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Essays on Economic Geography

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Biographical note

Jorge Manuel Saraiva Pereira was born on 22 February 1995. He has ever lived in his hometown of Porto, where he got both his Bachelor's in Economics (2016) and Master's in Economics (2018) at the School of Economics and Management (FEP) of the University of Porto.

His Master's dissertation "Volunteering - Incentives and Social Reputation" was supervised by Professors João Correia-da-Silva and Sofia B. S. D. Castro.

He enrolled on the Doctoral programme in Economics at FEP in 2018, also under the supervision of Professors João Correia-da-Silva and Sofia B. S. D. Castro.

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During his PhD path, he was awarded a scholarship grant from FCT and presented his research at some international conferences in the field of Economic Geography.

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Abstract

This thesis is composed of four essays and addresses two topics in Economic Geography. The first topic is the relationship between education choices and the spatial distribution of economic activity, which is explored in the first three essays. The second topic is the effect of a unilateral withdrawal from an economic union on the spatial distribution of economic activity and social welfare, which is studied in the fourth essay.

In the first essay, we study how regional asymmetries in firms' productivity affect the spatial distribution of economic activity and social welfare. We introduce an exogenous regional asymmetry in the two-region quasi-linear log utility footloose entrepreneur model to explore how the regional framework alone can affect firms' productivity that would otherwise be homogeneous across regions. We find that the agglomeration of entrepreneurs in the most productive region is stable as long as transportation costs are not too high. We also find that while the concentration of most entrepreneurs in the least productive region may be stable, the concentration of most entrepreneurs in the most productive region is always stable when it occurs. Finally, we conclude that the spatial distribution of an economy moves closer to the one that ensures optimal social welfare as the preference for variety increases and that, in general, society would be better off if the most productive region did not concentrate as many entrepreneurs.

In the second essay, we study how entrepreneurs make their education decisions when they live in an economy with multiple regions. We introduce a simple education mechanism, which acts as a proxy for productivity level, in the two-region Cobb-Douglas utility footloose entrepreneur model to explore how agents decide their optimal productivity level in three different types of societies – a regulated economy, an unionised economy, and a decentralised economy. We find that the highest productivity level occurs in regulated economies and the lowest in unionised economies. We also find that education yields a positive externality for the whole economy due to price decreases. Finally, we conclude that individual and average education levels are strategic substitutes. Hence, agents have the incentive to free-ride as society's average education level rises.

In the third essay, we study how entrepreneurs make spatial and educational decisions endogenously. We implement the two-region quasi-linear log utility footloose entrepreneur model in an overlapping generations model to explore how forward-looking agents decide where to live and whether to study. The agents may follow one of four different life paths, and we find that qualified and unqualified workers can become segregated between regions. We conclude that when the productivity gains from education are sufficiently high, everyone chooses to study. However, we also conclude that even for relatively low productivity gains, it may be optimal to study due to price decreases. Finally, we also find that the equilibrium seems invariant to changes in economic conditions, except for changes in productivity gains.

In the last essay, we study how the unilateral withdrawal of a region from an economic union affects the spatial distribution of economic activity and social welfare. We explore the three-region quasi-linear log utility footloose entrepreneur model under the assumption that this dissent can be expressed as a higher transportation cost between the leaving party and the remaining union members. We find that a spatial distribution in which entrepreneurs are equally shared between the three regions is no longer possible and that asymmetric equilibria – in which the dissident region has the lowest share of entrepreneurs – arise. We also find that it is not stable for entrepreneurs to distribute themselves only between the remaining regions in the union. We conclude that the leaving region's share of entrepreneurs is higher, the lower the differential in transportation costs is, and the higher the mobility of workers between regions is. Finally, we also conclude that, from a global social welfare point of view, the economy as a whole attains its maximum well-being when most entrepreneurs do not live in the dissident region.

Keywords Economic Geography, Education Economics, Agglomeration, Productivity, Education, Individual Decisions, Social Decisions, Social Welfare, Brexit, Economic Union
JEL Classification C62, D70, D80, F20, F53, I21, I26, J24, R10

Resumo

Esta tese é composta por quatro ensaios e aborda dois tópicos da área da Economia Geográfica. O primeiro tópico, que é explorado nos três primeiros ensaios, é a relação entre escolhas de educação e a distribuição espacial da atividade económica. O segundo tópico, que é explorado no quarto ensaio, é o efeito da decisão unilateral de saída de uma união económica, por parte de um estado-membro, sobre a distribuição espacial da atividade económica e o bem-estar social.

No primeiro ensaio, estudamos de que forma é que assimetrias regionais na produtividade das empresas afetam a distribuição espacial da atividade económica e o bem-estar social. Introduzimos uma assimetria regional exógena no modelo *footloose entrepreneur* com duas regiões e utilidade logarítmica quase-linear para explorar como é que o enquadramento regional pode afetar a produtividade das empresas, que de outra forma seria homogénea entre regiões. Descobrimos que a aglomeração de empresas na região mais eficiente é estável desde que os custos de transporte não sejam demasiado elevados. Também descobrimos que, embora a concentração da maioria das empresas na região menos eficiente possa ser estável, a concentração da maioria das empresas na região mais eficiente é estável sempre que ocorre. Finalmente, concluímos que a distribuição espacial de uma economia aproxima-se daquela que garante que o bem-estar social é ótimo à medida que a preferência dos consumidores por variedade aumenta e que, em geral, a sociedade estaria melhor se a região mais eficiente não concentrasse tantas empresas.

No segundo ensaio, estudamos como é que os empresários tomam as suas decisões de educação quando vivem numa economia com várias regiões. Introduzimos um mecanismo de educação simples, que funciona como *proxy* do nível de produtividade, no modelo *footloose entrepreneur* com duas regiões e utilidade Cobb-Douglas para explorar como é que os agentes decidem o seu nível ótimo de educação em três diferentes tipos de sociedades – uma economia regulada, uma economia sindicalizada e uma economia descentralizada. Descobrimos que o maior nível de produtividade acontece na economia regulada e que o menor acontece na economia sindicalizada. Também descobrimos que a educação gera externalidades positivas

para toda a sociedade devido à descida dos preços. Finalmente, concluímos que os níveis individuais e médios de educação são substitutos estratégicos. Assim, os agentes têm incentivos para adotar comportamentos de *free-riding* à medida que a educação média aumenta.

No terceiro ensaio, estudamos como é que os empresários tomam as suas decisões de educação e migração de forma endógena. Implementamos o modelo *footloose entrepreneur* com duas regiões e utilidade logarítmica quase-linear no modelo de gerações sobrepostas para explorar como é que agentes com expectativas racionais decidem onde viver e se devem estudar. Os agentes podem seguir um de quatro diferentes caminhos de vida e descobrimos que os empresários qualificados e não qualificados podem acabar segregados entre as regiões. Concluímos que quando os ganhos de produtividade gerados pela educação são suficientemente grandes, todos os agentes decidem estudar. No entanto, também concluímos que, mesmo para ganhos de produtividade relativamente baixos, pode ser ótimo estudar devido à descida dos preços. Finalmente, também descobrimos que o equilíbrio parece ser invariante a mudanças nas condições económicas, com exceção de alterações nos ganhos de produtividade.

No último ensaio, estudamos de que forma é que a decisão unilateral de saída de uma união económica, por parte de um estado-membro, afeta a distribuição espacial da atividade económica e o bem-estar social. Exploramos o modelo *footloose entrepreneur* com três regiões e utilidade logarítmica quase-linear sob a hipótese de que esta dissensão pode ser expressa através de um maior custo de transporte entre a região que abandonou a união e os restantes membros. Descobrimos que a distribuição espacial na qual os empresários estão igualmente divididos entre as regiões deixa de ser possível e que equilíbrios assimétricos – nos quais a região dissidente tem a menor percentagem de empresários – podem surgir. Também descobrimos que não é estável os empresários dividirem-se apenas entre as regiões que permanecem na união económica. Concluímos que a percentagem de empresários na região dissidente é tanto maior quanto menor for o diferencial de custos de transporte e quanto maior for a mobilidade dos trabalhadores entre regiões. Finalmente, também concluímos que a economia alcança o maior bem-estar social global possível quando a maioria dos empresários não vive na região dissidente.

Palavras-chave Economia Geográfica, Economia da Educação, Aglomeração, Produtividade, Educação, Decisões Individuais, Decisões Sociais, Bem-estar Social, Brexit, União Económica

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Chapter 1

Introduction

This thesis aims to contribute to the Economic Geography literature by proposing novel approaches and providing new insights regarding two topics – education and economic unions.

We explore these topics from two main perspectives. On the one hand, we study how the economy reacts to changes in individual agents' characteristics. On the other hand, we explore how institutional and political frameworks affect the economy.

From the agents' point of view, we address the education problem by allowing the agents to study and become more productive, hence pulling apart from a static economy with homogeneous and constant productivity. This perspective is developed essentially in chapters 3 and 4.

From the regions' point of view, we address the problem of a shift in institutional and political frameworks between regions. This perspective is developed throughout all essays, with particular relevance in chapters 2, 3, and 5.

As Gaspar (2018) explores, theoretical Economic Geography literature is still constrained by some of the initial assumptions made by Krugman (1991b). Gaspar points out some research avenues that should help "breaking through the strait-jacket" and advocates that exploring those avenues is vital to further developing the current Economic Geography research. Therefore, our study is aligned with the ideas explored by Gaspar, particularly by introducing knowledge linkages, the subsequent heterogeneity of the agents' productivity, and working in multi-regional frameworks.

In chapter 2, "On regional productivity asymmetries and agglomeration", we first explore the relationship between education and the spatial distribution of economic activity. We study how regional asymmetries in firms' productivity affect the spatial distribution of industry and agents' welfare. We use the two-region Pflüger (2004) footloose entrepreneur model and introduce the novelty that each region has a different productivity level. This allows us to introduce some degree of heterogeneity in the economy, as our model features regions with varying levels of efficiency.

We find that it is an equilibrium for all the agents to be agglomerated in any region but not to be evenly distributed between them. Moreover, it may be possible for the least productive region to concentrate most entrepreneurs as long as the productivity gap between the regions is not too large. When the agglomeration of all workers in the least productive region is stable, so is the agglomeration in the most productive one. Agglomeration in the most productive region is unstable only if transportation costs are too high. The concentration of most agents in the least productive region may be stable, but the concentration of most workers in the most productive region is always stable when it occurs. We extend our approach to include a third region and find that regions with an intermediate productivity level tend to end up depleted of industry. Finally, we conclude that an economy moves closer to optimal social welfare as the preference for variety increases and that, in general, society would be better off if the most productive region did not concentrate as many mobile workers.

In chapter 3, "On optimal education choices", we continue to explore the relationship between education and the spatial distribution of economic activity. We study how agents make their education decisions when they live in an economy with multiple regions and how these decisions depend on the political context. We extend the two-region Forslid and Ottaviano (2003) footloose entrepreneur model by introducing a novel and simple education mechanism, which acts as a proxy for productivity. This allows us to study how different types of societies – a regulated economy, a highly unionised economy and a totally decentralised economy – make decisions regarding their optimal productivity level.

Solving the maximisation problem that defines optimal education decisions, we conclude that the highest optimal productivity level occurs in the regulated economy, while the lowest occurs in the unionised one. We find how economic conditions and the spatial agglomeration of economic activity drive education decisions. We also conclude that education has positive externalities for all society due to the decrease in prices. Further, given the economic conditions, we can estimate this effect, which is bigger the more society spends on industrial goods and the less population prefers variety. Finally, we also find that individual and average education levels are strategic substitutes, which can induce free-riding behaviour.

In chapter 4, "On endogenous education and agglomeration dynamics", we conclude our exploration of the relationship between education and the spatial distribution of economic activity. We study how education and spatial decisions are made endogenously within a setup that mixes the two-region Pflüger (2004) footloose entrepreneur model with an overlapping generations framework. This novel conceptualisation represents a breakthrough in analysing spatial issues in a way that classical Economic Geography models usually do not allow. We combine elements from chapters 2 – the concept of the regional number of varieties and the formulation of wages and price indices with different productivity levels – and 3 – the broad concept of optimal education decisions – to develop a model that features forward-looking agents that decide where to live and whether to study.

The construction of the model is such that any agent has four possible life paths that they may follow, and we find that qualified and unqualified workers can become segregated between regions. Moreover, when education induces elevated productivity increases, everyone wants to become qualified. However, we also find that even low productivity gains may be enough for agents to qualify due to the specific life paths that education offers. Finally, we conclude that the equilibrium does not seem to be affected by economic conditions apart from productivity gains.

In chapter 5, "On the disentanglement of an economic union", we explore the second research line of this thesis – the study of economic unions, particularly their breakup. We study how the unilateral withdrawal of one member from an established economic union affects the spatial distribution of industry and the welfare of the agents. We use the three-region Pflüger (2004) footloose entrepreneur model, and our novelty is to allow one region to exit an established economic union and consider that the aftermath of this dissent can be expressed as an increase in the transportation costs between the leaving party and the remaining members of the economic union.

We find that an even distribution of mobile workers amongst the three regions is no longer possible and that we may have totally asymmetric spatial distributions – in which the dissident region has the lowest share of workers – when transportation costs are high enough. We also find that configurations with mobile workers distributed only between the remaining members of the union are not stable. Using a numerical simulation, we conclude that the lower the difference between transportation costs and the higher the mobility of industrial workers is, the more mobile workers live in the leaving region. Finally, we also conclude that it is never socially optimal that the leaving region concentrates more than one-third of the industry.

In chapter 6, we conclude by making some final remarks regarding our major conclusions, address the use of the Forslid and Ottaviano (2003) model and the Pflüger (2004) model as a baseline, and hypothesise about possible exciting paths for future work.

Chapter 2

On regional productivity asymmetries and agglomeration

2.1 Introduction

Imagine that an entrepreneur wants to set up a firm and may choose where to locate it. Suppose that it is common knowledge that, in location A, the transportation network is very efficient, while in location B, some deficiencies make it suboptimal. Then, even though the internal structure of the firm is the same regardless of where it is located, it will operate more efficiently in location A, as the regional framework favours it. Therefore, this essay deals with the question of, given the choice, whether an entrepreneur should ever choose to locate a firm in any other location than the most efficient one.

In particular, this motivates us to study how regional asymmetries in firms' productivity affect the spatial distribution of economic activity and the agents' welfare. More precisely, we are interested in studying how the gap in productivity between regions generates imbalances in the spatial distribution of industry.

To achieve our goal, we use a quasi-linear log utility footloose entrepreneur model with two regions and consider that each region has a specific productivity common to all of its firms. Thus, our novelty is to allow for an exogenous regional asymmetry in productivity.

Note that, while we are introducing some degree of heterogeneity, we are not considering firm heterogeneity *per se*. Our conceptualisation assumes that all the firms in the economy are homogeneous. In particular, any firm – regardless of its location – has the same input requirement of skilled workers. Therefore, regional heterogeneity emerges from the fact that regions have different frameworks – either economic, political, judicial or social, for example. It is the combination of all these factors that determine whether a region is more or less efficient. Hence, firms are able to produce more in the regions that offer them the best framework.

We study the migration dynamics under the regional productivity asymmetry and find that an even distribution of population between both regions is no longer an equilibrium but that interior equilibria may exist and even be multiple. We also find that it is always an equilibrium to agglomerate in any region.

Focusing on the stability of the former equilibria, we find that stability of agglomeration in the most productive region is more easily achieved and that stability of agglomeration in the least productive region always implies stability of agglomeration in the other. Moreover, we also conclude that interior equilibria in which most mobile workers live in the most productive region are always stable.

We extend our analysis to three regions using a numerical approach. We assume that each region has a different productivity level, and we conclude that regions do not want intermediate productivity levels since it generally implies that they always end up depleted of industry.

Finally, we explore the agents' welfare and find that immobile workers never achieve their maximum welfare, while mobile ones enjoy the highest well-being when agglomerating in the most productive region. Furthermore, we also find that the spatial distribution that ensures maximum social welfare is the one that endogenously occurs when the preference for variety is high.

When New Economic Geography started, with Krugman's seminal contributions (Krugman, 1991a, 1991b), the central focus was on agglomeration – what causes populations to move closer together or to go on separate ways. Krugman's core-periphery model was later revisited by Forslid and Ottaviano (2003) and Pflüger (2004), and Economic Geography as a whole was also the subject of Fujita et al. (1999), Baldwin et al. (2003), and Combes et al. (2008).

Unquestionably, these works have opened the path for the research of spatial aspects in economics, both in theoretical and empirical fields, but there is still room for deeper exploration.

Baldwin et al. (2003) state that the analytical complexity of the core-periphery model becomes a significant hurdle when we move away from its simplifying assumptions. However, as acknowledged by Gaspar et al. (2018), it is poignant that research moves in that direction and tries to break the "strait-jacket", namely by introducing heterogeneity in productivity and skills. We move in this direction by allowing firms' productivity in different regions to be different.

In particular, our interpretation that regional asymmetries may surge due to differences in transportation and communication networks is something that has already been addressed in the literature. Roller and Waverman (2001) investigate the relationship between telecommunications infrastructure and economic growth and find that the two are significantly correlated. The authors conclude that a better telecommunications infrastructure, particularly if its adoption is almost universal, leads to a higher GDP per capita. Hong et al. (2011) conclude that improvements in transportation infrastructure in China played a great role in regional development, thus linking better regional transportation infrastructure with higher economic performance.

While some contributions in the literature tackle the heterogeneity of firms, to the best of our knowledge, there is no contribution whose main focus is on region productivity asymmetries and their effect on migrations. Moreover, note that Baldwin and Okubo (2005) discuss that the usual assumption of identical firms is "neither necessary nor innocuous", pointing out that the sorting and selection effects that arise from considering heterogeneous firms qualitatively affect the migration decisions of the agents.

On the one hand, Demidova (2008), Okubo (2009), Okubo et al. (2010), Von Ehrlich and Seidel (2013), Pflüger and Südekum (2013), and Tabuchi et al. (2018) study firm heterogeneity following the stochastic approach developed by Melitz (2003), but they do not objectively explore regional asymmetries. On the other hand, Sidorov and Zhelobodko (2013) discuss that one of the aspects of Economic Geography that hinder a more widespread discussion in mainstream economics is the rigidity of some assumptions, particularly those of symmetry of the regions and homogeneity of the agents, but they tackle this issue by introducing regional agricultural asymmetries rather than productivity ones. Therefore, our work contributes to the literature by introducing regional productivity asymmetries, which is in line with Baldwin and Okubo (2005) suggestions.

Demidova (2008) considers an economy *à la Meltiz* with the particularity that the productivity distribution in the regions is different. However, there may exist some overlapping in the productivity levels of both regions. Therefore, our focus is more directed towards a clear productivity gap between the regions, such that no two firms in different regions can have the same productivity. Moreover, the contribution of Demidova does not explore migrations but only the effects of falling trade costs on welfare. Okubo et al. (2010) make an interesting contribution by considering an economy *à la Meltiz* within a footloose capital model. Therefore, this makes it possible for firms to sort themselves between the regions, creating a regional productivity gap. While their results are remarkable, mainly due to the endogenous dynamic, the regional disparity is a result, not a hypothesis. Thus, our approach adds to the literature by showing how regional productivity asymmetries affect not only the welfare but also the spatial distribution of industry.

Von Ehrlich and Seidel (2013) examine the relationship between firm heterogeneity and regional agglomeration using an economy *à la Meltiz* and conclude that introducing firm heterogeneity in the core-periphery model changes the role of technological progress, making it favour the agglomeration of industry. Tabuchi et al. (2018) achieve a similar result as they explore the relationship between technological progress and regional disparities and conclude that increased productivity may lead to the concentration of industry in some regions. Thus, even though we use a different approach, our main conclusion is similar. There is a strong tendency for agents to be located in the most productive region. Moreover, Tabuchi et al. state that it would be interesting to extend their work to account for endogenous technological progress, which the authors argue is "place specific". Hence, while we do not explicitly address the endogeneity of productivity, the regional setting that we consider is somewhat aligned with this concern from the authors.

2.2 The productivity problem

Our model is an extension of the footloose entrepreneur model with quasi-linear log utility (Gaspar et al., 2018; Pflüger, 2004). We extend the baseline model to accommodate for exogenous regional asymmetries in industrial firms' productivity. In other words, the input requirement of skilled labour is different between regions but equal within regions. We assume that firms do not incur any costs related to the regional productivity level.

Our objective is to study how asymmetric regional productivities affect the spatial distribution of industry and the agents' welfare.

2.2.1 Economic model

In this economy, there are L unskilled workers – equally divided between the two regions – that are immobile between regions, and H homogeneous skilled workers – H_i in region $i = \{1, 2\}$ – that are mobile between regions.

The preferences of all agents are defined by

$$U = \mu \ln M + A, \tag{2.1}$$

where $\mu \in (0, 1)$ is the expenditure share in the industrial good, A is the consumption of the agricultural good, and M is the consumption of the usual CES composite of differentiated varieties of the industrial good, defined by

$$M = \left[\int_{s \in S} d(s)^{\frac{\sigma-1}{\sigma}} \mathrm{d}s \right]^{\frac{\sigma}{\sigma-1}}, \tag{2.2}$$

where $d(s)$ is the consumption of variety s, S is the mass of varieties and $\sigma > 1$ is the constant elasticity of substitution between varieties.

Let $p_{ij}(s)$ represent the delivered price in region i of variety s produced in region j and $d_{ij}(s)$ its demand. Then, the regional price index associated with the composite good (2.2) in region i is

$$P_i = \left[\int_{s \in S} p_{ij}(s)^{1-\sigma} \mathrm{d}s \right]^{\frac{1}{1-\sigma}}. \tag{2.3}$$

Every agent in region i maximises their utility subject to the budget constraint, given by

$$P_i M + A = y_i,$$

where y_i represents the nominal income of the agent ($y_i = w_i$ if skilled and $y_i = 1$ otherwise), P_i is given in (2.3) and the price of the agricultural good is normalised to one. Thus, the demand functions are given by

$$d_{ij}(s) = \mu \frac{p_{ij}(s)^{-\sigma}}{P_i^{1-\sigma}}, \quad M = \frac{\mu}{P_i}, \quad A = y_i - \mu. \tag{2.4}$$

From (2.1) and (2.4) we derive the indirect utility function in region i, which is given by

$$V_i = y_i - \mu \ln P_i + \mu(\ln \mu - 1). \tag{2.5}$$

The production of the agricultural good uses one unit of unskilled labour per unit produced and has no transportation costs. Thus, $p_1^A = p_2^A = p^A$, which lead us to choose this good as *numeraire* ($p^A = 1$). Since the agricultural market is perfectly competitive, marginal cost pricing implies that the nominal wage of unskilled workers is the same everywhere and, in particular, equal to p^A. Hence, $w_i^L = p^A = 1$.

We assume that the non-full-specialisation (NFS) condition (Baldwin et al., 2003; Gaspar et al., 2018) holds[1], so we have

$$\lambda > \frac{\mu \frac{\sigma-1}{\sigma}}{\frac{1}{2} - \mu \frac{\sigma-1}{\sigma}},$$

where $\lambda = L/H$ represents the global immobility ratio.

[1] It is straightforward but cumbersome to show that the weighted average nominal wage of skilled workers is the same as in Gaspar et al. (2018) – $\bar{w} = \frac{\mu}{\sigma}(1+\lambda)$ –, and so is the NFS condition.

As for the production of the industrial good, both skilled and unskilled labour is used. In particular, each unit produced requires α/ϵ_i units of skilled labour in region i and β units of unskilled labour. Thus, the regional productivity asymmetry we introduce affects the input requirement of skilled labour by assuming that $\epsilon_i \neq \epsilon_j, \ \forall i \neq j$. Therefore, the production cost of an industrial firm in region i is

$$PC_i(x_i) = \frac{\alpha}{\epsilon_i} w_i + \beta x_i.$$

Hence, an industrial firm in region i that produces variety s maximises the profit function

$$\pi_i(s) = \sum_{j=1}^{2} d_{ij}(s) \left(H_j + L/2\right) \left[p_{ij}(s) - \tau_{ij}\beta\right] - \frac{\alpha}{\epsilon_i} w_i, \tag{2.6}$$

where $\tau \in (1, +\infty)$ represents the usual iceberg transportation cost between regions regarding the industrial good. Note that $\tau_{ij} = \tau$ whenever $i \neq j$ and $\tau_{ij} = 1$ otherwise.

Therefore, profit maximisation of (2.6) yields the optimal prices

$$p_{ij}(s) = \tau_{ij}\beta \frac{\sigma}{\sigma - 1}. \tag{2.7}$$

Considering that a firm produces one unit of industrial good by using α units of skilled labour in a region with a unitary productivity level and that it produces the same unit of industrial good by using α/ϵ_i units of skilled labour in a region i with a ϵ_i productivity level, we have that the number of industrial varieties produced in region i is $H_i/(\alpha/\epsilon_i)$.

Then, using (2.7) and the fact that the number of industrial varieties produced in region i is $H_i/(\alpha/\epsilon_i)$, the regional price index of the composite good (2.3) becomes

$$P_i = \frac{\beta\sigma}{\sigma - 1} \left[\frac{1}{\alpha} \sum_{j=1}^{2} \phi_{ij} \epsilon_j H_j\right]^{\frac{1}{1-\sigma}}, \tag{2.8}$$

where $\phi_{ij} \equiv \tau_{ij}^{1-\sigma} \in (0, 1]$ represents the freeness of trade (or the inverse of trade costs) between regions, regarding the industrial good. Note that $\phi_{ij} = \phi$ whenever $i \neq j$ and $\phi_{ij} = 1$ otherwise.

Given the monopolistic competition setup in the industrial market, the free entry condition implies zero profits in equilibrium. Using (2.4), (2.7), and (2.8) into $\pi_i(s) = 0$, the equilibrium wages that skilled workers earn are given by

$$w_i = \epsilon_i \frac{\mu}{\sigma} \sum_{j=1}^{2} \phi_{ij} \frac{H_j + L/2}{\sum_{m=1}^{2} \phi_{mj} \epsilon_m H_m}. \tag{2.9}$$

Note that w_i could be rewritten as $w_i = \epsilon_i \hat{w}_i$. This evidences two aspects. First, the wage in each region heavily depends on its productivity. Second, there is a baseline wage ($\hat{w}_i$) that depends on the conditions of the economy as a whole.

Thus, by defining the share of skilled workers in region 1 as $h_1 = h = H_1/H$, in region 2 as $h_2 = 1 - h = H_2/H$, and the global immobility ratio as $\lambda = L/H$ it is possible to express the nominal wage (2.9) as a function of h and $\boldsymbol{\epsilon} = (\epsilon_1, \epsilon_2)$, which yields

$$w_i(h, \boldsymbol{\epsilon}) = \epsilon_i \frac{\mu}{\sigma} \sum_{j=1}^{2} \phi_{ij} \frac{h_j + \lambda/2}{\sum_{m=1}^{2} \phi_{mj} \epsilon_m h_m}. \tag{2.10}$$

The regional price index (2.8) can also be rewritten as

$$P_i(h, \boldsymbol{\epsilon}) = \frac{\beta\sigma}{\sigma - 1} \left(\frac{H}{\alpha}\right)^{\frac{1}{1-\sigma}} \left[\sum_{j=1}^{2} \phi_{ij} \epsilon_j h_j\right]^{\frac{1}{1-\sigma}}. \tag{2.11}$$

Therefore, by replacing (2.10) and (2.11) in (2.5) the indirect utility of a skilled agent is now

$$V_i(h, \boldsymbol{\epsilon}) = \epsilon_i \frac{\mu}{\sigma} \sum_{j=1}^{2} \left[\phi_{ij} \frac{h_j + \lambda/2}{\sum_{m=1}^{2} \phi_{mj} \epsilon_m h_m}\right] + \frac{\mu}{\sigma - 1} \ln \left[\sum_{j=1}^{2} \phi_{ij} \epsilon_j h_j\right] + \eta,$$

where $\eta = \mu(\ln \mu - 1) + \frac{\mu}{\sigma-1} \ln \left[\frac{H}{\alpha}\right] - \mu \ln \left[\frac{\beta\sigma}{\sigma-1}\right]$ is a constant[2].

2.2.2 Migrations

In the long-run, agents choose to reside in the region that offers them the highest indirect utility. Agents' migration decisions are governed by the replicator dynamics (Sandholm, 2010; Taylor & Jonker, 1978), given by

$$\dot{h} = h \left[V_1(h, \boldsymbol{\epsilon}) - \bar{V}(h, \boldsymbol{\epsilon})\right] = h\,(1 - h)\left[V_1(h, \boldsymbol{\epsilon}) - V_2(h, \boldsymbol{\epsilon})\right],$$

where $\bar{V}(h, \boldsymbol{\epsilon}) = hV_1(h, \boldsymbol{\epsilon}) + (1 - h)V_2(h, \boldsymbol{\epsilon})$.

A spatial distribution $h \equiv h^* \in [0, 1]$ is said to be an equilibrium if $\dot{h} = 0$. An equilibrium h^* is locally stable if, after a small perturbation due to an exogenous migration, the equilibrium h^* is restored.

Without loss of generality[3], let us assume that ϵ_1 is normalised to unity and that

[2]Note that, in any expression, simply considering $\boldsymbol{\epsilon} = 1$ would recover the original equations from the Pflüger (2004) model.

[3]Suppose $\alpha/\epsilon_1 = \alpha_1$ and $\alpha/\epsilon_2 = \alpha_2$. Then, expressing the number of industrial varieties in region 2 relative to the number of varieties in region 1 yields $(H_2/\alpha_2)/(H_1/\alpha_1) = \frac{1-h}{h}\frac{\epsilon_2}{\epsilon_1}$, which implies that the number of industrial varieties depends only on the ratio of regional productivities

only $\epsilon \equiv \epsilon_2 \in (0,1)$ varies, thus $\boldsymbol{\epsilon} = (1, \epsilon)$. This simplification implies that region 1 represents the most efficient production possible, hence serving as the benchmark for region 2 whose productivity can be seen as a percentage of that of region 1. Therefore, our problem is simplified[4] to

$$\dot{h} = h\,(1-h)\,[V_1(h,\epsilon) - V_2(h,\epsilon)]\,,$$

Solving the differential equation $\dot{h} = 0$ always yields the full agglomeration ($h = 0$ or $h = 1$) equilibrium. However, the symmetric dispersion ($h = 0.5$) as an ever-existing equilibrium is excluded due to the regional asymmetry of productivity.

2.2.2.1 Agglomerations

Full agglomeration in region i is only stable if no one wants to migrate to region j. This occurs as long as $V_i > V_j,\ \forall j \neq i$. Moreover, due to the regional asymmetry of productivity, agglomeration in region 1 and in region 2 have different stability regions.

Proposition 2.1. *Agglomeration in region 1 is stable if*

$$(1+\lambda) - \epsilon\frac{\lambda + (\lambda+2)\phi^2}{2\phi} - \frac{\sigma}{\sigma - 1}\ln\left[\phi\right] > 0.$$

Proof. Cumbersome but straightforward algebraic manipulation of $V_1(1,\epsilon) > V_2(1,\epsilon)$ yields the result. □

Proposition 2.2. *Agglomeration in region 2 is stable if*

$$(1+\lambda) - \frac{1}{\epsilon}\frac{\lambda + (\lambda+2)\phi^2}{2\phi} - \frac{\sigma}{\sigma - 1}\ln\left[\phi\right] > 0.$$

Proof. Cumbersome but straightforward algebraic manipulation of $V_2(0,\epsilon) > V_1(0,\epsilon)$ yields the result. □

Note that $1/\epsilon > 1 > \epsilon$, which implies that if the inequality in Proposition 2.2 is verified, so is the inequality in Proposition 2.1. Hence, whenever agglomeration in region 2 is stable, so is agglomeration in region 1.

While Gaspar et al. (2018) used a closed solution to represent the stability of full agglomeration in terms of the mobility of workers, we make use of the same strategy but focus on our main variable of interest – productivity.

and not on their levels.

[4] We also simplify notation as $V_i(h,(1,\epsilon)) = V_i(h,\epsilon)$.

Corollary 2.3. *Agglomeration in region 1 is stable if*

$$\epsilon < \epsilon_{s_1} \equiv \frac{(1+\lambda) - \frac{\sigma}{\sigma-1}\ln[\phi]}{\frac{\lambda+(\lambda+2)\phi^2}{2\phi}}.$$

Proof. Cumbersome but straightforward algebraic manipulation of $V_1(1,\epsilon) > V_2(1,\epsilon)$ yields the result. □

Corollary 2.4. *Agglomeration in region 2 is stable if*

$$\epsilon > \epsilon_{s_2} \equiv \frac{\frac{\lambda+(\lambda+2)\phi^2}{2\phi}}{(1+\lambda) - \frac{\sigma}{\sigma-1}\ln[\phi]} = \frac{1}{\epsilon_{s_1}}.$$

Proof. Cumbersome but straightforward algebraic manipulation of $V_2(0,\epsilon) > V_1(0,\epsilon)$ yields the result. □

Figure 2.1 illustrates Corollaries 2.3 and 2.4 and helps visualise the conclusion that whenever agglomeration in region 2 is stable, so is agglomeration in region 1.

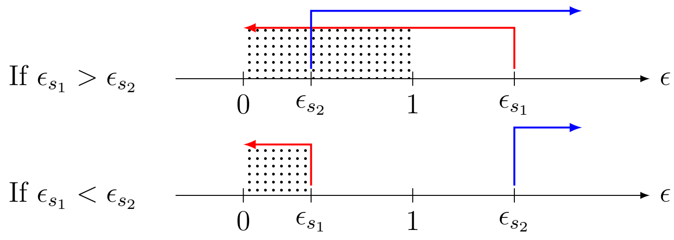


Figure 2.1: The figure shows the two possible configurations for the relative position of ϵ_{s_1} (in red) and ϵ_{s_2} (in blue) in the productivity scale.

Furthermore, as functions ϵ_{s_1} and ϵ_{s_2} depend on the freeness of trade, it is useful to plot them in the (ϕ, ϵ) space, as shown in Figure 2.2.

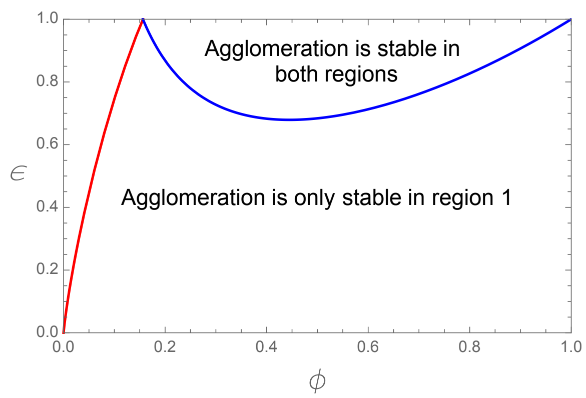


Figure 2.2: The figure shows the functions ϵ_{s_1} (in red) and ϵ_{s_2} (in blue) for $\sigma = 2$ and $\lambda = 2$. It also shows the range of parameters for which each type of agglomeration is stable in the (ϕ, ϵ) space. Note that no agglomeration is stable in the region to the left of the red curve.

Note that there is a range of values for ϵ that ensure that agglomeration is stable in both regions – the region above the blue curve. Numerical simulation shows that this range becomes more narrow as labour mobility across regions increases.

Moreover, it is also clear that stability of agglomeration in region 1 occurs more easily and for a wider range of productivity levels than in region 2. In this case, for agglomeration in region 2 to be stable, the productivity gap needs not be too large.

2.2.2.2 Interior equilibria

Existence

Beyond the invariant pattern of full agglomeration, interior equilibria, in which $h \in (0,1)$, may exist. Interior equilibria occur if the indirect utility of living in any region is the same, that is, $V_i(h) = V_j(1-h), \ \forall j \neq i$.

Proposition 2.5. *The distribution* $(h, 1-h)$*, with* $h \in (0,1)$*, is an interior equilibrium if and only if*

$$\left[(1-\epsilon\phi)\frac{h+\lambda/2}{h+\epsilon\phi(1-h)} + (\phi-\epsilon)\frac{(1-h)+\lambda/2}{\phi h+\epsilon(1-h)}\right] + \frac{\sigma}{\sigma-1}\ln\left[\frac{h+\epsilon\phi(1-h)}{\phi h+\epsilon(1-h)}\right] = 0.$$

Proof. Cumbersome but straightforward algebraic manipulation of $V_1(h,\epsilon) = V_2(h,\epsilon)$ yields the result. □

Given the condition of Proposition 2.5, a necessary and sufficient condition for interior equilibria to exist is that $\lambda = \lambda_{\mathcal{I}} > 0$, where $\lambda_{\mathcal{I}}$ is given by

$$\lambda_{\mathcal{I}}(h,\phi,\epsilon) = -2\frac{h\dfrac{1-\epsilon\phi}{h+\epsilon\phi(1-h)} + (1-h)\dfrac{\phi-\epsilon}{\phi h+\epsilon(1-h)} + \dfrac{\sigma}{\sigma-1}\ln\left[\dfrac{h+\epsilon\phi(1-h)}{\phi h+\epsilon(1-h)}\right]}{\dfrac{1-\epsilon\phi}{h+\epsilon\phi(1-h)} + \dfrac{\phi-\epsilon}{\phi h+\epsilon(1-h)}}.$$

Lemma 2.6. *If* $\phi > \epsilon > \frac{h}{1-h}$*, then there is no interior equilibrium.*

Proof. Straightforward inspection of $\lambda_{\mathcal{I}}(h,\phi,\epsilon)$ for $\epsilon < \phi$ and $\epsilon > \frac{h}{1-h}$ yields that $\lambda_{\mathcal{I}}(h,\phi,\epsilon) < 0$. □

Multiple interior equilibria

For some values of λ, interior equilibria may not be unique due to the nonlinearity of $\lambda_{\mathcal{I}}$. Note that $\lambda_{\mathcal{I}}$ has a vertical asymptote when

$$\frac{1-\epsilon\phi}{h+\epsilon\phi(1-h)} + \frac{\phi-\epsilon}{\phi h+\epsilon(1-h)} = 0.$$

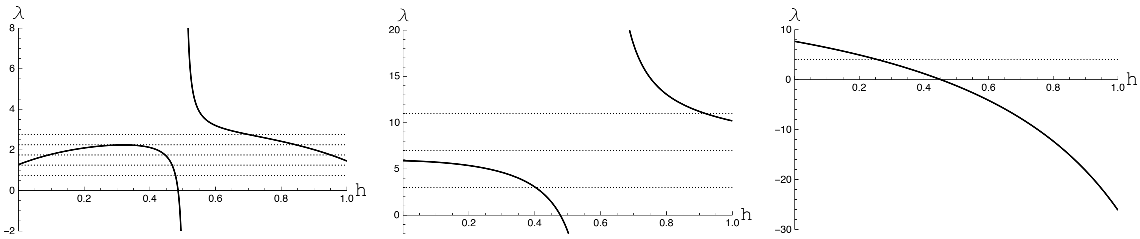
Given that $1-\epsilon\phi > 0$, the vertical asymptote only exists if $\epsilon > \phi$ – which implies that the regional asymmetry is not too big. This asymptote is then located at

$$h_A = \frac{1}{2}\left(1 - \frac{1}{1-\epsilon\phi} - \frac{\epsilon}{\phi - \epsilon}\right).$$

It follows from $\epsilon > \phi$ that $h_A > 1/2$. Since $h \in (0,1)$, for $h_A \in (1/2, 1)$ we need to impose $\epsilon > \frac{2\phi}{1+\phi^2}$, which is always bigger than ϕ.

Moreover, we have that $\lim_{h \to h_A^-} \lambda_{\mathcal{I}} = -\infty$ and $\lim_{h \to h_A^+} \lambda_{\mathcal{I}} = +\infty$. Hence, for $h \in (h_A, 1)$, there exists a value for λ that implies the existence of an interior equilibrium in which the partial core is located in region 1.

Figure 2.3 shows the three qualitative distinct configurations that $\lambda_{\mathcal{I}}(h,\phi,\epsilon) > 0$ may display. An equilibrium occurs when an horizontal line – which represents a value for λ – intersects $\lambda_{\mathcal{I}}(h,\phi,\epsilon)$. We have multiple interior equilibria if a single horizontal line has several intersections.



(a) $\phi = 0.1$ (b) $\phi = 0.4$ (c) $\phi = 0.8$

Figure 2.3: This figure illustrates $\lambda_{\mathcal{I}}(h,\phi,\epsilon)$ for $\epsilon = 0.95$, $\sigma = 2$, and three different values of ϕ.

As stated before, let h_A be the vertical asymptote of $\lambda_{\mathcal{I}}$. Furthermore, let h_Z be the zero of $\lambda_{\mathcal{I}}$. Then, Figure 2.3 shows how multiple interior equilibria occur.

First, for $h \in (0, h_Z)$, there is a value for λ that corresponds to, at most, two interior equilibria. Second, for $h \in (h_Z, h_A)$, there is no value for λ that corresponds to an interior equilibrium. Third, for $h \in (h_A, 1)$, there is a value for λ that corresponds to a unique interior equilibrium.

Therefore, we may have, at most, three interior equilibria for a given value for λ. Moreover, note that when multiple interior equilibria exist, most[5] of its spatial distributions feature the majority of mobile workers living closer to the least productive region.

Note that

$$\lambda_{\mathcal{I}}(0,\phi,\epsilon) = -2\frac{\frac{\phi-\epsilon}{\epsilon} + \frac{\sigma}{\sigma-1}\ln[\phi]}{\frac{1-\epsilon\phi}{\epsilon\phi} + \frac{\phi-\epsilon}{\epsilon}} \text{ and } \lambda_{\mathcal{I}}(1,\phi,\epsilon) = -2\frac{(1-\epsilon\phi) - \frac{\sigma}{\sigma-1}\ln[\phi]}{(1-\epsilon\phi) + \frac{\phi-\epsilon}{\epsilon}}.$$

[5] With the sole exception of the single instance where two interior equilibria exist in which one spatial distribution occurs for a high h and the other for a low h.

It is clear that $\lambda_{\mathcal{I}}(1,\phi,\epsilon) > 0$ only occurs for $\epsilon > \frac{2\phi}{1+\phi^2}$. Therefore, $\lambda_{\mathcal{I}}(1,\phi,\epsilon)$ is positive if and only if the vertical asymptote exists. Hence, in these conditions, $\lambda_{\mathcal{I}}(0,\phi,\epsilon) > 0$ is always verified.

Since $\frac{\partial\lambda_{\mathcal{I}}(0,\phi,\epsilon)}{\partial\epsilon} > 0$, as the productivity gap narrows, the global mobility of workers required for interior equilibria to exist becomes wider, which implies that even low values of λ may be enough.

Lemma 2.7. *When the productivity gap between the regions is too big, the mobile agents will always choose to be agglomerated in one of the regions.*

Proof. Since $\frac{\partial\lambda_{\mathcal{I}}(0,\phi,\epsilon)}{\partial\epsilon} > 0$, as the productivity gap widens, the value of $\lambda_{\mathcal{I}}(0,\phi,\epsilon)$ decreases and, eventually, becomes negative, which implies that no interior equilibria exists. □

Moreover, $\lambda_{\mathcal{I}}(0,\phi,\epsilon) = 0$ implies that $\epsilon = \frac{\phi}{1-\frac{\sigma}{\sigma-1}\ln[\phi]}$. Therefore, as we have already established that $\frac{\partial\lambda_{\mathcal{I}}(0,\phi,\epsilon)}{\partial\epsilon} > 0$ and since $\frac{\phi}{1-\frac{\sigma}{\sigma-1}\ln[\phi]} < \frac{2\phi}{1+\phi^2}$ – which implies that no vertical asymptote exists –, we conclude that $\epsilon < \frac{\phi}{1-\frac{\sigma}{\sigma-1}\ln[\phi]}$ make interior equilibria impossible.

Finally, let h_Z be the zero of $\lambda_{\mathcal{I}}(h,\phi,\epsilon)$. Then, h_Z is implicitly given by

$$h_Z\frac{1-\epsilon\phi}{h_Z+\epsilon\phi(1-h_Z)} + (1-h_Z)\frac{\phi-\epsilon}{\phi h_Z+\epsilon(1-h_Z)} + \frac{\sigma}{\sigma-1}\ln\left[\frac{h_Z+\epsilon\phi(1-h_Z)}{\phi h_Z+\epsilon(1-h_Z)}\right] = 0.$$

Even though the signs of all the terms in the previous sum are straightforward, it is not analytically possible to determine h_Z explicitly. However, we can specify the general conditions in which interior equilibria exist.

We have already subdivided the range of productivity into $\epsilon \in \left(0, \frac{\phi}{1-\frac{\sigma}{\sigma-1}\ln[\phi]}\right)$, $\epsilon \in \left(\frac{\phi}{1-\frac{\sigma}{\sigma-1}\ln[\phi]}, \frac{2\phi}{1+\phi^2}\right)$, and $\epsilon \in \left(\frac{2\phi}{1+\phi^2}, 1\right)$. Then, we can also subdivide the range of the share of mobile agents living in region 1 into $h \in (0,h_Z)$, $h \in (h_Z,h_A)$, and $h \in (h_A,1)$.

Table 2.1 summarises the existence of interior equilibria in the specified ranges.

	$h \in (0,h_Z)$	$h \in (h_Z,h_A)$	$h \in (h_A,1)$
$\epsilon \in \left(0, \frac{\phi}{1-\frac{\sigma}{\sigma-1}\ln[\phi]}\right)$		Impossible	
$\epsilon \in \left(\frac{\phi}{1-\frac{\sigma}{\sigma-1}\ln[\phi]}, \frac{2\phi}{1+\phi^2}\right)$	Possible	Impossible	
$\epsilon \in \left(\frac{2\phi}{1+\phi^2}, 1\right)$	Possible	Impossible	Possible

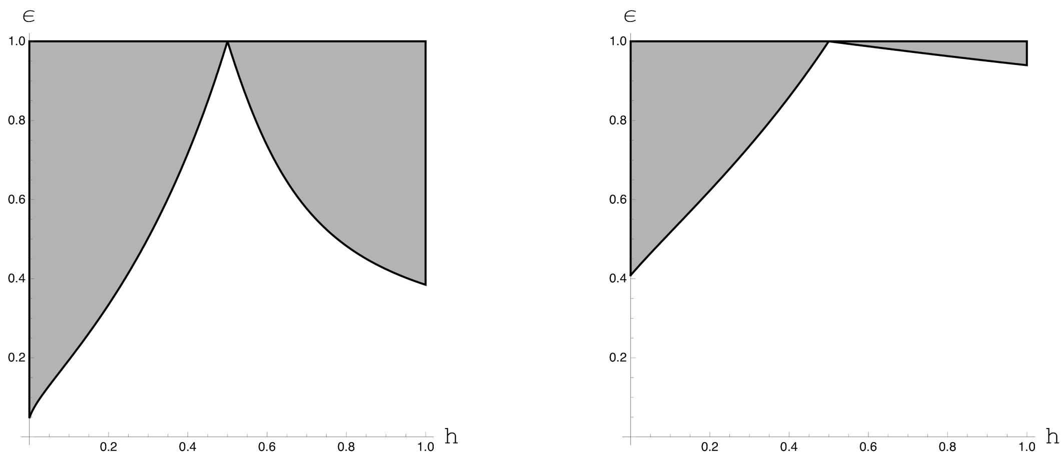
Table 2.1: The table summarises the existence of interior equilibria in the relevant ranges of the variables. An empty cell means that the intersection of both ranges is the empty set.

From Table 2.1, we conclude that interior equilibria in which the majority of the agents live in the most productive region are only possible whenever the differences in productivity are small enough (high ϵ), given the transportation costs.

Moreover, it also shows that interior equilibria in which the majority of the agents live in the least productive region are only possible whenever differences in productivity are intermediate at most (medium or high ϵ), given the transportation costs.

Finally, it is also remarkable that more equitable spatial distributions – in which the mobile agents are almost evenly distributed – are never feasible.

Figure 2.4 illustrates the summary of Table 2.1 in the (h, ϵ) space and depicts the regions in which interior equilibria exist – that is when $\lambda_{\mathcal{I}}(h, \phi, \epsilon) > 0$.



(a) $\phi = 0.2$ (b) $\phi = 0.7$

Figure 2.4: This figure illustrates the regions in which $\lambda_{\mathcal{I}}(h, \phi, \epsilon) > 0$ for $\sigma = 2$.

First, note that Figure 2.4 shows two different regions of parameters in which interior equilibria exist. The bigger region, on the left, is delimited by h_Z, while the smaller region, on the right, is delimited by h_A.

Figure 2.4 is helpful to show that as the productivity gap widens, spatial distributions near symmetric dispersion become impossible. Moreover, it also shows that bigger productivity gaps lead to the agglomeration of mobile workers in the most productive region.

Stability

Interior equilibria are only stable as long as migrations between regions are worthless so that even if someone migrated, they would then return to their departure region. This implies that the Jacobian matrix evaluated at an interior equilibrium must be negative for an interior equilibrium to be stable.

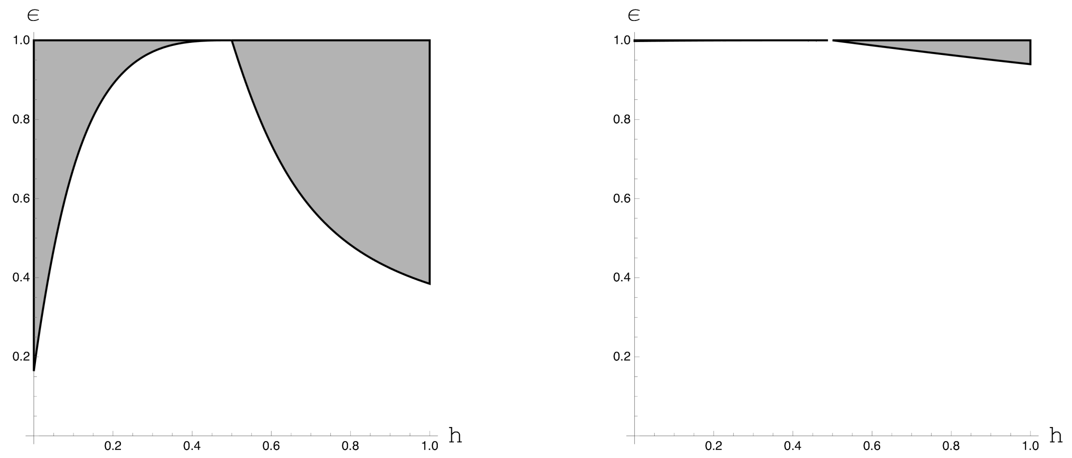
Proposition 2.8. *An interior equilibrium is stable if*

$$\left(\frac{\sigma}{\sigma-1}+1\right)\left(\frac{1-\epsilon\phi}{h+\epsilon\phi(1-h)}-\frac{\phi-\epsilon}{\phi h+\epsilon(1-h)}\right)-(1-\epsilon\phi)^2\left(\frac{h+\frac{\lambda(h,\phi,\epsilon)}{2}}{[h+\epsilon\phi(1-h)]^2}\right)-(\phi-\epsilon)^2\left(\frac{(1+h)+\frac{\lambda(h,\phi,\epsilon)}{2}}{[\phi h+\epsilon(1-h)]^2}\right)<0.$$

Proof. Cumbersome but straightforward algebraic manipulation of $\frac{\partial V_1(h,\epsilon)}{\partial h}-\frac{\partial V_2(h,\epsilon)}{\partial h}<0$ evaluated at $\lambda=\lambda_{\mathcal{I}}$ yields the result. □

Even though the last two terms in the condition for interior stability are indubitably negative, the first one is always positive. Hence, the stability of interior equilibria will hinge on $\lambda_{\mathcal{I}}$, which implies that explicit solutions are unattainable as before.

Figure 2.5 shows the region of stability of the interior equilibria depicted in Figure 2.4.



(a) $\phi = 0.2$

(b) $\phi = 0.7$

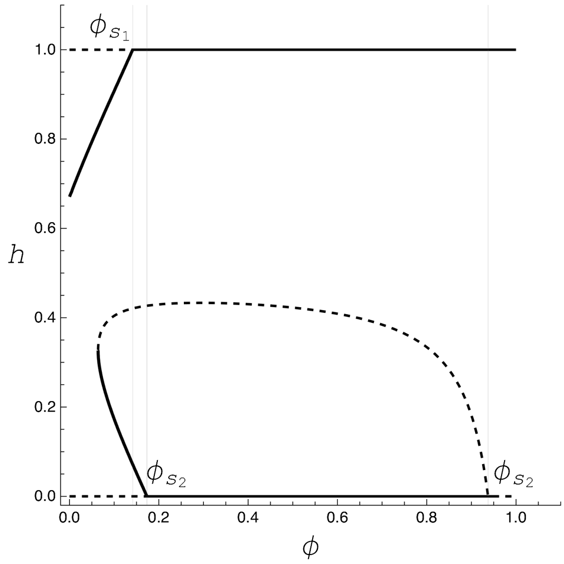
Figure 2.5: This figure illustrates the regions in which interior equilibria exists and is stable for $\sigma = 2$.

An overlay of both Figures 2.4 and 2.5 shows that interior equilibria in which the majority of mobile workers live in the most productive region are always stable. However, when most of the skilled workers live in the least productive region, stability is not guaranteed.

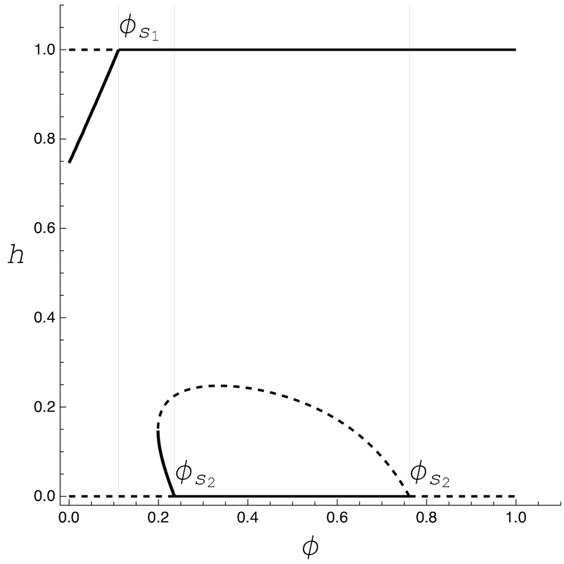
In fact, as the productivity gap widens, spatial distributions with mobile agents living in the least productive region are only stable if almost everyone lives there, thus benefiting from a higher market size instead of the lower prices that more productive firms offer. Moreover, this effect only occurs if transportation costs are high (low ϕ). Otherwise, importing the industrial good from the most productive region is always best as the lower price is enough to compensate for the small transportation cost.

2.2.2.3 Migration dynamics

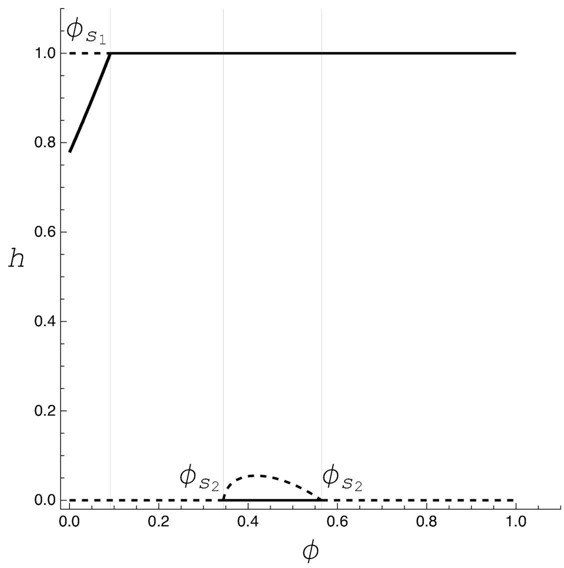
So far, we have established the existence and stability of agglomeration and interior equilibria with a particular focus on the effects of productivity. Now, in Figure 2.6, we present the full picture of this phenomenon using bifurcation diagrams in the (ϕ, h) space to show the four qualitatively different possibilities for mobile workers to be spatially distributed.



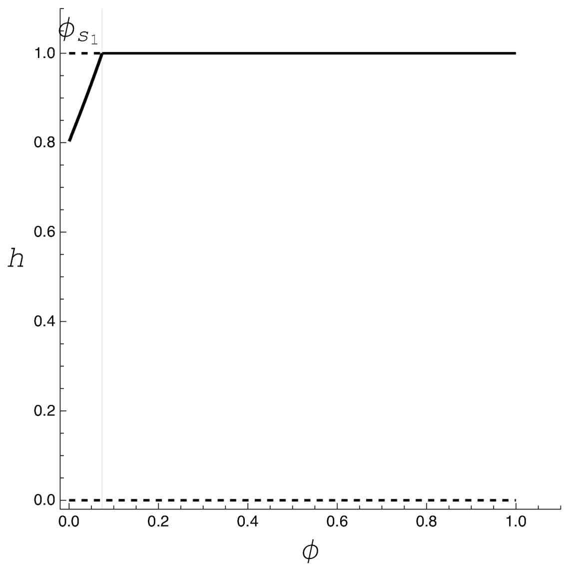
(a) $\epsilon = 0.95$



(b) $\epsilon = 0.80$



(c) $\epsilon = 0.70$



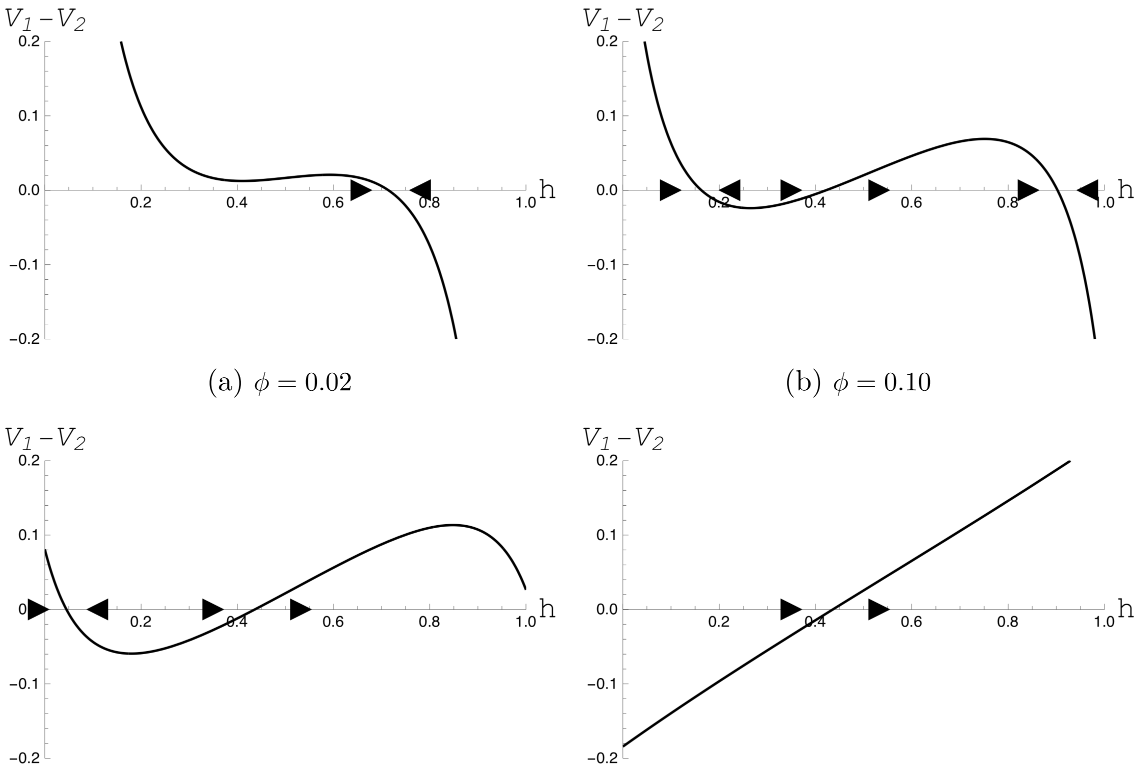
(d) $\epsilon = 0.60$

Figure 2.6: This figure presents the bifurcation diagram for $\lambda = 2$, $\sigma = 2$, $\mu = 0.3$, and four different values for ϵ. Dashed lines represent unstable equilibria, and solid lines represent stable equilibria. ϕ_{s_1} represents the threshold of ϕ for agglomeration in region 1 to be stable, as shown in Proposition 2.1. ϕ_{s_2} represents the superior and inferior thresholds of ϕ between which agglomeration in region 2 is stable, as shown in Proposition 2.2.

Following what we have studied previously, we verify that when agglomeration in the least productive region is stable, so is agglomeration in the most productive one.

Moreover, we also see that the existence of interior equilibria reduces when the productivity gap widens and that multiple interior equilibria are possible when the productivity gap is relatively low. Last, we also observe that interior equilibria with most mobile workers living in the least productive are stable when the transportation costs are elevated.

Figure 2.7 shows the general qualitative different regions for spatial configurations' existence and stability, depending on the transportation costs.



(a) $\phi = 0.02$ (b) $\phi = 0.10$

(c) $\phi = 0.15$ (d) $\phi = 0.50$

Figure 2.7: This figure illustrates $V_1(h, \epsilon) - V_2(h, \epsilon)$ for $\epsilon = 0.95$, $\lambda = 2$, $\sigma = 2$, $\mu = 0.3$, and four different values for ϕ. We manually added the arrows on the horizontal axis, around the zeros of the function, to evidentiate the direction of the movement – if a pair of arrows points to each other, it represents a stable equilibrium, and if the pair points in the same direction, it represents an unstable equilibrium.

Finally, plotting $V_1(h, \boldsymbol{\epsilon}) - V_2(h, \boldsymbol{\epsilon})$ with respect to h allows us to study cross-sections of the bifurcation diagrams of Figure 2.6. While Figure 2.7 shows the four more interesting regions of the bifurcation diagram of Figure 2.6a, it can be generalised to any of the bifurcation diagrams. The zeros of these functions represent a spatial equilibrium, and the arrows represent its (un)stability.

2.2.3 Extension to three regions

A rather interesting question is to ask what happens if we have three instead of two regions with asymmetric productivity. We can gather some insight on this subject if we extend our approach to three regions and consider that now we have $\boldsymbol{\epsilon} = (\epsilon_1, \epsilon_2, \epsilon_3)$ and that, once again, $\epsilon_1 = 1$. Then, let $\epsilon_2 \neq \epsilon_3$ and, in particular, $\epsilon_2 > \epsilon_3$. So, we have that region 1 is the most productive and region 3 is the least productive.

Even though the analytical complexity of this extension is high, it is straightforward to run a numerical simulation and understand how the population migrates. While one may think that the population should distribute itself between the three regions or simply not live in the least productive region, Figure 2.8 shows otherwise.

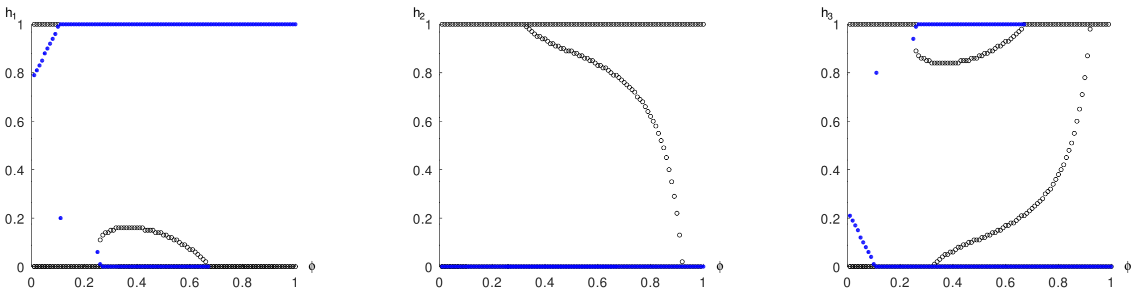


Figure 2.8: This figure presents the bifurcation diagrams of the three regions for $\lambda = 3$, $\sigma = 2$, $\mu = 0.3$, and $\boldsymbol{\epsilon} = (1, 0.9, 0.8)$. On the left panel, we have region 1, which is the most efficient. On the middle panel, we have region 2, which features an intermediate productivity level. On the right panel, we have region 3, which is the least efficient. Black circles represent unstable equilibria, and blue dots represent stable equilibria.

Note that, excluding region 2, the bifurcation diagrams are qualitatively very similar to some of those in Figure 2.6. However, it is surprising that, generally, it is never stable for anyone to live in a region with an intermediate productivity level. Hence, we conclude that firms do not want to be located in regions with intermediate productivity levels, which implies that policymakers do not want to promote policies that lead their regions to that intermediate level.

2.2.4 Welfare

Apart from studying the possible spatial distribution of workers and its stability, it is also relevant to study the associated welfare of the agents. To achieve this objective, we use a utilitarian criterion that considers average indirect utility as the welfare measure (Gaspar et al., 2018; Pflüger & Südekum, 2008).

Since the weighted average nominal wage of entrepreneurs is the same as in Gaspar et al. (2018), $\bar{w} = hw_1(h, \boldsymbol{\epsilon}) + (1-h)w_2(h, \boldsymbol{\epsilon}) = \frac{\mu}{\sigma}(1+\lambda)$, we have that the average welfare of a skilled agent is given by

$$\bar{V} = \frac{\mu}{\sigma}(1+\lambda) + \frac{\mu}{\sigma-1}\left(h \ln\left[h + \epsilon\phi(1-h)\right] + (1-h)\ln\left[\phi h + \epsilon(1-h)\right]\right) + \eta.$$

Proposition 2.9. *Skilled workers attain maximum welfare when they agglomerate in the most productive region.*

Proof. $\frac{\partial \bar{V}}{\partial h} = 0 \Leftrightarrow h \in (0, 1/2)$, $\frac{\partial^2 \bar{V}}{\partial h^2} > 0$, and $\bar{V}(h=1) > \bar{V}(h=0)$. □

As for the average welfare of an unskilled agent, we have that it is given by

$$\bar{V}^L = 1 + \frac{\mu}{\sigma-1}\left(\frac{1}{2}\ln\left[h + \epsilon\phi(1-h)\right] + \frac{1}{2}\ln\left[\phi h + \epsilon(1-h)\right]\right) + \eta.$$

Proposition 2.10. *Unskilled workers attain maximum welfare when skilled agents are distributed such that $h = \frac{1}{2}\left(1 - \frac{1}{1-\epsilon\phi} - \frac{\epsilon}{\phi-\epsilon}\right)$.*

Proof. $\frac{\partial \bar{V}^L}{\partial h} = 0 \Leftrightarrow h = \frac{1}{2}\left(1 - \frac{1}{1-\epsilon\phi} - \frac{\epsilon}{\phi-\epsilon}\right)$ and $\frac{\partial^2 \bar{V}^L}{\partial h^2} < 0$. □

Note that this configuration corresponds to the vertical asymptote h_A, which we have already concluded is only interior if $\epsilon > \phi$. Moreover, by definition, a spatial distribution such that $h = h_A$ cannot be an equilibrium. Hence, immobile agents never achieve their maximum welfare.

Finally, we can define social welfare as a weighted average of skilled and unskilled agents' welfare, that is

$$W = \frac{\bar{V} + \lambda \bar{V}^L}{1+\lambda}.$$

While $\frac{\partial W}{\partial h} = 0$ represents the optimal social welfare points, it is not possible to express the resulting spatial distribution explicitly. However, we can use the same strategy we used to find the solution to the condition of Proposition 2.5. Thus, let $\lambda = \lambda_W$ be the solution for $\frac{\partial W}{\partial h} = 0$, where λ_W is given by

$$\lambda_W(h,\phi,\epsilon) = -2\frac{h\dfrac{1-\epsilon\phi}{h+\epsilon\phi(1-h)} + (1-h)\dfrac{\phi-\epsilon}{\phi h + \epsilon(1-h)} + \ln\left[\dfrac{h+\epsilon\phi(1-h)}{\phi h + \epsilon(1-h)}\right]}{\dfrac{1-\epsilon\phi}{h+\epsilon\phi(1-h)} + \dfrac{\phi-\epsilon}{\phi h + \epsilon(1-h)}}.$$

The expression for λ_W implicitly defines the spatial distributions that maximise social welfare. It is clear that λ_W and $\lambda_{\mathcal{I}}$ are quite similar. In fact, the only difference is that the elasticity of substitution between varieties – which can be interpreted as the preference for variety – does not influence λ_W. Therefore, since $\frac{\sigma}{\sigma-1}$ tends to 1 as σ tends to positive infinity, we conclude that the spatial distribution chosen by mobile agents converges to the socially optimal one as the preference for variety increases. Figure 2.9 shows the comparison between the actual interior equilibria and the corresponding social optimal one.

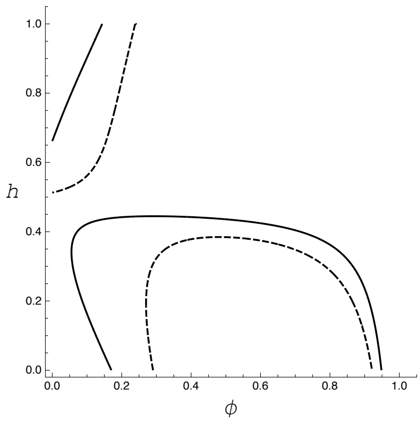


Figure 2.9: This figure illustrates the interior equilibria (solid lines) and the optimal social welfare points (dashed lines) for $\epsilon = 0.95$, $\lambda = 2$, $\sigma = 2$, and $\mu = 0.3$.

Then, we can conclude that, generally, society would be better off if the most productive region did not concentrate as many mobile workers.

2.3 Concluding remarks

In this essay, we study how regional asymmetries in firms' productivity affect the spatial distribution of economic activity. In particular, we focus on how the gap in productivity between regions generates imbalances in the landscape of an economy and how it affects the agents' welfare.

We develop an extension to the quasi-linear log utility footloose entrepreneur model with two regions that accounts for exogenous regional asymmetries in the productivity of industrial firms. We consider that regional asymmetries can occur due to transportation networks' efficiency differences between regions or due to different bureaucratic processes between the regions. Thus, firms within the same region are homogeneous regarding productivity but are heterogeneous between regions. Moreover, without loss of generality, we assume that the firms in one of the regions have the maximum possible productivity, whereas the firms in the other region only enjoy a percentage of that maximum productivity. Hence, we have an efficient region and a relatively inefficient one.

There are three major straightforward implications from considering the regional productivity asymmetry compared to the baseline model. First, the wages in the most productive region increase, while the wages in the least productive one decrease.

Second, prices decrease in both regions, especially in the most productive region. The decrease in prices implies that the optimal consumption of the industrial good increases, which is consistent with the higher productivity level – skilled agents take less time to produce, hence they can produce more. Third, an even distribution of mobile agents between the regions is no longer an equilibrium.

We find that agglomeration in either region is always possible. However, stability is more likely to be achieved in the most productive region. For agglomeration in the least productive region to be stable, the productivity gap needs not be too large. Moreover, we also conclude that whenever agglomeration in the least productive region is stable, so is the agglomeration in the most productive one.

We study interior equilibria and find that multiple equilibria may exist simultaneously. When multiple equilibria exist, the norm is that either of them – should there be two – or that the majority of them – should there be three – have a low share of mobile workers in the most efficient region. Moreover, the bigger the productivity gap, the less probable that spatial distributions are close to being symmetric and the more probable that industry is agglomerated in the most efficient region.

Regarding the stability of these equilibria, we find that interior equilibria with most industrial workers living in the most productive region are always stable and that spatial distributions with mobile workers living in the least productive region are only stable if almost everyone lives there.

We resort to numerical simulation to study the effects of considering a third region and find that the region with an intermediate productivity level tends to up depleted of industry.

Finally, we study the economy's welfare and find several interesting conclusions. First, skilled workers enjoy maximum well-being when they all agglomerate in the most productive region. Second, unskilled workers can never achieve their maximum welfare. Third, the spatial distribution that assures that welfare from the society is maximum is the same as the spatial distribution chosen by mobile agents when the preference for variety is very high. Fourth, more generally, society would be better off if the most productive region did not concentrate as many mobile workers.

From a policy perspective, this essay may help guide investments in education, more efficient networks of transport, communication, and also in more efficient fiscal and judiciary systems, as our conclusions indubitably point to the supremacy of more efficient regions.

Moreover, in a multiple-region setup, policymakers whose regions feature an intermediate productivity level may be incentivised to lower its level rather than increase it, which is puzzling and opens a line for future research.

Chapter 3

On optimal education choices

3.1 Introduction

The purpose of this essay is to understand how agents formulate their education decisions. In particular, we seek to explore how the spatial distribution of economic activity and the political framework of an economy affect the optimal education level.

Since the political and cultural backgrounds condition each country's specific approach, it is reasonable that different types of societies make different choices regarding how they seek to maximise the potential of their human capital. Moreover, it is also reasonable to expect that the agglomeration of industry also plays a role in education decisions, as the scarcity of workers in some locations should drive the wages up.

To achieve our goal, we introduce a simple education mechanism in the Forslid and Ottaviano (2003) model. We consider skilled and unskilled workers within a two-region setup. The education choice is costly due to the opportunity cost of time and the effort an agent must make to become more productive. Thus, our novelty is to allow for changes in the productivity level of skilled individuals while maintaining both the skilled and unskilled populations constant.

We determine the optimal education choice by maximising the indirect utility for three scenarios. In the first scenario, we have a society ruled by an education minister aiming to maximise the utility of all workers. In another scenario, we have a society in which skilled workers are unionised and whose union aims to maximise the utility of all members. In the last scenario, we have a totally decentralised society in which every skilled individual aims to maximise their expected utility.

Solving the maximisation problems above yields the optimal productivity levels in each type of society. We further analyse these levels and find that the optimal productivity level increases with the share of expenditure on the industrial good and with the proportion of immobile to mobile agents and decreases with the unitary cost of education and with the elasticity of substitution between industrial varieties. Moreover, this choice of productivity depends on spatial distribution and transportation costs. A highly unionised society is the least affected by spatial features. Also, higher agglomeration levels generally lead to lower optimal productivity levels. However, when transportation costs tend to zero, this effect dissipates, and the optimal choice becomes independent of spatial agglomeration.

The education level differs for the three types of societies we study. However, there is a specific order between the three – the education minister always chooses the highest productivity levels among the three, and the union chooses the lowest. In contrast, the completely decentralised scenario level is between both extremes.

We unravel education externalities and conclude that education positively affects all individuals – even unskilled ones – due to the decrease in prices caused by a more productive skilled population.

We explore the strategic profile of individual and average education levels and find that they are strategic substitutes. Hence, the more productive the society is, the least an individual agent wants to be, which may generate an incentive for agents to free-ride.

Finally, to verify the validity of our results, we implement our education mechanism to the Pflüger (2004) model and find no qualitative differences. However, the Forslid and Ottaviano (2003) model offers more in-depth explanations and is not as restrictive as the Pflüger model, despite the more complex expressions, thus justifying its choice for the main implementation of our education mechanism.

To the best of our knowledge, the only contribution that develops an analytical model that introduces human capital in Economic Geography is that of Toulemonde (2006). In his work, Toulemonde focuses on the possibility that low-skilled workers learn and become high-skilled ones, which allows them to seek jobs in the industrial sector with higher wages. Toulemonde then applies his model to study how the government can subsidise skills acquisition. Toulemonde states a circular causality that explains why firms would want to be agglomerated as education rises. Our work adds to the literature as our mechanism differs from the one introduced by Toulemonde – we consider that the agents that partake in education remain in the same work sector, whereas Toulemonde considers that the agents go from the agricultural sector to the industrial one –, and is more general, allowing it to be further developed and explored in future research.

Moretti (2004) surveys the literature on education externalities to explore how the level of human capital affects the real economy and, in particular, its effects when geography is in play. The author starts by exploring how human capital is distributed across cities, then presents some theories on the social returns of education, and finally summarises some estimation strategies to measure the externalities. Although Moretti considers that education externalities have widespread positive effects, he also states that its measurement is not yet much explored in literature. Our conclusions are aligned with those of Moretti and contribute to enriching this strand of literature, particularly by introducing a simple measure of education externalities, which is something that Moretti deems necessary.

Proost and Thisse (2019) survey the literature and intend to dissect the forces in action in Spatial Economics. One of these forces, an agglomerative one, is the acquisition of skills. The authors consider workers as immobile and capital as mobile, allowing unskilled workers to study and become skilled. They argue that education causes a snowball effect that leads to further agglomeration and higher levels of education and that this effect – the mobility from the unskilled sector to the skilled one – replaces the usual spatial mobility of workers.

Fujita and Mori (2005) survey the state of the art of Economic Geography with a particular focus on relevant underdeveloped topics. One of the aspects studied is the heterogeneity of workers, or the lack of it in most mainstream models, with Tabuchi and Thisse (2002), Murata (2003), and Mori and Turrini (2005) amongst the main exceptions. In our work, even though we consider homogeneous agents, the fact that their productivity may change can be loosely thought of as introducing some individual heterogeneity.

Moreover, Fujita and Mori (2005) stress the importance of the so-called "K-linkages", which are the transmission channels through which knowledge is created and transferred, and state that the literature lacks a micro-founded approach that includes it into Economic Geography models so it can be a new type of agglomeration force. In this essay, we show how the fundamentals of the economy affect education.

Like Fujita and Mori (2005), Gaspar (2018) also stresses the relevance of both the heterogeneity of workers and the "K-linkages" when discussing new directions for future developments in Economic Geography literature.

In Picard and Toulemonde (2004), all the agents are immobile between regions but have the option to study so they can pursue better wages in their own region. Thus, the spatial distribution occurs due to firms moving between regions to find better matches with employees, namely to reduce their costs. Furthermore, Picard and Toulemonde find that a higher elasticity of substitution between industrial varieties decreases the education level, which is in line with our findings.

Redding and Schott (2003) develop a model that builds upon Fujita et al. (1999) and that introduces human capital accumulation. Their work, however, differs from ours in two fundamental points. First, the model is not a classical Economic Geography one, at least not in the long-run point of view, whereas our approach retains the usual Economic Geography framework. Second, while we consider the possibility that high-skill workers increase their productivity, the authors consider that all workers are unskilled and may choose to study to become skilled. Even though these differences exist, Redding and Schott discuss the effects of some drivers in educational demand and state that education is higher, the higher the expenditure share in industrial goods is and the lower the cost of education is, which is in line with our findings.

Candau and Dienesch (2015) develop a footloose entrepreneur model that follows Redding and Schott (2003), in which unskilled workers may invest in education and become skilled. However, they remain immobile between regions, as in Picard and Toulemonde (2004), since the authors consider that these workers develop a home bias towards their own region. Furthermore, the education mechanism differs from ours since each worker has a different chance that their investment in education is successful, whereas in our model this heterogeneity, or uncertainty, is not present. The authors explore how spatial distribution and trade integration affect education levels. We also address this point and find that, generally, a higher level of integration leads to more education and that more agglomeration leads to lower education levels. While our results do not contradict those of Candau and Dienesch, we find a particular detail – when the choice is individual, agglomeration within an extremely low integration level leads to higher levels of education.

Blanchard and Olney (2017) develop an econometric model that aims at understanding some factors that drive educational attainment and note that there are still some "drivers of human capital investment that are still not well understood". We close this gap by answering this question within the framework of our model and its parameters. Namely, we find that education increases with the share of expenditure on the industrial good and with the proportion of immobile to mobile agents, and that it decreases with the unitary cost of education and with the elasticity of substitution between industrial varieties.

Ghose (2021) develops an empirical approach in which the agents are forward-looking and decide whether or not to study based on wages and education costs within a multi-regional setting. The author concludes that skill acquisition generates welfare gains but that education costs hinder those gains. In this essay, we show that education does increase real wages and that higher education costs lead to lower levels of education, which implies lower increases in real wages. Therefore, our findings seem to corroborate those of Ghose.

Bertinelli and Zou (2008) show that higher levels of agglomeration lead to higher levels of education, even though this effect is non-linear. In particular, there may exist an "under-urbanisation trap" below which further agglomeration induces lower education, which the authors suggest may be due to industry-cities where most workers are unskilled. The findings of Bertinelli and Zou are particularly interesting since our results corroborate their nuances but also are somehow contradictory since Bertinelli and Zou consider that agglomeration is education-enhancing as the rule and the "under-urbanisation trap" to be an exception. In contrast, we show the opposite – agglomeration induces education as an exception.

Thisse (2018) argues that cities are important for both the development of economies and the learning process of individuals, which is in line with Bertinelli and Zou (2008), but conflictual with our conclusions, as discussed before.

De Blasio and Di Addario (2005) develop an econometric approach to study the effects of industrial agglomeration on workers' welfare. The authors also study what role education plays in this setting and find that agglomeration hinders higher education levels, which is corroborated by our findings, as discussed before.

3.2 The education problem

Our model is an extension of the footloose entrepreneur model with Cobb-Douglas utility (Forslid & Ottaviano, 2003; Gaspar et al., 2020). We extend the baseline model to account for the educational decisions made by mobile agents, which serve as a proxy for their productivity level.

We allow for a setup in which the indirect utility of an agent depends both on their productivity (ϵ_k) and the average productivity level in society (ϵ). Even though each agent can choose their own productivity level, the fact that agents are homogeneous makes it reasonable that the optimal solution is such $\epsilon_k = \epsilon_{k+1} = ... = \epsilon$, which implies that every agent chooses the same productivity level. However, it would not make sense to restrict the decision *a priori* as we want to derive the individual optimal productivity level.

Since the opportunity cost of time is positive, education is always costly[6]. We consider that the cost an agent bears is the effort exerted during the qualification process and, therefore, not the monetary tuition fees. Also, to keep things as simple as possible, we consider that there is no discount rate, that there exists only one pe-

[6] See Borjas (2013, Chapter 6) for a microfounded discussion on education costs.

riod of time, that education is regarded as instantaneous, and that the cost involved in studying is embodied in a cost function common to both regions,

$$C(\epsilon_k) : [1, +\infty) \mapsto [0, +\infty), \quad C(1) = 0, \quad \frac{dC(\epsilon_k)}{d\epsilon_k} > 0, \quad \frac{d^2C(\epsilon_k)}{d\epsilon_k^2} \geq 0.$$

The cost function is assumed to be strictly increasing, twice-differentiable and weakly convex. We normalise $C(1) = 0$ so that an agent who does not invest in education has unitary productivity. Changing the root of the cost function would merely shift the innate value of productivity.

3.2.1 Economic model

In this economy, there are L unskilled workers – equally divided between the two regions – that are immobile between regions, and H skilled workers – H_i in region $i = \{1, 2\}$ – that are mobile between regions.

The preferences of every agent k are defined by

$$U_k = \left(\frac{M}{\mu}\right)^{\mu} \left(\frac{A}{1-\mu}\right)^{1-\mu} - C(\epsilon_k), \tag{3.1}$$

where $\mu \in (0, 1)$ is the expenditure share in the industrial good, A is the consumption of the agricultural good, and M is the consumption of the usual CES composite of differentiated varieties of the industrial good, defined by

$$M = \left[\int_{s \in S} d(s)^{\frac{\sigma-1}{\sigma}} \mathrm{d}s\right]^{\frac{\sigma}{\sigma-1}}, \tag{3.2}$$

where $d(s)$ is the consumption of variety s, S is the mass of varieties and $\sigma > 1$ is the constant elasticity of substitution between varieties.

Let $p_{ij}(s)$ represent the delivered price in region i of variety s produced in region j and $d_{ij}(s)$ its demand. Then, the regional price index associated with the composite good (3.2) in region i is

$$P_i = \left[\int_{s \in S} p_{ij}(s)^{1-\sigma} \mathrm{d}s\right]^{\frac{1}{1-\sigma}}. \tag{3.3}$$

Every agent k in region i maximises its utility subject to the budget constraint given by

$$P_i M + A = y_{ki},$$

where y_{ki} represents the nominal income of agent k in region i ($y_{ki} = \epsilon_k w_i$ if skilled and $y_{ki} = 1$ otherwise), P_i is given in (3.3) and the price of the agricultural good is normalised to one. Thus, the demand functions are given by

$$d_{ij}(s) = \mu \frac{p_{ij}(s)^{-\sigma}}{P_i^{1-\sigma}} Y_i, \quad M = y_{ki} \frac{\mu}{P_i}, \quad A = y_{ki}(1-\mu), \tag{3.4}$$

where $Y_i = w_i \epsilon H_i + L/2$ represents the regional income. From (3.1) and (3.4) we derive the indirect utility function in region i for every agent k, which is given by

$$V_i(\epsilon_k) = \frac{y_{ki}}{P_i^{\mu}} - C(\epsilon_k). \tag{3.5}$$

The former expression represents the real wage of an agent net of education costs. Note that education costs are not affected by the price index since they represent an effort cost rather than a monetary one.

The production of the agricultural good uses one unit of unskilled labour per unit produced and has no transportation costs. Thus, $p_1^A = p_2^A = p^A$, which leads us to choose this good as *numeraire* ($p^A = 1$). Since the agricultural market is perfectly competitive, marginal cost pricing implies that the nominal wage of unskilled workers is the same everywhere and, in particular, equal to p^A. Hence, $w_i^L = p^A = 1$.

We assume that the baseline non-full-specialisation (NFS) condition (Baldwin et al., 2003; Gaspar et al., 2020) holds[7], so we have

$$\mu < \frac{\sigma}{2\sigma - 1}.$$

As for the production of the industrial good, both skilled and unskilled labour is used. In particular, each unit produced requires α units of skilled labour and β units of unskilled labour. Therefore, the production cost of an industrial firm in region i is

$$PC_i(x_i) = \alpha w_i + \beta x_i. \tag{3.6}$$

Hence, an industrial firm in region i that produces variety s maximises the profit function

$$\pi_i(s) = \sum_{j=1}^{2} d_{ij}(s) \left[p_{ij}(s) - \tau_{ij}\beta \right] - \alpha w_i, \tag{3.7}$$

where $\tau \in (1, +\infty)$ represents the usual iceberg transportation cost between regions regarding the industrial good. Note that $\tau_{ij} = \tau$ whenever $i \neq j$ and $\tau_{ij} = 1$ otherwise.

[7]Since the total production of the agricultural good in one region is $\lambda/2$ and the total consumption of the agricultural good in the whole economy is $(1-\mu)\left(\frac{\lambda\mu}{\sigma-\mu} + \lambda\right)$, simple algebraic manipulation shows that the NFS condition is $\mu < \sigma/(2\sigma - 1)$, which is precisely the baseline one.

Therefore, profit maximisation of (3.7) yields the optimal prices

$$p_{ij}(s) = \tau_{ij}\beta\frac{\sigma}{\sigma - 1}. \tag{3.8}$$

Considering that a firm produces one unit of industrial good by using α units of skilled labour endowed with a unitary productivity level, each agent k whose productivity is ϵ_k is now able to work in ϵ_k firms, hence producing ϵ_k industrial varieties and earning ϵ_k nominal wages[8]. Thus, since in region i there are H_i agents with ϵ_k productivity, we have that the number of industrial varieties produced is $\epsilon_k H_i/\alpha$. As those agents are homogeneous, we can further simplify and state that the number of industrial varieties produced in region i is $\epsilon H_i/\alpha$, where $\epsilon = \epsilon_k$ represents the average productivity of the economy.

Then, using (3.8) and the fact that the number of industrial varieties produced in region i is $\epsilon H_i/\alpha$, the regional price index of the composite good (3.3) becomes

$$P_i = \frac{\beta\sigma}{\sigma - 1}\left[\frac{\epsilon}{\alpha}\sum_{j=1}^{2}\phi_{ij}H_j\right]^{\frac{1}{1-\sigma}}, \tag{3.9}$$

where $\phi_{ij} \equiv \tau_{ij}^{1-\sigma} \in (0, 1]$ represents the freeness of trade (or the inverse of trade costs) between regions, regarding the industrial good. Note that $\phi_{ij} = \phi$ whenever $i \neq j$ and $\phi_{ij} = 1$ otherwise.

Given the monopolistic competition setup in the industrial market, the free entry condition implies zero profits in equilibrium. Using (3.4), (3.8), and (3.9) into $\pi_i(s) = 0$, the equilibrium wages that skilled workers earn are

$$w_i = \frac{\mu}{\sigma}\sum_{j=1}^{2}\phi_{ij}\frac{\epsilon H_j w_j + L/2}{\sum_{m=1}^{2}\phi_{mj}\epsilon H_m}. \tag{3.10}$$

Thus, by defining the share of skilled workers in region 1 as $h_1 = h = H_1/H$, in region 2 as $h_2 = 1 - h = H_2/H$, and the global immobility ratio as $\lambda = L/H$, it is possible to express the nominal wage per efficiency unit of labour (3.10) as a function of h and ϵ, which yields[9]

$$w_i(h, \epsilon) = \frac{\lambda}{2\epsilon}\frac{\frac{\mu}{\sigma}}{1 - \frac{\mu}{\sigma}}\frac{2h_i\phi + h_j\left[\left(1 - \frac{\mu}{\sigma}\right) + \left(1 + \frac{\mu}{\sigma}\right)\phi^2\right]}{\phi\left[h^2 + (1 - h)^2\right] + h(1 - h)\left[\left(1 - \frac{\mu}{\sigma}\right) + \left(1 + \frac{\mu}{\sigma}\right)\phi^2\right]}. \tag{3.11}$$

[8]Note that we consider ϵ_k a real number and are not concerned that it may be fractional.

[9]Equation (3.10) leads to a system of two equations with two unknowns which can be explicitly solved.

The regional price index (3.9) can also be rewritten as

$$P_i(h,\epsilon) = \epsilon^{\frac{1}{1-\sigma}} \frac{\beta\sigma}{\sigma-1}\left(\frac{H}{\alpha}\right)^{\frac{1}{1-\sigma}} [\phi + (1-\phi)h_i]^{\frac{1}{1-\sigma}} . \tag{3.12}$$

Therefore, by replacing (3.11) and (3.12) in (3.5) the indirect utility of a skilled agent is now

$$V_i^H(h,\epsilon,\epsilon_k) = \epsilon_k \epsilon^{\frac{\mu}{\sigma-1}-1} \frac{\frac{\lambda}{2}\frac{\frac{\mu}{\sigma}}{1-\frac{\mu}{\sigma}} \frac{2h_i\phi + h_j\left[\left(1-\frac{\mu}{\sigma}\right)+\left(1+\frac{\mu}{\sigma}\right)\phi^2\right]}{\phi\left[h^2+(1-h)^2\right]+h(1-h)\left[\left(1-\frac{\mu}{\sigma}\right)+\left(1+\frac{\mu}{\sigma}\right)\phi^2\right]}}{\left[\frac{\beta\sigma}{\sigma-1}\left(\frac{H}{\alpha}\right)^{\frac{1}{1-\sigma}}[\phi+(1-\phi)h_i]^{\frac{1}{1-\sigma}}\right]^{\mu}} - C(\epsilon_k),$$

and, considering $y_{ki} = 1$ instead of $y_{ki} = \epsilon_k w_i(h,\epsilon)$, is equal to

$$V_i^L(h,\epsilon) = \epsilon^{\frac{\mu}{\sigma-1}} \left[\frac{\beta\sigma}{\sigma-1}\left(\frac{H}{\alpha}\right)^{\frac{1}{1-\sigma}}[\phi+(1-\phi)h_i]^{\frac{1}{1-\sigma}}\right]^{-\mu},$$

if the agent is unskilled[10]. The notation $V_i^H(h,\epsilon,\epsilon_k)$ implies that the indirect utility of skilled agents depends not only on their individual productivity level but also on the average productivity level in the society. As each skilled agent produces ϵ_k industrial varieties earning ϵ_k nominal wages and only controls their own productivity level, their decision directly determines how many varieties they produce and their cost of education. However, both the nominal wage per efficiency unit of labour and the regional price index depend on the average productivity level in society.

As for unskilled agents, $V_i^L(h,\epsilon)$ implies that only the average productivity level in the society affects their indirect utility.

Note that $V_i^H(h,\epsilon,\epsilon_k) = \epsilon_k \epsilon^{\frac{\mu}{\sigma-1}-1}\frac{w_i(h,1)}{[P_i(h,1)]^{\mu}} - C(\epsilon_k) = \epsilon_k \epsilon^{\frac{\mu}{\sigma-1}-1}\omega_i(h) - C(\epsilon_k)$, where $w_i(h,1)$ and $P_i(h,1)$ are equivalent to the functions from the baseline Forslid and Ottaviano (2003) model. Therefore, $\omega_i(h)$ can be interpreted as the real wage of a skilled individual in region i in the baseline model – featuring the same behaviour regarding spatial agglomeration – and $\epsilon_k \epsilon^{\frac{\mu}{\sigma-1}-1}\omega_i(h)$ can be interpreted as the real wage per efficiency unit of labour. It is straightforward to conclude that the latter increases with the individual level of education and decreases with the average level of education in the society.

We assume that the baseline no-black-hole (NBH) condition (Gaspar et al., 2020) holds[11], so we have $\sigma > 1 + \mu$.

[10] Note that, in any expression, simply considering $\epsilon = \epsilon_k = 1$ would recover the original equations from the Forslid and Ottaviano (2003) model.

[11] The NBH condition guarantees that the symmetric dispersion ($h = 1/2$) will be stable for

3.2.2 Optimal education level

We start by considering that our two-region economy is ruled by an education minister whose sole decision is the productivity level that all skilled agents must have. All remaining decisions are decentralised. In particular, free migration of citizens is allowed. Let us assume that the education minister's objective is to maximise a welfare function (W^R) that measures the total utility of all the agents in the economy, both skilled and unskilled. This function[12] is then

$$W^R = hHV_1^H(h,\epsilon) + (1-h)HV_2^H(h,\epsilon) + \frac{L}{2}V_1^L(h,\epsilon) + \frac{L}{2}V_2^L(h,\epsilon).$$

The education minister's optimisation problem is then $\max_\epsilon W^R$.

Proposition 3.1. *The socially optimal productivity level satisfies*

$$\frac{\mu}{\sigma-1}\left(h\left[\frac{w_1(h,1)}{[P_1(h,1)]^\mu}\right] + (1-h)\left[\frac{w_2(h,1)}{[P_2(h,1)]^\mu}\right] + \frac{\lambda}{2}\left[\frac{1}{[P_1(h,1)]^\mu} + \frac{1}{[P_2(h,1)]^\mu}\right]\right) = \frac{dC(\epsilon)}{d\epsilon}\epsilon^{-\left(\frac{\mu}{\sigma-1}-1\right)}. \quad (3.13)$$

Proof. See Appendix B. □

The result of the education minister's first-order condition (FOC) consists of two elements, and spatial features (h and ϕ) impact the outcome. First, on the left-hand side, we have the marginal increase in society's welfare measured by the weighted average real wages of all agents. Second, we have the marginal cost of education. Thus, the FOC represents the usual marginal benefit rule in which marginal benefit equals marginal cost at the optimum. Next, we illustrate these solutions considering the constant marginal cost scenario.

Corollary 3.2. *The socially optimal productivity level in the constant marginal cost* ($C(\epsilon) = \gamma(\epsilon - 1)$) *scenario is*

$$\epsilon^R = \left(\frac{\mu}{\sigma-1}\frac{h\left[\frac{w_1(h,1)}{[P_1(h,1)]^\mu}\right] + (1-h)\left[\frac{w_2(h,1)}{[P_2(h,1)]^\mu}\right] + \frac{\lambda}{2}\left[\frac{1}{[P_1(h,1)]^\mu} + \frac{1}{[P_2(h,1)]^\mu}\right]}{\gamma}\right)^{\frac{\sigma-1}{\sigma-(1+\mu)}},$$

and it is higher than one if

$$\gamma < \frac{\mu}{\sigma-1}\left(h\left[\frac{w_1(h,1)}{[P_1(h,1)]^\mu}\right] + (1-h)\left[\frac{w_2(h,1)}{[P_2(h,1)]^\mu}\right] + \frac{\lambda}{2}\left[\frac{1}{[P_1(h,1)]^\mu} + \frac{1}{[P_2(h,1)]^\mu}\right]\right). \quad (3.14)$$

Proof. See Appendix B. □

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some range of transportation costs, thus not precluding it. Since the indirect utilities of our model are a linear combination of the baseline indirect utilities, the NBH condition is not affected.

[12] Since all the agents will have the same productivity chosen by the education minister, we simplify $V_i^H(h,\epsilon,\epsilon)$ as $V_i^H(h,\epsilon)$.

Now, we consider a new scenario in which our two-region economy is no longer a regulated regime but now exists a union that represents all the skilled workers – that willingly adhere to it and consider that the union's decisions are beneficial to all of them. Similarly, we assume that this union's objective is to maximise a welfare function (W^U) that measures the total utility of all the skilled agents in the economy. This function[13] is then

$$W^U = hHV_1^H(h,\epsilon) + (1-h)HV_2^H(h,\epsilon).$$

Therefore, the union's optimisation problem is $\max_\epsilon W^U$.

Proposition 3.3. *The productivity level chosen by the union satisfies*

$$\frac{\mu}{\sigma-1}\left(h\left[\frac{w_1(h,1)}{[P_1(h,1)]^\mu}\right] + (1-h)\left[\frac{w_2(h,1)}{[P_2(h,1)]^\mu}\right]\right) = \frac{dC(\epsilon)}{d\epsilon}\epsilon^{-\left(\frac{\mu}{\sigma-1}-1\right)}. \tag{3.15}$$

Proof. See Appendix B. □

The result of the union's FOC consists of two elements, and spatial features impact the outcome. First, on the left-hand side, we have the marginal increase in the union's welfare measured by the weighted average real wages of all union members. Last, we have the marginal cost of education. The lack of the immobile workers' weighted average real wage is due to the union not caring about the unskilled population. Next, we illustrate these solutions considering the constant marginal cost scenario.

Corollary 3.4. *The optimal unionised productivity level in the constant marginal cost ($C(\epsilon) = \gamma(\epsilon - 1)$) scenario is*

$$\epsilon^U = \left(\frac{\mu}{\sigma-1}\frac{h\left[\frac{w_1(h,1)}{[P_1(h,1)]^\mu}\right] + (1-h)\left[\frac{w_2(h,1)}{[P_2(h,1)]^\mu}\right]}{\gamma}\right)^{\frac{\sigma-1}{\sigma-(1+\mu)}},$$

and it is higher than one if

$$\gamma < \frac{\mu}{\sigma-1}\left(h\left[\frac{w_1(h,1)}{[P_1(h,1)]^\mu}\right] + (1-h)\left[\frac{w_2(h,1)}{[P_2(h,1)]^\mu}\right]\right). \tag{3.16}$$

Proof. See Appendix B. □

In the last scenario, we consider that our two-region economy is now totally decentralised. Thus, the choice of productivity level is made individually by each

[13] The union's decision regarding the productivity level will translate into the individual choices, thus, once again, $\epsilon_k = \epsilon$, which allows us to simplify notation as before.

agent. Since each agent knows that it is possible to change locations at any time after choosing the productivity level but that they are myopic[14] and too small to change the general outcome for the average ϵ, we will assume that each mobile worker's objective is to maximise an individual welfare function (W^I) that weights the probability of ending up in either region. This function is then

$$W^I = hV_1^H(h,\epsilon,\epsilon_k) + (1-h)V_2^H(h,\epsilon,\epsilon_k).$$

Therefore, the individual's optimisation problem is $\max_{\epsilon_k} W^I$.

Proposition 3.5. *The productivity level chosen by each individual satisfies*

$$h\left[\frac{w_1(h,1)}{[P_1(h,1)]^\mu}\right] + (1-h)\left[\frac{w_2(h,1)}{[P_2(h,1)]^\mu}\right] = \frac{dC(\epsilon)}{d\epsilon}\epsilon^{-\left(\frac{\mu}{\sigma-1}-1\right)}. \tag{3.17}$$

Proof. See Appendix B. □

The result of the individual agent's FOC is far simpler than that of the education minister and the union. As before, this result does depend on spatial features. This FOC relates the weighted average real wage of a skilled individual and their marginal cost of education. The lack of the ratio of expenditure share in the industrial good and its elasticity of substitution occurs due to the fact that, unlike the minister of education and the union, an individual is unable to change the education decision of their peers, hence not influencing the average level of education in the society. Next, we illustrate these solutions considering the constant marginal cost scenario.

Corollary 3.6. *The optimal individual productivity level in the constant marginal cost ($C(\epsilon) = \gamma(\epsilon-1)$) scenario is*

$$\epsilon^I = \left(\frac{h\left[\frac{w_1(h,1)}{[P_1(h,1)]^\mu}\right] + (1-h)\left[\frac{w_2(h,1)}{[P_2(h,1)]^\mu}\right]}{\gamma}\right)^{\frac{\sigma-1}{\sigma-(1+\mu)}},$$

and it is higher than one if

$$\gamma < h\left[\frac{w_1(h,1)}{[P_1(h,1)]^\mu}\right] + (1-h)\left[\frac{w_2(h,1)}{[P_2(h,1)]^\mu}\right]. \tag{3.18}$$

Proof. See Appendix B. □

[14]Given that we assume that the agents are not forward-looking and that the education opportunity occurs before the migration decisions take place, we consider that the optimisation is done considering the average spatial distribution. Even though it is clear that after the migration occurs, the agents could be better off with a different productivity level, only their conceptualisation as forward-looking would allow that clairvoyance.

3.2.2.1 Education drivers

Given the optimal decisions in the three types of society we have just derived, an analysis of their partial derivatives allows us to determine that the optimal productivity level increases with the share of expenditure in the industrial good (μ) and with the proportion of immobile to mobile agents (λ). Moreover, it decreases with the unitary cost of education (γ) and with the elasticity of substitution between industrial varieties (σ).

Regarding spatial features, Figure 3.1 gives some clues on its effect on the optimal levels. The optimal level for the union is almost invariant to spatial features. The one for the regulated economy is influenced both by the agglomeration level and the transportation costs, with the latter being more prominent. Both spatial features also influence the optimal level for an individual, but the effects are more balanced.

Regarding the effect of agglomeration on the optimal levels, we have that higher agglomeration leads to lower productivity levels. Even though, when the decision is individual, there is a range of extreme values for h and ϕ that lead to higher productivity levels instead. Moreover, as the transportation costs tend to zero ($\phi \to 1$), the decision seems to become independent of spatial agglomeration.

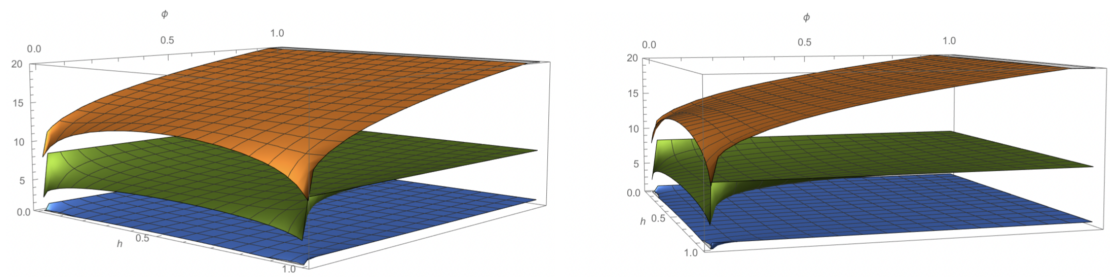


Figure 3.1: The plot shows the optimal productivity level for the three cases studied in two perspectives. The parameters used are $\sigma = 2.4$, $\mu = 0.55$, $\lambda = 5.5$, and $\gamma = 0.45$. In orange we have ϵ^R, in blue we have ϵ^U, and in green we have ϵ^I.

3.2.3 Social decisions on optimal education levels

We can compare all the optimal decisions to understand how they can be ordered.

Proposition 3.7. *The decisions about the optimal productivity level are ordered as* $\epsilon^R > \epsilon^I > \epsilon^U$*, should the no-black-hole condition hold.*

Proof. Since the right-hand side of (3.13), (3.15), and (3.17) are the same, the values of ϵ in equality satisfy the ordering as that of the left-hand side. It is clear that $\epsilon^R > \epsilon^U$ since the difference between the two is the immobile workers' average weighted real wage for a unitary productivity level, which is clearly positive. The analytical solution for $\epsilon^R > \epsilon^I$ seems to be impossible, due to the complexity of considering both h and ϕ variable simultaneously, however extensive numerical simulations show that it is always true. Finally, $\epsilon^I > \epsilon^U$ is equivalent to the NBH condition ($\sigma > 1 + \mu$). □

In the constant marginal cost scenario – as we have detailed in conditions (3.14), (3.16), and (3.18) –, optimal productivity levels above one are feasible, but not guaranteed.

From conditions (3.14), (3.16), and (3.18), it is easy to verify that the threshold for the unitary cost of productivity (γ) that ensures a higher than one productivity level for the regulated economy is the highest of the three[15]. The lowest one is the one associated with the union scenario. Therefore, we conclude that the education minister is the decision-maker willing to accept the highest costs – that will be burdened by the skilled agents forced to increase their qualifications to the desired level ruled by the education minister. Conversely, the union is only willing to accept the lowest education costs.

Should we rearrange conditions (3.14), (3.16), and (3.18) with respect to the expenditure share in the industrial good (μ), it would be easy to verify that the threshold for μ that ensures a higher than one productivity level for the regulated economy is the lowest of the three[16]. The highest one is the one associated with the union scenario. Therefore, we conclude that a low level of expenditure on the industrial good is sufficient for investment in productivity to occur in a regulated economy. On the contrary, if a union exists for the industrial workers, investment in productivity only happens with a high expenditure on the industrial good.

3.2.3.1 Externalities

As we have discussed, when an individual agent decides to partake in the education sector, their scope of influence is limited to himself[17]. Meanwhile, we can state that the social aggregate change due to an average variation in the optimal productivity level – caused by several individual decisions – or, more broadly, in the human capital level (Moretti, 2004), is the definition of externality, or social return.

In fact, it is straightforward to conclude that the indirect utilities of unskilled agents ($V_i^L(h, \epsilon)$) increase as the productivity level rises. Even though immobile workers do not engage in the education sector, they still benefit from the mobile agents who do – thus, education has a spillover effect.

[15] The ordering of the thresholds is the same as that of the optimal productivity levels.

[16] The ordering of the thresholds is the inverse of that of the optimal productivity levels.

[17] In the words of Lucas (1988, p. 18), “though all benefit from it, no individual human capital accumulation decision can have an appreciable effect on the average human capital level, so no one will take it into account”.

Proposition 3.8. *Education generates a positive externality that affects all the agents in the economy.*

Proof. To evaluate the social return of education, we compute the difference between the indirect utility of a society with an average productivity level ϵ and a unitary one. Therefore, we are essentially analysing $W^R(h,\epsilon) - W^R(h,1)$.

$$\begin{aligned}
&\left[hHV_1^H(h,\epsilon) + (1-h)HV_2^H(h,\epsilon) + \frac{L}{2}V_1^L(h,\epsilon) + \frac{L}{2}V_2^L(h,\epsilon)\right] \\
&- \left[hHV_1^H(h,1) + (1-h)HV_2^H(h,1) + \frac{L}{2}V_1^L(h,1) + \frac{L}{2}V_2^L(h,1)\right] \\
= &\left[h\left(\epsilon^{\frac{\mu}{\sigma-1}}\tfrac{w_1(h,1)}{[P_1(h,1)]^\mu} - C(\epsilon)\right) + (1-h)\left(\epsilon^{\frac{\mu}{\sigma-1}}\tfrac{w_2(h,1)}{[P_2(h,1)]^\mu} - C(\epsilon)\right) + \tfrac{\lambda}{2}\left(\tfrac{\epsilon^{\frac{\mu}{\sigma-1}}}{[P_1(h,1)]^\mu} + \tfrac{\epsilon^{\frac{\mu}{\sigma-1}}}{[P_2(h,1)]^\mu}\right)\right] \\
&- \left[h\left(\frac{w_1(h,1)}{[P_1(h,1)]^\mu}\right) + (1-h)\left(\frac{w_2(h,1)}{[P_2(h,1)]^\mu}\right) + \frac{\lambda}{2}\left(\frac{1}{[P_1(h,1)]^\mu} + \frac{1}{[P_2(h,1)]^\mu}\right)\right] \\
= &\ \epsilon^{\frac{\mu}{\sigma-1}} W^R(h,1) - C(\epsilon) - W^R(h,1) = \left(\epsilon^{\frac{\mu}{\sigma-1}} - 1\right) W^R(h,1) - C(\epsilon)
\end{aligned}$$

Therefore, since the cost of education is private, we conclude that education has positive externalities that affect the whole society because $\epsilon^{\frac{\mu}{\sigma-1}} - 1 > 1$ as long as $\epsilon > 1$. □

Corollary 3.9. *For each unit percentage increase in productivity, social welfare increases by approximately $\frac{\mu}{\sigma-1}$ per cent.*

Proof. Straightforward logarithmisation of $\epsilon^{\frac{\mu}{\sigma-1}} - 1$ from Proposition 3.8 yields the result, whose approximation accuracy is high for relative increases of productivity that are not too large. □

Since an education externality corresponds to the social aggregate change due to an average variation in the human capital level, education has positive externalities due to price reductions and consequent real wage increases. Thus, all the agents in the economy enjoy an increase in welfare due to higher levels of education, which implies that society as a whole is better off.

This explains why the optimal education decisions can be ordered as $\epsilon^R > \epsilon^I > \epsilon^U$. The education minister will aim at a high education level to increase the benefit of those agents who do not study, thus totally internalising this effect. The union will use this knowledge about market influence to choose an education level that maximises the welfare of its members while keeping their education costs as low as possible, thus partially internalising this effect. As for the individual agent, the inability to internalise this effect due to their atomicity will make him overshoot the union's optimum – which would be better for him – and burden a higher education cost.

3.2.4 Strategic profile of education

An interesting remark regarding individual (ϵ_k) and average (ϵ) education levels is whether they reinforce or offset each other.

Proposition 3.10. *Individual and average education levels are strategic substitutes.*

Proof. We have that the individual education level and the average education level are strategic substitutes if $\frac{\partial^2 V_i^H}{\partial \epsilon_k \partial \epsilon} < 0$. So, $\frac{\partial V_i^H}{\partial \epsilon_k} = \epsilon^{\frac{\mu}{\sigma-1}-1} \frac{w_i(h,1)}{[P_i(h,1)]^\mu} - \frac{dC(\epsilon_k)}{d\epsilon_k}$, $\frac{\partial^2 V_i^H}{\partial \epsilon_k \partial \epsilon} = \epsilon^{\frac{\mu}{\sigma-1}-2} \left(\frac{\mu}{\sigma-1} - 1\right) \frac{w_i(h,1)}{[P_i(h,1)]^\mu}$, which is clearly negative since $\mu \in (0,1)$ and $\sigma > 1$. □

Since individual and average education levels are strategic substitutes, the incentives for society and individuals are opposite. In particular, the higher the average level of education in the economy is, the least an individual wants to study.

3.2.5 Robustness

Finally, we test the robustness of our model by re-evaluating it, considering now the footloose entrepreneur model with quasi-linear (QL) log utility (Gaspar et al., 2018; Pflüger, 2004). This model, unlike the Forslid and Ottaviano (2003) one, does not feature an income effect, thus being interesting to test the robustness of our results. For brevity[18], we only present the most relevant expressions of the model. All omitted expressions are the same and have the same interpretation as before. In particular, the regional price index function is common to both models.

In the QL model, the utility function of every agent k in the economy is

$$U_{QL} = \mu \ln M_{QL} + A_{QL} - C(\epsilon_k), \quad 0 < \mu < 1,$$

the wages of skilled agents are given by

$$w_{i_{QL}}(h, \epsilon) = \frac{\mu}{\sigma} \sum_{j=1}^{2} \phi_{ij} \frac{h_j + \lambda/2}{\epsilon \left[\phi + (1-\phi)h_j\right]},$$

and the indirect utility functions are

$$V_{i_{QL}}^H(h, \epsilon, \epsilon_k) = \epsilon_k w_{i_{QL}}(h, \epsilon) + \frac{\mu}{\sigma - 1} \ln\left[\phi + (1-\phi)h_i\right] + \frac{\mu}{\sigma - 1} \ln(\epsilon) - C(\epsilon_k) + \eta,$$

$$V_{i_{QL}}^L(h, \epsilon) = 1 + \frac{\mu}{\sigma - 1} \ln\left[\phi + (1-\phi)h_i\right] + \frac{\mu}{\sigma - 1} \ln(\epsilon) + \eta,$$

where $\eta = -\mu \ln\left(\beta \frac{\sigma}{\sigma-1}\right) + \frac{\mu}{\sigma-1} \ln\left(\frac{H}{\alpha}\right) + \mu(\ln \mu - 1)$ is a constant.

[18] For a complete derivation of this model, see Appendix A.

Considering the same welfare functions (W^R, W^U, and W^I) as before, we have that the education minister's, the union's, and the individual's FOCs are, respectively,

$$\frac{\mu}{\sigma-1}(1+\lambda) = \frac{dC(\epsilon)}{d\epsilon}\epsilon, \quad \frac{\mu}{\sigma-1} = \frac{dC(\epsilon)}{d\epsilon}\epsilon, \quad hw_1(h,1) + (1-h)w_2(h,1) = \frac{dC(\epsilon)}{d\epsilon}\epsilon.$$

From these FOCs, we conclude that the marginal benefit rule still applies. However, the marginal benefit in welfare is only transmitted by prices in a regulated or unionised society. In these scenarios, the education externality is fully internalised and wages are unaffected. When the decision is individual, though, the agents are not able to anticipate their impact on the aggregate price index and the marginal welfare is seen as an income one, even though their nominal wage remains constant[19] and only the real wage increases.

These key differences can be traced back to the income effect in the Forslid and Ottaviano (2003) model. Since the consumption of the industrial good depends on nominal wage, increases in productivity lead to increases in consumption that are transmitted not only by prices but also by wages.

Using the same methods as before to find the three optimal levels of education yields

$$\epsilon^R_{QL} = \frac{(1+\lambda)\frac{\mu}{\sigma-1}}{\gamma}, \quad \epsilon^U_{QL} = \frac{\frac{\mu}{\sigma-1}}{\gamma}, \quad \epsilon^I_{QL} = \frac{(1+\lambda)\frac{\mu}{\sigma}}{\gamma}.$$

Finally, it is straightforward to show that no spatial feature (h or ϕ) influences the optimal education level under the QL model and that the education drivers all have the same qualitative effect as before. Moreover, it is also evident that $\epsilon^R_{QL} > \epsilon^I_{QL} > \epsilon^U_{QL}$, should the no-black-hole condition $\left(\lambda > \frac{\sigma}{\sigma-1}\right)$ hold, as well as the existence of a positive externality of education. Last, verifying that individual and average education levels remain strategic substitutes in this specification is also trivial.

3.3 Concluding remarks

In this essay, we study how agents make their education decisions and how the spatial distribution of industry and the economic framework affect this choice.

We build a simple education mechanism, which acts as a proxy for productivity, and focus on the choice of the optimal education level while also accounting for the cost of education – which we consider to be the agent's effort to become qualified.

[19] Note that $hw_1(h,1) + (1-h)w_2(h,1)$ is simply the weighted average nominal wage ($\bar{w}$) shown by Gaspar et al. (2018). Hence, $\bar{w} = (1+\lambda)\frac{\mu}{\sigma}$, which is constant.

In this derivation, which is a maximisation problem, we analyse three different types of society. First, the agents are within a regulated economy, in which the education minister chooses the productivity level that all workers must follow. Second, the agents are members of a union that decides their productivity level, which they strictly follow. Last, the agents live in a totally decentralised economy in which they are free to choose the productivity level they deem fit.

Following the results of this maximisation, we conclude how the conditions of the economy drive the optimal education level. In particular, the education level increases with the share of expenditure on the industrial good and with the proportion of immobile to mobile agents, while it decreases with the unitary cost of education and with the elasticity of substitution between industrial varieties. Therefore, one can expect that an economy which spends much on the industrial good, has relatively more unskilled workers, prefers less variety, and in which education is cheap, to have a high productivity level.

Our results on the education drivers are particularly important since we fill the gap stated by Blanchard and Olney (2017). Furthermore, our conclusions are aligned with those of Picard and Toulemonde (2004) and Redding and Schott (2003).

Apart from the education drivers, solving this maximisation problem also allows us to conclude that spatial features – that is, the level of agglomeration and the transportation costs – influence education decisions. We find that a highly unionised society is the least affected by spatial features, while the regulated economy is the one in which spatial features have the highest range of influence. Higher agglomeration levels, as do higher transportation costs, generally lead to lower optimal productivity levels. When transportation costs tend to zero, the optimal choice becomes independent of spatial agglomeration, which makes sense since a society like that can be considered regionless.

Generally, even though a higher agglomeration level leads to lower education levels, there are different qualitative behaviours in the optimal choices of the education minister and an individual agent. In the regulated scenario, agglomeration's effect on the education level is linear. Hence, more agglomeration means lower levels of education. On the opposite end, when the decision is decentralised, the effect of agglomeration in the education level is not linear since there is a range of extremely high transportation costs that inverts the general pattern. Thus, within this range, more agglomeration means higher levels of education.

We find that the optimal decisions of the three different types of society we study have a specific order – the education minister always chooses the highest productivity levels amongst the three, whereas the union chooses the lowest. This order follows a simple rationale. For the regulated economy, since the education minister equally cares about all the agents and that education has positive externalities for everyone, he has the incentive to choose higher education levels to increase the total welfare in the economy by maximising the externalities. Conversely, since the union is only concerned with its members, it wants to maximise their private returns. Hence, given the cost of education, its choice is the lowest. Finally, an independent agent must balance their private return, education cost, and location choice. Since they cannot account for externalities – unlike the education minister and the union – and are not sure where they will live after qualifying, they choose an intermediate education level that acts as insurance on their expected welfare.

We explore whether education exerts a positive spillover in the community and find that it does, even for the immobile agents who do not have the possibility to study. Moreover, we can quantify this externality and conclude that it is more significant the less the agents prefer variety and the more they spend on industrial goods. While this effect is not unheard of in the literature, we endogenously include it in an Economic Geography model and present its value explicitly. Thus, filling the gap stated by Moretti (2004).

We study the strategic profile of education and find that an individual's education level and society's average education level are strategic substitutes. This means that the more skilled a society is, the less skilled a particular agent wants to be. Thus, having an almost free-riding behaviour by desiring to enjoy the decrease in prices that higher levels of education generate without having to incur education costs.

Finally, we address the robustness of our specification by implementing the same education mechanism in the Pflüger (2004) model. Even though quantitative differences are inevitable, all our previous conclusions remain sound and are qualitatively equivalent between the two models.

The approach we develop in this essay does not allow us to study the subsidising policies that regions can implement and is still only of partial equilibrium – we study how agglomeration affects education, but not how education affects agglomeration. However, this work opens an interesting line for future research – to consider an endogenous general equilibrium setting that analyses both decisions simultaneously.

Furthermore, our insight into education externalities can open a debate regarding the social aspects of education funding for policymakers – since the qualifications that only some agents pursue have positive spillovers for everyone in society, should students support the tuition fees? Or should this cost be diluted and supported by everyone in the economy – through higher taxes, for example – since everyone benefits from higher levels of education?

Chapter 4

On endogenous education and agglomeration dynamics

4.1 Introduction

The purpose of this essay is to study how the spatial and educational decisions of entrepreneurs influence each other endogenously. In particular, we study the transfer of knowledge through formal education. Therefore, while we keep the creation of knowledge as a black box, we study how learning affects the real wage of individuals and, ultimately, how it influences their migration decisions.

To achieve our goal, we combine the two-region quasi-linear log utility footloose entrepreneur model with an overlapping generations model. Thus, our novelty is to construct a new approach to analyse spatial problems while maintaining some core properties of classical Economic Geography models when agents are forward-looking.

Since our focus is on formal education, we consider that agents live for two periods and have the opportunity to study in the first one. While this assumption leaves out the learning-by-doing referred to by Lucas (1988) and the possibility of studying and working simultaneously, it is important for the tractability of the model. Even so, this assumption lets us explore one additional geographical feature – the migrations of qualified and unqualified workers separately.

We also consider that the agents' education decision is one of quality and not quantity. Thus, students do not choose how many years to study or how many degrees they take. They only choose whether to study, given the quality of the university. Finally, we consider that there are no capacity restrictions at the university.

While we already achieve some interesting results, more research is needed, particularly in fully characterising the different types of equilibria since the behaviour of the equilibria is rather complex. Even so, our definition of the equilibrium conditions allows us to make some remarks.

We find that the productivity gains from education do not necessarily lead qualified workers to earn a higher lifetime nominal income than unqualified ones. However, even when the nominal income for qualified workers is lower than for unqualified ones, education can still occur and be optimal, given the different life paths that open up for qualified agents that are not available for unqualified ones.

In particular, we conclude that when lifetime income is the same for both groups, apart from the scenarios in which both types of workers are totally concentrated, qualified agents agglomerate themselves in the region that offers a university, while the majority of the unqualified workers choose to live in the other region. In this situation, the education rate never exceeds fifty per cent, and it lowers as more unqualified workers choose to live in the region that offers a university.

Note that our conceptualisation of education within an Economic Geography model is aligned with the concerns of Fujita (2007) in suggesting that it would probably be an important force in explaining the agglomeration of economic activity. As Fujita puts it, developed countries consider that the creation of knowledge plays a central role in their prosperity. Moreover, when assessing Economic Geography, Fujita notes that while dispersion forces are easily explained within the literature and the Economic Geography models, the agglomeration forces are essentially pecuniary externalities. This discussion leads to Fujita's suggestion of developing microfounded knowledge-linkages (K-linkages) within Economic Geography models, thus moving closer to constructing a comprehensive theory of geographical economics in which knowledge plays a role. Fujita also states that the creation and transfer of knowledge should be clearly distinguished.

In a survey that attempts to map the frontiers of the Economic Geography field, Fujita and Mori (2005) also advocate for the inclusion of agent heterogeneity, such as skills, and for the research of K-linkages to achieve a more generalised framework for the study of spatial problems.

Ottaviano (1999) and Oyama (2009) make two noteworthy contributions to the literature by implementing models with forward-looking agents that trace the path for its future use. While Ottaviano work is more straightforward and essentially a seminal approach to rational expectations, the study of Oyama is remarkable not only for its technical detail but also for the conclusion that, under rational expectations, several equilibria that exist in classical Economic Geography models with myopic agents cease to be stable and only one equilibrium subsists.

Faggian and McCann (2009) explore the link between the location of universities and the mobility of human capital and discuss the migration flows home-university and university-first job. They find that graduated workers have a high propensity to move far away from where they studied.

Abel and Deitz (2011) conduct an empirical study on the impact of the existence of universities on human capital and conclude that it can increase the regional stock of human capital due to the increase of both supply and demand for skills.

Although the results of Faggian and McCann (2009) and Abel and Deitz (2011) seem conflictual, as the former states that universities drive qualified workers away from its region, while the latter states the opposite, it may be the case that both are correct. Even though our results are more diffuse, we find evidence corroborating both works. On the one hand, we find the existence of interior equilibria that links higher productivity gains with a lower share of the qualified population working in their university's region. On the other hand, we also find that higher productivity gains can lead to the segregation of qualified and unqualified workers, in particular, making qualified workers agglomerated in the university's region. However, note that this last effect eventually fades out, which implies that, for very significant productivity gains, the thesis of Abel and Deitz would lose support.

Marré and Rupasingha (2020) study the relationship between school quality and migrations – particularly to rural areas – and find that better schools can indeed drive families to those peripheral areas. Thus, we can argue that if we added other universities, the results of our model would be similar to those of Marré and Rupasingha, as agents would look for the best education opportunities, given the cost.

4.2 The education problem

We study a two-region economy with two production sectors and a quasi-linear log utility (Gaspar et al., 2018; Pflüger, 2004) within an overlapping generations framework. We consider that every agent in this economy lives for two periods and that every generation has the same size. Therefore, in any period, we have one generation living in the first stage of their lives and another living in the second. We assume that all individuals are homogeneous upon birth regarding their preferences and skill level.

For simplicity, we ignore the first years in the life of an agent and consider that the first relevant period of life starts when they conclude mandatory schooling, which is equally available for all. Therefore, the assumption of the agents' homogeneity upon birth still stands true at this point.

When an agent begins this first period, they face a decision that will influence the rest of their life. Every individual must choose between pursuing an academic degree (and always completing it successfully) or starting to work immediately. The rules of this choice are common knowledge to everyone.

If the decision is to integrate the workforce in the first period, the agent becomes an unqualified high-skilled worker (H^W) with a unitary productivity level for both the first and second periods, earning the one market nominal wage (w) in each period. These agents are free to choose where to live in each period.

Instead, if the decision is to pursue a higher degree, the agent becomes a student (S_1) in the first period and does not earn any income during that time. Moreover, we assume that only region 1 offers a university, so every student knows where they live in the first period. The productivity gain (ψ) offered by a higher degree means that a student has a productivity level $\epsilon = 1 + \psi > 1$ by the end of the first period.

When the first stage of life closes, students become qualified high-skilled workers (H^S) and earn $\epsilon > 1$ times the market nominal wage w. Moreover, these agents can freely choose where they want to work.

Finally, note that, for simplification and without loss of generality, we assume that students consume even though they have no income. We consider that they can accurately anticipate their future second-stage income and will borrow, as needed, in an interest-free loan.

To summarise, the decision process of an individual at the start of their first life period is as follows.

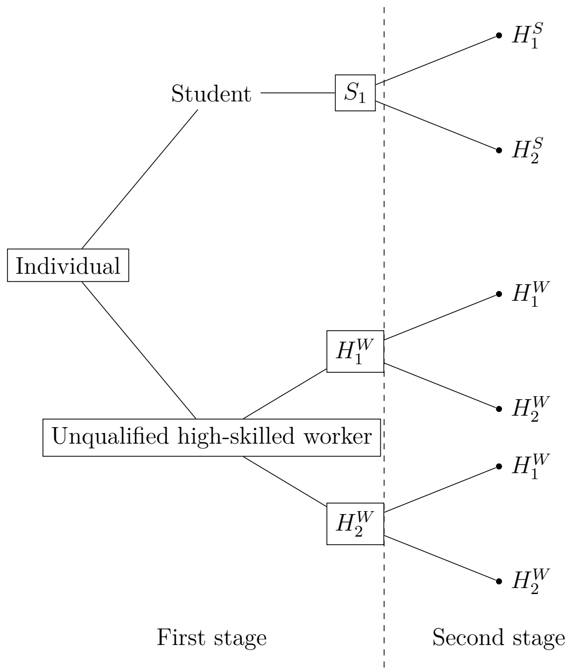


Figure 4.1: The figure shows the decisions and possible life paths of any high-skilled agent. Boxed nodes indicate that the agents can make a choice. Subscripts represent the region, while the superscript differentiates high-skilled workers between those who studied (S) and those who did not (W).

4.2.1 Economic model

In this economy, there are L unskilled workers – equally divided between the two regions – that are immobile between regions, H^W high-skilled workers – H_i^W in region $i = \{1, 2\}$ – that are mobile between regions, H^S qualified high-skilled workers – H_i^S in region i – that are also mobile between regions, and S_1 students in region 1.

Since the agents live for two periods, let us consider, without loss of generality, that those periods are $t = \{1, 2\}$, which implies that the skilled agents may work or study in $t = 1$ and may only work in $t = 2$.

The preferences of all agents in period t are defined by

$$U^t = \mu \ln M^t + A^t, \tag{4.1}$$

where $\mu \in (0, 1)$ is the expenditure share in the industrial good, A^t is the consumption of the agricultural good in period t, and M^t is the consumption of the usual CES composite of differentiated varieties of the industrial good in period t, defined by

$$M^t = \left[\int_{z\in Z^t} d^t(z)^{\frac{\sigma-1}{\sigma}} \mathrm{d}z\right]^{\frac{\sigma}{\sigma-1}}, \tag{4.2}$$

where $d^t(z)$ is the consumption of variety z in period t, Z is the mass of varieties in period t and $\sigma > 1$ is the constant elasticity of substitution between varieties.

Let $p_{ij}^t(z)$ represent the delivered price in region i of variety z produced in region j in period t and $d_{ij}^t(z)$ its demand. Then, the regional price index associated with the composite good (4.2) in region i in period t is

$$P_i^t = \left[\int_{z\in Z^t} p_{ij}^t(z)^{1-\sigma} \mathrm{d}z\right]^{\frac{1}{1-\sigma}}. \tag{4.3}$$

The global preferences[20] of all agents over their whole life are

$$U = U^1 + U^2 = \mu \ln \left[M^1 M^2\right] + \left(A^1 + A^2\right),$$

and every agent maximises their utility subject to the intertemporal[21] budget constraint, given by

$$P_j^1 M_j^1 + P_i^2 M_i^2 + A_j^1 + A_i^2 = y_{ji},$$

where y_{ji} represents[22] the nominal income of the agent ($y_{ji} = \epsilon_j w_i$ if qualified,

[20]For simplicity, we ignore a time preference parameter.

[21]For simplicity, we ignore a discount rate.

[22]The subscript ji in nominal income and indirect utility means that an agent lives in region j in the first period and in region i in the second. Note that we do not exclude that $j = i$.

$y_{ji} = w_j + w_i$ if high-skilled, and $y_{ij} = 1$ otherwise), P_i is given in (4.3), and the price of the agricultural good is normalised to one. Thus, the demand functions are given by

$$d_{ij}^t(z) = \mu \frac{p_{ij}(z)^{-\sigma}}{P_i^{1-\sigma}}, \quad M_i^t = \frac{\mu}{P_i^t}, \quad A_j^1 + A_i^2 = y_{ji} - 2\mu. \tag{4.4}$$

From (4.1) and (4.4) we derive the global indirect utility function from living in regions i and j sequentially, which is given by

$$V_{ji}(h) = y_{ji} - \mu \ln \left[P_j^1 P_i^2 \right] + 2\mu \left(\ln [\mu] - 1 \right).$$

Note that, in equilibrium, the decisions of each generation are perpetuated. This implies that the choices made by the second generation are the same as those made by the first, and so on. Thus, we have that $P_i^1 = P_i^2 = P_i$.

Hence, we have that the global indirect utility of a qualified high-skilled worker $\left(V_{ji}^{H^S} \right)$ is given by

$$V_{ji}^{H^S} = \epsilon_j w_i - \mu \ln \left[P_j P_i \right] + 2\mu \left(\ln [\mu] - 1 \right),$$

and that the global indirect utility of an unqualified high-skilled worker $\left(V_{ji}^{H^W} \right)$ is given by

$$V_{ji}^{H^W} = w_j + w_i - \mu \ln \left[P_j P_i \right] + 2\mu \left(\ln [\mu] - 1 \right).$$

The production of the agricultural good uses one unit of unskilled labour per unit produced and has no transportation costs. Thus, $p_1^A = p_2^A = p^A$, which leads us to choose this good as *numeraire* ($p^A = 1$). Since the agricultural market is perfectly competitive, marginal cost pricing implies that the nominal wage of unskilled workers is the same everywhere and, in particular, equal to p^A. Hence, $w_i^L = p^A = 1$.

As for the production of the industrial good, both skilled and unskilled labour is used. In particular, each unit produced requires α/ϵ units of skilled labour and β units of unskilled labour. Thus, the individual productivity heterogeneity we introduce affects the input requirement of skilled labour by assuming that qualified individuals have a productivity $\epsilon > 1$ and unqualified ones $\epsilon = 1$. Therefore, the production cost of an industrial firm in region i is

$$PC_i(x_i) = \frac{\alpha}{\epsilon} w_i + \beta x_i.$$

Hence, an industrial firm in region i that produces variety z maximises the profit function

$$\pi_i(z) = \sum_{j=1}^{2} d_{ij}(z) \left(S_j + H_j^S + H_j^W + L/2\right) [p_{ij}(z) - \tau_{ij}] - \frac{\alpha}{\epsilon} w_i, \tag{4.5}$$

where $\tau \in (1, +\infty)$ represents the usual iceberg transportation cost between regions regarding the industrial good. Note that $\tau_{ij} = \tau$ whenever $i \neq j$ and $\tau_{ij} = 1$ otherwise.

Therefore, profit maximisation of (4.5) yields the optimal prices

$$p_{ij}(z) = \tau_{ij} \beta \frac{\sigma}{\sigma - 1}. \tag{4.6}$$

Considering that H^W firms produce one unit of industrial good by using α units of skilled labour and H^{S_1} firms only need α/ϵ due to the higher productivity of its entrepreneur, we have that the number of industrial varieties produced in region i is given by $\frac{\epsilon H_i^S + H_i^W}{\alpha}$.

Then, using (4.6) and the fact that the number of industrial varieties produced in region i is given by $\frac{\epsilon H_i^S + H_i^W}{\alpha}$, the regional price index of the composite good (4.3) becomes

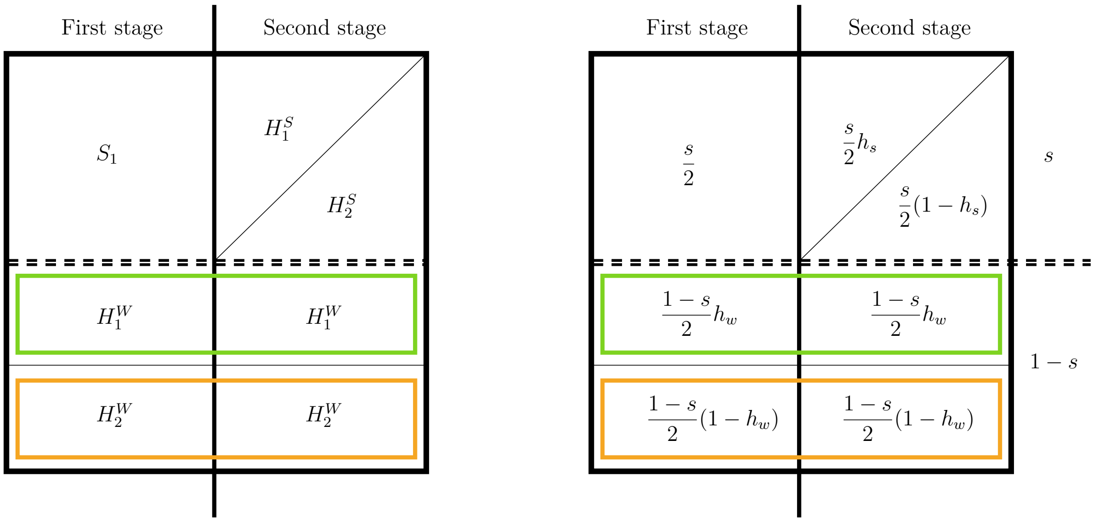
$$P_i = \frac{\beta \sigma}{\sigma - 1} \left(\frac{1}{\alpha}\right)^{\frac{1}{1-\sigma}} \left[\sum_{j=1}^{2} \phi_{ij} \epsilon H_j^S + \sum_{j=1}^{2} \phi_{ij} H_j^W\right]^{\frac{1}{1-\sigma}}, \tag{4.7}$$

where $\phi_{ij} \equiv \tau_{ij}^{1-\sigma} \in (0, 1]$ represents the freeness of trade (or the inverse of trade costs) between regions, regarding the industrial good. Note that $\phi_{ij} = \phi$ whenever $i \neq j$ and $\phi_{ij} = 1$ otherwise.

Given the monopolistic competition setup in the industrial market, the free entry condition implies zero profits in equilibrium. Using (4.4), (4.6), and (4.7) into $\pi_i(s) = 0$, the equilibrium wages that skilled workers earn are given by

$$w_i = \frac{\mu}{\sigma} \sum_{j=1}^{2} \phi_{ij} \frac{S_j + H_j^S + H_j^W + L/2}{\sum_{m=1}^{2} \phi_{mj} \epsilon H_m^S + \sum_{m=1}^{2} \phi_{mj} H_m^W}. \tag{4.8}$$

Finally, we want to describe the spatial distribution of industry with the share of qualified (h_s) and unqualified (h_w) entrepreneurs living in each region, given the share of students (s). Figure 4.2 is helpful to show who lives where at any given period of time and how that absolute distribution can be translated into relative shares of the population.



(a) Absolute population

(b) Shares of population

Figure 4.2: The figure represents the general distribution of the population. In both cases, the thick square represents the totality of the H agents. Since we assume equal generation sizes, both horizontal sides of the square have the same area. The dashed line separates the agents who study from those who do not. Solid thin lines separate the region where each type of agent lives. On the left panel, we have the absolute population diagram. On the right panel, we have that same population expressed as a share of the total H agents, which implies that the sum of all terms is equal to one.

Thus, by defining the share of qualified high-skilled workers in region i as $h_{s_i} = H_i^S/H^S$, $h_{s_1} = h_s$ and $h_{s_2} = 1 - h_s$, the share of unqualified high-skilled workers in region i as $h_{w_i} = H_i^W/H^W$, $h_{w_1} = h_w$ and $h_{w_2} = 1 - h_w$, and the global immobility ratio as $\lambda = L/H$, it is possible to express the nominal wage (4.8) as a function of s, h_s, and h_w, which yields

$$w_i(s, h_s, h_w) = \frac{\mu}{\sigma} \sum_{j=1}^{2} \phi_{ij} \frac{\frac{\chi_j}{2} + \frac{s}{2} h_{s_j} + (1-s) h_{w_j} + \lambda/2}{\epsilon \frac{s}{2} \left[\phi + h_{s_j}(1-\phi)\right] + (1-s)\left[\phi + h_{w_j}(1-\phi)\right]},$$

where $\chi_1 = s$ and $\chi_2 = 0$. The regional price index (4.7) can also be rewritten as

$$P_i(s, h_s, h_w) = \frac{\beta\sigma}{\sigma - 1} \left(\frac{H}{\alpha}\right)^{\frac{1}{1-\sigma}} \left[\epsilon \frac{s}{2} [\phi + h_{s_i}(1-\phi)] + (1-s)[\phi + h_{w_i}(1-\phi)]\right]^{\frac{1}{1-\sigma}}.$$

4.2.2 Optimal generational decisions

Notice that, in equilibrium, the unqualified high-skilled workers' decision regarding where they live will be the same in both periods. Since, in equilibrium, the environment that surrounds these workers is the same in both periods – one generation of qualified high-skilled workers, one generation of students, and one generation of unqualified high-skilled workers –, the region that they decide to live in in the first period is also the same region that they choose in the second period, given that no variable has changed from their point of view. Therefore, their indirect utility function can be simplified[23] as

$$V_{ii}^{HW}(s, h_s, h_w) = 2w_i - \mu \ln [P_i P_i] + 2\mu (\ln [\mu] - 1) .$$

Moreover, since we know that an agent can only study in region 1, we can further simplify V_{ji}^{HS} as

$$V_{1i}^{HS}(s, h_s, h_w) = \epsilon w_i - \mu \ln [P_1 P_i] + 2\mu (\ln [\mu] - 1) .$$

Thus, as detailed in the simplified decision tree in Figure 4.3, every agent may follow one of four life paths.

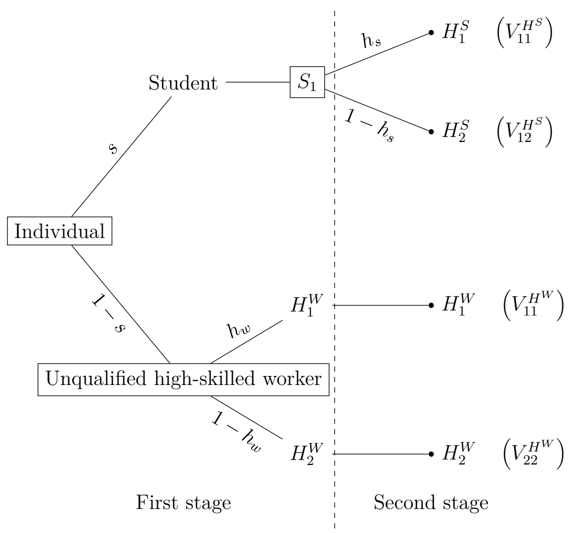


Figure 4.3: The figure shows the decisions and possible life paths of any high-skilled agent. Boxed nodes indicate that the agents can make a choice. Subscripts represent the region, while the superscript differentiates high-skilled workers between those who studied (S) and those who did not (W).

4.2.3 Long-run equilibrium

Each agent takes two decisions in their life – first, whether to study or not and, second, a spatial decision. Hence, we solve the model by backward induction.

We start by studying the second decision – the spatial one. So, we study how qualified and unqualified high-skilled agents decide where to work.

Afterwards, we study the first decision – the education one. So, we study how an agent decides whether to study or not.

[23] We omit the arguments (s, h_s, h_w) of functions w_i and P_i to ease the notation.

4.2.3.1 Spatial decision

Note that we already know that students can only live in region 1 and that the spatial decisions of unqualified high-skilled workers are the same in both periods. Thus, regardless of being qualified or not, when faced with their spatial decision, any high-skilled agent will choose to live in region 1 if their indirect utility is higher than that of living in region 2.

Thus, given the four life paths described before, a qualified agent chooses to live in region 1 if

$$V_{11}^{H^S} > V_{12}^{H^S} \quad \Leftrightarrow \quad \epsilon w_1 - \mu \ln [P_1] > \epsilon w_2 - \mu \ln [P_2] \, ,$$

while an unqualified agent chooses to live in region 1 if

$$V_{11}^{H^W} > V_{22}^{H^W} \quad \Leftrightarrow \quad w_1 - \mu \ln [P_1] > w_2 - \mu \ln [P_2] \, .$$

Therefore, considering that unqualified workers have a productivity level such that $\epsilon = 1$, we can simply state that any mobile worker chooses to live in region i if

$$\epsilon(w_i - w_j) > \mu \ln \left[\frac{P_i}{P_j} \right] , \quad j \neq i. \tag{4.9}$$

Thus, both types of agents decide very similarly, regardless of their productivity level – any agent chooses to live in the region that provides the higher real wage.

Furthermore, remember that we are analysing the choices of an individual. However, since all qualified workers are homogeneous, and all unqualified workers are also homogeneous, the choice of a single one corresponds to each group's choice. For example, if a qualified worker chooses to work in region 1, then all qualified workers choose the same. Therefore, this condition is also the stability condition for agglomeration in region i.

4.2.3.2 Education decision

Any high-skilled agent will choose to study if their indirect utility of following a study-work life path is higher than that of a work-work life path.

Thus, an agent decides to study if

$$\max \left\{ V_{11}^{H^S}, V_{12}^{H^S} \right\} > \max \left\{ V_{11}^{H^W}, V_{22}^{H^W} \right\} .$$

Note that this decision is rather complex since we can have multiple types of equilibria. Should the payoff from any of the life paths always be superior to the others, the equilibrium would be that everyone chooses that path. Likewise, should the life paths of a student always yield a greater (lesser) payoff than those who choose to work right away, in equilibrium, everyone studies (works).

For instance, we may have $s = 0$ if the productivity gains (ψ) are very low and do not compensate the opportunity costs, which implies that nobody studies and our conclusions would be qualitatively similar to the baseline Pflüger (2004) model.

On the opposite end, we may have $s = 1$ if ψ is high enough, which implies that everybody studies and our conclusions would be qualitatively similar to the Pflüger (2004) model with increased productivity[24].

Hence, the most interesting equilibria occur when $s \in (0,1)$, which effectively implies that the two groups of high-skilled workers coexist, in which case we have

$$\max\left\{V_{11}^{H^S}, V_{12}^{H^S}\right\} = \max\left\{V_{11}^{H^W}, V_{22}^{H^W}\right\}. \tag{4.10}$$

Therefore, we define an equilibrium as the combination (s, h_s, h_w) that guarantees that no one wants to choose a different life path, where s is the share of qualified agents, h_s is the share of qualified high-skilled agents in region 1, and h_w the share of unqualified high-skilled agents in region 1.

Note that the fact that equilibria in the $s \in (0,1)$ range lead to the same real wage for both qualified and unqualified workers is due to the underlying assumption of perfect competition. As we have already stated, if productivity gains are elevated, everyone wants to study and become more productive. Thus, for equilibria to occur when $s \in (0,1)$, agents have to be indifferent between studying or working in the first period, which is only true if their real wages are the same. Intuitively, this is similar to what happens in the perfect competition model in the long-run, when firms have zero profits in equilibrium.

4.2.3.3 Types of equilibrium

For a combination (s, h_s, h_w) to be an equilibrium, two criteria must be met. First, condition (4.9) must hold[25] for both groups of agents – the spatial decision. Second, the condition (4.10) must hold – the education decision.

[24] Note that due to the construction of the model, it is not an equilibrium for the first-ever born generation to choose $s = 1$ since that would mean that no industrial goods would be produced in the first period, which is not optimal. However, any subsequent generation can make this choice as there will already be high-skilled workers living in the second stage of their lives who can produce. Therefore, $s = 1$ can occur in the steady state.

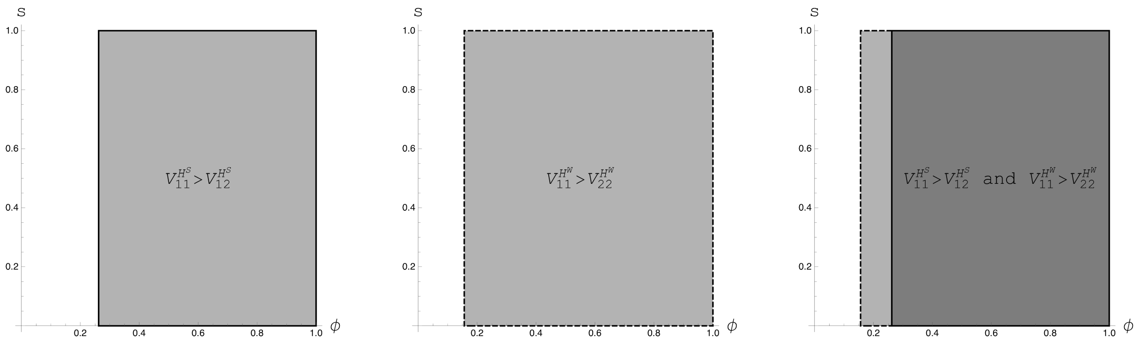
[25] Note that condition (4.9) should be evaluated at equality when we are analysing an interior spatial distribution.

Full agglomeration equilibria

In the full agglomeration equilibria, we have a situation in which both qualified and unqualified agents agglomerate. Therefore, in equilibrium, the indirect utilities of qualified and unqualified workers are equal and, within each group, are always bigger in the core region.

There are four possible full agglomeration scenarios – $h_s = 1$ and $h_w = 1$, $h_s = 0$ and $h_w = 0$, $h_s = 1$ and $h_w = 0$, and $h_s = 0$ and $h_w = 1$.

First, if $h_s = 1$ and $h_w = 1$, condition (4.9) holds when $V_{11}^{H^S} > V_{12}^{H^S}$ and $V_{11}^{H^W} > V_{22}^{H^W}$. Figure 4.4 shows, in the (ϕ, s) space, the region in which condition (4.9) holds, given $h_s = 1$ and $h_w = 1$.



(a) Qualified agents (b) Unqualified agents (c) Intersection

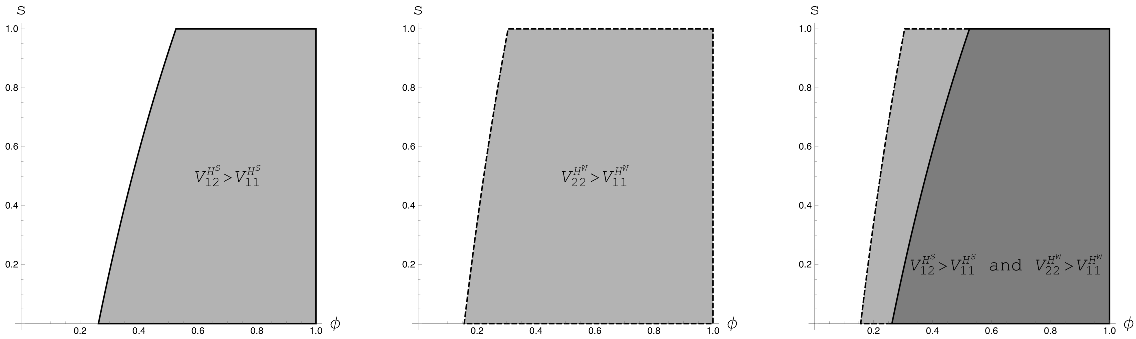
Figure 4.4: The figure shows the region in which the full agglomeration equilibrium ($h_s = 1, h_w = 1$) is stable. In the left panel, we have the region corresponding to the stability of agglomeration of the qualified agents. In the middle panel, we have the region corresponding to the stability of agglomeration of the unqualified agents. In the right panel, in a darker shade, we have the intersection of both regions of stability and, thus, the stability region for the $h_s = 1$ and $h_w = 1$ equilibrium. The parameters used are $\sigma = 2$, $\lambda = 2$, and $\epsilon = 2$.

Figure 4.4 helps us conclude that the stability of the spatial distribution $h_s = 1$ and $h_w = 1$ does not depend on the percentage of students and occurs as long as the transportation costs are not too high. Furthermore, numerical simulation shows that it is the stability condition of the qualified agents that always determines the stability and that the frontier of this region moves to the right as either the preference for variety or the mobility of the agents increase. Increases in productivity gains rotate the frontier counter-clockwise.

Moreover, if $h_s = 1$ and $h_w = 1$, condition (4.10) holds when $V_{11}^{H^S} = V_{11}^{H^W}$. Then, the solution for $V_{11}^{H^S}(s, 1, 1) = V_{11}^{H^W}(s, 1, 1)$ is $\psi^{A1} = 1$. Therefore, any $\psi > \psi^{A1} = 1$ is enough for everyone to opt for studying and any $\psi < \psi^{A1} = 1$ drives all agents directly to the workforce. With $\psi = \psi^{A1} = 1$, any $s \in (0, 1)$ is possible as agents are indifferent between studying or working in the first period.

Thus, we conclude that the spatial distribution $h_s = 1$ and $h_w = 1$ is a stable equilibrium if $\psi = \psi^{A1} = 1$ and if we are in the darker region of Figure 4.4c.

Second, if $h_s = 0$ and $h_w = 0$, condition (4.9) holds when $V_{11}^{H^S} < V_{12}^{H^S}$ and $V_{11}^{H^W} < V_{22}^{H^W}$. Figure 4.5 shows, in the (ϕ, s) space, the region in which condition (4.9) holds, given $h_s = 0$ and $h_w = 0$.



(a) Qualified agents (b) Unqualified agents (c) Intersection

Figure 4.5: The figure shows the region in which the full agglomeration equilibrium ($h_s = 0, h_w = 0$) is stable. In the left panel, we have the region corresponding to the stability of agglomeration of the qualified agents. In the middle panel, we have the region corresponding to the stability of agglomeration of the unqualified agents. In the right panel, in a darker shade, we have the intersection of both regions of stability and, thus, the stability region for the $h_s = 0$ and $h_w = 0$ equilibrium. The parameters used are $\sigma = 2$, $\lambda = 2$, and $\epsilon = 2$.

Figure 4.5 helps us conclude that the stability of the spatial distribution $h_s = 0$ and $h_w = 0$ depends on the percentage of students and occurs as long as the transportation costs are not too high. Furthermore, numerical simulation shows that it is the stability condition of the qualified agents that always determines the stability and that the frontier of this region moves to the right as either the preference for variety or the mobility of the agents increase. Increases in productivity gains rotate the frontier counter-clockwise.

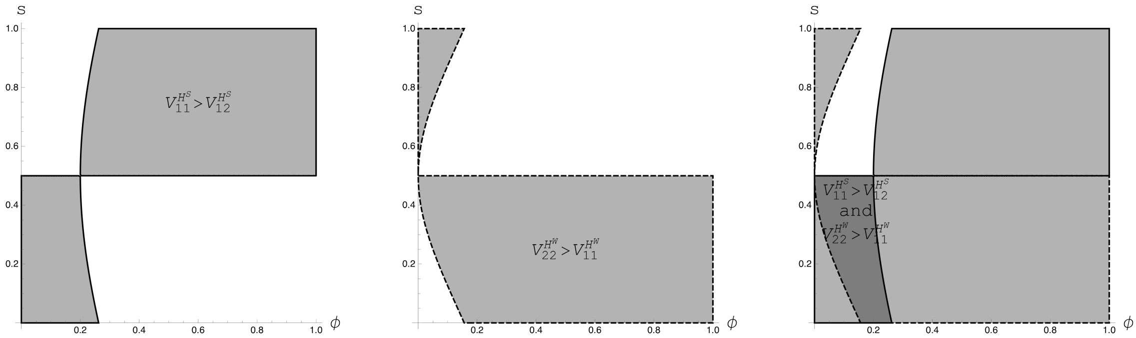
Moreover, if $h_s = 0$ and $h_w = 0$, condition (4.10) holds when $V_{12}^{H^S} = V_{22}^{H^W}$. Then, $V_{12}^{H^S}(s, 0, 0) = V_{22}^{H^W}(s, 0, 0)$ implies that, in equilibrium,

$$(\epsilon - 2) w_2(s, 0, 0) = \frac{\mu}{\sigma - 1} \ln \left[\frac{1}{\phi} \right].$$

Note that the right-hand side is always positive and that $w_2(s, 0, 0)$ is also positive. Hence, it is obvious that any $\epsilon < 2$ makes this equilibrium impossible and nobody studies. Let ψ^{A2} be the solution for $V_{12}^{H^S}(s, 0, 0) = V_{22}^{H^W}(s, 0, 0)$. Then, for this equilibrium to hold, it is necessary that $\psi = \psi^{A2} \geq 1$. Therefore, any $\psi > \psi^{A2}$ is enough for everyone to opt for studying and any $\psi < \psi^{A2}$ drives all agents directly to the workforce. With $\psi = \psi^{A2}$, any $s \in (0, 1)$ is possible as agents are indifferent between studying or working in the first period.

Thus, we conclude that the spatial distribution $h_s = 0$ and $h_w = 0$ is a stable equilibrium if $\psi = \psi^{A2}$ and if we are in the darker region of Figure 4.5c.

Third, if $h_s = 1$ and $h_w = 0$, condition (4.9) holds when $V_{11}^{H^S} > V_{12}^{H^S}$ and $V_{11}^{H^W} < V_{22}^{H^W}$. Figure 4.6 shows, in the (ϕ, s) space, the region in which condition (4.9) holds, given $h_s = 1$ and $h_w = 0$.



(a) Qualified agents (b) Unqualified agents (c) Intersection

Figure 4.6: The figure shows the region in which the full agglomeration equilibrium $(h_s = 1, h_w = 0)$ is stable. In the left panel, we have the region corresponding to the stability of agglomeration of the qualified agents. In the middle panel, we have the region corresponding to the stability of agglomeration of the unqualified agents. In the right panel, in a darker shade, we have the intersection of both regions of stability and, thus, the stability region for the $h_s = 1$ and $h_w = 0$ equilibrium. The parameters used are $\sigma = 2$, $\lambda = 2$, and $\epsilon = 2$.

Figure 4.6 helps us conclude that the stability of the spatial distribution $h_s = 1$ and $h_w = 0$ depends on the percentage of students and occurs for a relatively small range of transportation costs. Furthermore, numerical simulation shows that the stability region moves to the right as either the preference for variety or the mobility of the agents increase. Increases in the productivity gains shrink the stability region towards the origin[26].

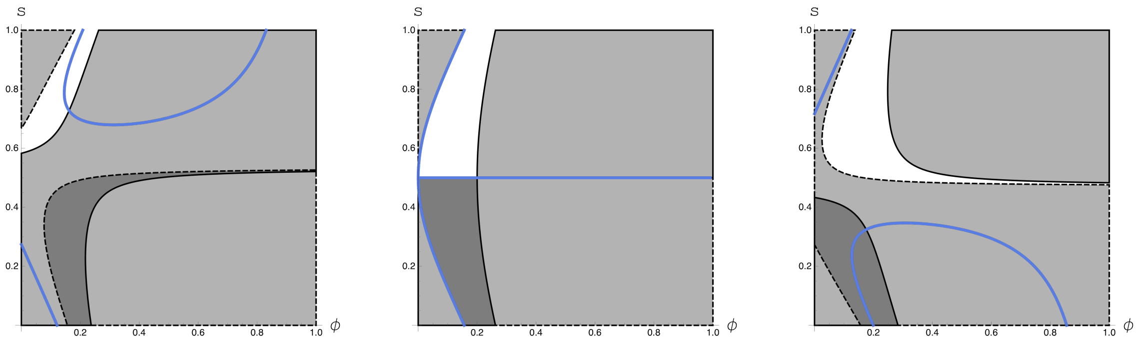
Moreover, if $h_s = 1$ and $h_w = 0$, condition (4.10) holds when $V_{11}^{H^S} = V_{22}^{H^W}$. Then, $V_{11}^{H^S}(s, 1, 0) = V_{22}^{H^W}(s, 1, 0)$ implies that, in equilibrium,

$$\epsilon w_1(s, 1, 0) - 2w_2(s, 1, 0) = 2\frac{\mu}{\sigma - 1} \ln \left[\frac{(1-s) + \frac{s\epsilon}{2}\phi}{(1-s)\phi + \frac{s\epsilon}{2}} \right].$$

Let ψ^{A3} be the solution for $V_{11}^{H^S}(s, 1, 0) = V_{22}^{H^W}(s, 1, 0)$. Then, for this equilibrium to hold, it is necessary that $\psi = \psi^{A3}$. Therefore, any $\psi > \psi^{A3}$ tendentially leads everyone to opt for studying and any $\psi < \psi^{A3}$ tendentially drives all agents directly to the workforce. With $\psi = \psi^{A3}$, the blue curve in Figure 4.7 shows the range of s for which agents are indifferent between studying or working in the first period.

[26] Figure 4.7 illustrates this.

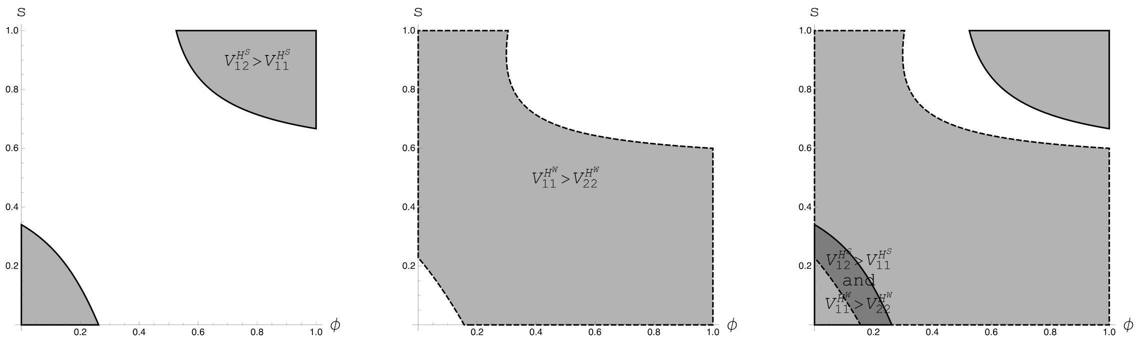
Numerical simulation, illustrated in Figure 4.7, shows that the spatial distribution $h_s = 1$ and $h_w = 0$ is a stable equilibrium if $\psi^{A3} \geq 1$ and if we are in the darker region of Figure 4.6c. Moreover, there is an upper limit to ψ^{A3} above which the equilibrium becomes unstable.



(a) $\psi = 1.7$ (b) $\psi = 2$ (c) $\psi = 2.3$

Figure 4.7: The figure shows the region in which the full agglomeration equilibrium $(h_s = 1, h_w = 0)$ is stable, as before. The blue curve represent the pairs (ϕ, s) that solve $V_{11}^{H^S}(s, 1, 0) = V_{22}^{H^W}(s, 1, 0)$, for three given values of ψ. Only when the blue curve is in the darker region, the equilibrium exists and is stable. The parameters used are $\mu = 0.3$, $\sigma = 2$, and $\lambda = 2$.

Finally, if $h_s = 0$ and $h_w = 1$, condition (4.9) holds when $V_{11}^{H^S} < V_{12}^{H^S}$ and $V_{11}^{H^W} > V_{22}^{H^W}$. Figure 4.8 shows, in the (ϕ, s) space, the region in which condition (4.9) holds, given $h_s = 0$ and $h_w = 1$.



(a) Qualified agents (b) Unqualified agents (c) Intersection

Figure 4.8: The figure shows the region in which the full agglomeration equilibrium $(h_s = 0, h_w = 1)$ is stable. In the left panel, we have the region corresponding to the stability of agglomeration of the qualified agents. In the middle panel, we have the region corresponding to the stability of agglomeration of the unqualified agents. In the right panel, in a darker shade, we have the intersection of both regions of stability and, thus, the stability region for the $h_s = 0$ and $h_w = 1$ equilibrium. The parameters used are $\sigma = 2$, $\lambda = 2$, and $\epsilon = 2$.

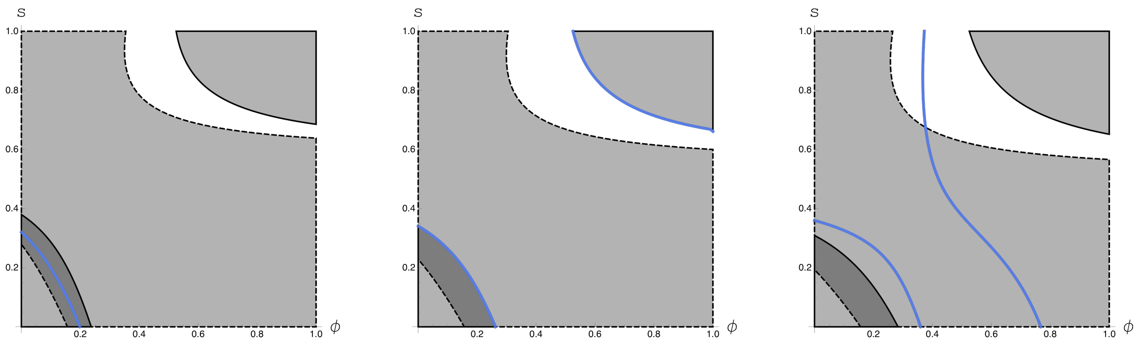
Figure 4.8 helps us conclude that the stability of the spatial distribution $h_s = 0$ and $h_w = 1$ depends on the percentage of students and occurs for a relatively small range of transportation costs. Furthermore, numerical simulation shows that the stability region moves to the right as either the preference for variety or the mobility of the agents increase. Increases in the productivity gains shrink the stability region towards the origin[27].

Moreover, if $h_s = 0$ and $h_w = 1$, condition (4.10) holds when $V_{12}^{H^S} = V_{11}^{H^W}$. Then, $V_{12}^{H^S}(s, 0, 1) = V_{11}^{H^W}(s, 0, 1)$ implies that, in equilibrium,

$$\epsilon w_1(s, 1, 0) - 2w_2(s, 1, 0) = 2\frac{\mu}{\sigma - 1} \ln \left[\frac{(1-s) + \frac{s\epsilon}{2}\phi}{(1-s)\phi + \frac{s\epsilon}{2}} \right].$$

Let ψ^{A4} be the solution for $V_{12}^{H^S}(s, 0, 1) = V_{11}^{H^W}(s, 0, 1)$. Then, for this equilibrium to hold, it is necessary that $\psi = \psi^{A4}$. Therefore, any $\psi > \psi^{A4}$ tend to lead everyone to opt for studying and any $\psi < \psi^{A4}$ tend to drive all agents directly to the workforce. With $\psi = \psi^{A4}$, the blue curve in Figure 4.9 shows the range of s for which agents are indifferent between studying or working in the first period.

Numerical simulation, illustrated in Figure 4.9, shows that the spatial distribution $h_s = 0$ and $h_w = 1$ is a stable equilibrium if $\psi^{A4} \neq 1$ and if we are in the darker region of Figure 4.8c. Moreover, there is a lower limit to ψ^{A4} below which the equilibrium becomes unstable.



(a) $\psi = 1.7$ (b) $\psi = 2$ (c) $\psi = 2.3$

Figure 4.9: The figure shows the region in which the full agglomeration equilibrium ($h_s = 0, h_w = 1$) is stable, as before. The blue curve represent the pairs (ϕ, s) that solve $V_{12}^{H^S}(s, 0, 1) = V_{11}^{H^W}(s, 0, 1)$, for three given values of ψ. Only when the blue curve is in the darker region, the equilibrium exists and is stable. The parameters used are $\mu = 0.3$, $\sigma = 2$, and $\lambda = 2$.

Our analysis of the full agglomeration equilibria allows us to draw two interesting conclusions. First, if $\psi = 1$, then we are sure that the four types of full agglomeration equilibria exist and are stable. Moreover, note that $\psi = 1$ has a very particular meaning – the productivity gains are such that a qualified agent is able to earn the same nominal wage in one period that an unqualified one would take two periods to earn, which implies that qualified agents can completely make up for the time spent

[27]Figure 4.9 illustrates this.

in the university. Second, in this scenario, segregation of qualified and unqualified workers – $h_s = 1$ and $h_w = 0$, or $h_s = 0$ and $h_w = 1$ – is possible. However, for $\psi \neq 1$, only one of the segregation configurations is stable – since it is clear from Figures 4.7 and 4.9 that when one is stable, the other is not – and it may be so that not even one can be stable if ψ is sufficiently different from one.

Partial dispersion and semi-agglomeration equilibria

Apart from the full agglomeration equilibria, we may have partial dispersion equilibria – in which both groups of high-skilled agents are distributed between the regions ($h_s \in (0,1)$ and $h_w \in (0,1)$) – and semi-agglomeration equilibria – in which one of the two groups of high-skilled agents is agglomerated in one region while the other is dispersed between them. Therefore, our analysis of the partial dispersion equilibria is sufficient to also study the semi-agglomeration equilibria, as the latter occurs when h_s and h_w tend to their limits.

Since we are now analysing interior equilibria, we evaluate condition (4.9) at equality, which is simply

$$\epsilon(w_i - w_j) = \mu \ln \left[\frac{P_i}{P_j} \right], \quad j \neq i.$$

First, if $h_s \in (0,1)$ and $h_w \in (0,1)$, condition (4.9) holds at equality when $V_{11}^{HS} = V_{12}^{HS}$, which implies that the qualified workers are indifferent between living in any region. Figure 4.10 shows, in the (h_s, h_w, s) space, the region in which condition (4.9) holds at equality, given $h_s \in (0,1)$ and $h_w \in (0,1)$.

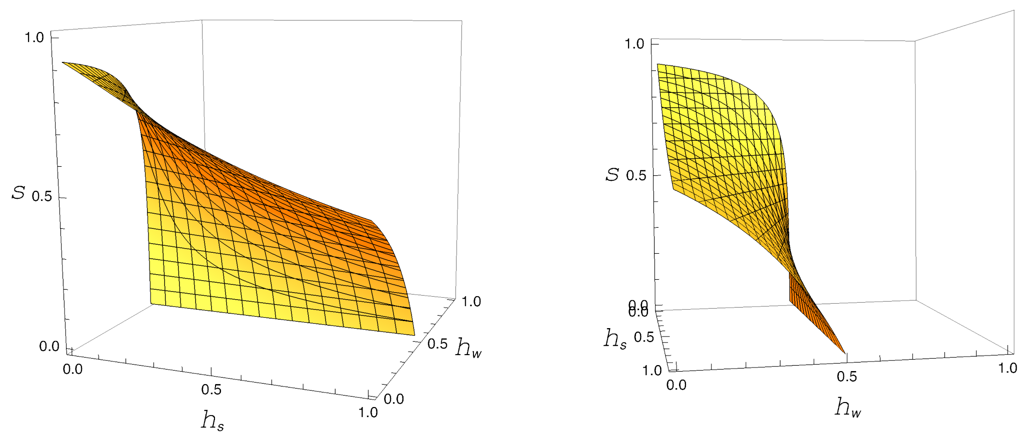


Figure 4.10: The plot shows the combinations (h_s, h_w, s) that make qualified workers indifferent between living in any region, in two perspectives. The parameters used are $\sigma = 2$, $\lambda = 2$, $\phi = 0.5$, and $\epsilon = 2$.

Second, if $h_s \in (0,1)$ and $h_w \in (0,1)$, condition (4.9) holds at equality when $V_{11}^{H^W} = V_{22}^{H^W}$, which implies that the unqualified workers are indifferent between living in any region. Figure 4.10 shows, in the (h_s, h_w, s) space, the region in which condition (4.9) holds at equality, given $h_s \in (0,1)$ and $h_w \in (0,1)$.

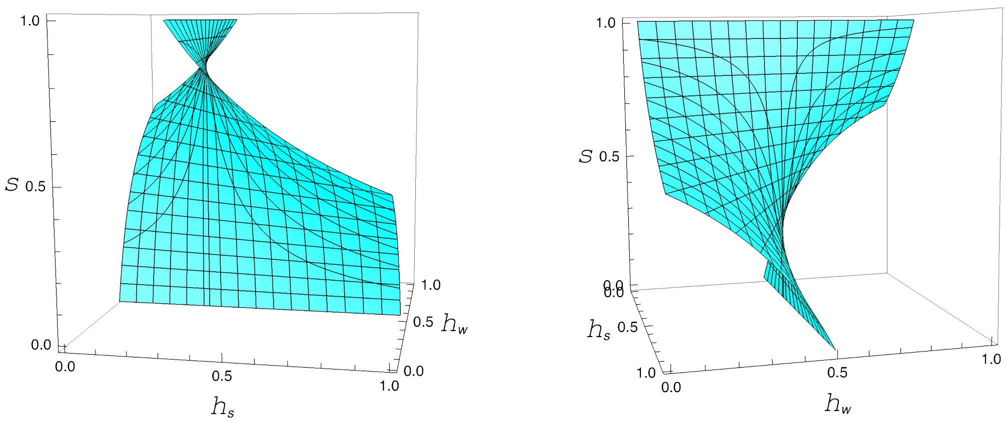


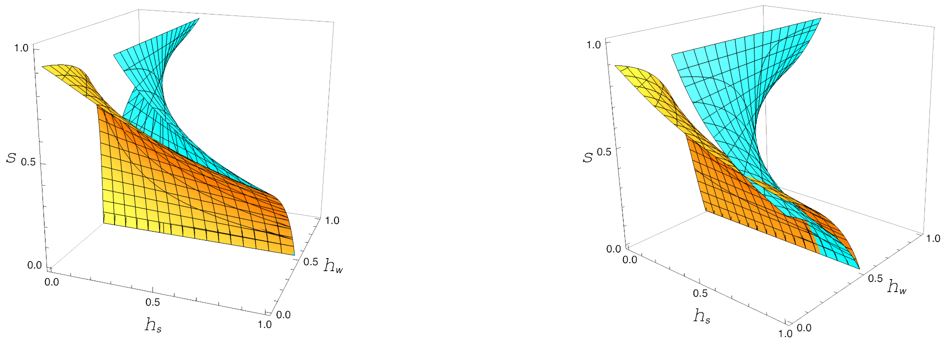
Figure 4.11: The plot shows the combinations (h_s, h_w, s) that make unqualified workers indifferent between living in any region, in two perspectives. The parameters used are $\sigma = 2$, $\lambda = 2$, $\phi = 0.5$, and $\epsilon = 2$.

Finally, if $h_s \in (0,1)$ and $h_w \in (0,1)$, condition (4.10) holds when $(V_{11}^{H^S} = V_{12}^{H^S}) = (V_{11}^{H^W} = V_{22}^{H^W})$, which implies that the agents are indifferent between studying or working in the first period. Then, since Figures 4.10 and 4.11 show $V_{11}^{H^S} = V_{12}^{H^S}$ and $V_{11}^{H^W} = V_{22}^{H^W}$, respectively, the intersection of both surfaces represent the points that verify condition $(V_{11}^{H^S} = V_{12}^{H^S}) = (V_{11}^{H^W} = V_{22}^{H^W})$.

Both conditions help us analyse how the two groups of high-skilled workers react to changes in the choices of the opposite group and society as a whole – represented by the percentage of agents who study (s). Note that, in both cases, when $s = 0$, the agents opt to disperse evenly between the regions, which is the usual symmetric dispersion equilibrium of the baseline model.

Therefore, the intersection represents the partial dispersion equilibrium points. Should any of them occur when either h_s or h_w is equal to zero or one, we have the semi-agglomeration equilibria. Figure 4.12a shows both conditions from Figures 4.10 and 4.11 in the same plot. Thus, we find that the interior equilibria in this scenario (given $\psi = 1$) only[28] occur for $h_s = 1$. Note that this type of equilibrium is a semi-agglomeration one.

[28]Since we are assuming $s \in (0,1)$. Otherwise, for $s = 0$, symmetric dispersion is an equilibrium, as stated before.



(a) $\psi = 1$ (b) $\psi = 1.8$

Figure 4.12: The plot shows the combinations (h_s, h_w, s) that make qualified (orange) and unqualified (blue) workers indifferent between living in any region. The intersection of both conditions represents the partial dispersion equilibrium points. The parameters used are $\sigma = 2$, $\lambda = 2$, and $\phi = 0.5$.

Figure 4.12b shows the interior equilibria in another scenario (given $\psi = 1.8$). This time, the equilibria is clearly interior, and, interestingly enough, it seems that equilibria is always orthogonal to the h_s axis.

Moreover, extensive numerical simulation seems to point to the invariance of the interior equilibria to economic conditions, except for productivity. Thus, while the surfaces that represent the conditions for the spatial decision change – depending on the love for variety and the mobility of the agents, for example –, their intersection, which represents the interior equilibria, remains unchanged. However, if the productivity level changes, then the interior equilibria also changes. In particular, this change is seemingly linear, with a tendency for h_s to drop as productivity increases.

4.3 Concluding remarks

In this essay, we study how agents make spatial and education decisions endogenously. In particular, we explore how the availability of formal education affects the spatial distribution of economic activity and the agents' decisions regarding whether to pursue an academic degree.

We implement the two-region quasi-linear log utility footloose entrepreneur model in an overlapping generations model to explore the spatial and educational decisions of forward-looking agents. We allow high-skilled agents to freely choose between studying or working in the first period of their lives, considering that only one of the regions offers a university.

We use backward induction to find the model's equilibrium conditions and determine an individual's possible life paths. Thus, given the share of qualified agents, we can state that a spatial distribution of qualified and unqualified workers is an equilibrium if no one wants to migrate nor change their education decision.

We find that the productivity gains from studying can make a qualified agent be in one of three scenarios regarding lifetime accumulated nominal wage – defined by the sum of the nominal wages in their life. First, they can earn a lower accumulated nominal wage than unqualified ones. Second, they can earn the same accumulated nominal wage. Third, they can earn a higher accumulated nominal wage.

When the nominal wage of qualified agents is lower than that of unqualified ones, the only possible equilibrium is the full agglomeration in which qualified and unqualified workers become segregated. In particular, qualified workers live in the region that does not offer a university, and unqualified workers live in the region that offers a university. This segregation occurs because qualified agents migrate to look for lower prices, while unqualified workers migrate to look for the bigger market, making it easier for low-productive firms to thrive.

When the nominal wage of qualified agents is equal to that of unqualified ones, there are two possible equilibria. First, we may have any full agglomeration equilibria, except for the one in which everyone agglomerates in the region that does not offer a university. Second, we may have the semi-agglomeration equilibria in which qualified workers agglomerate in the region that offers a university and the unqualified ones are dispersed between the two regions.

When the nominal wage of qualified agents is higher than that of unqualified ones, there are two possible equilibria. First, we may have the full agglomeration equilibria in which both types of agents agglomerate in the region that does not offer a university or the full agglomeration equilibria in which qualified and unqualified workers become segregated – in particular, qualified workers live in the region that offers a university, and unqualified workers live in the region that does not offer a university. Second, we may have the partial dispersion equilibria in which both types of agents are dispersed between the two regions.

Although our analysis already allows us some insightful conclusions, more research is needed, particularly in the study of the partial dispersion equilibria, to more accurately define the different types of equilibria and how the economic conditions affect them, which is something we intend to pursue in future work.

While our model seems rather complex, we find that the backward induction technique allows us to achieve several straightforward conclusions already. Even so, we believe this conceptualisation seems to have the potential to bring a new spark to the literature by proposing a new approach for analysing more profound subjects that typically are challenging to study within classical Economic Geography models.

Chapter 5

On the disentanglement of an economic union*

5.1 Introduction

After the Second World War, the integration of European countries was seen as a possibility to maintain peace and avoid extreme nationalist movements. In 1946, Winston Churchill delivered a speech in which he advocated the creation of the United States of Europe. Eleven years later, with the Treaty of Rome, the European Economic Community (EEC) was established with a primary goal of creating a customs union with a common external tariff. Since then, several enlargements have increased the number of members, and the EEC also changed its name to the European Union (EU) with the Treaty of Lisbon. Another important landmark for the EU is the creation of the Eurozone, which lead to several of its members sharing the same common currency.

However, a country opposed joining the Eurozone and insisted on keeping its currency – the United Kingdom (UK). Eventually, in 2016, a referendum was held, and its result dictated the withdrawal of the UK from the EU. It is commonly referred to as Brexit and was effectively consummated on the 1st of January 2021.

More recently, a report from ReWAGE and the Migration Observatory at the University of Oxford has shown that the labour mobility restrictions introduced by Brexit have contributed to the current labour shortages in the UK.

This motivates us to study the issues surrounding the relationship between an established economic union and one dissident country. In particular, we seek to find

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– 4th International Workshop "Market Studies and Spatial Economics", Brussels, 5 April 2022 (online)
– 11th European Meeting of the Urban Economics Association, London, 30 April 2022

out the potential consequences of Brexit on the spatial configuration of industry in the UK and the EU and on the welfare of the different agents in the economy.

The approach we propose is valid for any three symmetric regions with a transportation cost structure as described in the next section. Even though our exposition revolves around Brexit from a narrative perspective, it does not consider the specificities of the countries involved.

To achieve our goal, we use a quasi-linear log model with three regions within an economy that features both skilled and unskilled workers (Gaspar et al., 2018; Pflüger, 2004) in which the regions are economically integrated. We consider that this economic union is translated as a high economic integration amongst all regions. Our novelty is to allow one region to withdraw from the established economic union, which we assume leads to a lower economic integration (or higher transportation cost) between the leaving region and the other two.

Note that we analyse a simplified version of the Brexit case by considering the leaving region to be the UK and the EU to consist of two regions, namely France (FRA) and Germany (GER), with all three being identical. By considering the UK-FRA-GER model, we are focusing our attention on the economic powerhouses of the Union – Eurostat data on imports and exports show that, in the period between 2011 and 2019, these three countries alone represent 42% of both exports and imports of the EU, with the three of them constantly on the top five in exports and always on the top three in imports.

The choice of the number of regions in the model is not arbitrary from a technical point of view. To begin with, considering only two regions, for example, the UK and the EU would imply exogenous regional asymmetries in the size of immobile labour between regions. Asymmetries such as these have been thoroughly studied in a two-region context by authors such as Forslid and Ottaviano (2003) and Berliant and Kung (2009), but in a three-region model would deem our analysis intractable. Other than that, it is now widely acknowledged that a two-region setup is unrealistic from an empirical point of view and can only account for a limited amount of predictions (see Gaspar et al. (2018)). Moreover, it would not allow us to account for the dynamics between the remaining regions in the union.

Therefore, at least three regions are needed. More than three regions, however, may be somewhat unwarranted for our purposes. First, if we include an arbitrary number of regions, each with its own characteristics, the problem under analysis becomes analytically intractable. Second, if we consider that all regions that remain in the union are symmetric in all respects, the qualitative properties of the model are likely to become invariant under a higher number of regions, as demonstrated by Gaspar et al. (2021) – who show that going from two to three regions while preserving the symmetry has qualitative implications that become less apparent as the number of regions increases beyond three. Therefore, three is the number of regions that allows us to convey our main messages by studying all the relevant spatial configurations while keeping the problem analytically tractable, to some extent, without sacrificing qualitative aspects.

We study how Brexit affects the possible spatial distributions between the three regions and find that the scenario in which the entrepreneurs are equally distributed amongst the three regions is no longer viable. More interesting, however, is that when transportation costs are high enough, spatial distributions emerge in which entrepreneurs' share differs from region to region.

We explore if the previously defined spatial distributions can be stable in the long term and find that spatial distributions in which the leaving region is totally depleted of mobile workers are not stable, except for agglomerations in the remaining members of the union. We also explore how mobile and immobile agents' welfare changes with the different spatial distributions and find that, for mobile agents, any agglomeration is optimal. In contrast, immobile agents prefer that the entrepreneurs become more dispersed as transportation costs with the UK increase.

Finally, we conduct a numerical analysis and find that the lower the difference between transportation costs and the higher the mobility of industrial workers, the more mobile workers live in the UK. This analysis also leads us to conclude that the welfare of the entire economy would be higher if, generally, a slightly higher percentage of entrepreneurs lived in the UK.

Albeit an exciting area in the geographical economy, while there are some empirical contributions to the Brexit phenomenon, theoretical Economic Geography works on Brexit are still scarce, with that of Commendatore et al. (2021) being, to the best of our knowledge, the only one.

In their work, Commendatore et al. (2021) focus on the disintegration effects and how the regions that remain integrated may move closer together. The authors particularly consider asymmetric transportation costs and symmetric regions, regarding their size, within a usual footloose entrepreneur model. The model is then studied via numerical simulation – for two scenarios of Brexit, soft and hard, depending on the increase of transportation costs – leading to some interesting conclusions, namely how the starting situation of the union may drastically alter the outcome after the breakup. Parametrisations aside, the main conclusions are that firms from the leaving region may move to the union regions, thus acting as a trade substitute; that the competition within the remaining union may also lead some companies to relocate to the leaving region; and that further integration between the remaining regions may weaken the trade bonds between the union and the leaving region.

Compared to our work, Commendatore et al. (2021) follow a slightly different approach as they consider that the two other regions are the centre of the EU and the periphery of the EU. Noteworthy on a technical level, our work focuses on general analytical results while Commendatore et al. rely solely on numerical computation. Besides that, we also consider continuous transportation costs and dynamics while Commendatore et al. have a discrete approach for both transportation costs and dynamics.

Brakman et al. (2018) study how the UK can minimise losses due to Brexit and, paradoxically, conclude that the solution would be a trade agreement with the EU.

Dhingra et al. (2017) explore the welfare effects of Brexit and conclude that every party loses. However, it is the UK that is the biggest loser. Sampson (2017) also shares this opinion. Our results seem to corroborate these findings since the economy as a whole attains its maximum social welfare when most entrepreneurs do not live in the UK.

Javorcik et al. (2019) explore how Brexit affects the labour market and conclude that circulation restrictions can potentially be more harmful than trade barriers.

There is also a survey by Busch and Matthes (2016) which revises some studies on the economic impact of Brexit and sheds some light on legal and bureaucratic issues. Finally, Dhingra et al. (2018) explore possible alternative deals outside the EU and discuss the implication of Brexit in the flow of Foreign Direct Investment (FDI) towards the UK and, indirectly, towards the EU. This last point is particularly relevant from an Economic Geography perspective since workers' mobility can be considered an FDI flow.

On a more theoretical – and general – strand of literature, Puga and Venables (1997) develop a model that studies Preferential Trade Agreements (PTA) within a geographical setup that precedes the classic Economic Geography models. Their main conclusion is that these agreements pull industry towards their members and apart from outside regions. Even so, Puga and Venables also refer that within the union there may exist imbalances. While it is true that our study focuses on the disentanglement of an established economic union, there are clear parallels to the creation of a PTA since one region will end up more remote than in the beginning. In fact, the broad conclusion that the PTA's members will benefit from a higher concentration of industry is something that we also conclude. The main difference we can point out is the starting point of the integration level. When creating a PTA – or an economic union – we assume that the objective is to bring the regions closer, therefore the starting integration level is low and the economy desires it to be higher. With the disentanglement, we move in the opposite direction – the starting integration level is high, but the region that leaves the agreements wishes it would be lower.

Behrens et al. (2007) also study PTA and conclude that the effects depend on whether changes are done in transport or non-transport – such as tariffs and regulations – frictions. They state that only the former allows for clear predictions of changes in industry location and welfare. This also helps justify why we consider that all the changes due to the disentanglement can be summarised in the transportation cost.

Finally, Mossay and Tabuchi (2015) conclude that a PTA increases the welfare of its members while reducing that of left-out regions. We also reach this conclusion as we show that the spatial distribution of entrepreneurs that maximises the welfare of the economy as a whole is such that the majority of them should not live in the dissident country, which is especially prejudicial for the immobile agents.

5.2 The disentanglement problem

Our model is based on the footloose entrepreneur model with quasi-linear log utility with three regions (Gaspar et al., 2018; Pflüger, 2004). In this economy, we consider that one of the three regions has unilaterally decided to leave an economic union that was previously established among them.

Our objective is to understand how the spatial distribution of industry and the welfare of the agents is affected following the disentanglement of an economic union.

We work within a three-region Economic Geography model and, without loss of generality, we assume that the leaving region is region 1, and we name[29] regions 1, 2, and 3 as the United Kingdom (UK), France (FRA), and Germany (GER), respectively. Moreover, we assume that all the consequences – whether they are commercial, political, or even regarding mobility barriers[30] for the agents, such as working visas – of leaving the economic union can be summarised as an increase in the bilateral transportation costs between the leaving and the remaining regions (τ_{UK}). In contrast, the remaining regions keep the initial level of transportation costs between them (τ_{EU}).

[29] As stated before, the naming of the regions proves helpful when referring to them further ahead. However, our results are general for any three regions we shall consider.

[30] Although more complex conceptualisations for mobility barriers can be adopted, that problem is out of the scope of our analysis.

5.2.1 Economic model

In this economy, there are L unskilled workers – equally divided between the three regions – that are immobile between regions, and H skilled workers – H_i in region $i = \{1, 2, 3\}$ – that are mobile between regions.

The preferences of all agents are defined by

$$U = \mu \ln M + A, \tag{5.1}$$

where $\mu \in (0, 1)$ is the expenditure share in the industrial good, A is the consumption of the agricultural good, and M is the consumption of the usual CES composite of differentiated varieties of the industrial good, defined by

$$M = \left[\int_{s \in S} d(s)^{\frac{\sigma-1}{\sigma}} \mathrm{d}s \right]^{\frac{\sigma}{\sigma-1}}, \tag{5.2}$$

where $d(s)$ is the consumption of variety s, S is the mass of varieties and $\sigma > 1$ is the constant elasticity of substitution between varieties.

Let $p_{ij}(s)$ represent the delivered price in region i of variety s produced in region j and $d_{ij}(s)$ its demand. Then, the regional price index associated with the composite good (5.2) in region i is

$$P_i = \left[\int_{s \in S} p_{ij}(s)^{1-\sigma} \mathrm{d}s \right]^{\frac{1}{1-\sigma}}. \tag{5.3}$$

Every agent in region i maximises its utility subject to the budget constraint given by

$$P_i M + A = y_i,$$

where y_i represents the nominal income of the agent ($y_i = w_i$ if skilled and $y_i = 1$ otherwise), P_i is given in (5.3) and the price of the agricultural good is normalised to one. Thus, the demand functions are given by

$$d_{ij}(s) = \mu \frac{p_{ij}(s)^{-\sigma}}{P_i^{1-\sigma}}, \quad M = \frac{\mu}{P_i}, \quad A = y_i - \mu. \tag{5.4}$$

From (5.1) and (5.4), we derive the indirect utility function in region i, which is given by

$$V_i = y_i - \mu \ln P_i + \mu(\ln \mu - 1). \tag{5.5}$$

The production of the agricultural good uses one unit of unskilled labour per unit produced and has no transportation costs. Thus, $p_1^A = p_2^A = p_3^A = p^A$, which leads us to choose this good as *numeraire* ($p^A = 1$). Since the agricultural market is perfectly competitive, marginal cost pricing implies that the nominal wage of unskilled workers is the same everywhere and, in particular, equal to p^A. Hence, $w_i^L = p^A = 1$.

We assume that the non-full-specialisation (NFS) condition (Baldwin et al., 2003; Gaspar et al., 2018) holds[31], so we have

$$\lambda > \frac{\mu \frac{\sigma-1}{\sigma}}{\frac{1}{3} - \mu \frac{\sigma-1}{\sigma}},$$

where $\lambda = L/H$ represents the global immobility ratio.

As for the production of the industrial good, both skilled and unskilled labour is used. In particular, each unit produced requires θ units of skilled labour and β units of unskilled labour. Therefore, the production cost of an industrial firm in region i is

$$PC_i(x_i) = \theta w_i + \beta x_i.$$

Hence, an industrial firm in region i that produces variety s maximises the profit function

$$\pi_i(s) = \sum_{j=1}^{3} d_{ij}(s) \left(H_j + L/3\right) \left[p_{ij}(s) - \tau_{ij}\beta\right] - \theta w_i, \tag{5.6}$$

where $\tau \in (1, +\infty)$ represents the usual iceberg transportation cost between regions regarding the industrial good. Note that $\tau_{ij} = \tau$ whenever $i \neq j$ and $\tau_{ij} = 1$ otherwise.

Therefore, profit maximisation of (5.6) yields the optimal prices

$$p_{ij}(s) = \tau_{ij}\beta \frac{\sigma}{\sigma - 1}. \tag{5.7}$$

Then, using (5.7) and the fact that the number of industrial varieties produced in region i is H_i/θ, the regional price index of the composite good (5.3) becomes

$$P_i = \frac{\beta\sigma}{\sigma - 1} \left[\frac{1}{\theta} \sum_{j=1}^{3} \phi_{ij} H_j\right]^{\frac{1}{1-\sigma}}, \tag{5.8}$$

where $\phi_{ij} \equiv \tau_{ij}^{1-\sigma} \in (0, 1]$ represents the freeness of trade (or the inverse of trade costs) between regions, regarding the industrial good. Note that $\phi_{ij} = \phi$ whenever $i \neq j$ and $\phi_{ij} = 1$ otherwise.

Furthermore, let us define $\boldsymbol{\tau}$ as the matrix of the transportation costs after the fall out and $\boldsymbol{\phi}$ as the corresponding matrix of the freeness of trade, whose elements

[31] It is straightforward but cumbersome to show that the weighted average nominal wage of skilled workers is the same as in Gaspar et al. (2018) – $\bar{w} = \frac{\mu}{\sigma}(1 + \lambda)$ –, and so is the NFS condition.

are such that $\phi_{ij} = \tau_{ij}^{1-\sigma}$. Note that, by definition, $\tau_{UK} > \tau_{EU}$, which implies that $\phi_{UK} < \phi_{EU}$. So, we have

$$\boldsymbol{\tau} = \begin{pmatrix} 1 & \tau_{UK} & \tau_{UK} \\ \tau_{UK} & 1 & \tau_{EU} \\ \tau_{UK} & \tau_{EU} & 1 \end{pmatrix}, \quad \boldsymbol{\phi} = \begin{pmatrix} 1 & \phi_{UK} & \phi_{UK} \\ \phi_{UK} & 1 & \phi_{EU} \\ \phi_{UK} & \phi_{EU} & 1 \end{pmatrix}.$$

Given the monopolistic competition setup in the industrial market, the free entry condition implies zero profits in equilibrium. Using (5.4), (5.7), and (5.8) into $\pi_i(s) = 0$, the equilibrium wages that skilled workers earn are

$$w_i = \frac{\mu}{\sigma} \sum_{j=1}^{3} \phi_{ij} \frac{H_j + L/3}{\sum_{m=1}^{3} \phi_{mj} H_m}. \tag{5.9}$$

Thus, by defining the share of skilled workers in each region i as $h_i = H_i/H$, the global immobility ratio as $\lambda = L/H$, and the set of spatial distribution as the 2-dimensional simplex $\mathit{\Delta} = \left\{ \boldsymbol{h} \in \mathbb{R}_+^3 : \sum_{i=1}^{3} h_i = 1 \right\}$, it is possible to express the nominal wages (5.9) as a function of $\boldsymbol{h}$, which yields

$$w_i(\boldsymbol{h}) = \frac{\mu}{\sigma} \sum_{j=1}^{3} \phi_{ij} \frac{h_j + \lambda/3}{\sum_{m=1}^{3} \phi_{mj} h_m}. \tag{5.10}$$

The regional price index (5.8) can also be rewritten as

$$P_i(\boldsymbol{h}) = \frac{\beta\sigma}{\sigma - 1} \left(\frac{H}{\theta} \right)^{\frac{1}{1-\sigma}} \left[\sum_{m=1}^{3} \phi_{mi} h_m \right]^{\frac{1}{1-\sigma}}. \tag{5.11}$$

Therefore, by replacing (5.10) and (5.11) in (5.5) the indirect utility of a skilled agent is now

$$V_i(\boldsymbol{h}) = \frac{\mu}{\sigma} \sum_{j=1}^{3} \left[\phi_{ij} \frac{h_j + \lambda/3}{\sum_{m=1}^{3} \phi_{mj} h_m} \right] + \frac{\mu}{\sigma - 1} \ln \left[\sum_{m=1}^{3} \phi_{mi} h_m \right] + \eta, \tag{5.12}$$

where $\eta = \mu(\ln \mu - 1) + \frac{\mu}{\sigma-1} \ln \left(\frac{H}{\theta} \right) - \mu \ln \left(\frac{\beta\sigma}{\sigma-1} \right)$ is a constant.

5.2.2 Long-run equilibria

In the long-run, agents choose to reside in the region (country) that offers them the highest indirect utility. Agents' migration decisions are governed by the replicator dynamics (Sandholm, 2010; Taylor & Jonker, 1978), given by

$$f_i = \dot{h}_i \equiv h_i \left(V_i(\boldsymbol{h}) - \bar{V}(\boldsymbol{h}) \right), \quad i = \{1, 2\}, \tag{5.13}$$

where $\bar{V}(\boldsymbol{h}) = \sum_{i=1}^{3} h_i V_i(\boldsymbol{h})$, $\boldsymbol{h} = (h_1, h_2, h_3)$, and the dynamics for the third region are given residually by $\dot{h}_3 = -\dot{h}_1 - \dot{h}_2$. Next, let us define the existence and stability of long-run equilibria.

Defintion 5.1. A spatial distribution $\boldsymbol{h} \equiv \boldsymbol{h}^*$ is said to be a long-run *equilibrium* if $f_i = 0$ $(i = 1, 2)$.

A complementary condition must hold, $V_i - \hat{V} \leq 0$, with $\hat{V}$ being the highest utility of the solution to this problem, ensuring that no agent can get a higher indirect utility from moving to another region.

A simple depiction of our simplex is given in Figure 5.1. This figure also features a qualitative description of the possible different types of equilibria and classifies them according to their location on the simplex Δ.

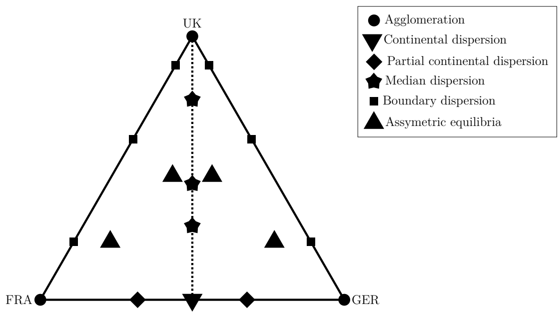


Figure 5.1: This figure represents the simplex that corresponds to our model. Each vertex represents one region, adequately named. There is a border connecting any two regions. The dashed line is a median that connects the UK region with the *continental dispersion*, which is also represented by an inverted triangle. The remaining points are the possible equilibria for $\mu = 0.3$, $\sigma = 6$ and $\lambda = 2$ for some pairs (ϕ_{EU}, ϕ_{UK}).

We have *agglomeration*, with all mobile agents residing in a single region; *continental dispersion* with mobile agents evenly dispersed between France and Germany; *partial continental dispersion*, whereby no mobile agent resides in the UK; *median dispersion*, with some mobile agents living in the UK and an even distribution of agents between France and Germany; *boundary dispersion*, with either France or Germany absent of mobile agents; and *asymmetric equilibria*, whereby all regions have a different number of mobile agents.

We shall see that agglomeration, continental dispersion, and partial continental dispersion are *invariant patterns* (Aizawa et al., 2020; Ikeda et al., 2012), that is, they are solutions to $f_i = 0$ $(i = 1, 2)$ in (5.13) for any range of the parameter values.

Thus, their existence is assured. All the other aforementioned equilibria exist only in a subset of parameter space and thus require a deeper investigation.

Next, we define local stability. An equilibrium $\boldsymbol{h}^*$ is locally *stable* if, after a small perturbation due to an exogenous migration, the new spatial distribution reverts back to $\boldsymbol{h}^*$. Formally, an equilibrium is stable if the two eigenvalues of the Jacobian matrix of (5.13) evaluated at $\boldsymbol{h}^*$ are negative[32].

The Jacobian matrix of the dynamic system (5.13) is given by

$$J = \begin{bmatrix} \dfrac{\partial f_1}{\partial h_1} & \dfrac{\partial f_1}{\partial h_2} \\ \\ \dfrac{\partial f_2}{\partial h_1} & \dfrac{\partial f_2}{\partial h_2} \end{bmatrix}.$$

As for the stability of the *agglomerations*, the sufficient condition is equivalent to requiring the indirect utility of the agglomerated region to be strictly larger than those of the other two regions. Indeed, under the replicator dynamics, the study of local stability of interior equilibria of Δ is equivalent to the study of the stability of equilibria of any boundary of Δ (Gaspar et al., 2021) – or, more generally, by inspection of the signs of the eigenvalues of the Jacobian matrix.

5.2.2.1 Agglomerations

We first study the simplest solutions of the dynamic system (5.13), which are the points at which each region has all the skilled agents. Spatial configurations of this form are called *agglomerations*. Since the UK is not a symmetric region in relation to either FRA or GER, because of the differentiated transportation cost, we subdivide the agglomerations into *agglomeration in the UK* and *continental agglomeration*. Thus, agglomeration in the UK is simply $\boldsymbol{h} = (1, 0, 0)$ and continental agglomeration can either be $\boldsymbol{h} = (0, 1, 0)$ or $\boldsymbol{h} = (0, 0, 1)$. Figure 5.2 highlights these configurations.

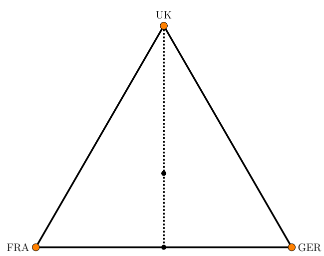


Figure 5.2: This figure highlights, in orange, the agglomerations.

[32]For a necessary and sufficient condition, we require the eigenvalues to be non-positive. If the equilibrium is non-hyperbolic (at least one eigenvalue is zero), it is called *irregular* (see, for example, Castro et al. (2022)). We do not investigate this case in the present work.

Existence

It is fairly standard to observe that agglomeration in the UK, and continental agglomeration are always solutions to $f_i = 0$ $(i = 1, 2)$ in (5.13), that is, they are invariant patterns.

Stability of agglomeration in the UK

To study the stability of the agglomeration in the UK, we need the indirect utility of the UK to be strictly larger than those of FRA and GER. Hence, we have $V_1 > V_3$ and $V_1 > V_2$. Since that, by symmetry, we know that $V_2 = V_3$, it is enough for the equilibrium to be stable to have $V_1 > V_3$.

Proposition 5.2. *The agglomeration in the UK is stable if $\Lambda^{AUK} < 0$, where Λ^{AUK} is given in Appendix C.1.*

Proof. Using (5.12) and simplifying $V_1 > V_3$ yields the result. □

Figure 5.3 illustrates the regions in which $\Lambda^{AUK} < 0$ holds.

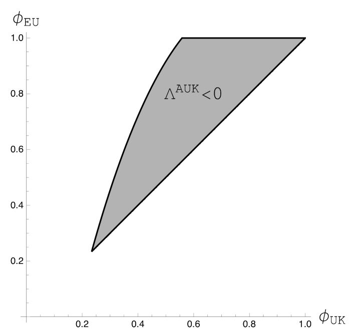


Figure 5.3: This figure illustrates $\Lambda^{AUK} < 0$ for $\lambda = 3$ and $\sigma = 6$. The area represents the combinations of (ϕ_{UK}, ϕ_{EU}) in which the agglomeration in the UK is stable, given the restriction $\phi_{EU} > \phi_{UK}$.

Figure 5.3 suggests that the agglomeration in the UK will only be stable for values of ϕ_{UK} and ϕ_{EU} that are not too small and that are relatively close. Therefore, since the shaded area represents the pairs (ϕ_{UK}, ϕ_{EU}) in which the indirect utility is higher in the UK than in any continental region, we conclude that the UK benefits from a high EU integration (ϕ_{EU}) and that the higher this integration is, the higher is the range of ϕ_{UK} that can sustain the stability of agglomeration in the UK. In other words, agglomeration in the UK benefits from a higher freeness of trade with the UK (ϕ_{UK}), but its range is greater for higher levels of the freeness of trade within the EU. Moreover, note that while agglomeration in the UK can still occur after the breakup, it will only be stable as long as the increase in transportation costs with the UK is not too high.

Stability of continental agglomeration

To study the stability of the continental agglomeration, we need the indirect utility of the agglomerated region to be strictly larger than those of the other two regions. Hence, we have $V_3 > \max\{V_1, V_2\}$.

Proposition 5.3. *The continental agglomeration is stable if* $\Lambda_1^{CA} < 0$ *and* $\Lambda_2^{CA} < 0$*, where* Λ_1^{CA} *and* Λ_2^{CA} *are given in Appendix C.1.*

Proof. Using (5.12) and simplifying $V_3 > V_1$ and $V_3 > V_2$ yields the results. □

Figure 5.4 illustrates the regions in which $\Lambda_1^{CA} < 0$ and $\Lambda_2^{CA} < 0$ hold.

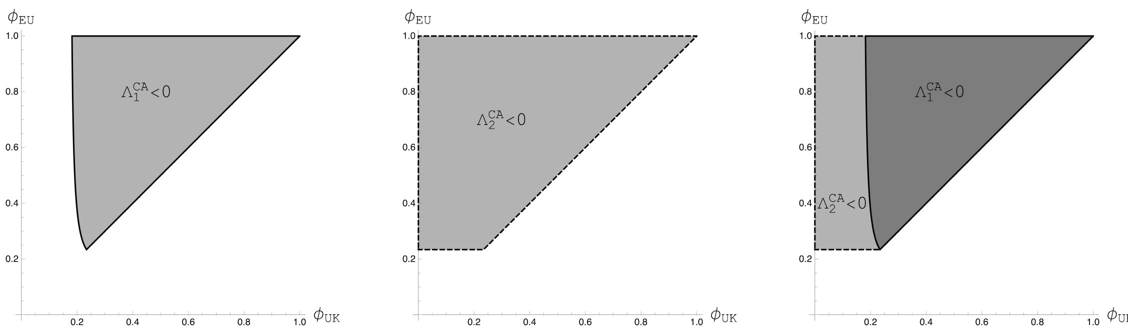


Figure 5.4: This figure illustrates $\Lambda_1^{CA} < 0$ and $\Lambda_2^{CA} < 0$ for $\lambda = 3$ and $\sigma = 6$, given the restriction $\phi_{EU} > \phi_{UK}$. The intersection area, whose shade is darker, represents the combinations of (ϕ_{UK}, ϕ_{EU}) in which the continental agglomeration is stable.

Figure 5.4 suggests that the continental agglomeration will be stable as long as neither the integration levels in the EU nor the integration levels in the UK are too small. Moreover, it is possible to show that, for realistically low values of λ, the stability is determined solely by Λ_1^{CA}, as Λ_2^{CA} envelops[33] it. Since the area $\Lambda_1^{CA} < 0$ represents the pairs (ϕ_{UK}, ϕ_{EU}) in which the indirect utility of GER is higher than that of the UK, we conclude that both EU and UK integration need not be too low. Therefore, high economic integration between the EU and the UK will drive people out of the UK towards GER. However, this is not enough as we also have to take into account the shaded area $\Lambda_2^{CA} < 0$, which represents the pairs (ϕ_{UK}, ϕ_{EU}) in which the indirect utility of GER is higher than that of FRA. In this situation, UK integration is irrelevant, and agents only prefer to live in GER if the transportation

[33] Let $\bar{\lambda} \equiv -\frac{3\phi_{EU}\phi_{UK}[(\sigma-1)(\phi_{EU}-\phi_{UK})+\sigma\ln(\phi_{EU}/\phi_{UK})]}{(\sigma-1)(\phi_{EU}-\phi_{UK})(\phi_{EU}\phi_{UK}+\phi_{UK}-1)}$ be the solution for $\Lambda_1^{CA} = \Lambda_2^{CA}$. Note that the numerator of $\bar{\lambda}$ is positive. Hence, $\bar{\lambda}$ is positive if, and only if, $\phi_{UK} \leq \frac{1}{2}$ or $\phi_{UK} > \frac{1}{2}$ and $\phi_{EU} < \frac{1-\phi_{UK}}{\phi_{UK}}$. Should these conditions be verified, we have that $\frac{\partial(\Lambda_1^{CA}-\Lambda_2^{CA})}{\partial\lambda} = -\frac{(\phi_{EU}-\phi_{UK})(\phi_{EU}\phi_{UK}+\phi_{UK}-1)}{3\sigma\phi_{EU}\phi_{UK}} > 0$. Therefore, $\Lambda_1^{CA} > \Lambda_2^{CA}$ if $\lambda > \bar{\lambda}$ and $\phi_{UK} \leq \frac{1}{2}$ or $\phi_{UK} > \frac{1}{2}$ and $\phi_{EU} < \frac{1-\phi_{UK}}{\phi_{UK}}$, which means that, under certain conditions for the pairs (ϕ_{UK}, ϕ_{EU}), stability is solely determined by Λ_1^{CA}.

costs within the EU are not extremely high. Combining both factors, we find that living in GER will only be truly beneficial if neither economic integration is too low.

Moreover, it is straightforward to show that Λ^{AUK} is always contained in Λ_1^{CA}. Thus, given the previous conditions for stability of continental agglomeration determined solely by Λ_1^{CA}, we can conclude that whenever agglomeration in the UK is stable, so is the continental agglomeration.

5.2.2.2 Median dispersion

Another equilibrium is the one that lies on the one-dimensional subspace of Δ defined by $\Delta_m = \left\{\boldsymbol{h} \in \Delta : h_1 = \alpha \in (0,1) \wedge h_2 = h_3 = \frac{1-\alpha}{2}\right\}$. Spatial configurations that lie on Δ_m and are equilibria are called *median dispersion* (MD). Figure 5.5 highlights these configurations.

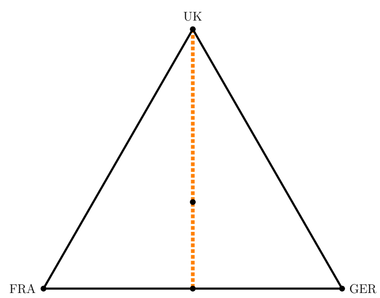


Figure 5.5: This figure highlights, in orange, the median dispersion.

Two things are noteworthy about Δ_m. First, notice that the total dispersion – in which all regions share the same number of entrepreneurs – belongs to Δ_m. Second, contrary to Gaspar et al. (2018) – in which all regions were symmetric in every respect – Δ_m is the unique *interior* invariant subspace in Δ in this model due to both FRA and GER being fully symmetric between each other, while the UK is different. This difference is trivially captured by the difference in the freeness of trade amongst the regions – ϕ_{UK} and ϕ_{EU}.

Existence

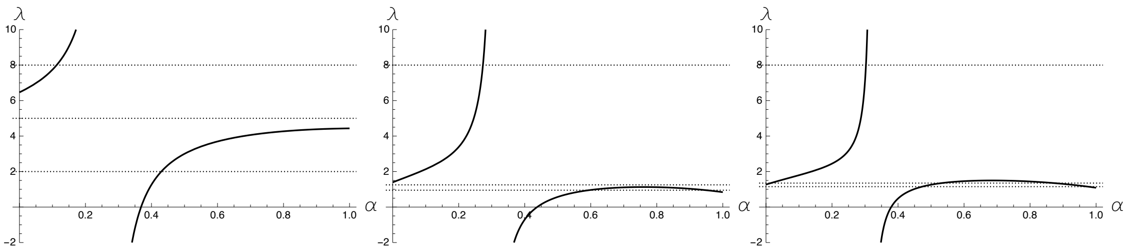
Not every configuration in Δ_m is a spatial equilibrium. Proposition 5.4 defines which points can be an equilibrium.

Proposition 5.4. *The spatial configuration* $\left(\alpha, \frac{1-\alpha}{2}, \frac{1-\alpha}{2}\right)$*, with* $\alpha \in (0,1)$*, is a solution to the system of equations* (5.13) *if* $\lambda = \lambda_m^*(\alpha)$*, where* $\lambda^*(\alpha)$ *is given in Appendix C.2.*

Proof. See Appendix C.2. □

In other words, a spatial configuration $\boldsymbol{h} = \left(\alpha, \frac{1-\alpha}{2}, \frac{1-\alpha}{2}\right)$ is an MD equilibrium if, and only if, there exists a value of $\lambda > 0$ such that $\lambda = \lambda_m^*(\alpha)$.

Moreover, for any given labour mobility value (λ), we may have a different scenario regarding the existence of MD equilibria. Figure 5.6 illustrates $\lambda_m^*(\alpha)$, its characteristics, and thresholds, and Proposition 5.5 formally defines the associated thresholds.



(a) $\phi_{EU} = 0.5$, $\phi_{UK} = 0.4$ (b) $\phi_{EU} = 0.6$, $\phi_{UK} = 0.1$ (c) $\phi_{EU} = 0.3$, $\phi_{UK} = 0.1$

Figure 5.6: This figure illustrates $\lambda_m^*(\alpha)$. In all panels, we have $\sigma = 6$. Panel (a) depicts the possibility that zero or one equilibrium points exist. Panel (b) depicts the possibility that zero, one, or two equilibrium points exist. Panel (c) depicts the possibility that one, two, or three equilibrium points exist.

Proposition 5.5. *Let $\alpha_{lm} \in (0, 1/3)$ be the vertical asymptote of $\lambda_m^*(\alpha)$ and $\alpha_{0m} \in (1/3, 1/2]$ its zero.*
For $\alpha \in (0, \alpha_{lm})$, there is a value for λ that corresponds to a unique MD equilibrium.
For $\alpha \in (\alpha_{lm}, \alpha_{0m})$, there is no value for λ that corresponds to a MD equilibrium.
For $\alpha \in (\alpha_{0m}, 1)$, there is a value for λ that corresponds to, at most, two MD equilibria.
Therefore, we may have, at most, three MD equilibria for a given value of λ.

Proof. See Appendix C.2. □

Whatever the case and parameter values, we can conclude that, either for a significantly industrialised or sufficiently deindustrialised UK, we can always find a value of λ such that an MD equilibrium exists.

When there are three simultaneous MD equilibria, one corresponds to a smaller UK than the other regions, and the other two correspond to the UK with more industry than FRA and GER.

As stated before, $\alpha = 1/3$ is a particular case of the MD in which the spatial distribution is evenly made across all the regions, hence $\boldsymbol{h} = (1/3, 1/3, 1/3)$. This spatial configuration is called *total dispersion*. Figure 5.7 highlights this configuration.

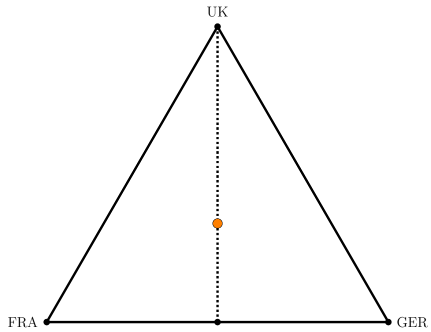


Figure 5.7: This figure highlights, in orange, the total dispersion.

Corollary 5.6. *The spatial configuration* $(1/3, 1/3, 1/3)$ *is never a solution to the dynamic system* (5.13)*.*

Proof. It follows from Proposition 5.5 that $\alpha = 1/3$ cannot be an equilibrium. □

Stability

To study the stability of the MD, as shown in Gaspar et al. (2021), it is sufficient that the eigenvalues of the Jacobian evaluated at any interior equilibrium belonging to Δ_m – which is the same as the interior invariant subspace $\mathcal{I}$ in Gaspar et al. (2021, p. 7) – are negative. Following the results in Lemma C.1 in Appendix C.2, the Jacobian evaluated at any equilibrium belonging to Δ_m has two single eigenvalues

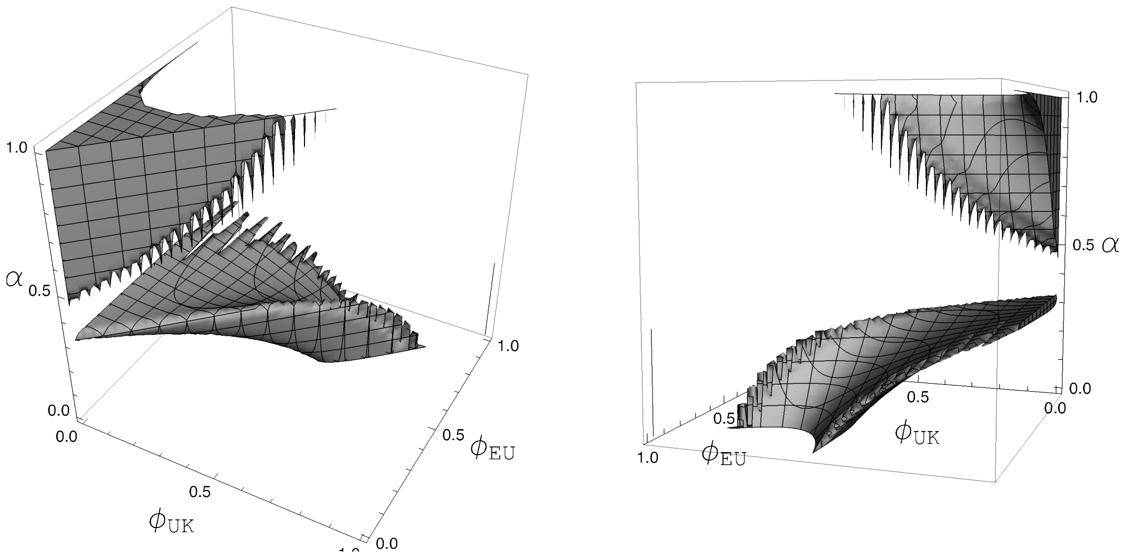
$$\Lambda_1^{MD} = V_1 - \bar{V} + \alpha \left(\frac{\partial V_1}{\partial h_1} - \frac{\partial \bar{V}}{\partial h_1} \right) \quad \text{and} \quad \Lambda_2^{MD} = \frac{1-\alpha}{2} \frac{\partial V_2}{\partial h_2}.$$

Proposition 5.7. *For* $\boldsymbol{h}^* \in \Delta_m$ *to be a stable MD equilibrium, it is sufficient that* $\Lambda_1^{MD} < 0$ *and* $\Lambda_2^{MD} < 0$*, where* Λ_1^{MD} *and* Λ_2^{MD} *are given in Appendix C.2.*

Proof. Using (5.12), computing the relevant partial derivatives, and simplifying Λ_1^{MD} and Λ_2^{MD} yields the results. □

Figures 5.8 and 5.9 illustrate the regions for which $\Lambda_1^{MD} < 0$ and $\Lambda_2^{MD} < 0$ hold, in the $(\phi_{UK}, \phi_{EU}, \alpha)$ space and in the cross-section (ϕ_{UK}, ϕ_{EU}) space, respectively.

Figure 5.8 is very useful in showing that there may be three different qualitative scenarios for MD equilibria. In the first place, there is a range in Δ_m in which no equilibrium is stable – which corresponds to the range of $\alpha \in (\alpha_{lm}, \alpha_{0m})$ for which there are no MD equilibria, as stated in Proposition 5.5. This occurs for values of α approximately between one-third and one-half in this example. Therefore, equilibria of this kind are only stable when the UK is the least industrialised region or if the UK outweighs the other two regions' industrialisation. Then, in the scenarios in which MD is stable, Figure 5.9 helps further narrow down the range of admissible integration levels.



(a) Front view

(b) Back view

Figure 5.8: This figure illustrates the regions in which, for $\lambda = \lambda_m^*$ and $\sigma = 6$, we have simultaneously $\Lambda_1^{MD} < 0$ and $\Lambda_2^{MD} < 0$. Both panels show the same area – one gives a front view, and the other gives a back view. The area represents the combinations of $(\phi_{UK}, \phi_{EU}, \alpha)$ in which the median dispersion exists and is stable, given the restriction $\phi_{EU} > \phi_{UK}$.

If the UK is the least industrialised region ($\alpha < 1/3$), the range of possible integration levels is relatively wide. In general, the less industry in the UK, the smaller the stability region in the $(\phi_{UK}, \phi_{EU}, \alpha)$ space, and it requires a combination of high EU integration and high UK integration. As industry in the UK increases, nearly any combination of EU and UK integration allows for stable equilibrium.

A spatial distribution in which most mobile workers live in the UK is only a stable outcome if both EU and UK integration are simultaneously low.

Finally, note that from Figure 5.9, it is the second eigenvalue that, generally, defines whether an MD equilibrium can be stable since it is largely contained in the first eigenvalue. As studied by Gaspar et al. (2021, pp. 16-17), in economic terms, while the first eigenvalue governs changes in indirect utilities between the UK and the other two regions, the second eigenvalue rules utility variations regarding FRA and GER, that is, migrations that are transversal to the invariant space Δ_m. If the latter migrations become favourable towards FRA or GER, the MD equilibria would breakdown.

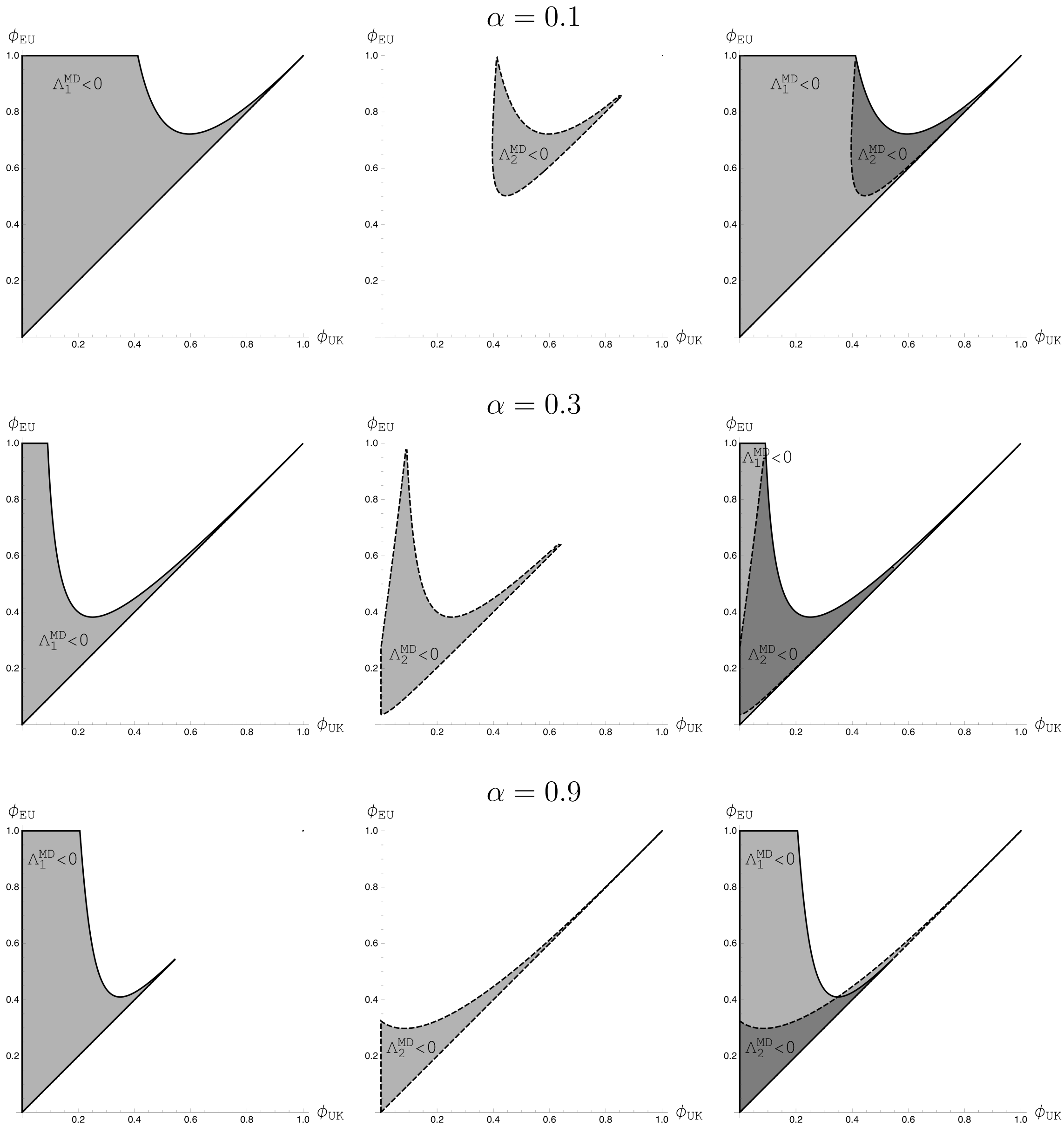


Figure 5.9: This figure illustrates three cross-sections of Figure 5.8 for $\alpha = 0.1$, $\alpha = 0.3$, and $\alpha = 0.9$, $\lambda = \lambda_m^*$, and $\sigma = 6$ – with the addition that both $\Lambda_1^{MD} < 0$ and $\Lambda_2^{MD} < 0$ are individually shown –, given the restriction $\phi_{EU} > \phi_{UK}$.The intersection area, whose shade is darker, represents the combinations of (ϕ_{UK}, ϕ_{EU}) in which the median dispersion exists and is stable.

5.2.2.3 Boundary dispersion

Without loss of generality, another invariant space is the border[34] of Δ that connects the UK with GER, defined by $\Delta_b = \{\boldsymbol{h} \in \Delta : h_1 = \alpha \in (0,1) \wedge h_2 = 0 \wedge h_3 = 1-\alpha\}$. Spatial configurations that lie on Δ_b and are equilibria are called *boundary dispersion* (BD). Figure 5.10 highlights these configurations.

[34] Given the symmetry of the problem, it is enough to explicitly only consider one border.

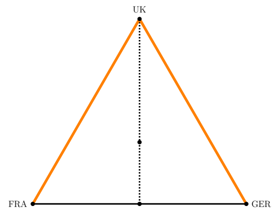


Figure 5.10: This figure highlights, in orange, the boundary dispersion.

Existence

Not every configuration in Δ_b is a spatial equilibrium. Proposition 5.8 defines which points can be an equilibrium.

Proposition 5.8. *The spatial configuration* $(\alpha, 0, 1-\alpha)$, *with* $\alpha \in (0,1)$, *is a solution to the system* (5.13) *if* $\lambda = \lambda_b^*(\alpha)$, *where* $\lambda_b^*(\alpha)$ *is given in Appendix C.3.*

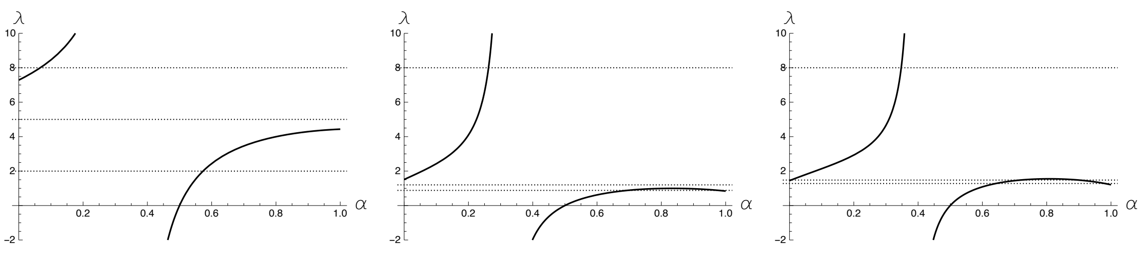
Proof. See Appendix C.3. □

In other words, a spatial configuration $\boldsymbol{h} = (\alpha, 0, 1-\alpha)$ is a BD equilibrium if, and only if, there exists a value of $\lambda > 0$ such that $\lambda = \lambda_b^*(\alpha)$.

Moreover, for any given labour mobility value (λ), we may have a different scenario regarding the existence of BD equilibria. Unfortunately, unlike in the case of MD, it is very hard to establish the multiplicity of BD equilibria analytically. Proposition 5.9 formally defines the associated thresholds, and Figure 5.11 illustrates $\lambda_b^*(\alpha)$, its characteristics and thresholds.

Proposition 5.9. *Let* $\alpha_{lb} \in (0, 1/2)$ *be the vertical asymptote of* $\lambda_b^*(\alpha)$.
For $\alpha \in (0, \alpha_{lb})$, *there is a value for* λ *that corresponds to a unique BD equilibrium.*
For $\alpha \in (\alpha_{lb}, 1/2)$, *there is no value for* λ *that corresponds to a BD equilibrium.*
For $\alpha \in (1/2, 1)$, *there is a value for* λ *that corresponds to, at most, two BD equilibria.*

Proof. See Appendix C.3. □



(a) $\phi_{EU} = 0.5$, $\phi_{UK} = 0.4$ (b) $\phi_{EU} = 0.6$, $\phi_{UK} = 0.1$ (c) $\phi_{EU} = 0.2$, $\phi_{UK} = 0.1$

Figure 5.11: This figure illustrates $\lambda_b^*(\alpha)$. In all panels, we have $\sigma = 6$. Panel (a) depicts the possibility that zero or one equilibrium points exist. Panel (b) depicts the possibility that zero, one, or two equilibrium points exist. Panel (c) depicts the possibility that one, two, or three equilibrium points exist.

Whatever the case and parameter values, we can conclude that either for a significantly industrialised or sufficiently deindustrialised UK, we can always find a value of λ such that a BD equilibrium exists.

When there are two simultaneous BD equilibria, both correspond to the UK with more industry than the non-empty region.

Stability

As in the MD, to study the stability of the BD, we need that the eigenvalues of the Jacobian evaluated at any equilibrium belonging to Δ_b be negative. Following the results in Lemma C.2 in Appendix C.3, the Jacobian evaluated at any equilibrium belonging to Δ_b has two single eigenvalues

$$\Lambda_1^{BD} = \alpha\left(\frac{\partial V_1}{\partial h_1} - \frac{\partial \bar{V}}{\partial h_1}\right) \quad \text{and} \quad \Lambda_2^{BD} = V_2 - V_1.$$

Proposition 5.10. *For $\boldsymbol{h}^* \in \Delta_b$ to be a stable BD equilibrium, it is sufficient that $\Lambda_1^{BD} < 0$ and $\Lambda_2^{BD} < 0$, where Λ_1^{BD} and Λ_2^{BD} are given in Appendix C.3.*

Proof. Using (5.12), computing the relevant partial derivatives, and simplifying Λ_1 and Λ_2 yields the results. □

Figures 5.12 and 5.13 illustrates the regions in which $\Lambda_1^{BD} < 0$ and $\Lambda_2^{BD} < 0$ hold, in the $(\phi_{UK}, \phi_{EU}, \alpha)$ space and in the cross-section (ϕ_{UK}, ϕ_{EU}) space, respectively.

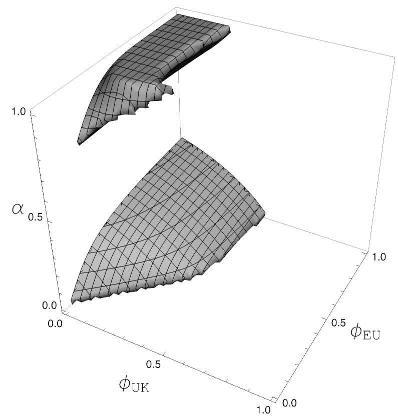


Figure 5.12: This figure illustrates the regions in which, for $\lambda = \lambda_b^*$ and $\sigma = 6$, we have simultaneously $\Lambda_1^{BD} < 0$ and $\Lambda_2^{BD} < 0$. The area represents the combinations of $(\phi_{UK}, \phi_{EU}, \alpha)$ in which the boundary dispersion exists and is stable, given the restriction $\phi_{EU} > \phi_{UK}$.

As in the MD case, Figure 5.12 also shows that there may be three different scenarios. Either the UK is the clear leader of the two regions in the boundary or the share of mobile workers living in the UK will be the smallest of both regions. Therefore, intermediate distributions are not stable.

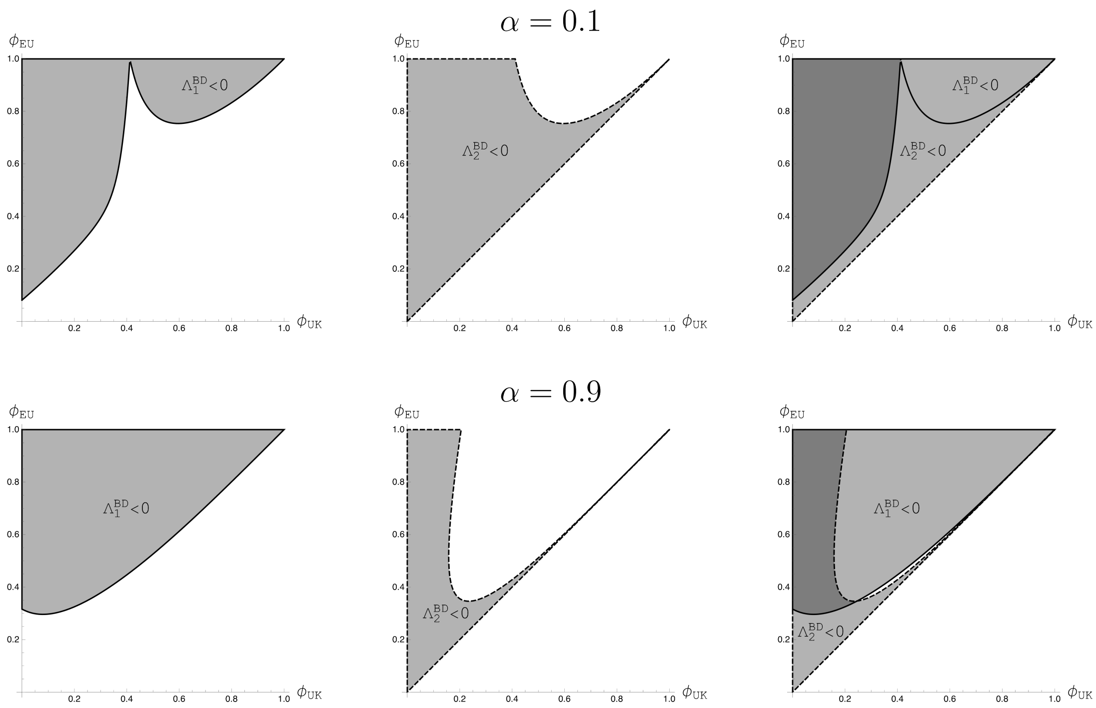


Figure 5.13: This figure illustrates two cross-sections of Figure 5.12 for $\alpha = 0.1$ and $\alpha = 0.9$, $\lambda = \lambda_b^*$, and $\sigma = 6$ – with the addition that both $\Lambda_1^{BD} < 0$ and $\Lambda_2^{BD} < 0$ are individually shown –, given the restriction $\phi_{EU} > \phi_{UK}$. The intersection area, whose shade is darker, represents the combinations of (ϕ_{UK}, ϕ_{EU}) in which the boundary dispersion exists and is stable.

Figure 5.13 is useful to assess further how EU integration and the integration between the EU and the UK determine the stability of any particular BD equilibrium. First, it is clear that the parameter range for a stable outcome decreases with industrialisation when the UK is not the leading region and increases if the UK is the clear leader. Furthermore, in both scenarios, the EU integration is the least relevant of the two, as long as it is not particularly small. Also, in both scenarios, the UK integration should be relatively low.

Therefore, for a BD equilibrium to be a stable outcome, regardless of the leading region, the transportation costs between the UK and the continental regions have to be relatively high, and the transportation costs between continental regions only need not be extremely high.

Finally, note that it is the second eigenvalue that, generally, defines whether a spatial distribution can be stable. As studied by Gaspar et al. (2021, pp. 16-17), in economic terms, while the first eigenvalue governs changes in indirect utilities between the UK and the other region in the boundary, the second eigenvalue rules utility variations regarding migrations that are transversal to the space Δ_b. If the latter migrations become favourable towards the empty region, the BD equilibria would breakdown.

5.2.2.4 Continental partial agglomeration

We now study the boundary in which the UK has no industry, which is equivalent to the one-dimensional subspace given by $\Delta_{b_{EU}} = \{\boldsymbol{h} \in \Delta : h_1 = 0 \wedge h_2 = \alpha \in (0,1) \wedge h_3 = 1-\alpha\}$. Spatial configurations that lie on $\Delta_{b_{EU}}$ and are equilibria are called *continental partial agglomeration* (CPA). Figure 5.14 highlights these configurations.

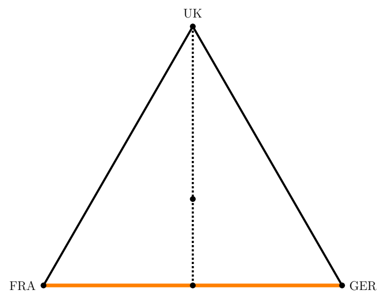


Figure 5.14: This figure highlights, in orange, the continental partial agglomeration.

Existence

Not every configuration in $\Delta_{b_{EU}}$ is a spatial equilibrium. Proposition 5.11 defines which points can be an equilibrium.

Lemma 5.11. *The spatial configuration* $(0, \alpha, 1-\alpha)$*, with* $\alpha \in (0,1)$*, is a solution to the system of equations* (5.13) *if* $\lambda = \lambda^*_{b_{EU}}(\alpha)$*, where* $\lambda^*_{b_{EU}}(\alpha)$ *is given in Appendix C.4.*

Proof. See Gaspar et al. (2021, Proposition 5). □

In other words, a spatial configuration $\boldsymbol{h} = (0, \alpha, 1-\alpha)$ is a CPA equilibrium if, and only if, there exists a value of $\lambda > 0$ such that $\lambda = \lambda^*_{b_{EU}}(\alpha)$.

Moreover, for any given labour mobility value (λ), we may have a different scenario regarding the existence of continental partial agglomeration equilibria. Proposition 5.12 formally defines the associated thresholds.

Lemma 5.12. *For* $\alpha \in (0, 1/2) \cup (1/2, 1)$*, there is a value for* λ *that corresponds to two symmetric continental partial agglomeration equilibria.*

Proof. See Gaspar et al. (2021, Proposition 5). □

The expression for $\lambda^*_{b_{EU}}(\alpha)$ is exactly the same as the expression in the proof of Proposition 5 in Gaspar et al. (2021), who also study the existence and stability of this kind of configuration – *partial agglomeration* in their work – in the Pflüger (2004) model with three completely symmetric regions. Notice how $\lambda^*_{b_{EU}}(\alpha)$ only depends on EU integration. This is because, along $\Delta_{b_{EU}}$, we have $h_1 = 0$, which is invariant for the dynamics. Hence the economy consists only of mobile workers that live either in FRA or in GER. This means that no manufactures are produced in the UK, and thus no consumer faces the respective transportation cost. Hence, in the particular restriction $h_1 = 0$, our model becomes qualitatively equivalent to Gaspar et al. (2018) and Gaspar et al. (2020), who consider equidistant regions.

Moreover, if the mobile agents evenly distribute themselves across the two continental regions, we end up with the spatial configuration $(0, 1/2, 1/2)$, which is a particular case of CPA in which $\alpha = 1/2$. This spatial configuration is called *continental dispersion* and is an invariant pattern. Figure 5.15 highlights this configuration.

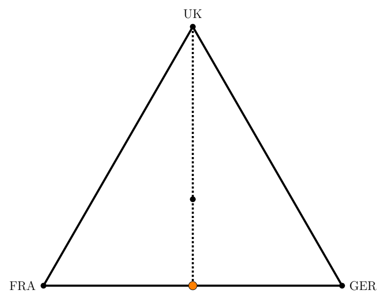


Figure 5.15: This figure highlights, in orange, the continental dispersion.

Lemma 5.13. *The spatial configuration* $\boldsymbol{h} = (0, 1/2, 1/2)$ *is a solution of the dynamic system* (5.13).

Proof. See Gaspar et al. (2021, Proposition 5). □

Stability

To study the stability of the continental partial agglomeration, we refer again to Gaspar et al. (2018) and Gaspar et al. (2021), since the one-dimensional subspace $\Delta_{b_{EU}}$ – the boundary such that no mobile workers live in the UK – is qualitatively equivalent to the one in those papers. This yields the following result.

Lemma 5.14. *Continental partial agglomeration and continental dispersion are always unstable.*

Proof. See Gaspar et al. (2021, Proposition 5). □

5.2.3 The disentanglement process

In Figures 5.16 and 5.17, we present two examples that shed some light on the dynamics of the disentanglement process. The parameters for the expenditure share in the industrial good and the elasticity of substitution between varieties are fixed at $\mu = 0.3$ and $\sigma = 6$, respectively. The spatial worker global mobility parameter increases from $\lambda = 1.5$ to $\lambda = 4$, representing a high and low global mobility situation, respectively.

Each example features three different cases regarding the level of economic integration within the remaining regions in the union, as measured by the variable ϕ_{EU}. The first row of each example is the high-integration case (high ϕ_{EU}), the second and third rows are intermediate-integration cases (intermediate ϕ_{EU}), and the last one is the low-integration case (low ϕ_{EU}).

Finally, each case is then divided into three specific scenarios, in which the only difference between them is the level of integration between the UK and the remaining regions in the union, as measured by the variable[35] ϕ_{UK}. The first column of each case is the high-integration case (high ϕ_{UK}), the second one is an intermediate-integration case (intermediate ϕ_{UK}), and the last one is the low-integration case (low ϕ_{UK}).

Starting with the high global mobility situation depicted in Figure 5.16, within a highly integrated union, we observe that as the integration between the dissident and the remaining regions declines, so does the likelihood of the UK concentrating all the mobile workers. Hence, if the continental regions have a deep integration, the Brexit scenario is unfavourable to the UK.

In case 2, we explore two levels of intermediate integration for the union, and we find that if the integration of the continental regions is towards the upper levels, the effects are similar to those of case 1.

When the intermediate integration is towards the bottom levels, increases in the transportation cost with the UK lead to a gradual exodus from the UK. We move from agglomeration in the UK to a median dispersion towards the UK and an asymmetric equilibrium towards the UK before collapsing to the boundary equilibrium like before.

Finally, when the union integration is low, no agglomeration is stable. However, when the integration with the UK is close to the integration in the EU, there are median dispersion equilibria towards the UK that eventually collapse to asymmetric equilibria towards the continental regions when the UK integration becomes too small. Note that these asymmetric equilibria are very close to the boundary equilibrium.

In the low global mobility situation depicted in Figure 5.17, the first case is

[35]Since $\phi_{EU} > \phi_{UK}$, the absolute values of ϕ_{UK} may represent high, low or intermediate levels of integration depending on the value of ϕ_{EU}.

qualitatively similar to that of example 1.

In case 2, if we consider the higher integration level of the union, we have qualitatively similar behaviour to that of case 1.

However, we observe a unique pattern with a lower continental integration level in which the stable equilibria are towards the continental regions. That can be the asymmetric equilibria if the UK integration is high enough or the median dispersion equilibria if the UK integration is not too high.

When the integration of the united regions is very low, transportation costs with the UK will not be highly significant, as the stable equilibrium will lie on the median dispersion, near-total dispersion, but with a slight advantage towards the continental regions.

Finally, a more general comparison between Figures 5.16 and 5.17 allows us to conclude that as the global mobility of industrial workers decreases (λ increases), so does the chance that the share of mobile workers in the UK after leaving the union is high. Hence, a higher global inter-regional mobility of workers mitigates the exodus of mobile workers from the UK after the disentanglement.

5.2.4 Welfare analysis

We follow the approaches of Pflüger and Südekum (2008) and Gaspar et al. (2018) regarding the study of welfare to understand how desirable all the spatial configurations we have just analysed are. To this end, we use a utilitarian criterion that considers average indirect utility as the welfare measure. Therefore, we look at the average indirect utility of entrepreneurs, agricultural workers, and the economy as a whole.

First, note that the weighted average nominal wage of entrepreneurs is the same as in Gaspar et al. (2018), that is, $\bar{w} = \frac{\mu}{\sigma}(1+\lambda)$. Thus, the well-being functions of entrepreneurs and agricultural workers are similar to those of Gaspar et al. Hence, the welfare of entrepreneurs is given by

$$\bar{V} = \frac{\mu}{\sigma}(1+\lambda) + \frac{\mu}{\sigma - 1}\sum_{i=1}^{3} h_i \ln\left[\sum_{m=1}^{3} h_m \phi_{mi}\right] + \eta.$$

Proposition 5.15. *The average indirect utility of entrepreneurs is convex in the spatial distribution of entrepreneurs* $\boldsymbol{h}$*, attaining a maximum at any agglomeration.*

Proof. Since

$$\frac{\partial^2}{\partial h_i^2}\left[h_i \ln\left(\sum_{m=1}^{3} h_m \phi_{mi}\right)\right] > 0, \quad i = \{1,2,3\},$$

is trivially satisfied, we show that $\bar{V}$ is convex as it is a composition of convex functions. Moreover, since

$$\frac{\partial}{\partial h_i}\left[h_i \ln\left(\sum_{m=1}^{3} h_m \phi_{mi}\right)\right]\Bigg|_{h_i=1} > 0, \quad i = \{1, 2, 3\},$$

and given the convexity of $\bar{V}$, we conclude that the agglomerations are the optimal welfare spatial distributions for mobile agents. Furthermore, note that $\bar{V}(1, 0, 0) = \bar{V}(0, 1, 0) = \bar{V}(0, 0, 1)$. Hence, mobile agents are indifferent regarding where they agglomerate. □

This result leads us to an interesting paradox that also arose in Gaspar et al. (2018) – entrepreneurs may end up dispersing themselves due to their short-sightedness, creating a situation similar to the prisoner's dilemma. For example, look at Figure 5.16e in which two BD equilibria exist near the UK. According to Proposition 5.15, if all the entrepreneurs moved to the UK, all of them would be better off. However, when only a marginal migration occurs towards the UK, these agents will return to the departure region since their indirect utility would be higher there and not in the UK.

As for the agricultural workers, their welfare is given by

$$\bar{V}^L = 1 + \frac{\mu}{3(\sigma - 1)} \sum_{i=1}^{3} \ln\left[\sum_{m=1}^{3} h_m \phi_{mi}\right] + \eta.$$

Proposition 5.16. *The average indirect utility of agricultural workers is concave in the spatial distribution of entrepreneurs* $\boldsymbol{h}$.

Proof. Since

$$\frac{\partial^2}{\partial h_i^2}\left[\ln\left(\sum_{m=1}^{3} h_m \phi_{mi}\right)\right] < 0, \quad i = \{1, 2, 3\},$$

is trivially satisfied, we show that $\bar{V}^L$ is concave as it is a composition of concave functions. □

Contrary to Gaspar et al. (2018), agricultural workers may not prefer a more dispersed distribution of entrepreneurs. Moreover, it is easily verifiable that $\bar{V}^L$ is higher for continental agglomerations than agglomeration in the UK. Therefore, numerical simulation shows that while sometimes agricultural workers do prefer a dispersed distribution of entrepreneurs – in particular, a distribution such that $h_1 < 1/3$ –, as transportation costs decrease, agricultural workers tend to prefer all the entrepreneurs to be concentrated in any of the continental regions.

Finally, we define the economy's social welfare as

$$\Omega(\boldsymbol{h}) = \frac{1}{1+\lambda}\left[\bar{V} + \lambda \bar{V}^L\right].$$

This can then be rewritten as

$$\Omega(\boldsymbol{h}) = \frac{\lambda(1+\eta) + \dfrac{\mu}{\sigma}(1+\lambda) + \eta}{1+\lambda} + \frac{\mu}{(1+\lambda)(\sigma-1)}\left[g(h_1) + g(h_2) + g(h_3)\right],$$

where $g(h_i) = \left(\dfrac{\lambda}{3} + h_i\right) \ln\left(\sum_{m=1}^{3} h_m \phi_{mi}\right)$. Thus, the optimisation plan for $\Omega(\boldsymbol{h})$ consists in maximising $\sum_{i=1}^{3} g(h_i)$, subject to $\sum_{i=1}^{3} h_i = 1$.

However, the symmetry argument used by Gaspar et al. (2018) – in which the optimal social welfare occurs when entrepreneurs are located in a median dispersion – is of no use in this problem due to the asymmetry caused by the different integration levels. Hence, an analytic solution cannot be achieved due to the complexity of these expressions. This is further supported by numerical simulations in which we verify that the social optimum spatial distribution can assume virtually all of the studied possibilities – either agglomeration, boundary or median dispersions, or even asymmetric distributions.

In Figures 5.18 and 5.19, we present the bifurcation diagram associated with each case studied in Figures 5.16 and 5.17 with a symbol code that makes the correspondence with the spatial descriptions in Figure 5.1. Moreover, we superimpose – with blue crosses – the economy's social optimum welfare points on these bifurcation diagrams, which represent the spatial distribution of mobile agents that maximise the social welfare in the economy. These figures offer new insight into how the reduction of integration with the UK changes spatial distribution and welfare outcomes.

Starting with case 1 of Figure 5.18, we conclude that there are only three stable equilibria. When integration with the UK is high enough, agglomeration in the UK may occur and be sustained. Agglomeration in FRA or GER only cannot be maintained if integration with the UK is too low, in which case a boundary dispersion scenario, with fewer mobile workers in the UK, arises. Welfare maximisation occurs for a boundary dispersion slightly more balanced towards the UK, by comparison with the stable equilibria, when integration with the UK is too low. As this integration increases, continental agglomeration is the economy's social optimum, which is also a stable equilibrium.

In particular, there is a level of integration with the UK below which agglomeration in the UK becomes unstable. Moreover, there is another level of integration with the UK – smaller than the former one – below which the only stable equilibria lie on the boundaries of the simplex, with most workers residing in one of the continental regions. Even though this phenomenon is not linear since the UK can agglomerate all the mobile workers if the new integration level is not too low, it eventually loses all the skilled workers as this integration decreases. Finally, if the integration is particularly low, the UK regains a small fraction of entrepreneurs, while at the same time, one continental region becomes depleted of them.

As for case 2, in the higher EU integration stance, the dynamics are very similar to case 1. However, when the EU integration drops, a new dynamic arises – when the integration with the UK is close to EU integration – the median dispersion, with the majority of mobile workers in the UK. The economy's welfare maximisation is also very similar to case 1 for both levels of intermediate EU integration.

Finally, case 3 has a different outcome in which no agglomeration equilibria exist. Either there is an asymmetric equilibrium with more mobile workers towards one of the continental regions or a median dispersion equilibrium towards the UK. Economy's welfare maximisation occurs for an asymmetric dispersion slightly more balanced towards the UK than stable equilibria.

As for Figure 5.19, cases 1 and 2 regarding the high EU integration are qualitatively similar to those of Figure 5.18. The major difference is the level of integration with the UK at which the type of equilibrium shifts. From the economy's welfare point of view, however, there are differences from example 1. Even though case 1 is qualitatively similar to that of example 1, case 2 is significantly different. In this case, with a high EU integration, the maximum social welfare of the economy is attained with a shifting type of spatial distribution – when the UK integration is very low, the economy's social optimum occurs for a median dispersion, then it becomes an asymmetric dispersion as the integration with the UK increases, and afterwards, it becomes a boundary dispersion one before collapsing to the continental agglomeration. In any of these distributions, the economy's social optimum implies a higher proportion of mobile agents living in the UK than those determined by the stable equilibria.

As for case 2 with a lower EU integration and case 3, the outcomes are qualitatively close. The main stable equilibrium is the median dispersion towards continental regions. Moreover, in case 2, asymmetric equilibria towards continental regions also become prevalent when the difference between the integration levels is small. As the EU integration decreases, the economy's optimal social distribution becomes constant, particularly a median dispersion that is very close to total dispersion, but in which the UK is the smaller region of the three. In fact, in case 3, the economy's optimal social distribution coincides with the stable median dispersion equilibria.

In a broad summary, we can infer that more favourable equilibria for the UK hinges on the integration levels, particularly on integration with the UK not being much lower than that within the EU. Moreover, as discussed, the cutoff at which this equilibrium shift is higher, the lower the global mobility of industrial workers (higher λ) is.

Also, note that there are some interesting, albeit unstable, regular curves of equilibria. Namely, a path for median dispersion equilibria towards the continental regions when the integration with the UK is low, and a C-shaped (or half-C-shaped) curve for boundary and median dispersion equilibria towards the UK that occurs almost simultaneously and, in some cases, may be linked by asymmetric equilibria.

Once again, the general comparison of both examples using the bifurcation diagrams of Figures 5.18 and 5.19 makes it even more evident that as the global mobility of industrial workers decreases (λ increases), the number of stable outcomes towards the UK becomes smaller. Since λ represents the proportion of immobile to mobile workers, decreases in the global mobility of industrial workers imply that there are relatively more agricultural workers – who cannot migrate – in the economy. This means that the immobile market size increases. Therefore, since the imbalance in integration levels pulls regions apart, the mobile workers will opt to migrate to move away from the UK to enjoy a broader immobile market.

As stated before, mobile agents always achieve their optimal welfare level in any of the agglomerations, while immobile agents prefer that the industrial activity become more concentrated in the continental regions as the integration level with the UK increases. In fact, the only spatial distribution of entrepreneurs that, at the same time, guarantees that both mobile and immobile agents enjoy their maximum welfare is the continental agglomeration, given that the integration with the UK is high enough.

Therefore, we conclude that, from an aggregate point of view, Brexit creates a social bias against living in the UK. Even though agglomeration in the UK still exists and may even be stable, as well as other distributions in which the majority of entrepreneurs live in the UK, for the welfare of the economy as a whole to be as high as possible, less than one-third of the mobile agents should live in the UK.

Example 1

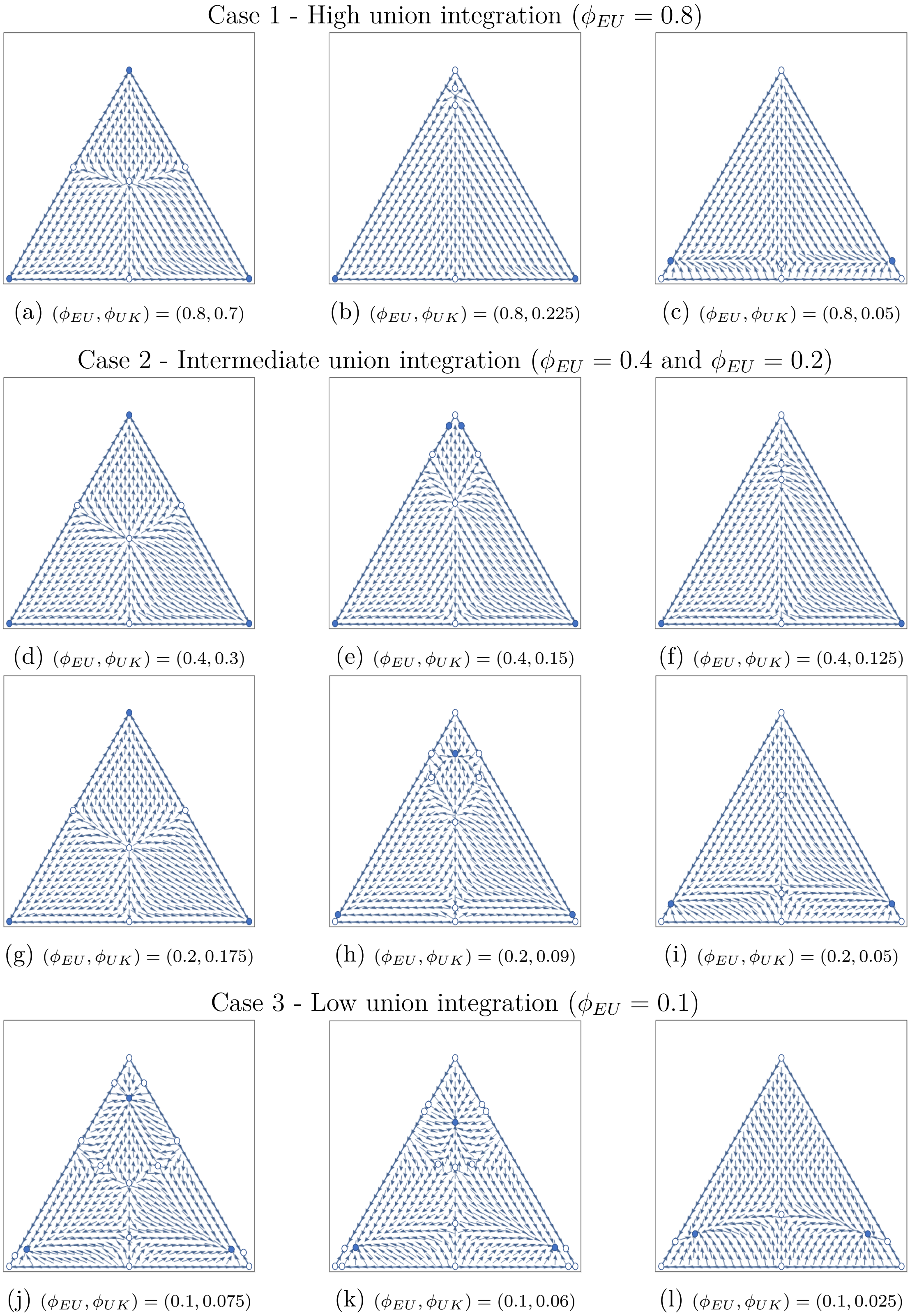
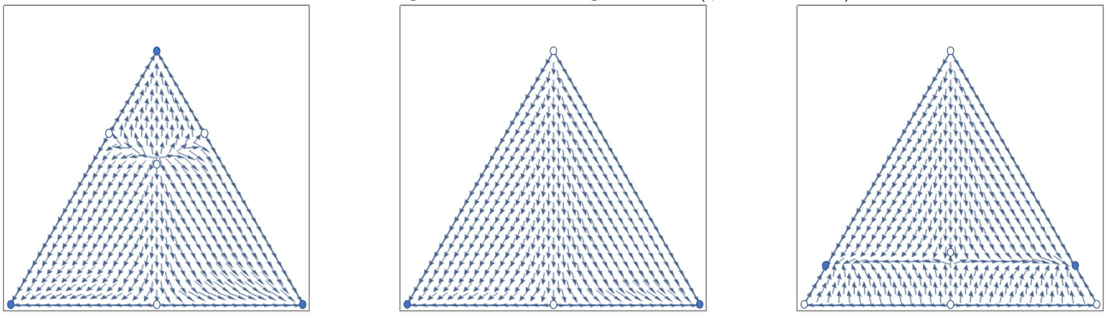


Figure 5.16: Simulation of spatial equilibria and stability in the simplex Δ. The parameters used are $\mu = 0.3$, $\sigma = 6$, $\lambda = 1.5$ and the pair (ϕ_{EU}, ϕ_{UK}) is described in each panel. Each row represents a case in which the ϕ_{EU} is fixed and the ϕ_{UK} decreases.

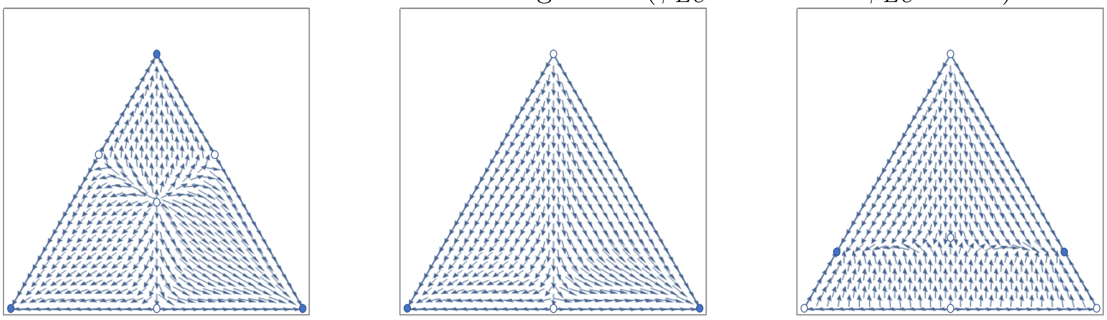
Example 2

Case 1 - High union integration ($\phi_{EU} = 0.8$)

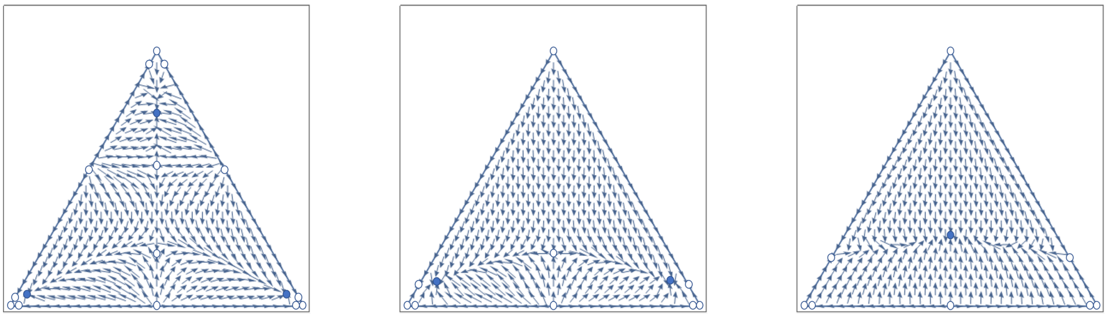


(a) $(\phi_{EU}, \phi_{UK}) = (0.8, 0.7)$ (b) $(\phi_{EU}, \phi_{UK}) = (0.8, 0.5)$ (c) $(\phi_{EU}, \phi_{UK}) = (0.8, 0.125)$

Case 2 - Intermediate union integration ($\phi_{EU} = 0.4$ and $\phi_{EU} = 0.3$)

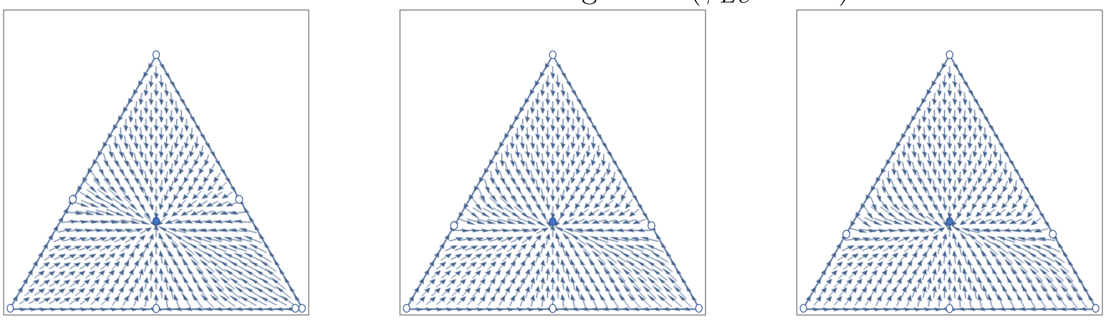


(d) $(\phi_{EU}, \phi_{UK}) = (0.4, 0.375)$ (e) $(\phi_{EU}, \phi_{UK}) = (0.4, 0.3)$ (f) $(\phi_{EU}, \phi_{UK}) = (0.4, 0.1)$



(g) $(\phi_{EU}, \phi_{UK}) = (0.3, 0.295)$ (h) $(\phi_{EU}, \phi_{UK}) = (0.3, 0.27)$ (i) $(\phi_{EU}, \phi_{UK}) = (0.3, 0.15)$

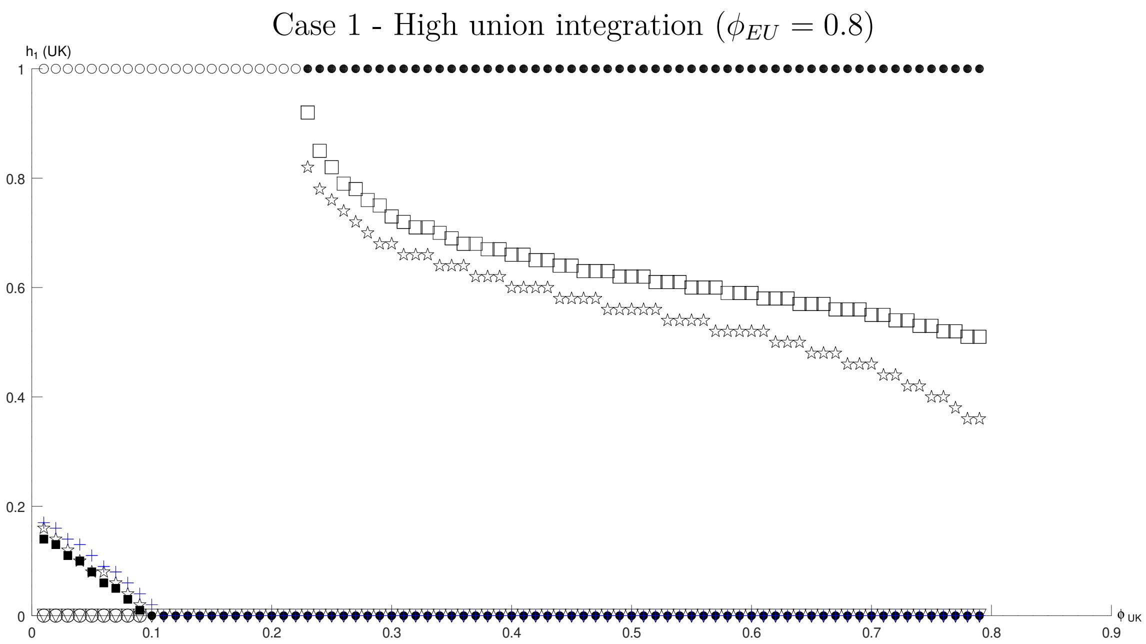
Case 3 - Low union integration ($\phi_{EU} = 0.1$)



(j) $(\phi_{EU}, \phi_{UK}) = (0.1, 0.085)$ (k) $(\phi_{EU}, \phi_{UK}) = (0.1, 0.04)$ (l) $(\phi_{EU}, \phi_{UK}) = (0.1, 0.01)$

Figure 5.17: Simulation of spatial equilibria and stability in the simplex Δ. The parameters used are $\mu = 0.3$, $\sigma = 6$, $\lambda = 4$ and the pair (ϕ_{EU}, ϕ_{UK}) is described in each panel. Each row represents a case in which the ϕ_{EU} is fixed and the ϕ_{UK} decreases.

Example 1



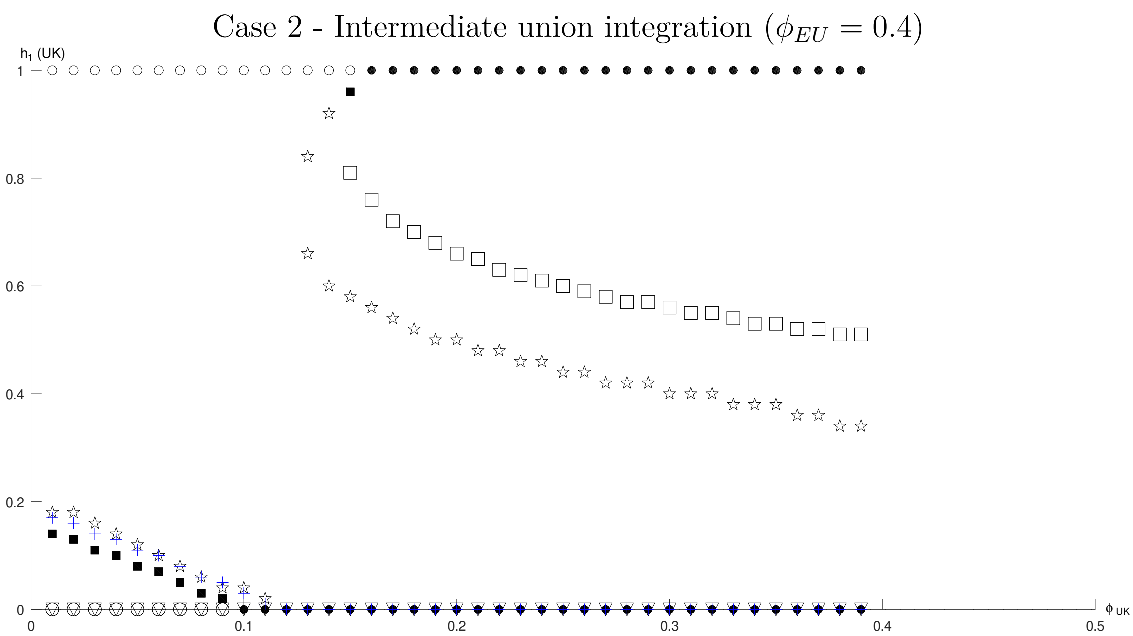
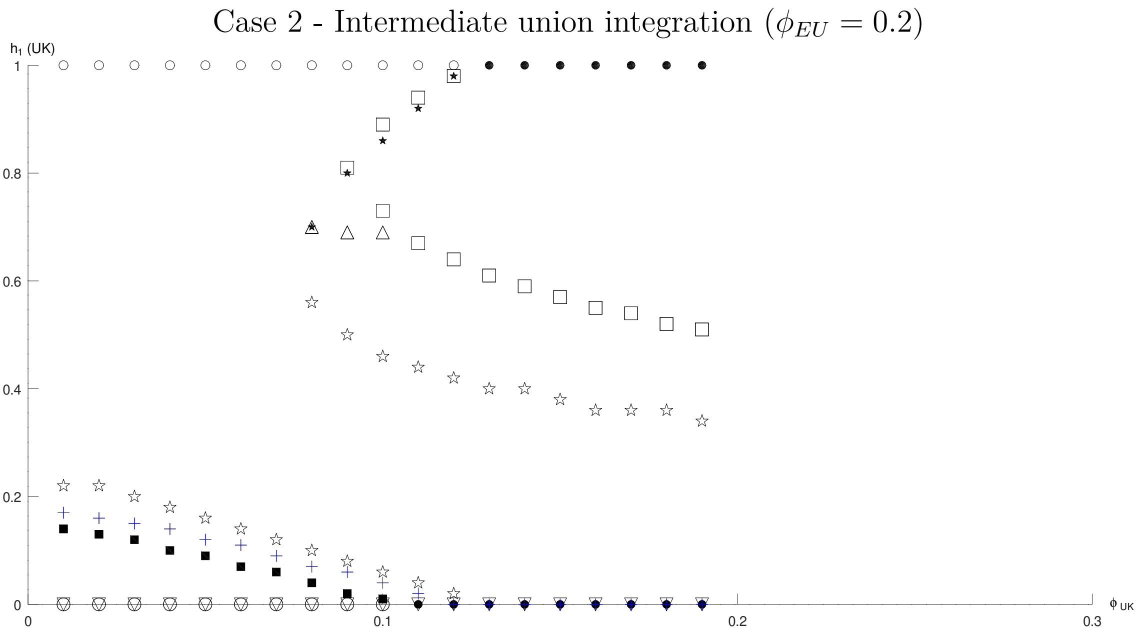


Figure 5.18: Bifurcation diagrams associated with the correspondent case in Figure 5.16 regarding the share of mobile workers in the UK. Each plotted point is a symbol with the same meaning as in Figure 5.1. The superimposed blue crosses represent the spatial distribution of mobile agents that maximise the economy's social welfare in the economy. *(cont.)*



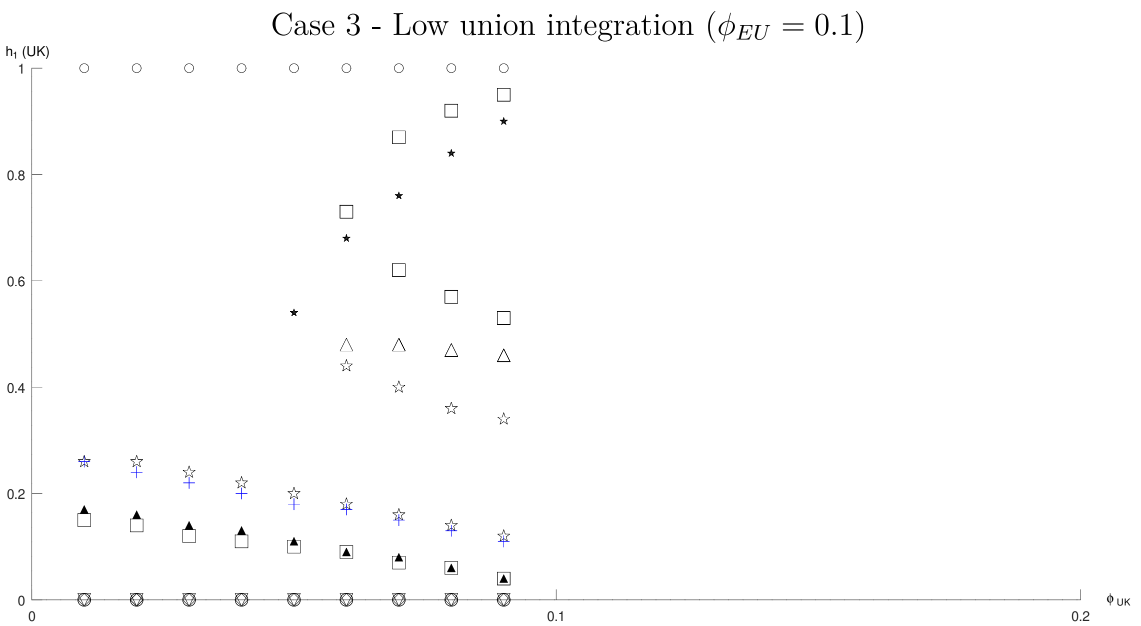
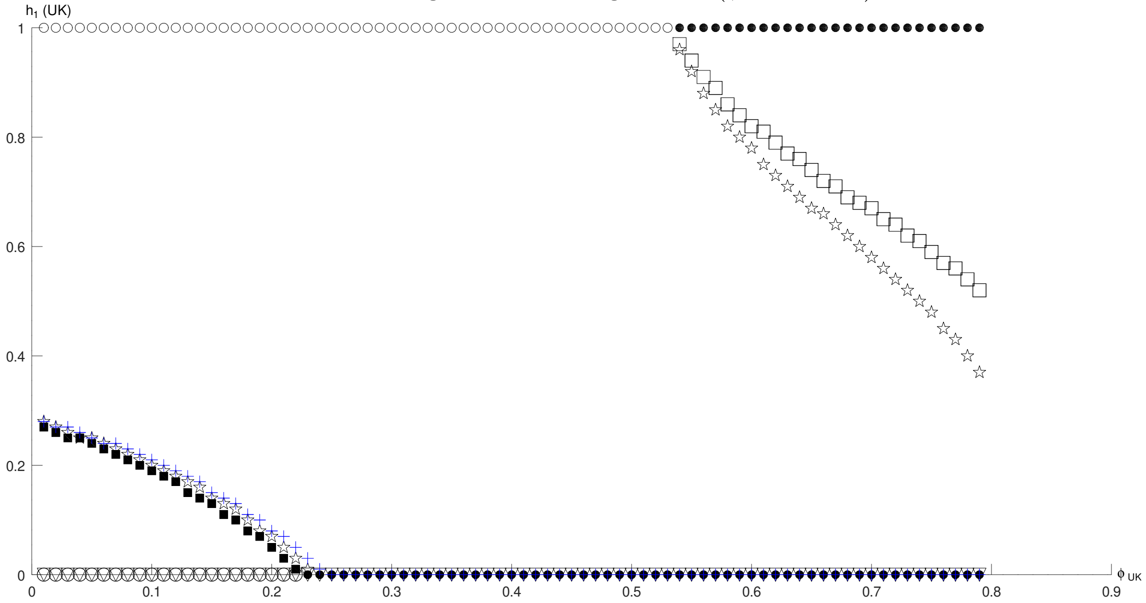


Figure 5.18: Bifurcation diagrams associated with the correspondent case in Figure 5.16 regarding the share of mobile workers in the UK. Each plotted point is a symbol with the same meaning as in Figure 5.1. The superimposed blue crosses represent the spatial distribution of mobile agents that maximise the economy's social welfare in the economy.

Example 2

Case 1 - High union integration ($\phi_{EU} = 0.8$)



Case 2 - Intermediate union integration ($\phi_{EU} = 0.4$)

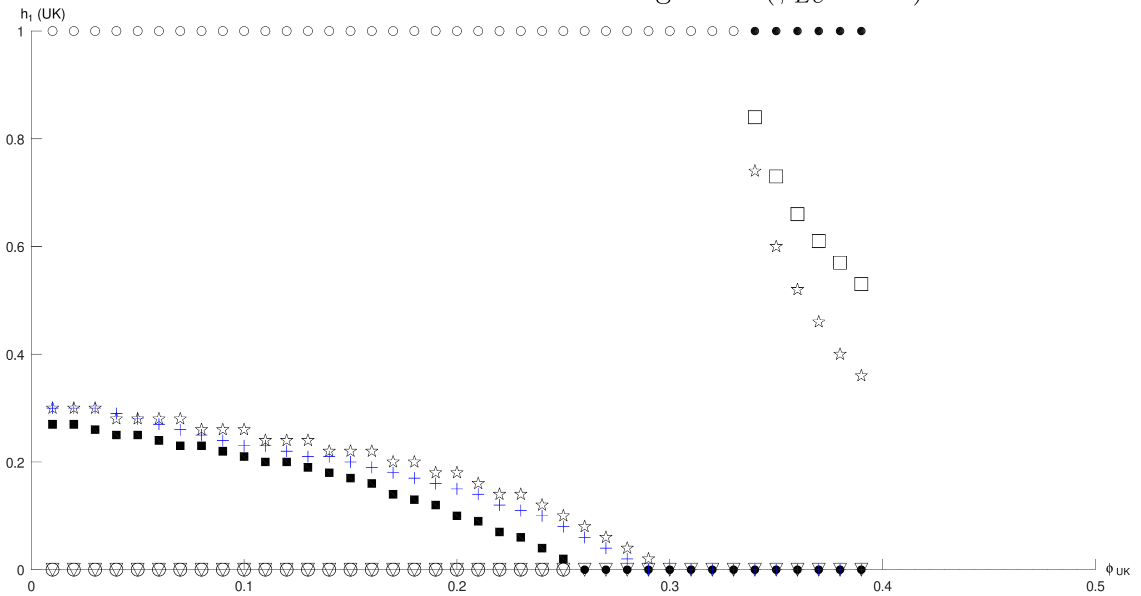
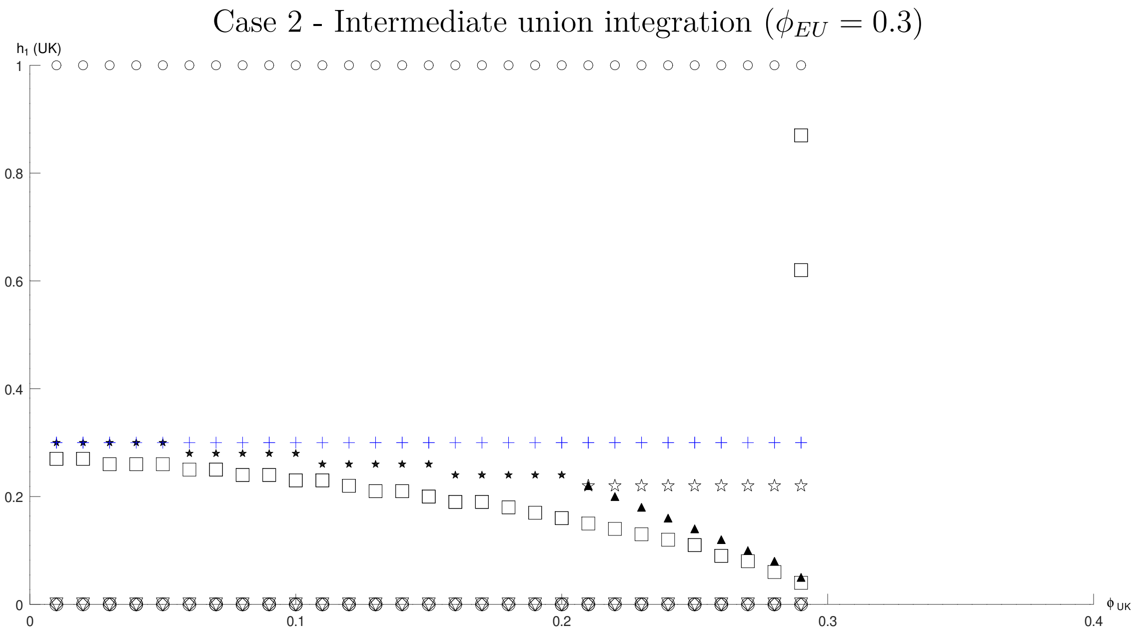


Figure 5.19: Bifurcation diagrams associated with the correspondent case in Figure 5.17 regarding the share of mobile workers in the UK. Each plotted point is a symbol with the same meaning as in Figure 5.1. The superimposed blue crosses represent the spatial distribution of mobile agents that maximise the economy's social welfare in the economy. *(cont.)*



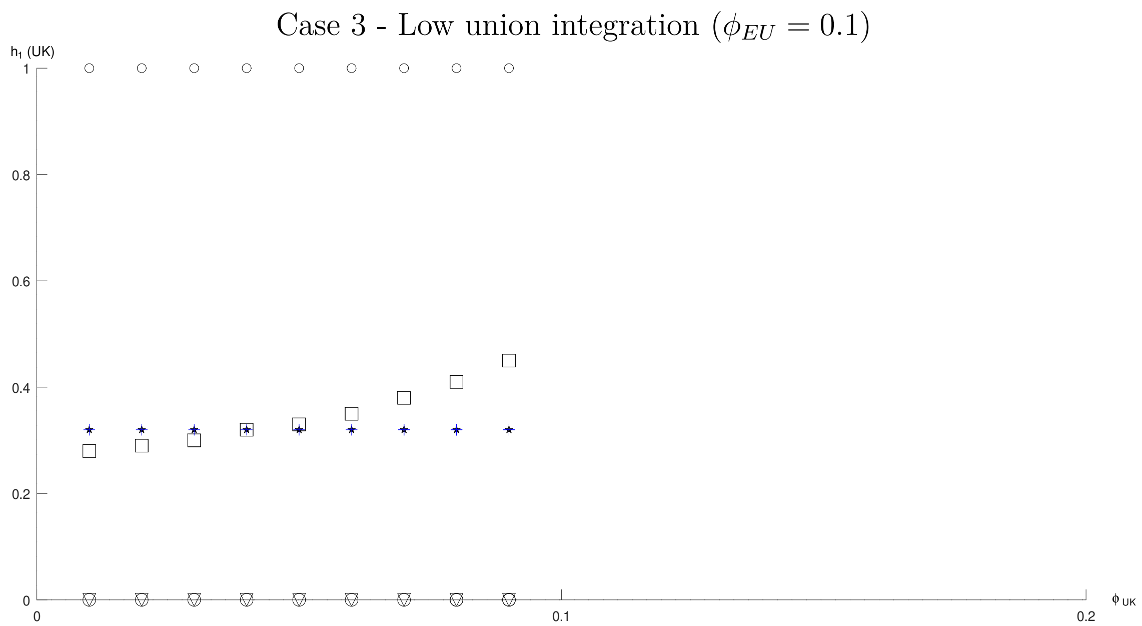


Figure 5.19: Bifurcation diagrams associated with the correspondent case in Figure 5.17 regarding the share of mobile workers in the UK. Each plotted point is a symbol with the same meaning as in Figure 5.1. The superimposed blue crosses represent the spatial distribution of mobile agents that maximise the economy's social welfare in the economy.

5.3 Concluding remarks

In this essay, we study how the disentanglement of an economic union affects both its remaining members and the leaving party. In particular, we focus on whether the dissident region ends up with the lead position regarding the share of industrial workers or with the short end of the stick and how it affects social welfare.

We develop an approach that uses a footloose entrepreneur model with three regions as a baseline. We assume that when a region wants to leave an economic union, the practical consequence of that decision is an increase in the transportation costs between the leaving region and its former allies. Note that even though our exposition revolves around Brexit, the model we propose is valid for any three symmetric regions with a structure of transportation costs as described.

We find five possible outcomes regarding spatial distribution – agglomeration in any region, median dispersion, boundary dispersion, continental partial agglomeration, and asymmetric equilibria. We also conclude that the total dispersion scenario is excluded since transportation costs are not equal between every region. Moreover, we find that an unusual spatial distribution arises, the asymmetric equilibria, in which each region has a different share of mobile workers than the other two.

As for the stability of the former equilibria, we find that the continental partial agglomeration equilibria – which is the spatial distribution with all industrial workers being divided only by the continental regions – are never stable. Hence, we conclude that, apart from the extreme scenarios of continental agglomerations, the leaving region never ends up in a situation in which it is totally depleted of industrial workers *ad aeternum*.

We also find the following. First, that agglomeration in the UK is stable as long as the integration level in the remaining regions is high. Second, the continental agglomeration is stable as long as the integration level in the remaining regions and with the UK are intermediate. Third, the median dispersion towards continental regions is stable as long as the integration level in the remaining regions and with the UK are high. Fourth, the median dispersion towards the UK is almost always stable if it exists, and the boundary dispersion is stable as long as the integration level in the remaining regions is intermediate and with the UK is low.

We conduct a numerical analysis in which we study how different levels of integration between the regions and different levels of mobility of industrial workers affect the spatial distribution. We conclude that the UK share of mobile workers is higher the lower the difference between transportation costs within the remaining regions and with the UK is. Moreover, the lower the global mobility of industrial workers is, the lower the chance that the UK's share of mobile workers is higher after leaving the union.

The former numerical analysis also allows us to study how welfare is affected by this disentanglement. We conclude that industrial workers enjoy their maximum well-being by agglomeration in any of the regions, while immobile agents prefer that the industry becomes more concentrated in the continental regions as the transportation costs with the UK decrease. Even so, when we look at the bigger picture, we find that the social optimum for the economy as a whole occurs for a spatial distribution of entrepreneurs that is always unfavourable towards the UK, to the point that it is never socially optimal that the UK concentrates more than one-third of the industry. Even so, when the social optimal spatial distribution does not coincide with the stable equilibria, it tends to mimic the same configuration of it – for example, if the stable equilibria were an MD, the social optimal spatial distribution of mobile agents would also be an MD –, and it predicts that it would be socially better to have a slightly higher percentage of industry in the UK than the one preconised by the stable equilibria distribution.

Since every day the world is becoming more globalised, an interesting line for future research – that we intend to pursue in future work – would be to bring a fourth region to our approach – the United States of America (USA) – to act as a commercial partner and help us figure out with more detail which are the dynamics that ensue from the disentanglement caused by falling out of one of the members of the economic union. In short, our concept is that of an economy similar to that studied in this essay with the addition of the USA that has a differentiated transportation cost with EU members[36]. Then, after Brexit, we assume that the UK will trade with the EU and the USA according to the initial USA-EU transportation cost, therefore leaving the continental regions with a differentiated transportation cost in this economy.

[36]Note that Commendatore et al. (2014) made a similar approach to this one. However, no migration ensues in their work.

Chapter 6

Concluding remarks

In this thesis, we study two topics – education and economic unions – from two main perspectives. From the agents' point of view, we explore how the economy reacts to changes in individual characteristics. From the regions' point of view, we analyse how institutional and political frameworks affect the economy. While the thematics of our two topics are quite different, the ultimate goal is similar – understanding agents' decision process when their decisions have a spatial dimension.

We aim to bring the Economic Geography literature forward by introducing novel approaches, such as the conceptualisation of economies with varying and different levels of productivity and the endogenous general equilibrium model that mixes the footloose entrepreneur model with the overlapping generations model.

In chapters 2 through 4, we explore the relationship between education and the spatial distribution of economic activity. In chapter 5, we explore the breakup of an economic union due to the unilateral withdrawal by one of its members.

In chapter 2, we study how regional asymmetries in firms' productivity affect the spatial distribution of economic activity and social welfare. In this chapter, the principal perspective is that of the regions, as we extend the two-region Pflüger (2004) footloose entrepreneur model to account for regional productivity asymmetries. This productivity asymmetry is transmitted to wages – wages in the most efficient region are higher – and prices – prices in the most efficient region are lower. We study the migrations of the agents and achieve several results. First, it is an equilibrium for the agents to be agglomerated in any of the regions but not to be evenly distributed between them. Second, while any agglomeration may be stable, stability in the least productive region is only possible if the productivity gap is small, and it also implies that agglomeration in the other region is stable. Third, multiple interior equilibria may occur, and spatial distributions in which most workers live in the efficient region are always stable. Fourth, we find that if more regions exist in the economy, those with intermediate productivity levels tend to become empty. Last, we conclude that mobile workers enjoy maximum welfare if they all agglomerate in the efficient region and that immobile ones never achieve maximum well-being.

In chapter 3, we study how agents make their education decisions when they live in an economy with multiple regions. In this chapter, we focus equally on the agents' and the regions' points of view since we explore how agents make their optimal education decisions depending on the framework they live in. We integrate a simple education mechanism in the two-region Forslid and Ottaviano (2003) footloose entrepreneur model and analyse how education decisions are made in different types of society. We find that the optimal productivity level is the highest in a regulated economy and the lowest in an unionised one. We also find how the economic conditions drive the optimal education level and that, generally, the more concentrated the agents are, the lower their optimal education level is. Moreover, we conclude that education has positive effects on the whole of society due to the decrease in prices. Finally, we also conclude that individual and average education levels are strategic substitutes, which may lead to free-riding behaviours as society's average education level increases.

In chapter 4, we study how agents make spatial and educational decisions endogenously. In this chapter, we focus equally on the agents' and the regions' points of view since we explore how formal education affects the spatial distribution of economic activity and the agents' decisions regarding whether to study under the assumption that only one region offers this possibility. We make use of the novelties introduced in chapters 2 and 3 – namely, the analytical framework to consider more than one productivity level and, broadly, the conceptualisation of finding the optimal education level – to construct a novel approach to the study of space – an endogenous general equilibrium model that mixes the footloose entrepreneur model with the overlapping generations model. We find that qualified and unqualified workers may become segregated between regions. We also find that while high productivity gains lead everyone to study, low productivity gains do not always preclude education. Finally, we conclude that the only economic condition that seems to influence the equilibrium is productivity gains.

Note that our choice to use as a baseline either the Forslid and Ottaviano (2003) model or the Pflüger (2004) model is not arbitrary. In chapter 2, the regional asymmetry we introduce would generate the same effects in any of the two models since the income effect would not subvert it. In fact, it can be shown that the migrational dynamics that would ensue in the Forslid and Ottaviano model are qualitatively similar to those of the Pflüger. Hence we opt for the more simple model that still conveys our message. In chapter 3, we explicitly present a robustness test that compares both models and finds no qualitative differences, except for the fact that spatial distribution of economic activity influences the optimal education level in the Forslid and Ottaviano model. This, along with the easier computation of education externalities, justifies our choice of using the Forslid and Ottaviano model in chapter 3. Finally, in chapter 4, our choice for the Pflüger model is essentially one of simplicity, especially since we endeavour to construct a new approach that already becomes analytically complex. Even though we have no reasons to believe that using the Forslid and Ottaviano model would render different qualitative results, it may also open a possibility for future research.

In chapter 5, we study how the disentanglement of an economic union affects the spatial distribution of economic activity and social welfare. In this chapter, we focus primarily on the perspective of the regions, as we consider that the principal consequence of this breakup is the increase in the transportation cost between the leaving party and the remaining union members. We use the three-region Pflüger (2004) footloose entrepreneur model applying the new transportation costs scheme and find that an even distribution of mobile workers amongst the three regions is no longer possible and that there exists a spatial distribution in which a different share of mobile workers in each region, which is a rather uncommon and interesting result. We also find that spatial configurations in which the dissident region is totally depleted of mobile workers are not stable, except for the continental agglomerations. We conduct a numerical analysis which indicates that there are more mobile workers in the leaving region when the difference between transportation costs is low and the mobility of industrial workers is high. Finally, we find that the social optimal spatial distributions are always unfavourable towards the leaving region as they preclude more than one-third of industry in the dissident region.

These essays are not without limitations. Both chapters 2 and 3 tackle the education problem from a partial equilibrium perspective since there is no simultaneous dependence on both educational and spatial decisions. This limitation is particularly challenging and leads us to develop the broader endogenous general equilibrium model of chapter 4 to study both decisions simultaneously.

Moreover, a clear general limitation that deters some of our work is the analytical complexity that arises. We acknowledge that some results may not be clear-cut and that numerical simulation is needed to get further insight as there are some points at which it is simply technically not possible to achieve analytical solutions.

From a policy perspective, as we discussed in chapter 2, our results may help guide investments not only directly in education but also in more efficient transportation networks and communications, for example. In chapter 3, we opened the discussion on who should fund the education system. Since education has positive externalities, is it fair that only students support its costs, or should everyone contribute towards that goal? While the answer to this question is highly dependent on political beliefs, we expect that our conclusion may help understand more clearly how a higher average education generates positive spillovers for everyone.

Even though we manage to reach a handful of interesting results, we find even more new and exciting research questions.

In chapter 2, we find the puzzle of why it is more beneficial for a region to be the least productive instead of the second most efficient, for example.

In chapter 4, we realise that our model still has much to offer, either within our topic – for example, considering a scenario in which the innate skills of the agents are completely heterogeneous – but also to allow other researchers to use this approach to explore subjects that are challenging to analyse within classical Economic Geography models.

In chapter 5, including a trade partner outside the economic union may also help understand the global dynamic – whether the regions that remain in the union become tighter or an approximation between the dissident region and the outside trade partner exists.

While these questions are interesting, the time is not enough for us to answer them in this thesis. Even so, we expect to continue pursuing them afterwards and help push the barriers of Economic Geography forwards.

Appendix A

Chapter 3 - Robustness check

In this economy, there are L unskilled workers – equally divided between the regions – that are immobile between the regions, and H skilled workers – H_i in region $i = \{1, 2\}$ – that are mobile between the regions.

The preferences of every agent k are defined by

$$U_{QL} = \mu \ln M_{QL} + A_{QL} - C(\epsilon_k), \tag{A.1}$$

where $\mu \in (0, 1)$ is the expenditure share in the industrial good, A is the consumption of the agricultural good, and M is the consumption of the usual CES composite of differentiated varieties of the industrial good, defined in (3.2), where $d(s)$ is the consumption of variety s, S is the mass of varieties and $\sigma > 1$ is the constant elasticity of substitution between varieties.

Let $p_{ij}(s)$ represent the delivered price in region i of variety s produced in region j and $d_{ij}(s)$ its demand. Then, the regional price index associated with the composite good (3.2) in region i is given in (3.3).

Every agent k in region i maximises its utility subject to the budget constraint given by

$$P_i M + A = y_{ki},$$

where y_{ki} represents the nominal income of agent k in region i ($y_{ki} = \epsilon_k w_i$ if skilled and $y_{ki} = 1$ otherwise), P_i is given in (3.3) and the price of the agricultural good is normalised to one. Thus, the demand functions are given by

$$d_{ij_{QL}}(s) = \mu \frac{p_{ij}(s)^{-\sigma}}{P_i^{1-\sigma}}, \quad M_{QL} = \frac{\mu}{P_i}, \quad A_{QL} = y_{ki} - \mu. \tag{A.2}$$

From (A.1) and (A.2) we derive the indirect utility functions in region i for every agent k, which is given by

$$V_{i_{QL}}(h,\epsilon,\epsilon_k) = y_{ki} - \mu \ln P_i - C(\epsilon_k) + \mu(\ln \mu - 1). \tag{A.3}$$

The former expression represents the real wage of an agent net of education costs. Note that education costs are not affected by the price index since they represent an effort cost rather than a monetary one.

The production of the agricultural good uses one unit of unskilled labour per unit produced and has no transportation costs. Thus, $p_1^A = p_2^A = p^A$, which leads us to choose this good as *numeraire* ($p^A = 1$). Since the agricultural market is perfectly competitive, marginal cost pricing implies that the nominal wage of unskilled workers is the same everywhere and, in particular, equal to p^A. Hence, $w_i^L = p^A = 1$.

We assume that the baseline non-full-specialisation (NFS) condition (Baldwin et al., 2003; Gaspar et al., 2018) holds[37], so we have

$$\lambda > \frac{\mu\frac{\sigma-1}{\sigma}}{\frac{1}{2} - \mu\frac{\sigma-1}{\sigma}}.$$

As for the production of the industrial good, both skilled and unskilled labour is used. In particular, each unit produced requires α units of skilled labour and β units of unskilled labour. Therefore, the production cost of an industrial firm in region i is given in (3.6).

Hence, an industrial firm in region i that produces variety s maximises the profit function

$$\pi_i(s) = \sum_{j=1}^{2} d_{ij}(s)\,(H_i + L/2)\,[p_{ij}(s) - \tau_{ij}\beta] - \alpha w_i, \tag{A.4}$$

where $\tau \in (1, +\infty)$ represents the usual iceberg transportation cost between regions regarding the industrial good. Note that $\tau_{ij} = \tau$ whenever $i \neq j$ and $\tau_{ij} = 1$ otherwise.

Therefore, profit maximisation of (A.4) yields the optimal prices

$$p_{ij}(s) = \tau_{ij}\beta\frac{\sigma}{\sigma - 1}. \tag{A.5}$$

Then, using (A.5) and the fact that the number of industrial varieties produced

[37] Since the total production of the agricultural good in one region is $\lambda/2$ and the total consumption of the agricultural good in the whole economy is $\frac{\mu}{\sigma}(\lambda+1) + \lambda - \mu(\lambda+1)$, simple algebraic manipulation shows that the NFS condition is $\lambda > \left(\mu\frac{\sigma-1}{\sigma}\right) / \left(\frac{1}{2} - \mu\frac{\sigma-1}{\sigma}\right)$, which is precisely the baseline one.

in region i is $\epsilon H_i/\alpha$, the regional price index of the composite good (3.3) becomes

$$P_i = \frac{\beta\sigma}{\sigma - 1}\left(\frac{\epsilon}{\alpha}\sum_{j=1}^{2}\phi_{ij}H_j\right)^{\frac{1}{1-\sigma}}, \tag{A.6}$$

where $\phi_{ij} \equiv \tau_{ij}^{1-\sigma} \in (0,1]$ represents the freeness of trade (or the inverse of trade costs) between regions, regarding the industrial good. Note that $\phi_{ij} = \phi$ whenever $i \neq j$ and $\phi_{ij} = 1$ otherwise.

Given the monopolistic competition setup in the industrial market, the free entry condition implies zero profits in equilibrium. Using (A.2), (A.5), and (A.6) into $\pi_i(s) = 0$, the equilibrium wages that skilled workers earn are

$$w_{i_{QL}} = \frac{\mu}{\sigma}\sum_{j=1}^{2}\phi_{ij}\frac{H_j + L/2}{\sum_{m=1}^{2}\phi_{mj}\epsilon H_m}. \tag{A.7}$$

Thus, by defining the share of skilled workers in region 1 as $h_1 = h = H_1/H$, in region 2 as $h_2 = 1 - h = H_2/H$, and the global immobility ratio as $\lambda = L/H$, it is possible to express the nominal wage per efficiency unit of labour (A.7) as a function of h and ϵ, which yields

$$w_{i_{QL}}(h,\epsilon) = \frac{\mu}{\sigma}\sum_{j=1}^{2}\phi_{ij}\frac{h_j + \lambda/2}{\epsilon\left[\phi + (1-\phi)h_j\right]}. \tag{A.8}$$

The regional price index (A.6) can also be rewritten as

$$P_i(h,\epsilon) = \epsilon^{\frac{1}{1-\sigma}}\frac{\beta\sigma}{\sigma-1}\left(\frac{H}{\alpha}\right)^{\frac{1}{1-\sigma}}\left[\phi + (1-\phi)h_i\right]^{\frac{1}{1-\sigma}}. \tag{A.9}$$

Therefore, by replacing (A.8) and (A.9) in (A.3) the indirect utility of a skilled agent is now

$$V_{i_{QL}}^{H}(h,\epsilon,\epsilon_k) = \epsilon_k w_{i_{QL}}(h,\epsilon) + \frac{\mu}{\sigma-1}\ln\left[\phi + (1-\phi)h_i\right] + \frac{\mu}{\sigma-1}\ln(\epsilon) - C(\epsilon_k) + \eta,$$

and, considering $y_{ki} = 1$ instead of $y_{ki} = \epsilon_k w_i(h,\epsilon)$, is equal to

$$V_{i_{QL}}^{L}(h,\epsilon) = 1 + \frac{\mu}{\sigma-1}\ln\left[\phi + (1-\phi)h_i\right] + \frac{\mu}{\sigma-1}\ln(\epsilon) + \eta,$$

if the agent is unskilled[38], where $\eta = \mu(\ln\mu - 1) + \frac{\mu}{\sigma-1}\ln\left(\frac{H}{\alpha}\right) - \mu\ln\left(\frac{\beta\sigma}{\sigma-1}\right)$ is a constant.

[38] Note that, in any expression, simply considering $\epsilon = \epsilon_k = 1$ would recover the original equations from the Pflüger (2004) model.

Considering the same welfare functions (W^R, W^U, and W^I) as before, we have that the education minister's, the union's, and the individual's FOCs are, respectively,

$$\frac{\mu}{\sigma-1}(1+\lambda) = \frac{dC(\epsilon)}{d\epsilon}\epsilon,$$

$$\frac{\mu}{\sigma-1} = \frac{dC(\epsilon)}{d\epsilon}\epsilon,$$

$$hw_1(h,1) + (1-h)w_2(h,1) = \frac{dC(\epsilon)}{d\epsilon}\epsilon.$$

Therefore, using the same methods as before to find the three optimal levels of education yields[39]

$$\epsilon_{QL}^R = \frac{(1+\lambda)\frac{\mu}{\sigma-1}}{\gamma}, \quad \epsilon_{QL}^U = \frac{\frac{\mu}{\sigma-1}}{\gamma}, \quad \epsilon_{QL}^I = \frac{(1+\lambda)\frac{\mu}{\sigma}}{\gamma}.$$

For these levels to be higher than one, we need that, respectively,

$$\gamma < (1+\lambda)\frac{\mu}{\sigma-1}, \quad \gamma < \frac{\mu}{\sigma-1}, \quad \gamma < (1+\lambda)\frac{\mu}{\sigma}.$$

Comparing the optimal levels of education we find that $\epsilon_{QL}^R > \epsilon_{QL}^I > \epsilon_{QL}^U$, since

$$\epsilon_{QL}^R > \epsilon_{QL}^U \Leftrightarrow \frac{(1+\lambda)\frac{\mu}{\sigma-1}}{\gamma} > \frac{\frac{\mu}{\sigma-1}}{\gamma} \Leftrightarrow 1+\lambda > 1 \Leftrightarrow \lambda > 0,$$

$$\epsilon_{QL}^R > \epsilon_{QL}^I \Leftrightarrow \frac{(1+\lambda)\frac{\mu}{\sigma-1}}{\gamma} > \frac{\frac{\mu}{\sigma}}{\gamma} \Leftrightarrow \sigma > \sigma-1 \Leftrightarrow 0 > -1,$$

$$\epsilon_{QL}^I > \epsilon_{QL}^U \Leftrightarrow \frac{(1+\lambda)\frac{\mu}{\sigma}}{\gamma} > \frac{\frac{\mu}{\sigma-1}}{\gamma} \Leftrightarrow 1+\lambda > \frac{\sigma}{\sigma-1}.$$

The education externality results from analysing $W^R(h,\epsilon) - W^R(h,1)$, then

$$\begin{aligned}
&\left[hHV_1^H(h,\epsilon) + (1-h)HV_2^H(h,\epsilon) + \frac{L}{2}V_1^L(h,\epsilon) + \frac{L}{2}V_2^L(h,\epsilon)\right] \\
&- \left[hHV_1^H(h,1) + (1-h)HV_2^H(h,1) + \frac{L}{2}V_1^L(h,1) + \frac{L}{2}V_2^L(h,1)\right] \\
= &\left[h\left(\epsilon w_{1_{QL}}(h,\epsilon) + \frac{\mu}{\sigma-1}\ln\left[\phi + (1-\phi)h\right] + \frac{\mu}{\sigma-1}\ln(\epsilon) - C(\epsilon) + \eta\right)\right. \\
&+(1-h)\left(\epsilon w_{2_{QL}}(h,\epsilon) + \frac{\mu}{\sigma-1}\ln\left[\phi + (1-\phi)(1-h)\right] + \frac{\mu}{\sigma-1}\ln(\epsilon) - C(\epsilon) + \eta\right) \\
&\left.+\frac{\lambda}{2}\left(1 + \frac{\mu}{\sigma-1}\ln\left[\phi + (1-\phi)h\right] + \frac{\mu}{\sigma-1}\ln(\epsilon) + \eta\right) + \frac{\lambda}{2}\left(1 + \frac{\mu}{\sigma-1}\ln\left[\phi + (1-\phi)(1-h)\right] + \frac{\mu}{\sigma-1}\ln(\epsilon) + \eta\right)\right]
\end{aligned}$$

[39] Note that $hw_1(h,1) + (1-h)w_2(h,1)$ is simply the weighted average nominal wage ($\bar{w}$) shown by Gaspar et al. (2018). Hence, $\bar{w} = (1+\lambda)\frac{\mu}{\sigma}$.

$$
\begin{aligned}
&-\Big[h\left(w_{1_{QL}}(h,1)+\frac{\mu}{\sigma-1}\ln\left[\phi+(1-\phi)h\right]+\eta\right)+(1-h)\left(w_{2_{QL}}(h,1)+\frac{\mu}{\sigma-1}\ln\left[\phi+(1-\phi)(1-h)\right]+\eta\right)\\
&\quad+\frac{\lambda}{2}\left(1+\frac{\mu}{\sigma-1}\ln\left[\phi+(1-\phi)h\right]+\eta\right)+\frac{\lambda}{2}\left(1+\frac{\mu}{\sigma-1}\ln\left[\phi+(1-\phi)(1-h)\right]+\eta\right)\Big]\\
&=(1+\lambda)\frac{\mu}{\sigma-1}\ln(\epsilon)-C(\epsilon).
\end{aligned}
$$

Therefore, since the cost of education is private, we conclude that education has positive externalities that affect the whole society because $(1+\lambda)\frac{\mu}{\sigma-1}\ln(\epsilon) > 1$ as long as $\epsilon > 1$. Moreover, the externality depends solely on the decrease in prices.

Regarding the strategic profile of education, we have that the individual education level and the average education level are strategic substitutes if $\frac{\partial^2 V^H_{i_{QL}}}{\partial\epsilon_k\partial\epsilon} < 0$. So, $\frac{\partial V^H_{i_{QL}}}{\partial\epsilon_k} = w_i(h,\epsilon) - \frac{dC(\epsilon_k)}{d\epsilon_k}$, $\frac{\partial^2 V^H_{i_{QL}}}{\partial\epsilon_k\partial\epsilon} = -w_i(h,\epsilon)/\epsilon^2$, which is clearly negative.

Appendix B

Chapter 3 - Proofs

Proof of Proposition 3.1. Applying the FOC to the optimisation problem $\max_\epsilon W^R = +hHV_1^H(h,\epsilon) + (1-h)HV_2^H(h,\epsilon) + \frac{L}{2}V_1^L(h,\epsilon) + \frac{L}{2}V_2^L(h,\epsilon)$ yields

$$\begin{aligned}
&\frac{\partial W^R}{\partial \epsilon} = 0 \\
\Leftrightarrow\quad & h\left[\epsilon^{\frac{\mu}{\sigma-1}-1}\frac{\mu}{\sigma-1}\frac{w_1(h,1)}{[P_1(h,1)]^\mu} - \frac{dC(\epsilon)}{d\epsilon}\right] + (1-h)\left[\epsilon^{\frac{\mu}{\sigma-1}-1}\frac{\mu}{\sigma-1}\frac{w_2(h,1)}{[P_2(h,1)]^\mu} - \frac{dC(\epsilon)}{d\epsilon}\right] \\
&+ \frac{\lambda}{2}\epsilon^{\frac{\mu}{\sigma-1}-1}\frac{\mu}{\sigma-1}\frac{1}{[P_1(h,1)]^\mu} + \frac{\lambda}{2}\epsilon^{\frac{\mu}{\sigma-1}-1}\frac{\mu}{\sigma-1}\frac{1}{[P_2(h,1)]^\mu} = 0 \\
\Leftrightarrow\quad & \frac{\mu}{\sigma-1}\left(h\left[\frac{w_1(h,1)}{[P_1(h,1)]^\mu}\right] + (1-h)\left[\frac{w_2(h,1)}{[P_2(h,1)]^\mu}\right] + \frac{\lambda}{2}\left[\frac{1}{[P_1(h,1)]^\mu} + \frac{1}{[P_2(h,1)]^\mu}\right]\right) = \frac{dC(\epsilon)}{d\epsilon}\epsilon^{-\left(\frac{\mu}{\sigma-1}-1\right)}.
\end{aligned}$$

Furthermore, to ensure that the extremum we found is, indeed, a maximum, we need W^R to be concave in ϵ. Since W^R is a linear combination of two main functions – $V_i^H(h,\epsilon)$ and $V_i^L(h,\epsilon)$ –, it is sufficient to prove that those functions are both concave. Therefore, we have that $V_i^H(h,\epsilon)$ is concave if

$$\frac{\partial^2 V_i^H(h,\epsilon)}{\partial \epsilon^2} \le 0 \quad \Leftrightarrow \quad \epsilon^{\frac{\mu}{\sigma-1}-2}\left(\frac{\mu}{\sigma-1} - 1\right)\frac{\mu}{\sigma-1}\frac{w_i(h,1)}{[P_i(h,1)]^\mu} - \frac{d^2C(\epsilon)}{d\epsilon^2} \le 0.$$

Thus, since $\mu \in (0,1)$, $\sigma > 1$, and that $\frac{d^2C(\epsilon)}{d\epsilon^2} > 0$, the left-hand side is clearly negative, which proves that the function is strictly concave. Moreover, we have that $V_i^L(h,\epsilon)$ is concave if

$$\frac{\partial^2 V_i^L(h,\epsilon)}{\partial \epsilon^2} \le 0 \quad \Leftrightarrow \quad \epsilon^{\frac{\mu}{\sigma-1}-2}\left(\frac{\mu}{\sigma-1} - 1\right)\frac{\mu}{\sigma-1}\frac{1}{[P_i(h,1)]^\mu} \le 0.$$

Thus, since $\mu \in (0,1)$ and $\sigma > 1$, the left-hand side is clearly negative, which proves that the function is strictly concave. □

Proof of Corollary 3.2. Considering the linear cost function $C(\epsilon) = \gamma(\epsilon - 1)$, we have that $\frac{dC(\epsilon)}{d\epsilon} = \gamma$. Therefore, replacing the value of the derivative in the first-order condition presented in Proposition 3.1, solving for ϵ and labelling the solution as ϵ^R yields

$$\epsilon^R = \left(\frac{\mu}{\sigma - 1} \frac{h\left[\frac{w_1(h,1)}{[P_1(h,1)]^\mu}\right] + (1-h)\left[\frac{w_2(h,1)}{[P_2(h,1)]^\mu}\right] + \frac{\lambda}{2}\left[\frac{1}{[P_1(h,1)]^\mu} + \frac{1}{[P_2(h,1)]^\mu}\right]}{\gamma} \right)^{\frac{1}{-\left(\frac{\mu}{\sigma-1}-1\right)}}.$$

Solving $\epsilon^R > 1$, with respect to γ, yields

$$\gamma < \frac{\mu}{\sigma - 1}\left(h\left[\frac{w_1(h,1)}{[P_1(h,1)]^\mu}\right] + (1-h)\left[\frac{w_2(h,1)}{[P_2(h,1)]^\mu}\right] + \frac{\lambda}{2}\left[\frac{1}{[P_1(h,1)]^\mu} + \frac{1}{[P_2(h,1)]^\mu}\right] \right). \qquad \square$$

Proof of Proposition 3.3. Applying the FOC to the optimisation problem $\max_\epsilon W^U = hHV_1^H(h,\epsilon) + (1-h)HV_2^H(h,\epsilon)$ yields

$$\begin{aligned} & \frac{\partial W^U}{\partial \epsilon} = 0 \\ \Leftrightarrow \quad & h\left[\epsilon^{\frac{\mu}{\sigma-1}-1} \frac{\mu}{\sigma-1} \frac{w_1(h,1)}{[P_1(h,1)]^\mu} - \frac{dC(\epsilon)}{d\epsilon}\right] + (1-h)\left[\epsilon^{\frac{\mu}{\sigma-1}-1} \frac{\mu}{\sigma-1} \frac{w_2(h,1)}{[P_2(h,1)]^\mu} - \frac{dC(\epsilon)}{d\epsilon}\right] = 0 \\ \Leftrightarrow \quad & \frac{\mu}{\sigma-1}\left(h\left[\frac{w_1(h,1)}{[P_1(h,1)]^\mu}\right] + (1-h)\left[\frac{w_2(h,1)}{[P_2(h,1)]^\mu}\right] \right) = \frac{dC(\epsilon)}{d\epsilon}\epsilon^{-\left(\frac{\mu}{\sigma-1}-1\right)}. \end{aligned}$$

Furthermore, to ensure that the extremum we found is, indeed, a maximum, we need W^U to be concave in ϵ. Since W^U is a linear combination of one main function – $V_i^H(h,\epsilon)$ –, it is sufficient to prove that that function is concave, which has already been shown in the proof of Proposition 3.1. $\square$

Proof of Corollary 3.4. Considering the linear cost function $C(\epsilon) = \gamma(\epsilon - 1)$, we have that $\frac{dC(\epsilon)}{d\epsilon} = \gamma$. Therefore, replacing the value of the derivative in the first-order condition presented in Proposition 3.3, solving for ϵ and labelling the solution as ϵ^U yields

$$\epsilon^U = \left(\frac{\mu}{\sigma - 1} \frac{h\left[\frac{w_1(h,1)}{[P_1(h,1)]^\mu}\right] + (1-h)\left[\frac{w_2(h,1)}{[P_2(h,1)]^\mu}\right]}{\gamma} \right)^{\frac{1}{-\left(\frac{\mu}{\sigma-1}-1\right)}}.$$

Solving $\epsilon^U > 1$, with respect to γ, yields

$$\gamma < \frac{\mu}{\sigma - 1}\left(h\left[\frac{w_1(h,1)}{[P_1(h,1)]^\mu}\right] + (1-h)\left[\frac{w_2(h,1)}{[P_2(h,1)]^\mu}\right] \right). \qquad \square$$

Proof of Proposition 3.5. Applying the FOC to the optimisation problem $\max_{\epsilon_j} W^I = hV_1^H(h,\epsilon,\epsilon_j) + (1-h)V_2^H(h,\epsilon,\epsilon_j)$ yields

$$\frac{\partial W^I}{\partial \epsilon_j} = 0 \quad \Leftrightarrow \quad h\left[\epsilon^{\frac{\mu}{\sigma-1}-1}\frac{w_1(h,1)}{[P_1(h,1)]^\mu} - \frac{dC(\epsilon_j)}{d\epsilon_j}\right] + (1-h)\left[\epsilon^{\frac{\mu}{\sigma-1}-1}\frac{w_2(h,1)}{[P_2(h,1)]^\mu} - \frac{dC(\epsilon_j)}{d\epsilon_j}\right] = 0.$$

Note that since all the agents are homogeneous, their decisions are all the same, thus $\epsilon_j = \epsilon$, which yields

$$h\left[\frac{w_1(h,1)}{[P_1(h,1)]^\mu}\right] + (1-h)\left[\frac{w_2(h,1)}{[P_2(h,1)]^\mu}\right] = \frac{dC(\epsilon)}{d\epsilon}\epsilon^{-\left(\frac{\mu}{\sigma-1}-1\right)}.$$

Furthermore, to ensure that the extremum we found is, indeed, a maximum, we need W^I to be concave in ϵ_j. Since W^I is a linear combination of one main function – $V_i^H(h,\epsilon,\epsilon_j)$ –, it is sufficient to prove that that function is concave. Therefore, we have that $V_i^H(h,\epsilon,\epsilon_j)$ is concave if

$$\frac{\partial^2 V_i^H(h,\epsilon,\epsilon_h)}{\partial \epsilon_j^2} \leq 0 \quad \Leftrightarrow \quad -\frac{d^2C(\epsilon)}{d\epsilon^2} \leq 0,$$

which is trivially satisfied, thus proving that the function is concave. □

Proof of Corollary 3.6. Considering the linear cost function $C(\epsilon) = \gamma(\epsilon - 1)$, we have that $\frac{dC(\epsilon)}{d\epsilon} = \gamma$. Therefore, replacing the value of the derivative in the first-order condition presented in Proposition 3.5, solving for ϵ and labelling the solution as ϵ^I yields

$$\epsilon^I = \left(\frac{h\left[\frac{w_1(h,1)}{[P_1(h,1)]^\mu}\right] + (1-h)\left[\frac{w_2(h,1)}{[P_2(h,1)]^\mu}\right]}{\gamma}\right)^{\frac{1}{-\left(\frac{\mu}{\sigma-1}-1\right)}}.$$

Solving $\epsilon^I > 1$, with respect to γ, yields

$$\gamma < h\left[\frac{w_1(h,1)}{[P_1(h,1)]^\mu}\right] + (1-h)\left[\frac{w_2(h,1)}{[P_2(h,1)]^\mu}\right].$$ □

Appendix C

Chapter 5 - Proofs

C.1 Agglomerations

We have that

$$\begin{aligned}
\Lambda^{AUK} &\equiv \frac{\lambda(1+\phi_{EU}-(3-\phi_{UK})\phi_{UK})-3(1-\phi_{UK})\phi_{UK}}{3\sigma\phi_{UK}}+\frac{\ln(\phi_{UK})}{\sigma-1},\\
\Lambda_1^{CA} &\equiv \frac{\phi_{UK}^2[(\lambda+3)\phi_{EU}+\lambda]-3(\lambda+1)\phi_{EU}\phi_{UK}+\lambda\phi_{EU}}{3\sigma\phi_{EU}\phi_{UK}}+\frac{\ln(\phi_{UK})}{\sigma-1},\\
\Lambda_2^{CA} &\equiv -\frac{(1-\phi_{EU})[3\phi_{EU}-\lambda(1-\phi_{EU})]}{3\sigma\phi_{EU}}+\frac{\ln(\phi_{EU})}{\sigma-1}.
\end{aligned}$$

C.2 Median dispersion

Proof of Proposition 5.4. To prove that $\left(\alpha, \frac{1-\alpha}{2}, \frac{1-\alpha}{2}\right)$ is an equilibrium, we need $\dot{h}_1 = 0 \wedge \dot{h}_2 = 0$ to be true. Thus, we have

$$\begin{aligned}
\dot{h}_1 &= \alpha\left(V_1(\boldsymbol{h})-\bar{V}(\boldsymbol{h})\right)=0,\\
\dot{h}_2 &= \frac{1-\alpha}{2}\left(V_2(\boldsymbol{h})-\bar{V}(\boldsymbol{h})\right)=0.
\end{aligned}$$

Therefore, we can equal both equations, which yields $V_1(\boldsymbol{h}) = V_2(\boldsymbol{h})$. As shown by Gaspar et al. (2018), in Proposition 4, it is possible to solve the former expression with respect to λ, as $\lambda_m^*(\alpha)$. Therefore, there exists an MD equilibrium if, and only if, there exists a value of $\lambda > 0$ such that $\lambda = \lambda_m^*(\alpha)$, where $\lambda_m^*(\alpha)$ is

$$\lambda_m^*(\alpha) = -3\frac{\gamma_1(\sigma-1)\phi_{UK}+\gamma_2\sigma\ln\left[\dfrac{\alpha(1-\phi_{UK})+\phi_{UK}}{\alpha\phi_{UK}+\frac{1}{2}(1-\alpha)(1+\phi_{EU})}\right]}{\gamma_3(\sigma-1)},$$

where γ_1, γ_2, and γ_3 are given in Appendix D. □

Proof of Proposition 5.5. Notice that $\lambda_m^*(\alpha)$ has a vertical asymptote if $\gamma_3(\sigma-1) = 0$, that is for

$$\alpha = \alpha_{lm} \equiv \frac{-3(\phi_{EU}+1)\phi_{UK} + \phi_{EU} + 4\phi_{UK}^2 + 1}{3(1-\phi_{UK})(\phi_{EU} - 2\phi_{UK} + 1)}.$$

We have that α_{lm} is lower than 1/3 since $\phi_{UK} < \phi_{EU}$. It is higher than zero if either (i) $\phi_{UK} < 1/2$, or (ii) $\phi_{UK} > 1/2$ and $\phi_{EU} < \frac{\phi_{UK}(4\phi_{UK}-3)+1}{3\phi_{UK}-1}$. If the former conditions are met, we have $\lim_{\alpha \to \alpha_{lm}^-} \lambda_m^*(\alpha) = +\infty$ and $\lim_{\alpha \to \alpha_{lm}^+} \lambda_m^*(\alpha) = -\infty$. Next, notice that if $\alpha \in [0, \alpha_{lm})$ we have $\gamma_1 < 0$, $\gamma_2 > 0$, $\gamma_3 > 0$ and the logarithm is negative. Hence, $\lambda_m^*(\alpha) > 0$ and there exists a value of λ such that $\alpha \in [0, \alpha_{lm})$ corresponds to an MD equilibrium. Next, notice that $\lambda_m^*(\alpha)$ has a zero given by

$$\alpha = \alpha_{0m} \equiv \frac{\phi_{EU} - 2\phi_{UK} + 1}{\phi_{EU} - 4\phi_{UK} + 3} \in \left(\frac{1}{3}, \frac{1}{2}\right].$$

It is straightforward to show that $\alpha_{lm} < \alpha_{0m}$. It can also be shown that $\lambda_m^*(\alpha) < 0$ for $\alpha \in (\alpha_{lm}, \alpha_{0m})$, which means that no positive value of λ exists such that an MD in that interval can correspond to a spatial equilibrium – specifically, check that $\gamma_1 > 0$, $\gamma_2 > 0$, $\gamma_3 > 0$, and that the logarithm of $\lambda_m^*(\alpha)$ is positive for $\alpha \in (\alpha_{lm}, \alpha_{0m})$. In other words, $\alpha \in (\alpha_{lm}, \alpha_{0m})$ can never be an equilbrium. For $\alpha > \alpha_{0m}$, it can be shown that $\gamma_1 > 0$, $\gamma_2 > 0$, $\gamma_3 < 0$ and that the logarithm of $\lambda_m^*(\alpha)$ is positive. Hence, $\lambda_m^*(\alpha) > 0$ for $\alpha \in (\alpha_{0m}, 1)$ and we can conclude that there exists a value of λ such that $\alpha \in (\alpha_{0m}, 1)$ corresponds to a spatial equilibrium. Now, let us first compute the first derivative of $\lambda_m^*(\alpha)$ with respect to α,

$$\frac{d\lambda_m^*(\alpha)}{d\alpha} = 3\frac{b_1 + \sigma b_2 b_3}{b_4},$$

where

$$\begin{aligned}
b_1 =& -\left(\phi_{EU} - 2\phi_{UK}^2 + 1\right)\left[\sigma\left(-3\alpha(1-\phi_{UK})(\phi_{EU} - 2\phi_{UK} + 1) - 5\phi_{EU}\phi_{UK}\right.\right. \\
&\left.\left. + \phi_{EU} + 1 + 6\phi_{UK}^2 - 3\phi_{UK}\right) + 2\phi_{UK}(\phi_{EU} - \phi_{UK})\right], \\
b_2 =& 3\alpha^2(\phi_{UK} - 1)^2(\phi_{EU} - 2\phi_{UK} + 1)^2 + 2\alpha(1-\phi_{UK})(\phi_{EU} - 2\phi_{UK} + 1)\times \\
&\times\left[\phi_{EU}(3\phi_{UK} - 1) + \phi_{UK}(3 - 4\phi_{UK}) - 1\right] + (\phi_{EU} + 1)^2 \\
&- 8(\phi_{EU} + 1)\phi_{UK}^3 + 3(\phi_{EU} + 1)^2\phi_{UK}^2 - 2(\phi_{EU} + 1)^2\phi_{UK} + 8\phi_{EU}^4, \\
b_3 =& \ln\left[\frac{\alpha(1-\phi_{UK}) + \phi_{UK}}{\alpha\phi_{UK} + \frac{1}{2}(1-\alpha)(\phi_{EU} + 1)}\right],
\end{aligned}$$

$$
\begin{aligned}
b_4 =& (\sigma - 1) \left[-3\alpha(1 - \phi_{UK})(\phi_{EU} - 2\phi_{UK} + 1) - 3(\phi_{EU} + 1)\phi_{UK} + \phi_{EU} + 1 \right. \\
& \left. + 4\phi_{UK}^2 \right]^2 .
\end{aligned}
$$

For $\alpha < \alpha_{lm}$, we can observe that $b_4 > 0$, so the sign of the derivative is the sign of its numerator, $N_1(\alpha) = b_1 + \sigma b_2 b_3$. First, we have

$$
N_1(\alpha_{lm}) = 2 \left(\phi_{EU} - 2\phi_{UK}^2 + 1 \right) \left[\left(\phi_{EU} - 2\phi_{UK}^2 + 1 \right) \Phi + 3(\sigma - 1)\phi_{UK}(\phi_{EU} - \phi_{UK}) \right],
$$

where

$$
\Phi = \sigma \ln \left[\frac{(\phi_{EU} - 2\phi_{UK}^2 + 1)(3\phi_{EU} - 6\phi_{UK} + 3)}{3 - 3\phi_{UK}} \right].
$$

All terms are positive, which means that $N_1(\alpha_{lm}) > 0$. Next, notice that

$$
\frac{dN_1}{d\alpha}(\alpha) = c_1 (c_2 + c_3 c_4),
$$

where

$$
\begin{aligned}
c_1 =& 3\sigma \left[-3\alpha(1 - \phi_{UK})(\phi_{EU} - 2\phi_{UK} + 1) - 3(\phi_{EU} + 1)\phi_{UK} + \phi_{EU} + 4\phi_{UK}^2 + 1 \right], \\
c_2 =& \left(\phi_{EU} - 2\phi_{UK}^2 + 1 \right) \frac{-2\alpha(1-\phi_{UK})(\phi_{EU}-2\phi_{UK}+1)-2(\phi_{EU}+1)\phi_{UK}+\phi_{EU}+2\phi_{UK}^2+1}{-[\alpha(1-\phi_{UK})+\phi_{UK}][\phi_{EU}+1-\alpha(\phi_{EU}-2\phi_{UK}+1)]}, \\
c_3 =& -2(1 - \phi_{UK})(\phi_{EU} - 2\phi_{UK} + 1), \\
c_4 =& \ln \left[\frac{\alpha\phi_{UK} + \frac{1}{2}(1 - \alpha)(\phi_{EU} + 1)}{\alpha(1 - \phi_{UK}) + \phi_{UK}} \right].
\end{aligned}
$$

Cumbersome yet standard inspection allows to show that, if $\alpha < \alpha_{lm}$, we have $c_1 > 0, c_2 < 0, c_3 < 0$ and $c_4 < 0$. Therefore, $dN_1(\alpha)/d\alpha < 0$, which means that $N_1(\alpha) > 0$ for $\alpha \in (0, \alpha_{lm})$ and, thus, $d\lambda^*(\alpha)/d\alpha > 0$. This implies that there exists at most one equilibrium $\alpha \in (0, \alpha_{lm})$. Now let us look at the case $\alpha \in (\alpha_{0m}, 1)$. Computing the second derivative, we get

$$
\frac{d^2\lambda_m^*(\alpha)}{d\alpha^2} = -\frac{d_1 (d_2 + d_3 d_4)}{d_5},
$$

where

$$
\begin{aligned}
d_1 =& 3 \left(\phi_{EU} - 2\phi_{UK}^2 + 1 \right), \\
d_2 =& 12(1 - \phi_{UK})\phi_{UK}(\phi_{EU} - 2\phi_{UK} + 1)(\phi_{EU} - \phi_{UK}) \\
& + \frac{\sigma}{[\alpha(1 - \phi_{UK}) + \phi_{UK}] [(1 - \alpha)(\phi_{EU} + 1) + 2\alpha\phi_{UK}]} \left\{ -2(\phi_{EU} + 1)^3 \phi_{UK} \right.
\end{aligned}
$$

$$
\begin{aligned}
&+(\phi_{EU}+1)^3-5(3\phi_{EU}+1)(\phi_{EU}+1)^2\phi_{UK}^2+32\phi_{UK}^6-8(\phi_{EU}+1)\phi_{UK}^5\\
&+4(\phi_{EU}+1)(\phi_{EU}(3\phi_{EU}+17)+8)\phi_{UK}^3-2(\phi_{EU}+1)(15\phi_{EU}+23)\phi_{UK}^4\\
&+2\alpha(1-\phi_{UK})(\phi_{EU}-2\phi_{UK}+1)\left[-6(3\phi_{EU}+1)\phi_{UK}^3-(\phi_{EU}+1)^2\right.\\
&\left.+4(\phi_{EU}+1)(3\phi_{EU}+4)\phi_{UK}^2-3(\phi_{EU}+1)(3\phi_{EU}+1)\phi_{UK}-4\phi_{UK}^4\right]\\
&\left.+3\alpha^2(\phi_{UK}-1)^2(\phi_{EU}-2\phi_{UK}+1)^2\left[\phi_{EU}(4\phi_{UK}-1)-2\phi_{UK}^2-1\right]\right\},\\
d_3 =&-4\sigma(1-\phi_{UK})(\phi_{EU}-2\phi_{UK}+1)\left(\phi_{EU}-2\phi_{UK}^2+1\right),\\
d_4 =&\ln\left[\frac{\alpha\phi_{UK}+\frac{1}{2}(1-\alpha)(\phi_{EU}+1)}{\alpha(1-\phi_{UK})+\phi_{UK}}\right],\\
d_5 =&(\sigma-1)\left[-3\alpha(1-\phi_{UK})(\phi_{EU}-2\phi_{UK}+1)-3(\phi_{EU}+1)\phi_{UK}+\phi_{EU}+4\phi_{UK}^2+1\right]^3.
\end{aligned}
$$

We have that $d_5 < 0$ for $\alpha > \alpha_{0m}$, which means that the sign of the derivative is the sign of its numerator given by $N_2(\alpha) = d_1(d_2 + d_3 d_4)$. First, note that

$$
\begin{aligned}
N_2(\alpha_{0m}) =&-6(\phi_{EU}-\phi_{UK})\left(\phi_{EU}-2\phi_{UK}^2+1\right)\times\\
&\left(-6(1-\phi_{UK})\phi_{UK}(\phi_{EU}-2\phi_{UK}+1)+\sigma\left\{\phi_{EU}^2\right.\right.\\
&\left.\left.+\phi_{EU}\left[\phi_{UK}(3-8\phi_{UK})+3\right]+\phi_{UK}\left[2\phi_{UK}(9\phi_{UK}-11)+3\right]+2\right\}\right),
\end{aligned}
$$

which is negative for $\alpha < \alpha_{0m}$. Next, we have

$$
\begin{aligned}
\frac{dN_2}{d\alpha}(\alpha) =&\frac{\sigma\left(\phi_{EU}-2\phi_{UK}^2+1\right)^3}{\left[\alpha(1-\phi_{UK})+\phi_{UK}\right]^2\left[(\alpha-1)(\phi_{EU}+1)-2\alpha\phi_{UK}\right]^2}(\phi_{EU}+1)\\
&\times\left(3\alpha(\phi_{UK}-1)(\phi_{EU}-2\phi_{UK}+1)-3(\phi_{EU}+1)\phi_{UK}+4\phi_{UK}^2\right)^2,
\end{aligned}
$$

which is negative, meaning that $N_2(\alpha) < 0$ for $\alpha \in [\alpha_{0m}, 1]$ and hence $\frac{d^2\lambda_m^*(\alpha)}{d\alpha^2} < 0$ for $\alpha \in (\alpha_{0m}, 1)$. This implies that $\lambda^*(\alpha)$ is concave for $\alpha > \alpha_0$ and thus there exist at most two median dispersion equilibria when $\alpha > \alpha_{0m}$. □

Lemma C.1. *The Jacobian matrix evaluated at any equilibrium $h^* \in \Delta_m$ is of the form*

$$
J = \begin{bmatrix} V_1-\bar{V}+\alpha\left(\dfrac{\partial V_1}{\partial h_1}-\dfrac{\partial \bar{V}}{\partial h_1}\right) & 0 \\ \\ (1-\alpha)\left(\dfrac{\partial V_2}{\partial h_1}-\dfrac{\partial \bar{V}}{\partial h_1}\right) & \dfrac{1-\alpha}{2}\dfrac{\partial V_2}{\partial h_2} \end{bmatrix}.
$$

Proof. Any configuration in Δ_m is such that $\boldsymbol{h} = (\alpha, \frac{1-\alpha}{2}, \frac{1-\alpha}{2})$ with $\alpha \in (0,1)$. For any such configuration, we have

$$\frac{\partial f_1}{\partial h_1} = V_1 - \bar{V} + \alpha \left(\frac{\partial V_1}{\partial h_1} - \frac{\partial \bar{V}}{\partial h_1} \right),$$

$$\frac{\partial f_1}{\partial h_2} = \alpha \left(\underbrace{\frac{\partial V_1}{\partial h_2}}_{0} - \underbrace{\frac{\partial \bar{V}}{\partial h_2}}_{0} \right) = 0,$$

$$\frac{\partial f_2}{\partial h_1} = (1-\alpha) \left(\frac{\partial V_2}{\partial h_1} - \frac{\partial \bar{V}}{\partial h_1} \right),$$

$$\frac{\partial f_2}{\partial h_2} = \underbrace{V_2 - \bar{V}}_{0} + \frac{1-\alpha}{2} \left(\frac{\partial V_2}{\partial h_2} - \underbrace{\frac{\partial \bar{V}}{\partial h_2}}_{0} \right).$$

Note that $V_2 - \bar{V} = 0$ since it is an equilibrium condition. Next, to prove that $\frac{\partial \bar{V}}{\partial h_2} = 0$, let us assume by way of contradiction that it is not. Suppose that $\frac{\partial \bar{V}}{\partial h_2} > 0$, then $\bar{V}(\alpha, \frac{1-\alpha}{2} + \epsilon, \frac{1-\alpha}{2} - \epsilon) > \bar{V}(\alpha, \frac{1-\alpha}{2} - \epsilon, \frac{1-\alpha}{2} + \epsilon)$. However $\bar{V}$ is invariant in the permutation of coordinates that interchanges populations in two regions since $\bar{V}(\alpha, \frac{1-\alpha}{2} + \epsilon, \frac{1-\alpha}{2} - \epsilon) = \bar{V}(\alpha, \frac{1-\alpha}{2} - \epsilon, \frac{1-\alpha}{2} + \epsilon)$, which is a contradiction. Hence it must be true that $\frac{\partial \bar{V}}{\partial h_2} = 0$. Finally, $\frac{\partial V_1}{\partial h_2} = 0$ due to the same symmetry argument that establishes invariance of payoffs in a region in the permutation of coordinates in different regions. See Gaspar et al. (2018) for more details. □

We have that

$$\Lambda_1^{MD} \equiv \frac{(1-\alpha)\gamma_4 + \sigma\gamma_5 \ln \left[\dfrac{\alpha(1-\phi_{UK}) + \phi_{UK}}{\alpha\phi_{UK} + \frac{1}{2}(1-\alpha)(\phi_{EU}+1)} \right]}{\gamma_6},$$

$$\Lambda_2^{MD} \equiv -2\mu(1-\phi_{EU}) \frac{\gamma_7 + \sigma\gamma_8 \ln \left[\dfrac{\alpha(1-\phi_{UK}) + \phi_{UK}}{\alpha\phi_{UK} + \frac{1}{2}(1-\alpha)(\phi_{EU}+1)} \right]}{\gamma_9}.$$

where γ_4, γ_5, γ_6, γ_7, γ_8, and γ_9 are given in Appendix D.

C.3 Boundary dispersion

Proof of Proposition 5.8. To prove that $(\alpha, 0, 1-\alpha)$ is an equilibrium, we need $\dot{h}_1 = 0 \wedge \dot{h}_2 = 0$ to be true. Thus, we have

$$\dot{h}_1 = \alpha \left(V_1(\boldsymbol{h}) - \bar{V}(\boldsymbol{h}) \right) = 0,$$
$$\dot{h}_2 = 0 \left(V_2(\boldsymbol{h}) - \bar{V}(\boldsymbol{h}) \right) = 0.$$

Therefore, we end up with $V_1(\boldsymbol{h}) = V_3(\boldsymbol{h})$. As shown by Gaspar et al. (2018), in Proposition 4, it is possible to solve the former expression with respect to λ, as $\lambda_b^*(\alpha)$. Therefore, there exists a BD equilibrium if, and only if, there exists a value of $\lambda > 0$ such that $\lambda = \lambda_b^*(\alpha)$, where $\lambda_b^*(\alpha)$ is

$$\lambda_b^*(\alpha) = -3\gamma_{10}\frac{\gamma_{11} + \gamma_{12}\ln\left[\frac{1-\alpha(1-\phi_{UK})}{\alpha(1-\phi_{UK})+\phi_{UK}}\right]}{(\sigma-1)\gamma_{13}},$$

where γ_{10}, γ_{11}, γ_{12}, and γ_{13} are given in Appendix D. □

Proof of Proposition 5.9. Notice that $\lambda_b^*(\alpha)$ has a vertical asymptote if $b_4 = 0$, that is for

$$\alpha = \alpha_{lb} \equiv \frac{2\phi_{EU}(\phi_{UK}-1)^2 - \phi_{UK}(\phi_{UK}-1)^2 + (1-\phi_{UK})e_1}{3(\phi_{UK}-1)^2(\phi_{EU}-\phi_{UK})},$$

where

$$e_1 = \sqrt{\phi_{EU}^2\left(\phi_{UK}^2 + \phi_{UK} + 1\right) - \phi_{EU}\phi_{UK}\left[\phi_{UK}(\phi_{UK}+4)+1\right] + \phi_{UK}^2\left(\phi_{UK}^2 + \phi_{UK} + 1\right)}.$$

It is possible to show that $\alpha_{lb} < 1/2$. It is positive if either (i) $\phi_{UK} < 1/2$, or (ii) $\phi_{UK} > 1/2$ and $\phi_{EU} < \frac{\phi_{UK}^2}{(3-\phi_{UK})\phi_{UK}-1}$. For $\alpha < \alpha_{lb}$, the logarithm of $\lambda_b^*(\alpha)$ is positive, and $\gamma_{10} > 0$, $\gamma_{11} < 0$, $\gamma_{12} < 0$, and $\gamma_{13} > 0$. Therefore, there exists a value of λ such that $\lambda_b^*(\alpha) > 0$ for $\alpha \in (0, \alpha_{lb})$. Next, notice that $\lambda_b^*(\alpha) = 0$ for $\alpha = 1/2$. We have that the logarithm is positive for $\alpha \in (\alpha_{lb}, 1/2)$, and $\gamma_{10} > 0$, $\gamma_{11} < 0$, $\gamma_{12} < 0$, and $\gamma_{13} < 0$. Therefore, $\lambda_b^*(\alpha) < 0$ and no equilibrium exists for $\alpha \in (\alpha_{lb}, 1/2)$. Finally, for $\alpha > 1/2$, the logarithm is negative, $\gamma_{10} > 0$, $\gamma_{11} > 0$, $\gamma_{12} < 0$ and $\gamma_{13} < 0$, which means that $\lambda_b^*(\alpha) > 0$ for $\alpha \in (1/2, 1)$. Thus, there exists a value of λ such that *boundary dispersion* with $\alpha \in (1/2, 1)$ is an equilibrium. □

Lemma C.2. *The Jacobian matrix evaluated at any equilibrium $h^* \in \Delta_b$ is of the form*

$$J = \begin{bmatrix} \alpha\left(\dfrac{\partial V_1}{\partial h_1} - \dfrac{\partial \bar{V}}{\partial h_1}\right) & \alpha\left(\dfrac{\partial V_1}{\partial h_2} - \dfrac{\partial \bar{V}}{\partial h_2}\right) \\ \\ 0 & V_2 - V_1 \end{bmatrix}.$$

Proof. Any configuration in Δ_b is such that $\boldsymbol{h} = (\alpha, 0, 1-\alpha)$ with $\alpha \in (0, 1)$. For any such configuration, we have

$$
\begin{aligned}
\frac{\partial f_1}{\partial h_1} &= \underbrace{V_1 - \bar{V}}_{0} + \alpha \left(\frac{\partial V_1}{\partial h_1} - \frac{\partial \bar{V}}{\partial h_1} \right), \\
\frac{\partial f_1}{\partial h_2} &= \alpha \left(\frac{\partial V_1}{\partial h_2} - \frac{\partial \bar{V}}{\partial h_2} \right), \\
\frac{\partial f_2}{\partial h_1} &= 0, \\
\frac{\partial f_2}{\partial h_2} &= V_2 - \bar{V} + 0 \times \left(\frac{\partial V_2}{\partial h_2} - \frac{\partial V_1}{\partial h_{\ 2}} \right) = V_2 - V_1.
\end{aligned}
$$

Note that $V_1 - \bar{V} = 0$ since it is an equilibrium condition. Thus, $V_1 = \bar{V}$. Next, we have $\frac{\partial f_2}{\partial h_1} = 0$ because $h_2 = 0$ at a BD equilibrium. This also determines the expression of $\frac{\partial f_2}{\partial h_2}$. □

We have that

$$
\begin{aligned}
\Lambda_1^{BD} &\equiv \frac{\mu}{(\sigma-1)\sigma\gamma_{15}} \begin{bmatrix} \sigma\gamma_{14}\ln(1-\alpha(1-\phi_{UK})) + \sigma\gamma_{15}\ln(\phi_{EU} - \alpha(\phi_{EU} - \phi_{UK})) \\ -\gamma_{16} - \gamma_{17}\ln(\alpha(1-\phi_{UK}) + \phi_{UK}) \end{bmatrix}, \\
\Lambda_2^{BD} &\equiv (\alpha-1)\alpha\mu \left[\sigma\gamma_{18} + \sigma\gamma_{19}\ln\left(\frac{1-\alpha(1-\phi_{UK})}{\alpha(1-\phi_{UK}) + \phi_{UK}} \right) + \gamma_{20} \right].
\end{aligned}
$$

where γ_{14}, γ_{15}, γ_{16}, γ_{17}, γ_{18}, γ_{19}, and γ_{20} are given in Appendix D.

C.4 Continental partial agglomeration

We have that

$$
\lambda_{b_{EU}}^*(\alpha) = \frac{3}{(\phi_{EU}-1)^2} \left[(1-\phi_{EU})\phi_{EU} + \gamma_{21} \frac{\ln\left[\frac{1-\alpha(1-\phi_{EU})}{\alpha(1-\phi_{EU})+\phi_{EU}}\right]}{(2\alpha-1)(\sigma-1)} \right].
$$

where γ_{21} is given in Appendix D.

Appendix D

Chapter 5 - Auxiliary terms

$$
\begin{aligned}
\gamma_1 =& \left[\alpha(\phi_{EU} - 4\phi_{UK} + 3) - \phi_{EU} + 2\phi_{UK} - 1\right],\\
\gamma_2 =& \left[\alpha(1 - \phi_{UK}) + \phi_{UK}\right]\left[(1-\alpha)(\phi_{EU} + 1) + 2\alpha\phi_{UK}\right],\\
\gamma_3 =& - 3\alpha(1 - \phi_{UK})(\phi_{EU} - 2\phi_{UK} + 1) - 3(\phi_{EU} + 1)\phi_{UK} + \phi_{EU} + 4\phi_{UK}^2 + 1,\\
\gamma_4 =& - \left(\phi_{EU} - 2\phi_{UK}^2 + 1\right)\left[\sigma\left(-3\alpha(1 - \phi_{UK})(\phi_{EU} - 2\phi_{UK} + 1) - 5\phi_{EU}\phi_{UK}\right.\right.\\
& \left.\left. +6\phi_{UK}^2 + \phi_{EU} - 3\phi_{UK} + 1\right) + 2\phi_{UK}(\phi_{EU} - \phi_{UK})\right],\\
\gamma_5 =& 3\alpha^2(\phi_{UK} - 1)^2(\phi_{EU} - 2\phi_{UK} + 1)^2 + 2\alpha(1 - \phi_{UK})(\phi_{EU} - 2\phi_{UK} + 1)\times\\
& \times\left[\phi_{EU}(3\phi_{UK} - 1) + \phi_{UK}(3 - 4\phi_{UK}) - 1\right] - 8(\phi_{EU} + 1)\phi_{UK}^3\\
& + 3(\phi_{EU} + 1)^2\phi_{UK}^2 - 2(\phi_{EU} + 1)^2\phi_{UK} + (\phi_{EU} + 1)^2 + 8\phi_{UK}^4,\\
\gamma_6 =& (\sigma - 1)\sigma(-\alpha(1 - \phi_{UK}) - \phi_{UK})[\phi_{EU} + 1 - \alpha(\phi_{EU} - 2\phi_{UK} + 1)]\times\\
& - 3\alpha(1 - \phi_{UK})(\phi_{EU} - 2\phi_{UK} + 1) - 3(\phi_{EU} + 1)\phi_{UK} + \phi_{EU} + 4\phi_{UK}^2 + 1,\\
\gamma_7 =& \sigma\left[-3\alpha(1 - \phi_{UK})(3\phi_{EU} - 4\phi_{UK} + 1) + \phi_{EU}(3 - 7\phi_{UK}) + \phi_{UK}(8\phi_{UK} - 5)\right.\\
& \left.+1\right] - 2[-3\alpha(1 - \phi_{UK}) - 2\phi_{UK} + 1](\phi_{EU} - \phi_{UK}),\\
\gamma_8 =& 2(1 - \phi_{EU})(\alpha(1 - \phi_{UK}) + \phi_{UK}),\\
\gamma_9 =& \sigma(\sigma - 1)\left[-(1-\alpha)(\phi_{EU} + 1) - 2\alpha\phi_{UK}\right]\left[-3\alpha(1 - \phi_{UK})(\phi_{EU} - 2\phi_{UK} + 1)\right.\\
& \left.+\phi_{EU} + 1 - 3(\phi_{EU} + 1)\phi_{UK} + 4\phi_{UK}^2\right],\\
\gamma_{10} =& \phi_{EU}(1 - \alpha) + \alpha\phi_{UK},\\
\gamma_{11} =& (2\alpha - 1)(\sigma - 1)(1 - \phi_{UK})\phi_{UK},\\
\gamma_{12} =& \sigma\left[\alpha(1 - \phi_{UK}) - 1\right](\alpha(1 - \phi_{UK}) + \phi_{UK}),\\
\gamma_{13} =& \phi_{EU}\left[3\alpha^2(\phi_{UK} - 1)^2 - 4\alpha(\phi_{UK} - 1)^2 - (3 - \phi_{UK})\phi_{UK} + 1\right] +\\
& \phi_{UK}\left[\phi_{UK} - \alpha(3\alpha - 2)(\phi_{UK} - 1)^2\right],\\
\gamma_{14} =& - \alpha^2(1 - \phi_{UK})\left[\phi_{EU}^2 - 3(\phi_{EU} + 1)\phi_{UK} + \phi_{EU} + 3\phi_{UK}^2 + 1\right]
\end{aligned}
$$

$$
\begin{aligned}
&+ \alpha \left[4(\phi_{EU}+1)\phi_{UK}^2 - 2(\phi_{EU}(\phi_{EU}+2)+2)\phi_{UK} + (\phi_{EU}+1)^2 - 2\phi_{UK}^3\right] \\
&+ (\phi_{EU}-\phi_{UK})(\phi_{EU}\phi_{UK}+\phi_{UK}-1), \\
\gamma_{15} =& \phi_{EU}\left[3\alpha^2(\phi_{UK}-1)^2 - 4\alpha(\phi_{UK}-1)^2 - (3-\phi_{UK})\phi_{UK}+1\right] \\
&+ \phi_{UK}\left[\phi_{UK} - \alpha(3\alpha-2)(\phi_{UK}-1)^2\right], \\
\gamma_{16} =& (1-\alpha-1)(\sigma-1)(1-\phi_{EU})(-3\alpha(1-\phi_{UK})-2\phi_{UK}+1)(\phi_{EU}-\phi_{UK}), \\
\gamma_{17} =& (1-\alpha)\sigma(\phi_{EU}-1)^2(\alpha(1-\phi_{UK})+\phi_{UK}), \\
\gamma_{18} =& -(2\alpha-1)\phi_{EU}\phi_{UK}\left(1-\phi_{UK}^2\right)\left[3\alpha^2(\phi_{UK}-1)^2 - 3\alpha(\phi_{UK}-1)^2\right. \\
&\left. -\phi_{UK}(\phi_{UK}+2)\right] + \phi_{EU}^2(1-\phi_{UK})(1-\alpha(1-\phi_{UK}))\left[3(\alpha-1)^2\phi_{UK}^2\right. \\
&\left. +(3-4\alpha)\phi_{UK} + 4\alpha - 3\alpha^2 - 1\right] + (1-\phi_{UK})\phi_{UK}^2(-\alpha(1-\phi_{UK})-\phi_{UK})\times \\
&\times\left[\alpha\left(-3\alpha\left(1-\phi_{UK}^2\right)+4\phi_{UK}+2\right)-\phi_{UK}\right], \\
\gamma_{19} =& \phi_{EU}^2\left[3\alpha^4(\phi_{UK}-1)^4 - 8\alpha^3(\phi_{UK}-1)^4 + ((\phi_{UK}-1)\phi_{UK}+1)^2\right. \\
&\left. +8\alpha^2(\phi_{UK}-1)^4 - 2\alpha(\phi_{UK}(2\phi_{UK}-3)+2)(\phi_{UK}-1)^2\right] + 2\phi_{EU}\phi_{UK}\times \\
&\times\left[-3\alpha^4(\phi_{UK}-1)^4 - \phi_{UK}^2 + 6\alpha^3(\phi_{UK}-1)^4 - 4\alpha^2(\phi_{UK}-1)^4\right. \\
&\left. +\alpha(\phi_{UK}-1)^4\right] + \phi_{UK}^2\left[3\alpha^4(\phi_{UK}-1)^4 - 4\alpha^3(\phi_{UK}-1)^4\right. \\
&\left. +2\alpha^2(\phi_{UK}-1)^4 + 2\alpha\phi_{UK}(\phi_{UK}-1)^2 + \phi_{UK}^2\right], \\
\gamma_{20} =& -(1-\phi_{UK})\phi_{UK}(\phi_{EU}-\phi_{UK})\left[\phi_{UK}\left(\alpha^2(1-2\phi_{EU})+\phi_{EU}\right)+\alpha^2\phi_{UK}^3\right. \\
&\left. +(\alpha-1)^2\phi_{EU} + \phi_{UK}^2(\alpha(2-\alpha)(2-\phi_{EU})+\phi_{EU}-1)\right], \\
\gamma_{21} =& \sigma\left[\alpha(1-\phi_{EU})-1\right]\left[\alpha(1-\phi_{EU})+\phi_{EU}\right].
\end{aligned}
$$

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