman and Hagerty (1995) show that mandatory disclosure of individual trading activities can lead to manipulation.

Information-based manipulation involves the release of false information or the spreading of rumors in order to achieve a favorable stock price. Vila (1989) presents a simple model of information-based manipulation in which the trader, after going short, releases false information in order to buy the stock back at a cheaper price. Benabou and Laroque

(1992) focus on the credibility of insiders and gurus who profitably manipulate prices by sometimes publicizing incorrect statements. They have an incentive to make false announcements in order to move the price in their favor. Since it is known that their information is noisy and thus manipulation cannot be detected with certainty, their reputation is not destroyed completely. Nevertheless their credibility is hurt, thereby rendering future information-based manipulations less effective.

Action-based manipulation results when corporate insiders entangle corporate decisions with their private stock market activities. They can take stock value enhancing or reducing actions within a firm with the objective of making private gains from speculation in the stock market.

Treynor and Ferguson (1985) address the problem faced by a trader who does not know whether his information is already known to all the other market participants or not. They demonstrate that the past price process can help the trader answer this question. That is, they illustrate the usefulness of technical analysis for this problem.

In my paper only the early-informed trader knows the extent to which the information revealed to the public is already reflected in the price. All other market participants try to infer this from the past price changes. In addition, this analysis focuses on the strategic behavior of the early-informed trader who trades for manipulative and speculative reasons. The complete analysis highlights the necessary conditions that generate the inference problem for the public. Furthermore, in contrast to most of the other models in the literature manipulative trading is derived without the imposition of any restrictions on the traders' order size.

3 Anal sis

3.1 Model Setup

There are two assets in the economy: a risky stock and a risk-free bond. For simplicity we normalize the interest rate of the bond to zero. Market participants include risk-neutral informed traders, liquidity traders and a market maker. Informed traders' sole motive for trading is to exploit their superior information about the fundamental value of the stock. Liquidity traders buy or sell shares for reasons exogenous to the model. Their demand typically stems from information which is not of common interest such as from their need to hedge against endowment shocks or private investment opportunities in an incomplete market setting.[2] A single competitive risk-neutral market maker observes the aggregate order flow and sets the price.

Traders submit their market orders to the market maker in two consecutive trading

[2] See Brunnermeier (1997) for a detailed discussion of the different reasons why liquidity traders trade, and for a discussion on the distinction between information of common versus private interest.

rounds taking into account the price impact of their orders. The market maker sets the price in each round after observing the aggregate order flow and trades the market clearing quantities. As in Kyle (1985) the market maker is assumed to set informationally efficient prices; thus his expected profit is zero. The underlying Bertrand competition with potential rival market makers is not explicitly modelled in this analysis.[3] Informed traders after the public announcement due to trader 2's information. In period three the true value of the stock $v = \delta_A + \delta_B$ is known to everybody. Liquidity traders do not receive any information and their aggregate trading activity is summarized by the random variables u_1 in period one and u_2 in period two. The information structure is common knowledge, i.e. I assume that all market participants know that trader 1 has received some noisy information about a forthcoming public announcement but they do not know its content.[4]

The information structure is summarized in the following table:

player i	in period $t=1$	in period $t=2$	in period $t=3$
market maker	X_1	δ_A, p_1, X_2	δ_B, p_2
trader 1	$\delta_A + \varepsilon^1$	δ_A, p_1	δ_B, p_2
trader 2	δ_B	δ_A, p_1	p_2

where $X_1 = x_1^1 + x_1^2 + u_1$ is the aggregated orderflow in $t = 1$ and $X_2 = x_2^1 + x_2^2 + u_2$ is the orderflow in $t = 2$. For notational simplicity I will denote the signal of trader i at time t by S_t^i. The random variables δ_A, δ_B, ε^1, u_1 and u_2 are independently normally distributed with mean zero. For symmetry reasons let $Var[\delta_A] = Var[\delta_B]$.

An analysis of price manipulation is ruled out in a Rational Expectations Equilibrium setting because all traders are assumed to be price-takers. In a Perfect Bayesian Nash equilibrium setting, however, all traders take the strategies of all other players as given. That is, they are aware that their trade affects the price. All traders submit their market orders, x_t^i, in each trading round to the market maker. After observing the aggregate net order flow the risk-neutral market maker sets the execution price p_t. The price is semi-strong informationally efficient, i.e. the price is the best estimate given the market maker's information. A different price would lead to an expected loss or an expected profit. The latter is ruled out because the market maker faces Bertrand competition from potential rival market makers. Informed traders maximize their expected capital gains. Their trading strategy involves assigning a corresponding order size for each possible information set. The measurability condition of strategies guarantees that players' strategies assign different actions to only those states that the player can distinguish. The state space by choice of nature is $\{\delta_A, \delta_B, \varepsilon^1, u_1, u_2\}$. All players have a common prior multivariate normal distribution over this space. In period two the information sets of all agents can be best illustrated in the space $\{\delta_A, \delta_B, \varepsilon^1, u_1, u_2, x_1^1, x_1^2, p_1\}$. Note that $x_t^i(\cdot)$, $p_t(\cdot)$ are functions of the state space if all market participants apply pure strategies. A trader's ability to distinguish between states depends on his signal structure. More formally, trader i's

[4] This problem can also be captured in a model with higher order uncertainty, i.e. information leakage occurs only with a certain probability. Trader 1 receives then two pieces of information. In addition to the actual signal he knows whether some information leaked or not. Trader 1's informational advantage at the time of the public announcement stems from his knowledge of whether he received an early signal or not. Such models were not pursued in this paper because they are either very simplistic or intractable and do not provide much additional insight.

pure strategy is the tuple of mappings $x^i = \{x_1^i(S_1^i), x_2^i(S_1^i, x_1^i, p_1, \delta_A)\}$, $i = 1, 2$.

Note that in period two, each trader i knows not only his signal, the price p_1 and the public information δ_A but also his demand in $t = 1$, x_1^i. For ease of exposition the strategy for the market maker, $p^0 = \{p_1(X_1), p_2(X_2; X_1, \delta_A)\}$, is exogenously specified. He has to set informationally efficient prices in equilibrium, i.e. $p_1 = E[v|X_1]$ and $p_2 = E[v|X_1, \delta_A, X_2]$ due to potential Bertrand competition.

A Perfect Bayesian Nash equilibrium of this trading game is given by strategy profile $\{\{x_1^{i,*}(\cdot), x_2^{i,*}(\cdot)\}_{i=\{1,2\}}, p_1^*(\cdot), p_2^*(\cdot)\}$ such that

(1) $x_2^{i,*} \in \arg\max_{x_2^i} E[x_2^i(v - p_2)|S_1^i, x_1^i, p_1, \delta_A]$ $\forall i$,
(2) $x_1^{i,*} \in \arg\max_{x_1^i} E[x_1^i(v - p_1) + x_2^{i,*}(v - p_2)|S_1^i]$ $\forall i$, and
prices $p_1^* = E[v|X_1^*]$ and $p_2^* = E[v|X_1^*, \delta_A, X_2^*]$,

where the conditional expectations are derived using Bayes' Rule to ensure that the beliefs are consistent with the equilibrium strategy.

A backward induction solution can be employed because a Perfect Bayesian Nash Equilibrium requires equilibrium strategies to be optimal for each information set under the given Bayesian rational belief system. Due to the continuous distributions of the signals and of the noise traders' demand any $p_1 \in \mathbb{R}$ is possible in equilibrium and Bayes' Rule can always be applied. Therefore, the equilibrium strategy defines a period-two-action for any possible p_1. There is no out of-equilibrium path which can be deduced as such. Consequently, a trader's deviation will not change the market maker's pricing rule, even though they move sequentially. Furthermore, no out-of-equilibrium beliefs for any continuation game need to be specified. Hence, in this setting, the Perfect Bayesian Nash equilibrium coincides with the sequential equilibrium or the trembling-hand perfect equilibrium. The impact of the noise traders can also be viewed as a certain form of tremble.

3.2 Characterization of Linear Equilibrium

Proposition 1 characterizes a Perfect Bayesian equilibrium in linear pure strategies. It has the nice feature that each trader's demand is the product of his trading intensity (or aggressiveness) and the difference in the trader's and market maker's expectations about the value of the stock. Linear strategies have the advantage that all random variables remain normally distributed. The trading (action) rule in $t = 2$ also has to specify how trader i reacts in $t = 2$ after he has deviated in $t = 1$ from x_1^i to $x_1^{i,di}$. In addition, the pricing rules are linear as a consequence of the Projection Theorem.[5] In period one the

[5]Since all variables are normally distributed the orthogonal projection of v on the space of linear-affine functions of S is equal to the projection of v (in the sense of $\mathcal{L}^2$) on the space $\mathcal{L}^2(S)$ of quadratic integrable functions of S. Consequently, $E[v|\mathbf{S}] = E[v] + (\mathbf{S} - E[\mathbf{S}])^{\tau}\mathbf{Var}^{-1}[\mathbf{S}]Cov[v, \mathbf{S}]$, which allows us

market maker's pricing rule is $p_1 = \lambda_1 X_1$ and in period two it is $p_2 = \delta_A + E[\delta_B|X_1, \delta_A] + \lambda_2 X_2$ in equilibrium. As in Kyle (1985) λ_t reflects the price impact of an increase in market order by one unit. This price impact restricts the trader's optimal order size. Kyle interpreted the reciprocal of λ_t as market depth. If the market is very liquid, i.e. λ_t is very low, then an increase in the trader's demand has only a small impact on the stock price. For expositional clarity I denote the regression coefficient of y on x by $\phi_x^y := \frac{Cov[x,y]}{Var[x]}$.

Proposition 1 *A Perfect Bayesian Nash equilibrium in which all pure trading strategies are of the* ***linear*** *form*

$x_1^i = \beta_1^i(S_1^i),$

$x_2^i = \beta_2^i(E[v|S_1^i, p_1, \delta_A] - E[v|p_1, \delta_A]) - \gamma_2^i \left(x_1^{i,di} - x_1^i\right),$

and the market maker's pricing rule

$p_1 = E[v|X_1] = \lambda_1 X_1,$

$p_2 = E[v|X_1, \delta_A, X_2] = \delta_A + \phi_{S_2^{p_1}}^{\delta_B} S_2^{p_1} + \lambda_2 X_2,$ *with* $S_2^{p_1} = \frac{X_1 - \beta_1^1 \delta_A}{\beta_1^2},$

is given by the fixed points of the following system of equations

$$\beta_1^{1,*} = \frac{1}{2\left(\lambda_1 - \lambda_2 \left(\gamma_2^1\right)^2 \phi_{S_1^1}^{\varepsilon^1}\right)} \phi_{S_1^1}^{\delta_A}$$

$$\beta_1^{2,*} = \frac{1}{2\lambda_1}\left(1 - 2\lambda_2 \beta_2^2 \gamma_2^2 \left(1 - \phi_{S_2^{p_1}}^{\delta_B}\right)\right)$$

where

$\lambda_1 = \frac{\beta_1^{1,*} Var[\delta_A] + \beta_1^{2,*} Var[\delta_B]}{Var[\beta_1^{1,*}(\delta_A + \varepsilon^1) + \beta_1^{2,*}(\delta_B) + u_2]} \qquad \lambda_2 = \frac{Cov[\delta_B, X_2|S_2^{p_1}]}{Var[X_2|S_2^{p_1}]}$

with

$\beta_2^1 = \frac{1}{2\lambda_2}\frac{1}{2}\frac{1}{1 - \frac{1}{4}\phi_w^{S_2^{1,w}} \phi_{S_2^{1,w}}^w} \phi_{S_2^{1,w}}^w \qquad \gamma_2^1 := \frac{\beta_2^1}{\beta_1^2}\frac{\phi_{S_2^{p_1}}^{\delta_B}}{\phi_{S_2^{1,w}}^w}$

$\beta_2^2 = \frac{1}{2\lambda_2}\frac{1 - \frac{1}{2}\phi_w^{S_2^{1,w}} \phi_{S_2^{1,w}}^w}{1 - \frac{1}{4}\phi_w^{S_2^{1,w}} \phi_{S_2^{1,w}}^w} \qquad \gamma_2^2 := \frac{1}{2\lambda_2}\frac{1}{\beta_1^2}\phi_{S_2^{p_1}}^{\delta_B} - \frac{1}{2}\frac{\beta_2^1}{\beta_1^2}\frac{\phi_{S_2^{p_1}}^{\delta_B}}{\phi_{S_2^{1,w}}^w}$

if the second order conditions $\lambda_2 > \lambda_1 \max\left\{\left[\frac{b_2^1}{b_1^2}\frac{\phi_{S_2^{p_1}}^{\delta_B}}{\phi_{S_2^{1,w}}^w}\right]^2, \left[\frac{1}{b_1^2}\phi_{S_2^{p_1}}^{\delta_B} - \frac{1}{2}\frac{b_2^1}{b_1^2}\frac{\phi_{S_2^{p_1}}^{\delta_B}}{\phi_{S_2^{1,w}}^w}\right]^2\right\}$, $\lambda_2 > 0$ *(with* $b_t^i := 2\lambda_t \beta_t^i$*) are satisfied.*

The interested reader is referred to the Appendix for a complete proof of the proposition. The proof makes use of backward induction. In order to solve the continuation game in $t = 2$, the information structure prior to trading in $t = 2$ has to be derived. For this purpose propose an arbitrary action rule profile, $\{\{\beta_1^i\}_{i=1,2}, p_1(X_1)\}$ for $t = 1$, which is mutual knowledge and is considered to be an equilibrium profile by all agents. In $t = 2$ all market participants can derive the aggregate order flow $X_1 = \beta_1^1(\delta_A + \varepsilon^1) + \beta_1^2 \delta_B + u_1$

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to calculate the conditional expectations.

from price p_1. After knowing δ_A the price signal is $S_2^{p_1} = \delta_B + \frac{\beta_1^1}{\beta_1^2}\varepsilon^1 + \frac{1}{\beta_1^2}u_1$. Since all market participants know $S_2^{p_1}$, it is useful to state each traders' information relative to the publicly known symmetric information, i.e. orthogonalize the signals with respect to $S_2^{p_1}$. The stock is split into an expected part $E[v|S_2^{p_1}, \delta_A] = \delta_A + E[\delta_B|S_2^{p_1}]$ and an unexpected part $w := \delta_B - E[\delta_B|S_2^{p_1}]$. This 'virtual' split of the stock v into a risk-less bond and a risky asset w is possible, without loss of generality, as long as all traders are risk-neutral or have exponential utility functions. Note that the stock split depends on the proposed action rule profile $\{\{\beta_1^i\}_{i=1,2}, p_1(X_1)\}$ for $t = 1$ and not on the one actually chosen. In other words, the stock split is not affected if some trader deviates. Trader 1's information is $S_2^{1,w} = w + \frac{1}{\beta_1^2}\vartheta_1$, where $\vartheta_1 = u_1 - E[u_1|S_2^{p_1}]$. Trader 2 knows the fundamental value of the risky component w but has to forecast trader 1's forecast $S_2^{1,w}$ in order to predict trader 1's order size in $t = 2$. Knowing trader 1's demand in $t = 2$ would help trader 2 predict the price p_2 at which his orders will be executed.[6] In $t = 2$ traders face a generalized static Kyle-trading-game with the usual trade-off. On the one hand, a risk-neutral trader wants to trade very aggressively in order to exploit the gap between his estimate of the fundamental value of the stock and the price of the stock. On the other hand, very aggressive trading moves the price at which his order will be executed towards his estimate of the asset's value since it allows the market maker to infer more of the trader's information from the aggregate order flow. This latter price impact reduces the value-price gap from which the trader can profit and restrains the traders from trading very aggressively.

Using backward induction one has to check whether a single player wants to deviate in $t = 1$ from the proposed action rule profile, $\{\{\beta_1^i\}_{i=1,2}, p_1(X_1)\}$. Trading in $t = 1$ affects not only the capital gains in $t = 1$ but also the future prospects for trading in $t = 2$. If trader i deviates in $t = 1$ from the proposed action rule β_1^i by trading more or less aggressively his expected future capital gains at $t = 2$ are affected in two ways. His optimal trading intensity at $t = 2$ is affected, as is the price in $t = 2$, p_2. The change in p_2 is due to the misperception by all other market participants. All other players, thinking that trader i did not deviate, still play their equilibrium strategy. Thus, they infer the wrong signal from the aggregate order flow in $t = 1$, X_1^{di} or $p_1^{di} = \lambda_1 X_1^{di}$, where the superscript di indicates that trader i deviated. This alters the other traders' order size in $t = 2$ and the market maker's price setting in $t = 2$. The impact of both effects on the price schedule is given by $2\lambda_2\gamma_2^i(x_1^i - x_1^{i,di})$. Consequently, the deviant adjusts his market order in $t = 2$ by $\gamma_2^i(x_1^i - x_1^{i,di})$ as stated in the proposition.

The value function

$$V_2^i(x_1^{1,d1}) = \kappa^i \left(S_2^{i,w}\right)^2 - \tau^i S_2^{1,w}\left(x_1^{i,di} - x_1^i\right) + \psi^i\left(x_1^{i,di} - x_1^i\right)^2 \forall i$$

[6] Note that the infinite regress problem discussed in Townsend (1983) can be avoided because of the linearity of the trading strategies $x_t^i = \beta_t^i S_t^i$, where S_t^i is the difference between trader i's and the market maker's expection of v. Linearity also preserves the normality distribution of the (conditional) random variables.

captures the impact of trading in $t = 1$ on the (maximal) expected gains in $t = 2$. Capital gains are the product of the optimal order size and the estimated value-price gap in $t = 2$. The value function is quadratic since deviation in $t = 1$ affects the optimal order size as well as the estimated value-price gap in $t = 2$ linearly. The value function can be decomposed into four parts. The first component is the expected capital gains in the proposed equilibrium captured by $\kappa^i \left(S_2^{i,w}\right)^2$. The second component reflects the impact of the deviation in $t = 1$ on the optimal order size in $t = 2$ while ignoring the impact on the execution price. The third component holds the order size fixed while taking into account the fact that a deviation in $t = 1$ changes the price in $t = 2$. This occurs because of the misperception of other market participants and the deviant's demand adjustment in $t = 2$. Finally, the fourth component isolates the effects which are solely due to the induced change in the optimal order size and in the price schedule not covered by the other effects. The second and third component are summarized by the coefficient τ^i and the fourth by coefficient ψ^i. An equilibrium is reached if no trader wants to deviate from the proposed action rule profile in $t = 1$. In other words, the Perfect Bayesian Nash Equilibrium is given by the fixed point described in Proposition 1.

Proposition 1 also presents two inequality conditions. They result from the second or-

In period two, only trader 1 knows the exact extent to which the price, p_1 already reflects the new public information, δ_A. It is interesting to note that the informational advantage of trader 1 in period two is a consequence of the technical analysis conducted by trader 2 and by the market maker. Both trader 2 and the market maker try to infer information in $t = 2$ from the past price p_1

Taking on a larger position in period one can result in higher profits today but also leads to worse prices for current and future trading rounds. Thus in a Kyle (1985) setting the insider restrains his trading activity with the objective of not trading his informational advantage away.

In contrast to the literature based on Kyle (1985), trader 1 in my model trades more aggressively in period one. He incurs myopically non-optimal excessive trades in period one and then recuperates the losses and makes additional profit in period two. Trading more aggressively in period one changes the price in such a way that his informational advantage in the next trading round is enhanced. Trading with the sole intention of increasing one's informational advantage in the next period is defined as *manipulative trading*. *Speculative trading* is defined as trading with the expectation to unwind one's position in the next period. The following definitions restate the two trading objectives:

Definition 1 *Speculative trading is carried out with the expectation of unwinding the acquired speculative position in the next period.*

Speculative trading can also be manipulative.

Definition 2 *Manipulative trading is intended to move the price in order to enhance the informational advantage in the next period.*

Manipulative trading is excessive in the sense that it is the component of trading intensity which exceeds the optimal myopic trading intensity, holding the other market participants' strategies fixed. The myopic trading intensity does not take into account the fact that by trading more aggressively trader 1 could enhance his informational advantage in period two.

Proposition 3 shows that trader 1 trades for speculative reasons since he expects to unwind part of his accomplished position in period two. Furthermore, he trades excessively with the objective of manipulating the price.

Proposition 3 *In period one, trader 1 trades conditional on his current information in order to build up a long-term position, and also for speculative and manipulative reasons.*
Speculative trading is given by $\gamma_2^1 \phi_{S_1^1}^{\varepsilon^1} \beta_1^1 S_1^1$.
Manipulative trading is given by $\lambda_2 \left(\gamma_2^1\right)^2 \phi_{S_1^1}^{\varepsilon^1} \beta_1^1 S_1^1$,
where the coefficients in front of S_1^1 *are strictly positive.*

The proof in the appendix shows that if trader 1 receives a positive signal, all trading objectives induce the trader to take a long position in the stock. Similarly, if trader 1 receives a negative signal he sells the stock.

All traders trade conditional on their signal in period one. Therefore, the price p_1 reflects not only the signal about δ_B but also the signal about $\delta_A + \varepsilon^1$. The main motive for technical analysis is to infer more information about δ_B and about the others' forecasts. All market participants can separate the impact of δ_A on p_1, but only trader 1 can deduct the impact of the ε^1 error term on p_1. Therefore, their inference about δ_B from the price p_1 is also perturbed by $\frac{\beta_1^1}{\beta_1^2}\varepsilon^1$. They overestimate (underestimate) δ_B if ε^1 is positive (negative). Since trader 1 can infer ε^1 in period two, he can make money by correcting trader 2's error. If ε^1 is positive (negative), trader 1 sells (buys) stock in period two. In period one not even trader 1 knows ε^1. His prediction of ε^1, given his signal $S_1^1 = \delta_A + \varepsilon^1$, $E[\varepsilon^1|S_1^1] = Var[\varepsilon^1](Var[\delta_A] + Var[\varepsilon^1])^{-1}S_1^1$ is always of the same sign as his trade in period one $x_1^1 = \beta_1^1 S_1^1$. Therefore, trader 1 expects to trade in period two in the opposite direction. 'On average', he partially unwinds his position in period two.

The purpose of manipulative trading is to extend the informational gap in the second trading round. By trading excessively in $t = 1$, trader 1 worsens the other market participants' price signal $S_2^{p_1}$ in $t = 2$ about the fundamental value δ_B. The reason is that by trading more aggressively the imprecision of trader 1's signal ε^1 has a larger impact on p_1. Consequently, the price signal $S_2^{p_1} = \delta_B + \frac{\beta_1^1}{\beta_1^2}\varepsilon^1 + \frac{1}{\beta_1^2}u_1$ reveals more information about ε^1 and less about the fundamental value δ_B. On the one hand, it increases trader 1's informational advantage in $t = 2$ with respect to the market maker. On the other hand, it allows trader 2 to better forecast trader 1's forecast of δ_B. Trader 2 knows already the fundamental value δ_B and his only purpose of conducting technical analysis is to achieve a better prediction of trader 1's market order and thus the execution price in $t = 2$, p_2. Therefore, he tries to infer the error term ε^1 from which trader 1's informational advantage stems. The competition between both traders is intensified by any additional knowledge that trader 2 can gain about trader 1's informational advantage. In short, if trader 1 trades more aggressive in period one, he builds up a larger informational advantage with respect to the market maker, but also reveals more of his informational advantage to his competitor trader 2, who will compete part of it away. Overall, more aggressive trading in period one increases trader 1's expected future capital gains. The proof in the appendix shows that in equilibrium the trading intensity of trader 1 is higher if he takes the impact on future expected capital gains into account, given the strategies of all other players. It is the expected knowledge of the ε^1-term in $t = 2$ which induces manipulative trading.

Speculative trading is also caused by the imprecision of trader 1' signal, ε^1. Consequently, an increase in trading intensity in period one due to manipulative behavior also leads to more speculation. The trader expects to unwind a larger position in $t = 2$.[7]

Hirshleifer, Subrahmanyam, and Titman (1994) appeal to traders' risk-aversion and thus provide a very distinct explanation for speculative behavior. In their model, early-informed traders receive the same piece of information one period prior to the late-informed traders, while the competitive risk-neutral market makers observe only the limit order book. Traders submit limit orders, in the form of whole demand schedules, to the risk-neutral market makers who set the price. All traders have to be risk-averse in their analysis. Furthermore, since they are also competitive price-takers, manipulative trading

[7]Note if δ_A and δ_B can be traded separately neither speculative nor manipulative trading would arise.

is ruled out. The intuition for speculative trading in their model is as follows. Since no risk premium is paid due to the market makers' risk-neutrality, risk-averse traders would be unwilling to take on any risky stock in the absence of any informational advantage. Early-informed investors are willing to take on risk since they receive a signal in period one. Their informational advantage, together with the existence of noise traders, compensates them for taking on the risky asset. However, the informational advantage of early-informed traders with respect to the late-informed traders vanishes in period two since both now receive the same signal. Thus, early-informed traders share the risk with late-informed traders in period two. In addition, the informational advantage of the early-informed traders with respect to the market makers shrinks as well since market makers can observe a second limit order book. Therefore, in period two, both these effects cause early-informed traders to partially unwind the position they built up in the previous period.

4 Impact of Information Leakage on Informational Efficienc

In the information structure analyzed above, trader 1 already received in period one some information about the forthcoming public announcement in period two. In other words, some news about the public announcement leaked to trader 1 before it was made public. This section compares this to the benchmark case where the announcer manages to keep the content of his public announcement secret. That is, no information leaks and thus trader 1 has no informational advantage in period one. It addresses the question of whether information leakage makes the price more or less informationally efficient.

A market is (strong-form) informationally efficient if the price is a sufficient statistic a world without asymmetric information. In this setting any price process is informationally efficient even though it is uninformative. The conditional variance of the stock value itself captures how informative the price (process) and the other public information are.[8] This variance is zero if all public information, including the price process, allows one to perfectly predict the liquidation value of the stock. In this case everybody knows the true stock value. This variance term, therefore, also measures the risk a liquidity trader faces when trading this stock.

The following definitions define both measures more formally.

Definition 3 *The reciprocal of the variance* $Var[E[v|\{p_t\}_{t\leq\tau}, S_\tau^{public}, \{S_\tau^i\}_{i\in\mathbb{I}}]|\{p_t\}_{t\leq\tau}, S_\tau^{public}]$ *conditional on the public information,* S_τ^{public}*, and the pool of private information up to time* τ *measures the degree of informational efficiency at time* τ*.*
The reciprocal of the conditional variance $Var[v|\{p_t\}_{t\leq\tau}, S_\tau^{public}]$ *measures how informative the price (process) and the public information are.*

Equipped with these measures, one can analyze how the information leakage of $\delta_A + \varepsilon^1$ to trader 1 affects informational efficiency and informativeness of the price (process). In addition it allows us to address the role of the imprecision of the rumor.

Since these definitions are time dependent, let us analyze informational efficiency and informativeness at the time after the first trading round, after the public announcement of δ_A, and after the second trading round. Let us assume for the following proposition that there is a sufficient amount of liquidity trading in $t = 1$. More precisely, $Var[u_1] > \frac{6}{25}\sqrt{\frac{2}{5}Var[u_2]Var[\delta_B]}$.

Proposition 4 *In* $t = 1$ *information leakage makes the price* p_1 *more informative but less informationally efficient if the information leakage is sufficiently precise. However, after the public announcement in* $t = 2$ *both informativeness as well as informational efficiency are reduced.*

Leakage of information makes the price p_1 in $t = 1$ more informative, if $Var[\varepsilon^1]$ is not too high. Trader 1 trades on his information $\delta_A + \varepsilon^1$ and thus price p_1 reveals information about not only δ_A but also about δ_B. Trader 1's market activity increases informed trading relative to liquidity trading. This allows the market maker as well as the public to infer more information from the aggregate order flow X_1. Note that for very high $Var[\varepsilon^1]$ this might not be the case since aggressive manipulative trading activity could increase the non-informative component of the aggregate order flow.

On the other hand, information leakage makes the market less informationally efficient in $t = 1$. If there is no leakage, p_1 reveals more about δ_B than it reveals about

[8] Note that all public information at the beginning of the trading game is incorporated in the common priors.

$E[v|\delta_B, \delta_A + \varepsilon^1] = \delta_B + \phi_{S_1^1}^{\delta_A}(\delta_A + \varepsilon^1)$ in the case of a leakage. The reason is that sufficiently precise information leakage leads to a higher λ_1 which reduces the trading intensity of trader 2, β_1^2. Therefore less information can be inferred about δ_B. In addition, $\delta_A + \varepsilon^1$ can only be partly inferred from the price p_1. Both effects together result in a lower informational efficiency for p_1 in the case of a precise leakage.

After the public announcement in $t = 2$ δ_A as well as δ_B are known by some traders in the economy, (i.e. the best forecast of v given the pooled information is v). Consequently, the measures of informational efficiency and informativeness coincide from that moment onwards. Since δ_A is common knowledge, the conditional variance stems solely from the uncertainty about δ_B. The proof in the appendix shows that sufficiently precise information leakage leads to a less liquid market, i.e. to a higher λ_1. As illustrated above, the leakage of information increases λ_1. This reduces β_1^2 and thus makes the price signal about δ_B less precise. In addition, the price signal $S_2^{p_1} = \delta_B + \frac{\beta_1^1}{\beta_1^2}\varepsilon^1 + \frac{1}{\beta_1^2}u_1$ is perturbed by the ε^1-error term. Therefore, information leakage makes the price p_1 after the public announcement less informative and less informationally efficient. The same is true after the second trading round for the price process $\{p_1, p_2\}$.

In summary, information leakage reduces informational efficiency at each point in time. It makes the price (process) more informative prior to the public announcement and less informative afterwards.

5 Extensions

5.1 Analyzing Mixed Strategy Equilibria

The propositions in Section 3 showed that trader's 1 informational advantage at the time of the public announcement as well as speculative and manipulative trading result from the imprecision of the rumor. This raises the question of whether the trader could generate some (additional) imprecision himself by trading above or below his optimal level in period one. Intuitively, the market maker will decrease the price, if the trader sells more today. Consequently, all other traders lower their evaluation of the stock tomorrow. This allows cheaper purchases tomorrow. Such manipulation was conducted by the Rothschild brothers during the Napoleonic wars. At the beginning of the 19th century, stock and bond prices in London depended crucially on news of the war. Despite knowing about Napoleon's fate at Waterloo, the Rothschild brothers sold English shares with the intent to drive prices down and repurchase cheaper shares later. Such a pure strategy cannot arise in equilibrium. In any Nash equilibrium, strategies of all players are mutual knowledge, i.e. all other traders would know the Rothschilds' true motivation for selling stocks.[9] It can, however, be the random realization of a mixed strategy.

[9] In a Kyle (1985) setting such behavior is not optimal as long as the second order conditions are satisfied.

Let us focus on mixed (or behavioral) strategies for trader 1 of the form $x_1^1 = \beta_1^1(\delta_A + \varepsilon^1) + \gamma_1^1\zeta^1$, i.e. trader 1 adds some noisy component $\gamma_1^1\zeta^1$ to his optimal demand. In order to preserve normality of all random variables, assume $\zeta^1 \sim \mathcal{N}(0,1)$. Adding random demand $\gamma_1^1\zeta^1$ in trading round one makes the market more liquid in $t=1$, but less liquid in $t=2$. This occurs because trader 1 trades in $t=2$ on information generated by $\gamma_1^1\zeta^1$. The changes in the liquidity measure, λ_t also alter the trading intensities, β_t^i. All this affects the new price signal $S_2^{p_1} = \delta_B + \frac{\beta_1^1}{\beta_1^2}\varepsilon^1 + \frac{\gamma_1^1}{\beta_1^2}\zeta^1 + \frac{1}{\beta_1^2}u_1$, which has the additional error term $\frac{\gamma_1^1}{\beta_1^2}\zeta^1$. This additional term is known to trader 1, but not to the other market participants. Therefore, trader 1's informational advantage in $t=2$ consists of his knowledge of $\frac{\beta_1^1}{\beta_1^2}\varepsilon^1$ as well as of $\frac{\gamma_1^1}{\beta_1^2}\zeta^1$. The two error terms differ in two respects. First, whereas trader 1 knows ζ^1 already in $t=1$, he learns the precise value of ε^1 only at the time of the public announcement. Second, if trader 1 wants to increase the importance of the error term $\frac{\beta_1^1}{\beta_1^2}\varepsilon^1$ by varying β_1^1, he must also trade more aggressively on his information in $t=1$. In contrast, trader 1 can control the impact of the error term $\frac{\gamma_1^1}{\beta_1^2}\zeta^1$ on the price signal $S_2^{p_1}$ separately by adjusting γ_1^1. The trade-off is that while on the one hand he acts like a noise trader in $t=1$ incurring trading costs, on the other hand he also increases his informational advantage in $t=2$.

The analysis of the continuation game in $t=2$ is analogous to the one in Proposition 1. The only difference stems from the less informative price signal $S_2^{p_1}$. This alters the stock split and trader 2's forecast of traders 1's forecast. Formally, $\phi^{\delta_B}_{S_2^{p_1}} = Var[\delta_B](Var[\delta_B] + (\frac{\beta_1^1}{\beta_1^2})^2 Var[\varepsilon^1] + (\frac{\gamma_1^1}{\beta_1^2}) + (\frac{1}{\beta_1^2})Var[u_1])^{-1}$ and $\phi_w^{S_2^{1,w}} = \frac{(\beta_1^1)^2 Var[\varepsilon^1] + (\gamma_1^1)^2}{(\beta_1^1)^2 Var[\varepsilon^1] + (\gamma_1^1)^2 + Var[u_1]}$ change due to the additional γ_1^1-terms. This affects β_2^i, γ_2^i and λ_2. In $t=1$ trader 1 expects a larger informational advantage for the second trading round due to randomization, $E[S_2^{1,w}|S_1^1] = -(\phi^{\delta_B}_{S_2^{p_1}} + \frac{1}{\beta_1^2}\phi^{u_1}_{S_2^{p_1}})\frac{1}{\beta_1^2}[\beta_1^1\phi^{\varepsilon^1}_{S_1^1}S_1^1 + \gamma_1^1\zeta^1]$. Trader 1's trading rule only exhibits the proposed form $x_1^1 = \beta_1^1 S_1^1 + \gamma_1^1\zeta^1$, if $\lambda_1 = \psi^1 = \lambda_2(\gamma_2^1)^2$. This implies $\beta_1^1 = \frac{1}{2\lambda_1}$.

For a mixed strategy to sustain in equilibrium, trader 1 has to be indifferent between any realized pure strategy, i.e. between any realization of ζ^1. Since the random variable ζ^1 can lead to any demand with positive probability, he has to be indifferent between any x_1^1 in equilibrium. This requires that the marginal trading costs in $t=1$ exactly offset the expected marginal gains in $t=2$. Trader 1's objective function consists of two parts: the expected capital gains in $t=1$ and the expected value function for $t=2$. They are illustrated in Figure 2.

Trader 1 is only indifferent between all realizations of ζ^1 if his $(x_1^{1,d1})^2[-\lambda_1 + \psi^1] + x_1^{1,d1}[\phi^{\delta_A}_{S_1^1}S_1^1 - \tau^1 E[S_2^{1,w}|S_1^1] - 2\psi^1 x_1^1] + C_1$ reduces to a constant, C_1. Trader 1 faces no additional trading costs in $t=1$ for the pure strategy given by the realization $\zeta^1 = 0$ but he still has an informational advantage in $t=2$. Therefore, the expected overall profits have to be strictly positive since he is indifferent between any realization of $\zeta^1 = 0$. Note

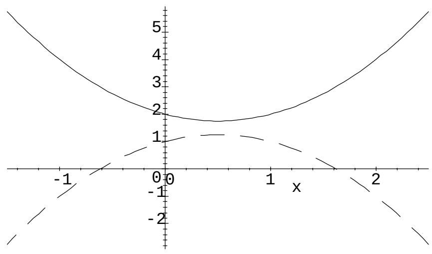

Figure 2: Components of Trader 1's Objective Function

that even if trader 1 receives no signal in $t = 1$ his informational advantage in $t = 2$ in the case of $\zeta^1 = 0$ is $S_2^{1,w} = E[\delta_B|\delta_B + \frac{1}{\beta_1^2}u_1] - E[\delta_B|S_2^{p_1}] = (\phi^{\delta_B}_{\delta_B+\frac{1}{\beta_1^2}u_1} - \phi^{\delta_B}_{S_2^{p_1}})(\delta_B + \frac{1}{\beta_1^2}u_1)$. In summary, the necessary conditions for a mixed strategy equilibrium are that $\lambda_1 = \psi^1$and $\phi^{\delta_A}_{S_1^1} S_1^1 - \tau^1 E[S_2^{1,w}|S_1^1] - 2\psi^1 x_1^1 = 0$. The second necessary condition simplifies to $1 - 2\lambda_2(\gamma_2^1)\beta_1^1 = 0$ and is equivalent to the first one.

Proposition 5 exploits the facts that in any mixed strategy equilibrium the second order condition of trader 1 is binding ($\lambda_1 = \psi^1$) and that the second order condition for trader 2 ($\lambda_1 > \psi^2$) also has to be satisfied. The proposition shows that no mixed strategy equilibrium exists except if the market is very liquid in $t = 2$.

Proposition 5 *There does not exists a mixed strategy equilibrium for sufficiently small* $Var[u_2]$.

See Appendix A.5 for a proof of this proposition. Note that the second order conditions also require that the trading round one is sufficiently liquid relative to trading round two, as stated in Proposition 1.

5.2 Increasing the Number of Traders

In reality there are many informed traders active in the market. One question which might arise is whether the results of Section 3 also hold in a setting with many informed traders. Before increasing the number of traders let us investigate what distinguishes trader 1 who received a signal about δ_A from trader 2 who received a signal about δ_B. In the setting described above, trader 1's prior knowledge about event A causes him to trade for speculative as well as manipulative reasons. However, trader 2 does not act speculatively or manipulatively in $t = 2$ despite his prior knowledge of the forthcoming public announcement about event B in $t = 3$. Neither the timing per se nor the fact that trader 2 got a precise signal about the public announcement in $t = 3$ can explain the difference. Trader 2 still would not speculate or try to manipulate the price even if his signal is imprecise, i.e. $\delta_B + \varepsilon^2$. This is in spite of the fact that the imprecision of trader 1's signal is necessary for trader 1's behavior. The distinctive feature is that when δ_A

is publicly announced in $t = 2$, p_1 still carries some information for market participants, which induces them to conduct technical analysis. This, in turn, makes it worthwhile for trader 1 to manipulate p_1. On the other hand, when δ_B is announced, neither p_1 nor p_2 carry any additional information. Since everybody knows the true value of the stock, $v = \delta_A + \delta_B$, nobody trades conditional on p_2. Thus trader 2 has no incentive to manipulate the price in $t = 2$.

Having understood this crucial distinction let us first analyze the impact of increasing the number of traders who receive some information about δ_B in $t = 1$, and then increase the number of traders who can potentially act manipulatively in equilibrium. If there are many informed δ_B-traders who receive different signals $\delta_B + \varepsilon^i$, $i \in \{2, ..., I\}$ they have an additional incentive to conduct technical analysis. In $t = 2$ they not only draw inference from price p_1 in order to improve their forecast of trader 1's forecast, they also try to learn more about the fundamental value δ_B. They try to infer each others' signal from p_1 although they know that the past price p_1 is perturbed by the ε^1-error term. This makes manipulation of p_1 even more effective and consequently trader 1 speculates and trades to manipulate the price.[10]

In the context of rumors, it may be hard to envision an information structure where only a single trader receives some vague information about a forthcoming public announcement. Instead, there could be many traders who receive some signal. In a setting in which all early-informed traders receive the same signal with a common noise component, $\delta_A + \xi$, manipulative trading and speculation still occur, but to a smaller extent. The reason is that all δ_A-traders try to free ride on the manipulative activity of the other manipulators. Manipulation is costly but benefits all other δ_A-traders in the second trading round. Such an information structure is, however, not very plausible since every recipient of a rumor can interpret it slightly differently. Even if all agree on the informational content of the rumor, they can still disagree on how it impacts the fundamental value of the stock. Therefore, the information structure that best fits the description of a rumor is one where many traders receive a signal $\delta_A + \xi + \varepsilon^i$ with a common and a private noise term. The private noise term ε^i alleviates the free rider problem. On the other hand, as the number of traders who hear about the rumor increases, the importance of the ε^i-terms diminishes. In addition, ε^i distorts trader i's estimate of ξ. This discussion suggests that rumors lead to more manipulative and speculative trading as long as they are not widely spread among many traders.

6 Conclusion

The objective of this paper is to model how traders respond to a public announcement. Traders have to figure out how much of this public information is really new relative to the information already reflected in the price. A trader who receives an imprecise signal prior to the public announcement trades on it and moves the price. He has an additional

[10]Proof of these propositions in a two δ_B-trader setting is available from the author on request.

informational advantage in the second trading round if the other market participants draw some inference from the past price. The early-informed trader can better interpret the past price. His trading also has a speculative feature. If he buys (sells) stocks when he receives his rumor, then he expects to sell (buy) it at the time when the news is made public. His strategy follows the well-known trading rule: "Buy on Rumors - Sell on News". By trading more aggressively prior to the public announcement, he can also affect the price and worsen the price signal for the other traders. This manipulative trading enhances his informational advantage at the time of the public announcement. Thus, his trading strategy is really to "Buy Aggressively on Rumors - Sell on News".

This paper adds to the literature by explaining the behavior of an early-informed trader who trades for both manipulative and speculative reasons. It also provides an alternative, rational explanation for the overactivism of traders. By demonstrating how rumors can reduce the informational efficiency of markets, the paper also provides support for the use of early public announcements as a tool for crisis management.

Some further extensions come to mind. A higher order uncertainty model could be used to address the same questions. However, these models tend to be either very simplistic or very intractable. The same analysis could also be conducted in a different framework where the market maker sets bid and ask prices before the order of a trader arrives, e.g. a setting á la Glosten (1989). Preliminary analysis suggests that such a setting would yield similar outcomes. Additional insights could be obtained by endogenizing the information acquisition process. For example, traders might like to commit themselves to purchase less precise signals. It would also be interesting to determine when it is more profitable to buy imprecise information about a forthcoming announcement and when it is more lucrative to acquire some long-lived information. The paper illustrates that information leakage reduces informational efficiency, but it does not make any normative welfare statements. In order to conduct a welfare analysis, one has to endogenize the trading acti

A Appendix

A.1 Proof of Proposition 1

Propose an arbitrary action rule profile for $t = 1$, $\{\{x_1^i(S_1^i)\}_{i=1,2}, p_1(X_1)\}$. This profile can be written as $\{\{\beta_1^i\}_{i=1,2}, p_1(X_1)\}$ since I focus on linear pure strategy Perfect Bayesian Nash Equilibria. Suppose that this profile is mutual knowledge among the agents and they all think it is an equilibrium profile.

Equilibrium in continuation game in $t = 2$.

Information structure in $t = 2$.

After δ_A is publicly announced, δ_B is the only uncertain component of the stock's value.

The *market maker* knows the aggregate order flow in $t = 1$, $X_1 = \beta_1^1(\delta_A + \varepsilon^1) + \beta_1^2(\delta_B) + u_2$ in addition to δ_A. His price signal $S_2^{p_1}$ (aggregate order flow signal, X_1) can be written as $S_2^{p_1} = \frac{X_1 - \beta_1^1 \delta_A}{\beta_1^2} = \delta_B + \frac{\beta_1^1}{\beta_1^2}\varepsilon^1 + \frac{1}{\beta_1^2}u_1$.

Since all market participants can invert the pricing function $p_1 = \lambda_1 X_1$ in $t = 2$, they all know $S_2^{p_1}$. For expositional clarity let us 'virtually' *split the stock* v into a risk-free bond with payoff $\delta_A + E[\delta_B|S_2^{p_1}]$ and a risky asset w. In equilibrium $E[w|S_2^{p_1}] = 0$ and $Var[w|S_2^{p_1}] = \left(1 - \phi_{S_2^{p_1}}^{\delta_B}\right) Var[\delta_B]$. The 'virtual' split of the stock v into a risk-free bond and a risky asset w is possible, without loss of generality, as long as all traders are risk neutral or have CARA utility functions.

Trader 1 can infer ε^1 in $t = 2$ and thus his price signal is $\delta_B + \frac{1}{\beta_1^2}u_1$. After orthogonalizing it to $S_2^{p_1}$, his signal can be written as $S_2^{1,w} := w + \frac{1}{\beta_1^2}\vartheta_1$, where $w = \delta_B - E[\delta_B|S_2^{p_1}]$ and $\vartheta_1 = u_1 - E[u_1|S_2^{p_1}]$. Trader 1's forecasts of the fundamental value of w is $E[w|S_2^{1,w}] = \phi_{S_2^{1,w}}^{w} S_2^{1,w}$, $\phi_{S_2^{1,w}}^{w} = \frac{Var[\delta_B]}{Var[\delta_B] + \frac{1}{\left(\beta_1^2\right)^2} Var[u_1]} = \phi_{\delta_B + \frac{1}{\beta_1^2}u_1}^{\delta_B}$. Trader 1's forecast of trader 2's forecast is also $E[w|S_2^{1,w}]$.

Trader 2 knows the fundamental value w. His forecast of trader 1's forecast is $E[w + \frac{1}{\beta_1^2}\vartheta_1 | w, S_2^{p_1}] = E[w + \frac{1}{\beta_1^2}\vartheta_1 | w] = \phi_w^{S_2^{1,w}} w$, where $\phi_w^{S_2^{1,w}} = \frac{\left(\beta_1^1\right)^2 Var[\varepsilon^1]}{\left(\beta_1^1\right)^2 Var[\varepsilon^1] + Var[u_1]}$.

Action (trading) rules in $t = 2$.

Due to potential Bertrand competition the risk-neutral *market maker* sets the price $p_2 = E[v|X_1, X_2] = \delta_A + E[\delta_B|S_2^{p_1}] + \lambda_2 X_2$. The first two terms reflect the value of the bond from the stock split and the last term $\lambda_2 X_2 =: p_2^w$ is the price for w. Note that $\lambda_2 = \frac{Cov[w, X_2|S_2^{p_1}]}{Var[X}$

for both traders' maximization problem is $\lambda_2 > 0$.

The *equilibrium* for a given action (trading) rule profile in $t = 1$ is given by

$$\beta_2^1 = \frac{1}{2\lambda_2}\frac{1}{2}\frac{1}{1-\frac{1}{4}\phi_w^{S_2^{1,w}}\phi^w_{S_2^{1,w}}}\phi^w_{S_2^{1,w}},\ \beta_2^2 = \frac{1}{2\lambda_2}\frac{1-\frac{1}{2}\phi_w^{S_2^{1,w}}\phi^w_{S_2^{1,w}}}{1-\frac{1}{4}\phi_w^{S_2^{1,w}}\phi^w_{S_2^{1,w}}},$$

$$\lambda_2 = \{\frac{1}{2}\frac{Var[\delta_B|S_2^{p1}]}{Var[u_2]}\left((b_2^1 + b_2^2) - \frac{1}{2}\left(b_2^1 + b_2^2\right)^2\right) + \frac{1}{2}\frac{Cov[\delta_B, u_1|S_2^{p1}]}{Var[u_2]}\left(\frac{b_2^1}{\beta_1^2}\left(1 - (b_2^1 + b_2^2)\right)\right)$$
$$-\frac{1}{4}\frac{Var[u_1|S_2^{p1}]}{Var[u_2]}\left(\frac{b_2^1}{\beta_1^2}\right)\}^{\frac{1}{2}},$$

where $b_2^i := 2\lambda_2\beta_2^i$. b_2^i depends only on the regression coefficients $\phi_w^{S_2^{1,w}}$ and $\phi^w_{S_2^{1,w}}$ which are determined by the proposed action rule profile in $t = 1$.

Equilibrium in $t = 1$.

The proposed arbitrary action rule profile is an equilibrium if no player wants to deviate given the strategies of the others.

The *market maker*'s pricing rule in $t = 1$ is always given by $p_1 = E[v|X_1] = \lambda_1 X_1$ with $\lambda_1 = \frac{Cov[v,X_1]}{Var[X_1]}$. He has to set an informationally efficient price due to (potential) Bertrand competition.

Trader 1's best response.

Deviation of trader 1 from $x_1^1(S_1^1) = \beta_1^1 S_1^1$ to $x_1^{1,d1}(S_1^1)$ will not alter the subsequent trading intensities of the other market participants, i.e. $\lambda_1, \beta_2^2, \lambda_2$. They still believe that trader 1 plays his equilibrium strategy since they cannot detect his deviation. Nor does his deviation change his own price signals since he knows the distortion his deviation causes. The definition of w is also not affected by this deviation.

Other market participants' misperception in $t = 2$.

Trader 1's deviation, however, distorts the other players price signal, $S_2^{p_1}$ to $S_2^{p_1,d1}$. This occurs because the other market participants attribute the difference in the aggregate order flow in $t = 1$ not to trader 1's deviation, but to a different signal realization or different noise trading. Deviation to $x_1^{1,d1}(\cdot)$ distorts the price signal by $S_2^{p_1,d1} - S_2^{p_1} = \frac{1}{\beta_1^2}\left(x_1^{1,d1} - x_1^1\right)$. Trader 2's signal prior to trading in $t = 2$ is not w but $w - \phi^{\delta_B}_{S^{p_1}}\left(S_2^{p_1,d1} - S_2^{p_1}\right)$. His market order in $t = 2$ is, therefore, $\beta_2^2 w - \beta_2^2\phi^{\delta_B}_{S^{p_1}}\frac{1}{\beta_1^2}\left(x_1^{1,d1} - x_1^1\right)$. Price p_2 is also distorted. The market maker's best estimate of w prior to trading in $t = 2$ is $\phi^{\delta_B}_{S^{p_1}}\left(S_2^{p_1,d1} - S_2^{p_1}\right)$ and after observing X_2^{d1}, $p_2^{w,d1} = \phi^{\delta_B}_{S^{p_1}}\frac{1}{\beta_1^2}\left(x_1^{1,d1} - x_1^1\right) + \lambda_2\left(x_2^{1,d1} + \beta_2^2 w - \beta_2^2\phi^{\delta_B}_{S^{p_1}}\frac{1}{\beta_1^2}\left(x_1^{1,d1} - x_1^1\right) + u_2\right)$. Since $\beta_2^1 = \frac{1}{2\lambda_2}\left(1 - \lambda_2\beta_2^2\right)\phi^w_{S_2^{1,w}}$, $p_2^{w,d1} = \lambda_2\left(x_2^{1,d1} + \beta_2^2 w + u_2\right) + 2\lambda_2\frac{\beta_2^1}{\beta_1^2}\frac{\phi^{\delta_B}_{S_2^{p_1}}}{\phi^w_{S_2^{1,w}}}\left(x_1^{1,d1} - x_1^1\right)$.

Trader 1's optimal trading rule in $t = 2$ after deviation in $t = 1$ results from the adjusted maximization problem $\max_{x_2^{1,d1}} E[x_2^{1,d1}\left(w - p_2^{w,d1}\right)|S_2^{1,w}]$. It is given by $x_2^{1,d1,*} = \beta_2^1 S_2^{1,w} - \gamma_2^1\left(x_1^{1,d1} - x_1^1\right)$, where $\gamma_2^1 := \frac{\beta_2^1}{\beta_1^2}\frac{\phi^{\delta_B}_{S_2^{p_1}}}{\phi^w_{S_2^{1,w}}}$, if the second order condition $\lambda_2 > 0$ is satisfied.

Trader 1's value function $V_2^1(x_1^{1,d1}) = x_2^{1,d1,*}E[w-p_2^w|S_2^{1,w}]$. After replacing $x_2^{1,d1,*}$ with $\beta_2^1 S_2^{1,w} - \gamma_2^1\left(x_1^{1,d1} - x_1^1\right)$ and noting that $(1-\lambda_2\beta_2^2) = 2\lambda_2\beta_2^1$ it simplifies to $V_2^1(x_1^{1,d1}) = \psi^1\left(x_1^{1,d1} - x_1^1\right)^2 - \tau^1 S_2^{1,w}\left(x_1^{1,d1} - x_1^1\right) + \kappa^1\left(S_2^{1,w}\right)^2$, with $\psi^1 = \lambda_2\left(\gamma_2^1\right)^2$, $\tau^1 = 2\lambda_2\beta_2^1\gamma_2^1$, $\kappa^1 = \lambda_2\left(\beta_2^1\right)^2$. In $t=1$, trader 1 forms expectations $E[V_2^2(x_2^{2,d2})|\delta_B]$ of the value function in $t=2$. $S_2^{1,w}$ is random in $t=1$. $E[S_2^{1,w}|S_1^1] = -\left(\phi^{\delta_B}_{S_2^{p_1}} + \frac{1}{\beta_1^2}\phi^{u_1}_{S^{p_1}}\right)\frac{\beta_1^1}{\beta_1^2}E[\varepsilon^1|S_1^1] = -\frac{\beta_1^1}{\beta_2^1}\gamma_2^1 E[\varepsilon^1|S_1^1] = -\frac{1}{\beta_2^1}\gamma_2^1\left(1-\phi^{\delta_A}_{S_1^1}\right)x_1^1$ since $\phi^{\delta_B}_{S^{p_1}} + \frac{1}{\beta_1^2}\phi^{u_1}_{S^{p_1}} = \frac{\phi^{\delta_B}_{S_2^{p_1}}}{\phi^{w}_{S_2^{1,w}}}$.

Trader 1's optimization problem in $t=1$ is thus $\max_{x_1^{1,d1}} E[x_1^{1,d1}\left(v-p_1^{d1}\right) + V_2^1(x_1^{1,d1})|S_1^1]$, where $p_1^{d1} = \lambda_1 X_1^{d1} = \lambda_1\left(x_1^{1,d1} + \beta_1^2 S_1^2 + u_1\right)$. Since S_1^1 is orthogonal to S_1^2 the first order condition is $E[\delta_A|S_1^1] - 2\lambda_1 x_1^{1,d1} + 2\psi^1\left(x_1^{1,d1} - x_1^1\right) - \tau^1 E[S_2^{1,w}|S_1^1] = 0$. Therefore, $x_1^{1,d1,*} = \frac{1}{2\left(\lambda_1 - \lambda_2\left(\gamma_2^1\right)^2\right)}(\phi^{\delta_A}_{S_1^1} + 2\lambda_2\left(\gamma_2^1\right)^2\beta_1^1)S_1^1$. The second order condition is $\lambda_1 > \lambda_2\left(\gamma_2^1\right)^2$. In Equilibrium $\beta_1^1 = \frac{1}{2\left(\lambda_1 - \lambda_2\left(\gamma_2^1\right)^2\phi^{\varepsilon^1}_{S_1^1}\right)}\phi^{\delta_A}_{S_1^1}$.

Trader 2's best response.

Other market participants' misperception in $t=2$.

Deviation from $x_1^2(S_1^2) = \beta_1^2 S_1^2$ to $x_1^{2,d2}(S_1^2)$ distorts the price signal by $S_2^{p_1,d2} - S_2^{p_1} = \frac{1}{\beta_1^2}\left(x_1^{2,d2} - x_1^2\right)$. Trader 1's signal prior to trading in $t=2$ is not $w + \frac{1}{\beta_1^2}\vartheta_1$ but $w - \phi^{\delta_B}_{S^{p_1}}\left(S_2^{p_1,d2} - S^{p_1}\right) + \frac{1}{\beta_1^2}\left(\vartheta_1 - \phi^{u_1}_{S^{p_1}}\left(S_2^{p_1,d2} - S^{p_1}\right)\right)$. His market order is, therefore, $x_2^{1,d2} = \beta_2^1\left(w + \frac{1}{\beta_1^2}\vartheta_1\right) - \beta_2^1\left(\phi^{\delta_B}_{S^{p_1}} + \frac{1}{\beta_1^2}\phi^{u_1}_{S^{p_1}}\right)\frac{1}{\beta_1^2}\left(x_1^{2,d2} - x_1^2\right)$. Price p_2 is also distorted. The market maker's best estimate of w prior to trading in $t=2$ is $\phi^{\delta_B}_{S^{p_1}}\left(S_2^{p_1,d2} - S_2^{p_1}\right)$ and after observing X_2^{d2}, $p_2^{w,d2} = \phi^{\delta_B}_{S^{p_1}}\left(S_2^{p_1,d2} - S_2^{p_1}\right) + \lambda_2\left(x_2^{1,d2} + x_2^{2,d2} + u_2\right)$. Let $\gamma_2^2 := \frac{1}{2\lambda_2}[\phi^{\delta_B}_{S^{p_1}} - \lambda_2\beta_2^1\left(\phi^{\delta_B}_{S_2^{p_1}} + \frac{1}{\beta_1^2}\phi^{u_1}_{S_2^{p_1}}\right)]\frac{1}{\beta_1^2}$ then $p_2^{w,d2} = \lambda_2\left(\beta_2^1\left(w + \frac{1}{\beta_1^2}\vartheta_1\right) + x_2^{2,d2} + u_2\right) + 2\lambda_2\gamma_2^2\left(x_1^{2,d2} - x_1^2\right)$.

Trader 2's optimal trading rule in $t=2$ *after deviation in* $t=1$ is the result of $\max_{x_2^{2,d2}} E[x_2^{2,d2}\left(w - p_2^{w,d2}\right)|S_2^{2,w}]$. The optimal order size in $t=2$ is $x_2^{2,d2,*} = \beta_2^2 S_2^{2,w} - \gamma_2^2\left(x_1^{2,d2} - x_1^2\right)$, if the second order condition $\lambda_2 > 0$ is satisfied. Note that if we replace β_2^1 with $\frac{1}{2\lambda_2}\left(1-\lambda_2\beta_2^2\right)\phi^w_{S_2^{1,w}}$, γ_2^2 simplifies to $\frac{1}{2}\frac{1}{\beta_1^2}\phi^{\delta_B}_{S_2^{p_1}}\frac{3-\phi_w^{S_2^{1,w}}\phi^w_{S_2^{1,w}}}{4-\phi_w^{S_2^{1,w}}\phi^w_{S_2^{1,w}}}$.

Trader 2's value function $V_2^2(x_1^{2,d2}) = x_2^{2,d2,*}E[w - p_2^w|S_2^{2,w}]$. After replacing $x_2^{2,d2,*}$ with $\beta_2^2 S_2^{2,w} - \gamma_2^2\left(x_1^{2,d2} - x_1^2\right)$ and noting that $(1-\lambda_2\beta_2^1\phi_w^{S_2^{1,w}}) = 2\lambda_2\beta_2^2$ it simplifies to $V_2^2(x_1^{2,d2}) = \psi^2\left(x_1^{2,d2} - x_1^2\right)^2 - \tau^2 S_2^{2,w}\left(x_1^{2,d2} - x_1^2\right) + \kappa^2\left(S_2^{2,w}\right)^2$, with $\psi^2 = \lambda_2\left(\gamma_2^2\right)^2$, $\tau^2 = 2\lambda_2\beta_2^2\gamma_2^2$, $\kappa^2 = \lambda_2\left(\beta_2^2\right)^2$. In $t=1$, trader 2 forms expectations $E[V_2^2(x_2^{2,d2})|\delta_B]$ of the value function in $t = 2$. $S_2^{2,w} = w$ is random in $t = 1$. The expectation of $S_2^{2,w}$ is given by $E[S_2^{2,w}|S_1^2] = E[w|\delta_B] = \left(1 - \phi^{\delta_B}_{S_2^{p_1}}\right)\delta_B$.

Trader 2's optimization problem in $t = 1$ is thus $\max_{x_1^{2,d2}} E[x_1^{2,d2}\left(v - p_1^{d2}\right) + V_2^2(x_1^{2,d2})|S_1^2]$, where $p_1^{d2} = \lambda_1 X_1^{d2} = \lambda_1\left(\beta_1^1 S_1^1 + x_1^{2,d2} + u_1\right)$. Since S_1^2 is orthogonal to S_1^1 the first order condition reduces to
$x_1^{2,d2,*} = \frac{1}{2\left(\lambda_1 - \lambda_2\left(\gamma_2^2\right)^2\right)}\left(\left(1 - 2\lambda_2\beta_2^2\gamma_2^2\left(1 - \phi^{\delta_B}_{S_2^{p_1}}\right)\right) - 2\lambda_2\left(\gamma_2^2\right)^2\beta_1^2\right)S_1^2$. The second order condition is $\lambda_1 > \lambda_2\left(\gamma_2^2\right)^2$.

Perfect Bayesian Nash Equilibrium is given by a fixed point in $(\beta_1^{1,*}, \beta_1^{2,*})$.

$$\beta_1^{1,*} = \frac{1}{2\left(\lambda_1 - \lambda_2\left(\gamma_2^1\right)^2\phi^{\varepsilon^1}_{S_1^1}\right)}\phi^{\delta_A}_{S_1^1}$$

$$\beta_1^{2,*} = \frac{1}{2\lambda_1}\left(1 - 2\lambda_2\beta_2^2\gamma_2^2\left(1 - \phi^{\delta_B}_{S_2^{p_1}}\right)\right),$$

where

$$\lambda_1 = \frac{\beta_1^{1,*}Var[\delta_A] + \beta_1^{2,*}Var[\delta_B]}{Var[\beta_1^{1,*}(\delta_A + \varepsilon^1) + \beta_1^{2,*}(\delta_B) + u_2]}$$

with

$$\beta_2^1 = \frac{1}{2\lambda_2}\frac{1}{2}\frac{1}{1 - \frac{1}{4}\phi_w^{S_2^{1,w}}\phi^w_{S_2^{1,w}}}\phi^w_{S_2^{1,w}} \qquad \gamma_2^1 := \frac{\beta_2^1}{\beta_1^2}\frac{\phi^{\delta_B}_{S_2^{p_1}}}{\phi^w_{S_2^{1,w}}}$$

$$\beta_2^2 = \frac{1}{2\lambda_2}\frac{1 - \frac{1}{2}\phi_w^{S_2^{1,w}}\phi^w_{S_2^{1,w}}}{1 - \frac{1}{4}\phi_w^{S_2^{1,w}}\phi^w_{S_2^{1,w}}} \qquad \gamma_2^2 := \frac{1}{2\lambda_2}\frac{1}{\beta_1^2}\phi^{\delta_B}_{S_2^{p_1}} - \frac{1}{2}\frac{\beta_2^1}{\beta_1^2}\frac{\phi^{\delta_B}_{S_2^{p_1}}}{\phi^w_{S_2^{1,w}}}$$

$$\lambda_2 = \{\frac{1}{2}\frac{Var[\delta_B|S_2^{p_1}]}{Var[u_2]}\left((b_2^1 + b_2^2) - \frac{1}{2}\left(b_2^1 + b_2^2\right)^2\right) + \frac{1}{2}\frac{Cov[\delta_B, u_1|S_2^{p_1}]}{Var[u_2]}\left(\frac{b_2^1}{\beta_1^2}\left(1 - (b_2^1 + b_2^2)\right)\right)$$
$$- \frac{1}{4}\frac{Var[u_1|S_2^{p_1}]}{Var[u_2]}\left(\frac{b_2^1}{\beta_1^2}\right)\}^{\frac{1}{2}},$$

where $b_2^i := 2\lambda_2\beta_2^i$ if the second order conditions

$\lambda_2 > \lambda_1 \max\{\left[\frac{b_2^1}{b_1^2}\frac{\phi^{\delta_B}_{S_2^{p_1}}}{\phi^w_{S_2^{1,w}}}\right]^2, \left[\frac{1}{b_1^2}\phi^{\delta_B}_{S_2^{p_1}} - \frac{1}{2}\frac{b_2^1}{b_1^2}\frac{\phi^{\delta_B}_{S_2^{p_1}}}{\phi^w_{S_2^{1,w}}}\right]^2\}$, $\lambda_2 > 0$ are satisfied. ■

A.2 Proof of Proposition 2

For the market maker as well as for trader 2 the p_1-price signal is $S_2^{p_1} = \delta_B + \frac{\beta_1^1}{\beta_1^2}\varepsilon^1 + \frac{1}{\beta_1^2}u_1$. Trader 1 can infer ε^1 and thus his price signal is more precise. Trader 1's informational advantage $\frac{\beta_1^1}{\beta_1^2}\varepsilon^1$ increases in β_1^1 and decreases in β_1^2. ■

A.3 Proof of Proposition 3

Speculative Trading

Trader 1 expects to trade $\beta_2^1 E[S_2^{1,w}|S_1^1]$ in $t = 2$.
Since $E[S_2^{1,w}|S_1^1] = -\left(\phi^{\delta_B}_{S_2^{p_1}} + \frac{1}{\beta_1^2}\phi^{u_1}_{S_2^{p_1}}\right)\frac{\beta_1^1}{\beta_1^2}\phi^{\varepsilon^1}_{S_1^1}S_1^1 = -\frac{\beta_1^1}{\beta_2^1}\gamma_2^1\phi^{\varepsilon^1}_{S_1^1}S_1^1$ and β_1^1, $\beta_2^1 > 0$, trader 1 expects to sell (buy) stocks in $t = 2$ if he buys (sells) stocks in $t = 1$.

Manipulative Trading

Trader 1 trades excessively for manipulative reasons if $\beta_1^1 > \beta_1^{1,\text{myopic}}$ (given the strategies of the other market participants).
$\beta_1^1 = \frac{1}{2\left(\lambda_1 - \lambda_2\left(\gamma_2^1\right)^2 \phi_{S_1^1}^{\varepsilon^1}\right)} \phi_{S_1^1}^{\delta_A}$ whereas $\beta_1^{1,\text{myopic}} = \frac{1}{2\lambda_1} \phi_{S_1^1}^{\delta_A}$. Thus manipulative trading is given by $\frac{\lambda_2\left(\gamma_2^1\right)^2 \phi_{S_1^1}^{\varepsilon^1}}{2\lambda_1\left(\lambda_1 - \lambda_2\left(\gamma_2^1\right)^2 \phi_{S_1^1}^{\varepsilon^1}\right)} \phi_{S_1^1}^{\delta_A} S_1^1$. The second order condition requires that $\lambda_1 > \lambda_2\left(\gamma_2^1\right)^2 \phi_{S_1^1}^{\varepsilon^1}$.

Note that for $Var[\varepsilon^1] = 0$, $\phi_{S_1^1}^{\varepsilon^1} = 0$ neither speculative nor manipulative trading will occur.

the inequality to $1 > \overline{b}_1^2 - \widetilde{b}_1^2$. This is always true.

Prior to trading in $t = 2$

If $\widetilde{\beta}_1^2 > \overline{\beta}_1^2$ then $\widetilde{S}^{p_1} = \delta_B + \frac{1}{\widetilde{\beta}_1^2} u_1$ is more informative than $\overline{S}^{p_1} = \delta_B + \frac{1}{\overline{\beta}_1^2} u_1$, even if $Var[\varepsilon^1] = 0$.

In the (β_1^2, λ_1)-space the equilibrium is determined by the intersection of $\lambda_1 = \frac{1/2}{\beta_1^2 + \frac{3}{4}\frac{1}{\beta_1^2}\phi^{\delta_B}_{S_2^{p_1}}\sqrt{(1-\phi^{\delta_B}_{S_2^{p_1}})\frac{Var[u_2]}{Var[\delta_B]}}}$ (1) with $\widetilde{\lambda}_1 = \frac{\widetilde{\beta}_1^2 Var[\delta_B]}{\left(\widetilde{\beta}_1^2\right)^2 Var[\delta_B] + Var[u_1]}$ (2) in the case where no information leaks and with $\overline{\lambda}_1 = \frac{\frac{1}{2\overline{\lambda}_1}Var[\delta_A] + \overline{\beta}_1^2 Var[\delta_B]}{\left(\frac{1}{2\overline{\lambda}_1}\right)^2 + \left(\overline{\beta}_1^2\right)^2 Var[\delta_B] + Var[u_1]}$ (3) in the case of information leakage. (3) can be simplified to
$\overline{\lambda}_1 = \frac{\overline{\beta}_1^2 Var[\delta_B] + \sqrt{\left(\overline{\beta}_1^2\right)^2 Var[\delta_B]^2 + \left(\overline{\beta}_1^2\right)^2 Var[\delta_B]Var[\delta_A] + Var[u_1]Var[\delta_A]}}{2\{\left(\overline{\beta}_1^2\right)^2 Var[\delta_B] + Var[u_1]\}}$. Note that we can restrict our attention to the positive root only because of the second order condition.

Claim 1: $\overline{\lambda}_1(\overline{\beta}_1^2) > \widetilde{\lambda}_1(\widetilde{\beta}_1^2)$ for all $\overline{\beta}_1^2 = \widetilde{\beta}_1^2$ follows immediately

Claim 2: $\lambda_1(\beta_1^2) = \frac{1/2}{\beta_1^2 + \frac{3}{4}\frac{1}{\beta_1^2}\phi^{\delta_B}_{S_2^{p_1}}\sqrt{(1-\phi^{\delta_B}_{S_2^{p_1}})\frac{Var[u_2]}{Var[\delta_B]}}}$ (1) is strictly decreasing in β_1^2 as long as $Var[u_1] > \frac{6}{25}\sqrt{\frac{2}{5}Var[\delta_B]Var[u_2]}$.

Its derivative is negative if the denominators' derivative is positive. The denominator can be rewritten as
$\beta_1^2 + \frac{3}{4}\left(\beta_1^2\right)^{-2}\left(Var[\delta_B] + \left(\beta_1^2\right)^{-2}Var[u_1]\right)^{-1.5}\left(Var[\delta_B]Var[u_1]Var[u_2]\right)^{0.5}$. Its derivative w.r.t. β_1^2 is $1 + \frac{3}{4}\frac{\sqrt{Var[\delta_B]Var[u_2]}}{Var[u_1]}\left(1 - \phi^{\delta_B}_{S_2^{p_1}}\right)^{1.5}\left(2 - 3\left(1 - \phi^{\delta_B}_{S_2^{p_1}}\right)\right)$. The global minimum for $\left(1 - \phi^{\delta_B}_{S_2^{p_1}}\right)^{1.5}\left(2 - 3\left(1 - \phi^{\delta_B}_{S_2^{p_1}}\right)\right)$ at $\phi^{\delta_B}_{S_2^{p_1}} = \frac{2}{5}$ is $-\frac{2}{5}\sqrt{\frac{2}{5}\frac{4}{5}}$. From this it follows immediately that for $Var[u_1] > \frac{6}{25}\sqrt{\frac{2}{5}Var[\delta_B]Var[u_2]}$, $\frac{\partial\lambda_1}{\partial\beta_2^1} < 0$.

Claim 3: $\widetilde{\lambda}_1(\widetilde{\beta}_1^2)$ is weakly increasing in $\widetilde{\beta}_1^2$, i.e. $\frac{\partial\widetilde{\lambda}_1}{\partial\widetilde{\beta}_2^1} \geq 0$.

$\frac{\partial\widetilde{\lambda}_1}{\partial\widetilde{\beta}_1^2} = \frac{Var[\delta_B]\left(Var[u_1] - \left(\widetilde{\beta}_1^2\right)^2 Var[\delta_B]\right)}{\left(\left(\widetilde{\beta}_1^2\right)^2 Var[\delta_B] + Var[u_1]\right)^2}$. $\frac{\partial\widetilde{\lambda}_1}{\partial\widetilde{\beta}_2^1} \geq 0$ if $Var[u_1] \geq \left(\widetilde{\beta}_1^2\right)^2 Var[\delta_B]$. Replacing $\widetilde{\beta}_1^2$ with $\frac{1}{2}\sqrt{\frac{Var[u_1]}{Var[\delta_B]\frac{1}{2}\widetilde{b}_1^2\left(1 - \frac{1}{2}\widetilde{b}_1^2\right)}}\widetilde{b}_1^2$ the condition simplifies to $\widetilde{b}_2^1 \leq 1$. This is always the case in equilibrium.

From Claim 1 to 3 it follows that $\overline{\lambda}_1 > \widetilde{\lambda}_1$ and $\overline{\beta}_1^2 < \widetilde{\beta}_1^2$ in the corresponding equilibria.

After trading in $t = 2$

The continuation game in $t = 2$ corresponds to a static Kyle (1985) model with a risky asset w. Since for lower β_1^2, the variance $Var[w] = Var[\delta_B | S_2^{p_1}]$ is higher, the price process $\{p_1, p_2\}$ reveals less information. ■

A.5 Proof of Proposition 5

In any mixed strategy equilibrium player 1 has to be indifferent between any x_1^1, i.e. $\lambda_1 = \lambda_2 \left(\gamma_2^1\right)^2$. In addition the second order condition of trader 2, $\lambda_1 \geq \lambda_2 \left(\gamma_2^2\right)^2$ must hold. Thus a necessary condition for a mixed strategy equilibrium is

$$\gamma_2^2 \leq \gamma_2^1$$

$$\frac{1}{2}\frac{1}{\beta_1^2}\phi^{\delta_B}_{S_2^{p_1}} \frac{3-\phi^{w}_{S_2^{1,w}}\phi_w^{S_2^{1,w}}}{4-\phi^{w}_{S_2^{1,w}}\phi_w^{S_2^{1,w}}} \leq \frac{1}{\beta_1^2}\phi^{\delta_B}_{S_2^{p_1}} \frac{1}{2\lambda_2} \frac{\frac{1}{2}}{1-\frac{1}{4}\phi^{w}_{S_2^{1,w}}\phi_w^{S_2^{1,w}}}$$

$$\lambda_2 \leq \frac{2}{3-\phi^{w}_{S_2^{1,w}}\phi_w^{S_2^{1,w}}} < 1,$$

$$\text{where } \phi^{w}_{S_2^{1,w}}\phi_w^{S_2^{1,w}} = \frac{\left(\beta_1^1\right)^2 Var[\varepsilon^1]+\left(\gamma_1^1\right)^2 Var[\zeta^1]}{\left(\beta_1^1\right)^2 Var[\varepsilon^1]+Var[u_1]+\left(\gamma_1^1\right)^2 Var[\zeta^1]} \frac{Var[\delta_B]}{Var[\delta_B]+\left(\frac{1}{\beta_1^2}\right)^2 Var[u_1]} < 1.$$

Since λ_2 is strictly decreasing in $Var[u_2]$ with $Var[u_2] \to 0 \Rightarrow \lambda_2 \to \infty$, for $Var[u_2]$ strictly smaller than the constant $C^*_{Var[u_2]}$ there exists no mixed strategy equilibrium. ■

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